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Localization of Leaks in Water Distribution Networks using Flow Readings

Javier Jiménez-Cabas * **Elena Romero-Fandiño** ** **Lizeth Torres** ***,**** **Marco Sanjuan** ** **Francisco Ronay López-Estrada** †

* *Departamento de Ciencias de la Informática y Electrónica, Universidad de la Costa.*
** *Departamento de Ingeniería Mecánica, Universidad del Norte.*
*** *Instituto de Ingeniería, Universidad Nacional Autónoma de México.*
**** *Cátedras CONACYT.*
† *Departamento de Ingeniería Electrónica, Insituto Tecnológico de Tuxtla Gutirrez.*

Abstract: This paper presents a novel approach to localize single and sequential leaks based on the lumped model of a water distribution network (WDN). The principal features of such a model are: a new friction term expressed as a power-law and a suitable representation expressed only in terms of the flow rate. From the response of this model and flow rate measurements at junctions of the pipelines composing the WDN, a set of residuals[1] is proposed for each pipeline. The residuals closest to zero will indicate the leak positions in the faulty pipelines. We present some simulation tests based on data from PipelineStudio® from Energy Solutions to illustrate the suitability of our method.

Keywords: Water distribution networks, pipelines, fault detection and isolation, leak diagnosis.

1. INTRODUCTION

The mean rates of non-revenue water (NRW) are estimated around 40% in developing and underdeveloped countries (Africa, Asia, Latin America and the Caribbean) and about 15% in United States (WHO (2000)). Actually, according to Kingdom et al. (2006): “Every year, more than 32×10^9 [m^3] of treated water physically leak(s) from urban water supply systems around the world, while 16×10^9 [m^3] are delivered to customers for zero revenue”. These loss in distribution networks are basically due to: overflowing service reservoirs, illegal connections and leaking pipes (valves and joints) (Multikanga et al. (2009)). With regard to leaking pipelines, there are collateral losses such as the energy used to compensate the pressure drops caused by the leaks. Moreover, there are risks associated to leaks such as landslides, contaminant infiltration into water distribution systems, property damage, among others.

Motivated for the above-mentioned facts, we present a method to detect and locate leaks in a WDN by only using flow rate measurements. Usually, flow rates are solely used to compute a mass balance to execute the leak detection but not the location. As example, in Buchberger and Nadimpalli (2004), flow rate readings were used for detecting the magnitude of leaks in small residential service zones (under 1,000 homes) of a drinking water distribution system. The methodology can be used to compare the magnitude of leakage among different service zones and, hence, provide a way to prioritize areas of the network needing water loss control measures.

Leaks diagnosis in WDN, i.e. detection and location, has been addressed by several researchers. For example, Pérez et al. (2009) proposed a diagnosis methodology based on the detection of discrepancies between pressure measurements and their estimations obtained from the simulation of a calibrated WDN model. Two years later, Pérez et al. (2011) proposed a leakage localization method based on the pressure measurements and a pressure sensitivity analysis of nodes in a network. The leakage localization methodology is founded in standard model-based fault diagnosis well established theory. In order to maximise the isolability with a reasonable number of sensors an optimal sensor placement methodology based on genetic algorithms is also proposed. The objective function in the minimisation process was the size of the maximum group discriminated. In Ponce et al. (2014) an extended time-horizon analysis of pressure sensitivities was considered as the base of a model-based leak diagnosis approach. Kim et al. (2016) introduced a new robust algorithm to detect leakage in WDN based on cumulative integral of shifted pressure data, floor function with three parameters followed by curvature function, and localization based on statistical estimation.

From a practical point of view, it is worthwhile to use pressure sensors to diagnose leaks in a WDN because they are cheaper, more operable and offer faster detection (Ostapkowicz (2016)). Our method is not intended to replace or compete with leak diagnosis methods based on

[1] Redundant relations which are equations with information from the model and the WDN

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pressure measurements such as the negative pressure wave methods (Silva et al. (1996)), the wave reflection methods, the pressure point analysis methods (Farmer (1989)) or the gradient method (Isermann (1984), Billmann and Isermann (1987)). Our method was designed to be used when pressure measurements are not available because the sensors are damaged or uncalibrated, or to support the diagnosis made with other kind of approaches.

