Economic MPC-LPV Control for the Operational Management of Water Distribution Networks [*]

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Abstract:
This paper presents an Economic Model Predictive Control (EMPC) for the operational management of water distribution networks (WDNs) with periodic operation based on embedding the nonlinearity of the model to the Linear Parameter Varying (LPV) model of WDNs. The performance of the WDN is identified by a set of difference-algebraic equations while the relation of hydraulic head/pressure and flow in connected pipes is nonlinear. In particular, the WDN model consists of two sections of nonlinear algebraic equations for bidirectional and unidirectional flows in pipes, respectively. The proposed algorithm is embedded the nonlinear algebraic equations into the LPV model. The proposed control approach allows the controller to accommodate the scheduling parameters. By computing the prediction of the state variables along a prediction time horizon, the system model can be modified according to the evaluation of the estimated state at each time instant. This iterative approach improves the implementation efficiency and reduces the computational burden compared to the solution of a non-linear optimization problem. Finally, the proposed strategy is applied to a well-known benchmark of the Richmond WDN. The closed-loop simulation results are shown and compared with other approaches by using several key performance indices.

Keywords: Economic Model Predictive Control, Linear Parameter Varying, Water Distribution Networks, Nonlinear MPC

1. INTRODUCTION

The management of the urban water cycle (UWC) is a subject of increasing interest taking into account its social, economic, and environmental impact (Ocampo-Martinez et al., 2013). Water distribution networks (WDNs) are complex and large-scale interconnected systems, which are designed to supply water to consumers in cities. By considering that water paucity is one of the main problems in many countries, benefits that operate WDNs are attracted to improving the reliability and efficiency of operation while conserving water. Consequently, there has been an incremented focus on optimal control of WDNs for developing the operational performance of the system. WDN is multivariable dynamic constrained systems that are characterized by the interrelationship of several subsystems (pumping stations including parallel pumps, pressure reducing valves, tanks, water demand nodes, sources as well as interconnected pipes). The static and dynamical behaviors of a WDN can be mathematically characterized by a set of nonlinear difference-algebraic equations (DAEs) in discrete-time (Puig et al., 2017). The system dynamics are described by difference equations while static relations including pressures and flows are defined by algebraic equations. Moreover, its optimal management, that can be formulated as a multi-criteria problem, is a complex challenge for water suppliers. To avoid these complications, Model Predictive Control (MPC) becomes an efficient method (Maciejowski, 2002).

In recent decades, MPC has started to attract the attention of both academia and industry due to the possibility of dealing with the conflicting power optimization and physical load reduction (Pour et al., 2018b). Accordingly, MPC can provide appropriate techniques to perform the operational control of water systems to develop their performance since it allows to compute optimal control strategies ahead of time for all the flow and pressure control elements (Cembrano et al., 2011). Different approaches reported in the literature highlight the importance and development of management the WDNs. Likewise, in Pascual et al. (2013) the optimal strategies are computed by optimizing a mathematical function describing the operational goals in a given time horizon and applying a significant model

of the network dynamics, as well as demand forecasts. In fact, solving a nonlinear optimization problem taking into account the full system dynamics and a large number of decision variables within a sampling time of to the WDN is the main challenge of applying MPC strategy for managing the WDN. Although sampling time for WDNs is usually chosen as one hour, but after a huge number of computational iterations can be obtained suboptimal solutions. One way to deal with non-linear MPC is to represent the process behavior by means of Linear Parameter Varying (LPV) models (Pour et al., 2017). The main advantage of LPV models is that the system nonlinearities are embedded in the varying parameters, which make the nonlinear system become a linear-like system with varying parameters (Pour et al., 2018a).

The main contribution of this paper is to provide an economic MPC-LPV strategy for the operational management of WDNs where the control-oriented model of WDNs uses a set of different and algebraic equations described in Wang et al. (2017). To this aim, the nonlinear algebraic equations are embedded into the LPV model. Then the constrained optimization problem for an LPV model is solved iteratively by a series of QP problem while the scheduling parameters are calculated at each time instant. The model is predicated on the horizon by using the previous sequence of scheduling variable and state of the model. In this case, the computational load is reduced and the implementation efficiency is improved. Finally, the proposed strategy is implemented to a real benchmark, namely the Richmond WDN, which is a medium-size network in the UK. Moreover, the proposed approach is compared with a nonlinear economic MPC and economic MPC strategy in (Wang et al., 2018). Further, evaluate the performance using defined key performance indicators (KPIs).