The method that we present is based on the model of a WDN described by a set of Liénard equations in terms of the flow rate (Torres et al. (2015, 2016); Jiménez et al. (2017)). Each Liénard equation represents a pipeline branch of the WDN and is expressed by a nonlinear partial differential equation (PDE) without analytical solution. Therefore, the finite difference method is used to discretized the one-dimensional space domain of the pipeline into space steps (sections). Once the discretization is done, the numerical solution of the resulting ordinary differential equation (ODE) system is calculated from the flow rates measured at the pipelines junctions (nodes) of the supervised WDN, which in fact act as boundary conditions. The numerical solution will provide internal discrete flows for every pipeline of the WDN. For pipeline branches free of leaks their discrete flow rates will be equals along each branch in steady state i.e. for every space section of the pipeline branch. In the case of a leak occurrence, the outflow of the leak will be distributed along the space sections of the pipe branch affected by the leak. For each space section the residuals can be calculated by subtracting the discrete flow rates from the pipe branch flow rate without leaks (the mean nominal flow). The residual corresponding to the section where the leak is occurring will be that close to zero.

For the application of our method, the following assumptions must be considered: (1) water demands are assumed to occur in the nodes and leaks between them (i.e. in the pipelines); (2) the pressure at the nodes must remain unaffected by the leaks. In order to test our method, the behavior of a WDN was recreated with the commercial software PipelineStudio® from Energy Solutions. The paper is organized as follows. Section 2 presents the core of the diagnosis methodology: the model to generate the residuals. Section 3 describes the proposed diagnostic method. Section 4 presents some simulation test results and Section 5 presents the corresponding conclusions.

2. FLOW RATE MODEL

In this article, we propose that every pipeline conforming the WDN can be modeled by the following partial differential equation:

$$\frac{\partial^2 Q(z,t)}{\partial t^2} + \underbrace{\frac{f\left(Q(z,t)\right)}{\phi A_r}|Q(z,t)|\frac{\partial Q(z,t)}{\partial t}}_{\text{Steady Friction Term}} - b^2\frac{\partial^2 Q(z,t)}{\partial z^2} = 0, \tag{1}$$

where $(z,t) \in [0,L]\times[0,\infty)$ gathers the space [m] and time [s] coordinates respectively, L is the length of the pipe, $Q(z,t)$ is the volumetric flow rate [m^3/s], b is the wave speed in the fluid [m/s], A_r is the cross-sectional area of the pipeline [m^2], ϕ is the inside diameter of the pipe [m] and f is the Darcy-Weisbach friction factor which depends on the Reynolds number: $\mathrm{Re} = Q(z,t)\phi/\nu A_r$ with ν as the kinematic viscosity.

2.1 Steady friction term

An expression which has become the acceptable standard for calculation of the friction factor in transitional and turbulent regimes is the relation known as the Colebrook-White equation (Colebrook and White (1937)). Such an equation is implicit with respect to f, and because of that it has to be solved by using iterative methods, which would increase the computational complexity of any leak diagnostic algorithms. For this reason, over the time a large number of studies developed several explicit approximations to the implicit Colebrook equation (Brkić (2011)). Among the explicit approximations to the Colebrook-White equation available, one finds the following power-law type equation proposed by Wood (1966):

$$f = a + b\mathrm{Re}^{-c}, \tag{2}$$

where

$$a = 0.53\left(\frac{\varepsilon}{\phi}\right) + 0.094\left(\frac{\varepsilon}{\phi}\right)^{0.225},\ b = 88\left(\frac{\varepsilon}{\phi}\right)^{0.44},$$

$$c = 1.62\left(\frac{\varepsilon}{\phi}\right)^{0.134},$$

with ε as the absolute roughness.

It is important to address that the estimated error of Wood approximation with respect to the Colebrook-White equation can reach a 49.51% (see Fig. 1) in the range $10^4 < \mathrm{Re} < 10^8$ for pipelines with relative roughness within the interval $1 \times 10^{-7} < \frac{\varepsilon}{\phi} < 0.05$. For the error computation the Colebrook-White equation was solved by used the method proposed by Clamond (2009).

To improve the approximation performance new coefficients for the Wood model were calculated by using the nonlinear regression MATLAB® function `nlinfit`. The improved Wood approximation results into

$$f = a + b\mathrm{Re}^{-c}, \tag{3}$$

where

$$a = 0.4133\left(\frac{\varepsilon}{\phi}\right) + 0.1110\left(\frac{\varepsilon}{\phi}\right)^{0.2598},\ b = 42.6463\left(\frac{\varepsilon}{\phi}\right)^{0.3273},$$

$$c = 1.3624\left(\frac{\varepsilon}{\phi}\right)^{0.1124}.$$

Once the coefficients were updated, the maximum error was reduced to 11.67% (see Fig. 2). Although this error is greater than that obtained with other approximations (Brkić (2011)), Wood equation is adequate for identification algorithms, especially for those based on governing equations, it can be easily integrated and differentiated.

In order to set system (1) in a state space representation, we use the Liénard transformation given by

$$\Phi : \begin{pmatrix} Q(z,t) \\ \dfrac{\partial Q(z,t)}{\partial t} \end{pmatrix} \to \begin{pmatrix} Q^a(z,t) \\ Q^b(z,t) \end{pmatrix} := \begin{pmatrix} Q(z,t) \\ \dfrac{\partial Q(z,t)}{\partial t} + F(Q(z,t)) \end{pmatrix},$$

where

$$F(\sigma(t)) = \int_0^{Q^a} \frac{f(\sigma)}{\phi A_r}\sigma|\sigma| d\sigma. \tag{4}$$

System (1) thus becomes

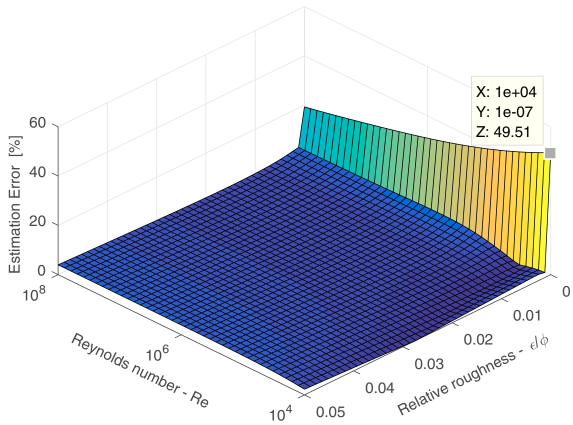

Fig. 1. Error of the original Wood approximation

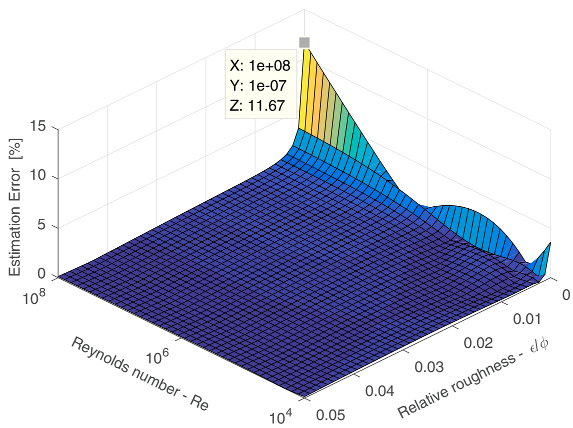

Fig. 2. Error of the improved Wood approximation

$$\frac{\partial Q^a(z,t)}{\partial t} = Q^b(z,t) - F\left(Q^a(z,t)\right),$$
$$\frac{\partial Q^b(z,t)}{\partial t} = b^2 \frac{\partial^2 Q^a(z,t)}{\partial z^2}, \tag{5}$$

where

$$F\left(Q^a(z,t)\right) = \alpha Q^a(z,t)|Q^a(z,t)| + \beta Q^a(z,t)|Q^a(z,t)| \tag{6}$$

is the steady friction term with

$$\alpha = \frac{\left(410(\frac{\varepsilon}{\phi}) + 111\phi\left(\frac{\varepsilon}{\phi}\right)^{\frac{13}{50}}\right)}{2000 A_r \phi^2}, \beta = -\frac{1075\left(\frac{\varepsilon}{\phi}\right)^{\frac{33}{100}}}{A_r\phi\left(\kappa - 50\right)(\mathrm{Re})^{\frac{\kappa}{25}}}$$
$$\kappa = 34\left(\frac{\varepsilon}{\phi}\right)^{\frac{14}{125}}.$$