The remainder of the paper is organized as follows. In Section 2 the control-oriented model and EMPC formulation of WDN is introduced. Then the proposed approach of EMPC-LPV based on the WDN is presented in Section 3. In Section 4, results of applying the proposed control strategy to the Richmond WDN are summarized. Finally,in Section 5 the conclusion of this work are drawn and some research lines for future work are proposed.

2. ECONOMIC MODEL PREDICTIVE CONTROL OF WDNS

2.1 System description and control-oriented model of WDNs

Several modeling methods dealing with WDNs have been proposed in the literature (see, e.g., (Cembrano et al., 2011; Mays, 2004)). In this paper, a control-oriented modeling approach that is based on the flow and hydraulic head relations for the different network components is considered. A WDN is consists of a set of fundamental elements: reservoirs/tanks, control valves, pump stations, nodes, and water demand sectors, each being characterized by means of flow-head relations (Puig et al., 2017).

Tanks: Water tanks satisfy and provide the entire WDN with the storage capacity of drinking water at suitable elevation levels to guarantee sufficient water pressure service to consumers. The mass balance expression relating the

stored volume x in the f-th tank can be formulated as the discrete-time difference equation

$$x_f(k+1) = x_f(k) + \Delta t \Big(\sum_i q_{r,f}^{in}(k) - \sum_j q_{h,f}^{out}(k) \Big), \quad (1)$$

where $q_f^{in}(k)$ indicates the manipulated inflows from the i-th element to the f-th tank, and $q_f^{out}(k)$ denotes the manipulated outflows from the f-th tank to the j-th element and Δt is the sampling time. The physical constraint associated to the range of acceptable storage volume in the f-th tank is presented as

$$\underline{x_f} \le x_f(k) \le \overline{x_f}, \quad (2)$$

for all $k \in \mathbb{Z}_+$, where $\underline{x_f}$ and $\overline{x_f}$ indicate the minimum and the maximum admissible storage capacity, respectively, given in m^3. Notice that $\underline{x_f}$ might correspond with an empty tank; in practice this value can be set as nonzero to maintain an emergency stored volume (Ocampo-Martinez et al., 2013).

Generally, in WDN the head model is formulated in terms of the hydraulic head that correlates the energy in an incondensable fluid to the height of an equivalent static column of that fluid. By using this implication, the head related to the m-th tank by respect to the volume of storage water inside can be obtained as

$$h_f(k) = \frac{x_f(k)}{S_f} + E_f, \quad (3)$$

for all $k \in \mathbb{Z}_+$, where S_f and E_f are the cross-sectional area of f-th tank and corresponds the f-th tank elevation, respectively.

Actuators: The control actuators of WDN are of two types: valves and pumps, or more accurately, complex pumping stations. The manipulated flows through the actuators describe the manipulated variables, indicated as q_u. Both pumping stations and valves have lower and upper physical limits, which are considered as system constraints. Hence, the physical constraint can be regarded as input constraints, which can be expressed as

$$\underline{q_{u_j}} \le q_{u_j}(k) \le \overline{q_{u_j}}, \quad (4)$$

for all $k \in \mathbb{Z}_+$, where $\underline{q_{u_j}}$ and $\overline{q_{u_j}}$ represent the minimum and the maximum flow capacity of the j-th actuator, respectively.

The hydraulic component of a pump is expressed by a nonlinear function correlated to head variables and the flow. Since according to a pump given in Fig.1, the hydraulic components are bounded by the following constraints:

$$\Delta h_p(k) = h_d(k) - h_s(k) \ge 0, \quad (5a)$$
$$h_d(k) \in [\underline{h_d}, \overline{h_d}], \quad h_s(k) \in [\underline{h_s}, \overline{h_s}], \quad (5b)$$

for all $k \in \mathbb{Z}_+$, where $h_s(k)$ and $h_d(k)$ indicate the delivery head and suction head at time instant k, sequentially, including the physical condition of $h_d(k) \ge h_s(k)$. Furthermore, $\underline{h_d}$, $\underline{h_s}$ and $\overline{h_d}$ and $\overline{h_s}$ denote the minimum and maximum values of the delivery and suction heads.