2.2 Lumped element model

If Eq. (5) is spatial-discretized by using the finite difference method the following lumped system is obtained:

$$\dot{Q}_i^a(t) = Q_i^b(t) - F\left(Q_i^a(t)\right), \quad i = 1, ..., n_\ell,$$
$$\dot{Q}_i{}^b(t) = b^2\left[\frac{Q_{i-1}^a(t) - 2Q_i^a(t) + Q_{i+1}^a(t)}{(\Delta z_i)^2}\right], \tag{7}$$

where Δz_i is the spatial step, n_ℓ is the total number of internal discrete flows and $F\left(Q_i^a(t)\right)$ is calculated via (6) for $Q_i^a(t)$. To find a numerical solution for (7), $Q(0,t) = Q_{in}(t)$ and $Q(L,t) = Q_{out}(t)$ are used as boundary conditions.

By applying equations (7) for each pipeline branch of the WDN, one has the following lumped element model for every pipeline as follows:

$$p^{th}\text{ Pipeline}\begin{cases} \dot{Q}_1^{a_p}(t) = Q_1^{b_p}(t) - F\left(Q_1^{a_p}(t)\right) \\ \dot{Q}_1^{b_p}(t) = b^2\left[\dfrac{Q_{in}^p(t) - 2Q_1^{a_p}(t) + Q_2^{a_p}(t)}{(\Delta z_1^p)^2}\right] \\ \dot{Q}_2^{a_p}(t) = Q_2^{b_p}(t) - F\left(Q_2^{a_p}(t)\right) \\ \dot{Q}_2^{b_p}(t) = b^2\left[\dfrac{Q_1^{a_p}(t) - 2Q_2^{a_p}(t) + Q_3^{a_p}(t)}{(\Delta z_2^p)^2}\right] \\ \vdots \\ \dot{Q}_{n_\ell^p}^{a_p}(t) = Q_{n_\ell^p}^{b_p}(t) - F\left(Q_{n_\ell^p}^{a_p}(t)\right) \\ \dot{Q}_{n_\ell^p}^{b_p}(t) = b^2\left[\dfrac{Q_{n_\ell^p-1}^{a_p}(t) - 2Q_{n_\ell^p}^{a_p}(t) + Q_{out}^p(t)}{\left(\Delta z_{n_\ell^p}^p\right)^2}\right] \end{cases}$$

where p denotes the index of each pipeline composing the WDN, $Q_{in}^p(t)$ and $Q_{out}^p(t)$ are the measured flows at the inlet and outlet of pipeline p, L^p is the length of pipeline p. The spatial step for pipeline p can be computed as $\Delta z_i^p = L^p/N_\ell^p$ with $N_\ell^p = n_\ell^p + 1$ as the total number of space steps (sections). n_ℓ^p is the total number of internal flows for pipeline p. Check Fig. 3 for a better conceptualization.

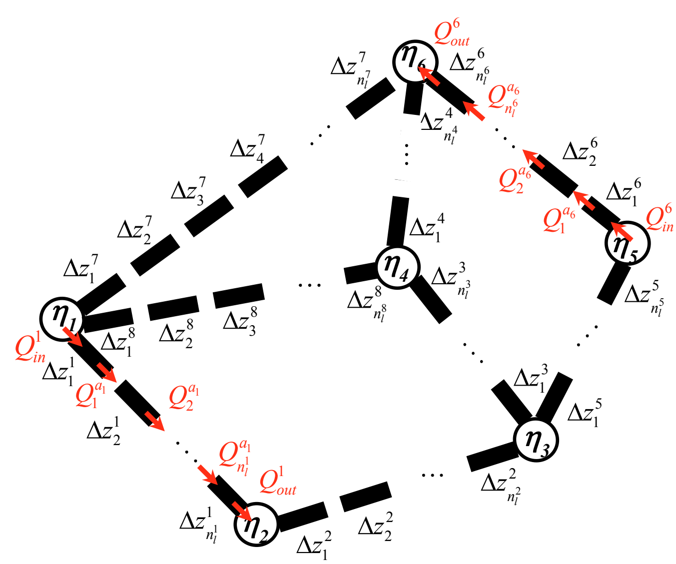

Fig. 3. Space discretization schema for a WDN

3. METHODOLOGY STRATEGY

Fig. 4 shows a flow diagram of the proposed methodology. The proposed approach relies on model (8), which is implemented in Matlab® and has as inputs the flow rates

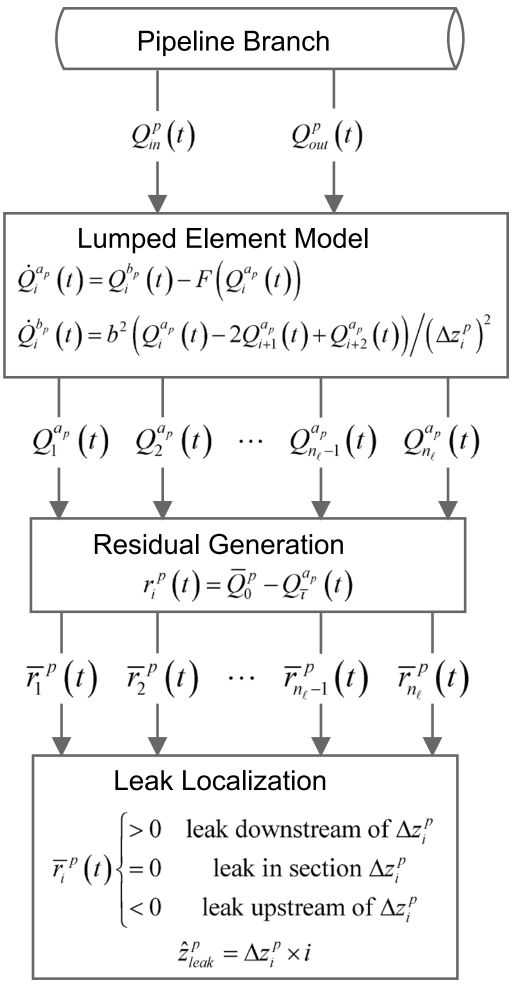

Fig. 4. Methodology Flow Diagram, Jiménez-Cabas et al. (2017)

measured at the ends of every pipeline branch composing the network, $Q_{in}^p(t)$ and $Q_{out}^p(t)$.

Since the unidimensional space is discretized into space steps (sections) of equal size, model (8) shall compute internal discrete flow rates corresponding to each section. If the pipeline branch is free of leaks, the discrete flow rates provided by the model will be equal in steady state, otherwise the leak outflow will be distributed along the discrete space of the pipeline branch.

If the internal discrete flows, calculated by the model after the leak, are subtracted from the mean nominal flow $\overline{Q}_0^p$ (the mean flow rate of the pipeline branch p without leaks), residuals corresponding to each section will be then obtained as follows:

$$r_i^p(t) = \overline{Q}_0^p - Q_{\overline{\iota}}^{a_p}(t), \forall i = 1, 2, ..., n_\ell^p, \quad \forall \overline{\iota} = n_\ell^p, n_{\ell-1}^p, ..., 2, 1. \tag{8}$$

where i is the index to enumerate the residuals, $\overline{\iota}$ is the index for the countdown of the flows and n_ℓ^p is the total number of residuals that matches with the total number of discrete flow rates calculated by the Liénard-type model (8).

Depending on the behavior of the discrete flow rates calculated by the lumped element model, for every section i, the mean value of the residuals will have the following behavior:

$$\overline{r}_i^p(t)\begin{cases} > 0, & \text{if there is a leak downstream of } \Delta z_i^p; \\ = 0, & \text{if there is a leak in section } \Delta z_i^p; \\ < 0, & \text{if there is a leak upstream of } \Delta z_i^p. \end{cases} \tag{9}$$

Remark 1: Notice that last section has not assigned a residual since the flow in this section is the downstream boundary condition $Q_{out}^p(t)$ of model (8) and not an internal discrete flow computed by model (8).
Remark 2: As a consequence, if a leak is placed in the last section, all the residuals will be then positive.
Remark 3: If the position of the leak does not match with the limits of each section, then $\overline{r}_j^p \approx 0$.