In terms of the actuator of valves, there are several options, such as flow variable control, non-return, pressure modulating, head control, pressure reducing and so on (Wang et al., 2017). Alike of pumps, the characteristic

of valves is hard to be modeled. Therefore, the static relationship between the flow and head of a valve is bounded as follows:

$$\Delta h_v(k) = h_{us}(k) - h_{ds}(k) \geq 0, \tag{6a}$$

$$h_{us}(k) \in [\underline{h}_{us}, \overline{h}_{us}], \quad h_{ds}(k) \in [\underline{h}_{ds}, \overline{h}_{ds}], \tag{6b}$$

for all $k \in \mathbb{Z}_+$, where $h_{ds}(k)$ and $h_{us}(k)$ indicate the heads at the nodes around the valve in the downstream and upstream at time instant k, respectively. $\underline{h}_{ds}$, $\underline{h}_{us}$ and $\overline{h}_{ds}$, $\overline{h}_{us}$ denote the minimum and maximum values of the downstream and upstream heads, respectively.

Nodes: These elements relate to the network points where water flows are split or merged. Therefore, water flow through each node of the network must satisfy the mass balance relations. The expression of the mass balance in these components can be expressed as

$$\sum_i q_{i,n}^{in}(k) = \sum_j q_{n,j}^{out}(k), \tag{7}$$

for all $k \in \mathbb{Z}_+$, where $q_{i,n}^{in}(k)$ and $q_{n,j}^{out}(k)$ represent the non-manipulated inflow and outflow from n-th node from the i-th element and the j-th element, respectively, given in $m^{3/s}$.

Demand Sectors: Demands from the controller point of view is considered as a system disturbance that can be measured at a certain time instant k. Thus, at specific time instant k, the consumed water in the m-th demand sector can be presented as $d_m(k)$. The optimal control strategy is considered as a predictive one. Hence, by utilizing a suitable demand forecasting method, the short-term demand forecasts can be obtained (Quevedo et al., 2014).

Pipes: The water inside pressurized pipes moves from the higher hydraulic head to the lower head. Hence, the head-flow relationship for a pipe can be described as

$$v_{i,j}(k) = \Phi_{i,j}(h_i(k) - h_j(k)), \tag{8}$$

where $\Phi_{i,j}$ is a non-linear relationship. Regularly it is obtained by an empirical equation, for instance by using the *Chezy-Manning* head-flow formula:

$$h_i(k) - h_j(k) = R_{i,j} v_{i,j}(k) |v_{i,j}(k)|, \tag{9}$$

where h_i and h_j coincide to the hydraulic heads at any two adjacent nodes, and $v_{i,j}$ is the correlated water flow. The parameter $R_{i,j}$ in the *Chezy-Manny formula* is given by

$$R_{i,j} = \frac{10.29 L_{i,j} C_{i,j}^2}{D_{i,j}^{5.33}}, \tag{10}$$

where $L_{i,j}$, $C_{i,j}$ and $D_{i,j}$ are the length, roughness and diameter coefficient of the corresponding pipe, respectively.

The generalized discrete-time DAE model can be written as follows:

$$x(k+1) = F(x(k), u(k), v(k), z(k), d(k)), \tag{11a}$$

$$0 = G(x(k), u(k), v(k), z(k), d(k)), \tag{11b}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the vector of hydraulic heads at the storage nodes (reservoirs/tanks) as differential states, $u(k) \in \mathbb{R}^{n_u}$ is the manipulated flow of the actuates (valves and pumps), $v(k) \in \mathbb{R}^{n_v}$ is the vector of mean non-manipulated flows through the interconnected pipes, $z(k) \in \mathbb{R}^{n_z}$ is the vector of hydraulic heads at the non-storage nodes as algebraic states and $d(k) \in \mathbb{R}^{n_m}$ is the

demanded flow as measured disturbances. $F(.)$ and $G(.)$ are vectors of mapping functions. Besides, (11a) is the discrete-time differential equation representing the system dynamics while (11b) is the discrete-time algebraic equation showing the static relations of elements in the WDN. According to that, the tanks are the only components with dynamics in the WDN, (11a) can be explicitly formulated as