Resuming the explanation of our methodology, the position of the single leak can be computed by using the following equation:

$$\hat{z}_{leak} = \Delta z_i^p \times i. \tag{10}$$

Thus, $\hat{z}_{leak} \to z_{leak}$ inasmuch $\Delta z_i^p \to 0$.

The magnitude of a single leak (the leak outflow) can be calculated by means of the mass balance

$$\hat{Q}_{leak}^p(t) = Q_{in}^p(t) - Q_{out}^p(t). \tag{11}$$

In case of sequential leaks, the leak outflow computed by (11) will increase with the addition of the outflow of each sequential leak. Each leak flow can be calculated by using the following equation:

$$\hat{Q}_{eq}^p(t) = \sum_{\kappa=1}^{M^p} \hat{Q}_{L_\kappa}^p(t), \tag{12}$$

where $\hat{Q}_{eq}^p$ is the *equivalent flow*, which is the total flow lost due to the leaks, $\hat{Q}_{L_\kappa}^p$ is the flow lost due to the κ-th sequential leak, which is located at the position $\hat{z}_{L_\kappa^p}$ and M^p is the total number of sequential leaks.

In case of sequential leaks, the residual close to zero will indicate the section involving the *equivalent position*. Hence, the leak position of the κ-th sequential leak can be obtained through the following equation:

$$\hat{z}_{eq}^p = \frac{\sum_{\kappa=1}^{M^p} \hat{Q}_{L_k}^p(t)\hat{z}_{L_\kappa}^p}{\hat{Q}_{eq}^p(t)}, \tag{13}$$

4. TESTS: LEAK DIAGNOSTIC

In this section the WDN shown in Figure 5 was considered. It consists of a source reservoir from which the product is pumped into a two-loop pipe network. There is also a pipe leading to a storage tank. Demands are assumed to occur in the nodes and leaks between them. In the figure the identification labels for the various components are shown. Pipeline properties are listed in Table 4.

One scenario regarding the application of the proposed method is presented: the diagnosis of independent leaks in different branches of the network. The pipeline network behavior was recreated with the commercial software PipelineStudio® from Energy Solutions, by considering as boundary conditions the upstream and downstream

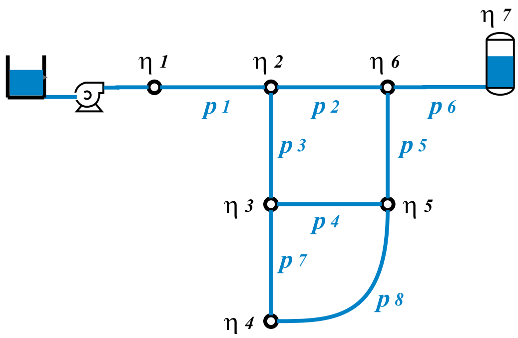

Fig. 5. Distribution Network

Pipe	Length [m]	Diameter [m]
$p = 1$	914.4	0.3556
$p = 2$	1524	0.3048
$p = 3$	1524	0.2032
$p = 4$	1524	0.2032
$p = 5$	1524	0.2032
$p = 6$	2133.6	0.254
$p = 7$	1524	0.1524
$p = 8$	2133.6	0.1524

Table 1. Network Pipe Characteristics

pressure heads in the nodes $\eta = 1$ and $\eta = 7$ (see Figure 5).

This simulator provides the flow rates to be injected to the lumped element model (8) as boundary conditions in order to get a numerical solution, i.e. to compute the discrete flows for the residual generation. Table 2 provides the parameters of the simulated WDN.

Table 2. Physical parameters

Symbol	Value	Units	Description
g	9.81	m/s^2	Gravitational acceleration
ε	$1.083 \times 10^{\ 3}$	m	Mean height of roughness
ν	$7.9822 \times 10^{\ 7}$	m^2/s	Kinematic viscosity

In this section three single leaks were induced in three different pipeline branches: p =3, p =5 and p =8. Table 3 shows details of the leaks scenario recreated, specifically, the pipeline in which the leak takes place, the leak position z_{leak}, the leak activation and deactivation times t^{on} and t^{off}. The mean values of the boundary conditions considered were $H_1(t) = 700$ [m] and $H_7(t) = 300$ [m]. The mean nominal flows obtained were about $\overline{Q}_{0_{p3}} = 0.1982$ [m^3/s], $\overline{Q}_{0_{p5}} = 0.2025$ [m^3/s] and $\overline{Q}_{0_{p8}} = 0.0776$ [m^3/s] for pipes branches $p = 3$, $p = 5$ and $p = 8$ respectively.

Table 3. Single Leaks Scenario

Pipe	z_{leak} [m]	t^{on} [s]	t^{off} [s]
p =3	150	200	600
p =5	725	300	700
p =8	1600.2	400	800

The lumped element model (8) was programmed in Matlab® by fixing a space step (section size) $\Delta z_i^p = L^p/N_\ell^p = L^p/21$ [m], where L^p is the length of the particular pipe branch (see Table 4). Since $N_\ell = 21$, 20 internal flows were calculated (i.e. $n_\ell = 20$), thereby 20 residuals were calculated ($r_1(t)$, $r_2(t)$, ..., $r_{20}(t)$). Fig. 6, Fig. 8 and Fig. 10 show the residuals calculated by using Eq. (8) for the three leaks considered respectively. The effects of the leak on the synthetic flows is clearly observed (once a leak occurs the leak outflow is distributed as several leaks in each discretization node). Fig. 6, Fig. 8 and Fig. 10 show the behavior of the residual in three dimensions. In these figures, we can appreciate the flow gradient along the space domain.