$$x(k+1) = Ax(k) + B_u u(k) + B_v v(k) + B_d d(k), \tag{12}$$

and by considering the static relations related to flow and head variables in (2), (7) and (9), the static equation (11b) can be explicitly rewritten as follows:

$$0 = E_u u(k) + E_v v(k) + E_d d(k), \tag{13a}$$

$$0 = P_x x(k) + P_z z(k) + \psi(v(k)). \tag{13b}$$

In addition, $A, B, B_v, B_d, E_u, E_v, E_d, P_x$ and P_z are time-invariant matrices of suitability dimensions dictated by the network topology, and $\psi(v) \in \mathbb{R}^{n_e}$ denotes a collection of nonlinear functions that describes the static relation of the hydraulic head and the water flow though interconnected pipes.

The system variables of the WDN (12)-(13) are constrained according to the operational limitations as follows:

$$x(k) \in \mathbb{X} \triangleq \{x(k) \in \mathbb{R}^{n_x} \,|\, \underline{x} \leq x(k) \leq \overline{x}\}, \quad \forall k \tag{14a}$$

$$u(k) \in \mathbb{U} \triangleq \{u(k) \in \mathbb{R}^{n_u} \,|\, \underline{u} \leq u(k) \leq \overline{u}\}, \quad \forall k \tag{14b}$$

where vectors $\underline{x} \in \mathbb{R}^{n_x}$ and $\overline{x} \in \mathbb{R}^{n_x}$ determine the minimum and maximum possible state values of the system, respectively. Similarly $\underline{u} \in \mathbb{R}^{n_u}$ and $\overline{u} \in \mathbb{R}^{n_u}$ determine the minimum and maximum possible value of manipulated variables, respectively.

2.2 Economic MPC formulation of WDNs

The main control objective of the WDN is to minimize the water distribution costs that involves water production costs and electrical costs associated with pumping. In fact, the goal is to penalize the economic cost by considering the periodic price signal sequence ρ along the MPC prediction horizon to preserve economic performance. Hence, the cost function associated to this objective can be formulated as

$$\ell_e(u(k), \rho_i) = \rho_i^T u(k), \qquad i = mod(k, T), \tag{15}$$

where, ρ is modeling a periodic variation of the electricity price with the period T as $\rho = \{\rho_1, \rho_2 ..., \rho_T\}$.

For determination of the preserving water supply in spite of the variation of water demands between two consecutive MPC sampling steps, a suitable safety volume for each storage tank is needed to be maintained. Therefore, the mathematical expression for this objective is developed in a quadratic way as

$$\ell_s(k) \triangleq \begin{cases} \|x(k) - x_s\|_2, & if x(k) \leq x_s \\ 0, & otherwise \end{cases} \tag{16}$$

where x_s denotes the vector of the safety heads for all the tanks. This cost function can be also obtained by means of a soft constraint with adding a slack variable ξ, which can be reformulated as

$$\ell_s(k) \triangleq \xi^\top(k)\xi(k), \tag{17}$$

where $x(k) \geq x_s - \xi(k)$ is available. Furthermore, to ensure the safety objective, the minimal pressure at the demand sectors can be obtained by setting the following constraint

on the hydraulic head z as $z(k) \geq \underline{z}$, $\forall k \in \mathbb{Z}_+$, and $\underline{z}$ guarantees the minimal required pressure for the nodes inside the WDN.

The EMPC controller design based on minimizing the finite horizon cost function along the MPC prediction horizon is represented as

$$J_T(x, u, \rho) = \sum_{i=0}^{T-1} (W_1 \ell_e(u(i), \rho_i) + W_2 \ell_s(x(i)), \qquad (18)$$

where W_1 and W_2 are prioritization weights and the network mathematical model (12)-(13) and the operational bounds (13) as the constraints.

3. ECONOMIC MPC-LPV CONTROL OF WDNS

In this section, an economic MPC-LPV scheme for WDN control is proposed by modelling the nonlinear part of WDN as LPV model with the aim of avoiding the nonlinear economic MPC problem. In particular, considering that the pipe resistance is function of v, Eq. (13b) can be rewritten as

$$0 = P_x x(i) + P_z z(i) + P_v(\theta) v(i). \qquad (19)$$

where the scheduling variable in the LPV model is defined as $\theta = v$.