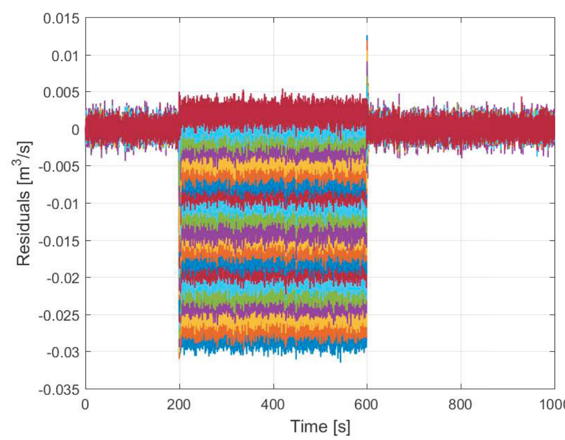

Fig. 6. Residuals for leak in pipe branch 3

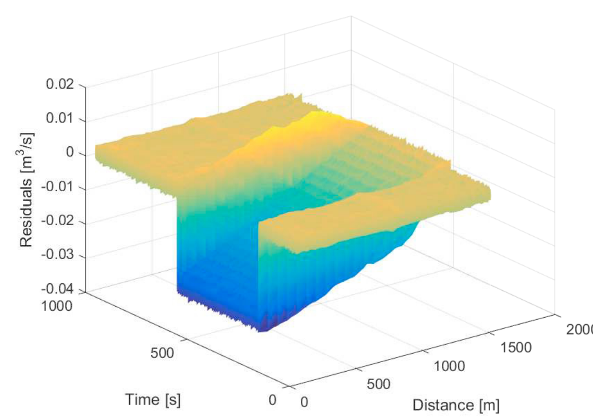

Fig. 7. Residuals for leak in pipe branch 3 in 3D

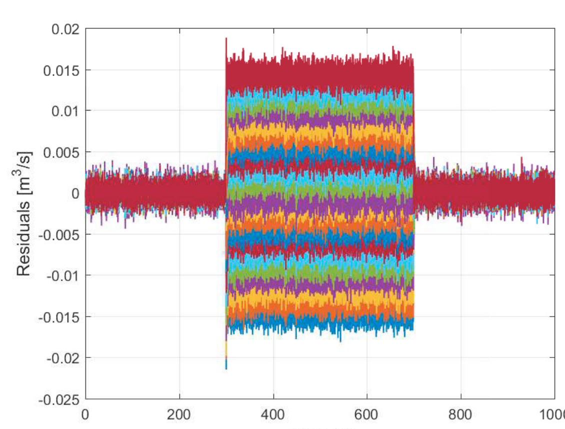

Fig. 8. Residuals for leak in pipe branch 5

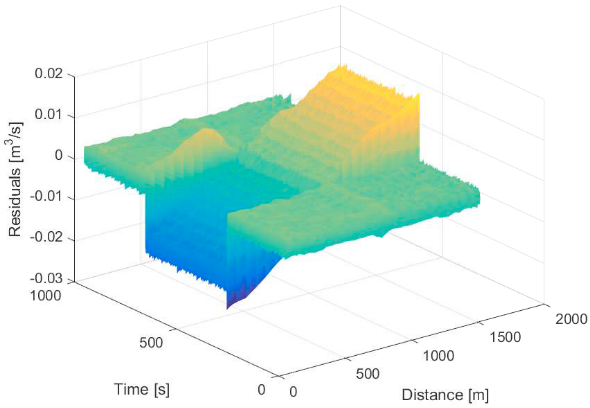

Fig. 9. Residuals for leak in pipe branch 5 in 3D

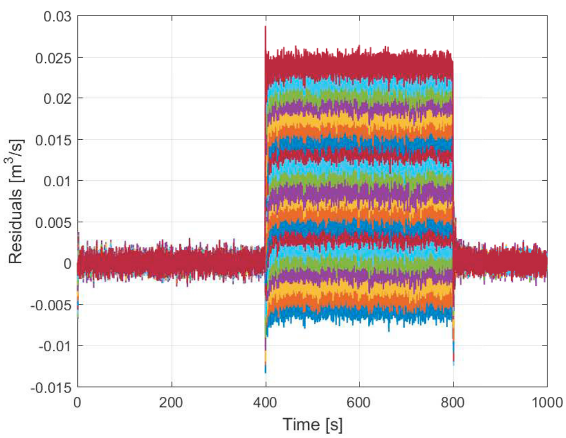

Fig. 10. Residuals for leak in pipe branch 8

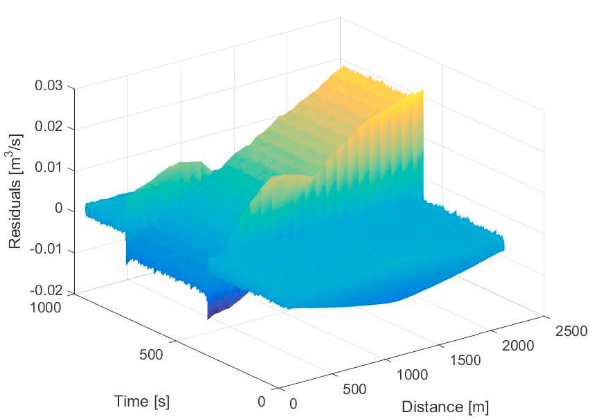

Fig. 11. Residuals for leak in pipe branch 8 in 3D

The residuals with mean value closer to zero were $r_2^3(t)$, $r_{11}^5(t)$ and $r_{17}^8(t)$ for the leaks considered respectively. The leak positions were estimated through (10). Table 4 summarizes the estimated leak positions $\hat{z}_{leak}^p$ and the estimation errors obtained for each one of three leaks considered.Notice that the estimation errors is calculated as $e = 100\left|\frac{z_{leak}^p - \hat{z}_{leak}^p}{L^p}\right|$.

Table 4. Diagnosis Results

z_{leak}^p [m]	$\hat{z}_{leak}^p$ [m]	Error [%]
150	138.55	0.75
725	762	2.43
1600.2	1648.7	0.0023

5. CONCLUSIONS

In this paper, we have presented a novel approach to diagnose single and sequential leaks in pipelines networks. In order to avoid the necessity of using pressure measurements, a representation of the pipeline dynamics under the form of a flow rate Liénard-type equation has been considered for the formulation of the proposed method. Provided simulations illustrated the good leak position estimation results obtained with the proposed methodology, which will be tested in short term in a lab water distribution network by considering unsteady conditions and variable water demands.

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