Considering the LPV model of the DWN, the idea is to find a solution to the problem MPC problem by formulating it as a QP problem. This can be done by transforming the exact economic MPC-LPV to a linear approximation of the economic MPC-LPV. This approximation is based on using an estimation of scheduling variables, $\hat{\theta}$ instead of using θ. It means that the scheduling variables in the prediction horizon are estimated and used to update the matrices of the model used by the MPC controller. In fact, for solving this problem, the sequence of the control nonmanipulated input is used to modify the system matrices of the model used in the prediction horizon. Thus, from the optimal control sequence $\mathbf{u}(k)$, it can be obtained the sequence of predicted parameters

$$\begin{aligned} \mathbf{x} &= [x(i|k), x(i+1|k), ..., x(T|k)], \\ \Theta &= [\hat{\theta}(i|k), \hat{\theta}(i+1|k), ..., \hat{\theta}(T-1|k)]. \end{aligned} \qquad (20)$$

The vector Θ includes parameters from time k to $k+T-1$ whilst the state prediction is accomplished for time $k+1$ to $k+T$. Hence, by using the (20), Eq. (19) can be reformulated as follows

$$0 = P_x x(i) + P_z z(i) + P_v(\Theta), \qquad (21)$$

By using (21) and augmented block diagonal weighting matrices $\tilde{W}_1 = diag_T(W_1)$ and $\tilde{W}_2 = diag_T(w_2)$, the cost function (18) can be rewritten in vector form as

$$\min_{\mathbf{u}, x, \boldsymbol{\xi}} \sum_{i=0}^{T-1} (\tilde{W}_1 \ell_e(u(i), \rho_i) + \tilde{W}_2 \ell_s(x(i)) + \tilde{W}_3 \ell_s(\gamma(i))), \qquad (22a)$$
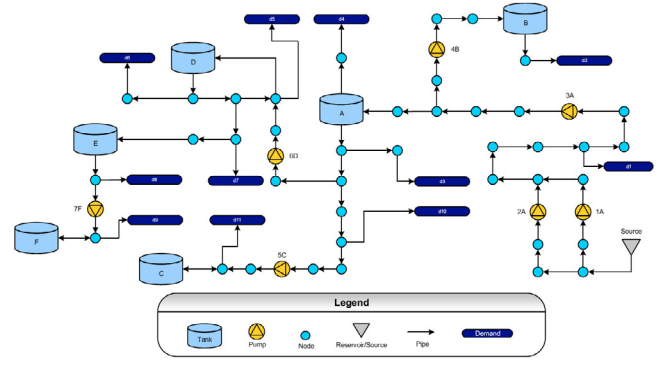
subject to:

Fig. 1. The topology of Richmond water distribution network.

$$\begin{aligned} x(i+1) &= Ax(i) + B_u u(i) + B_v v(i) + B_d d(i), & (22b) \\ 0 &= E_u u(i) + E_v v(i|k) + E_d d(i), & (22c) \\ 0 &= P_x x(i) + P_z z(i) + P_v(\Theta), & (22d) \\ x(i) &\geq x_s - \xi(i), \quad x(0) = x(T), & (22e) \\ P_v(\Theta) &= P_v(\Theta)\gamma(i) & (22f) \\ u(i) &\in \mathbb{U}, \qquad x(i), \in \mathbb{X}, & (22g) \\ \xi(i) &\geq 0 \qquad \gamma(i) \geq 0 & (22h) \\ x(j) &= \hat{x}(k), \quad j = mod(k, T), & (22i) \end{aligned}$$

where γ denotes a slack variable for the P_v, and $\tilde{W}_3$ is the weighting matrix related to the slack. Since the predicted $\Theta(k)$ in (21) are linear in controller, the optimization problem can be solved as a QP problem, that is significantly further easier than solving a nonlinear optimization problem.

4. APPLICATION

In this section, the proposed EMPC-LPV algorithm and control strategy is applied to a realistic benchmark network called the Richmond WDN (This benchmark is available from the link http: //emps.exeter.ac.uk/engineering/research/cws/resources/benchmarks. Richmond WDN is a medium-size network and the control-oriented model used in EPANET is expressed in the form of (12)-(13).

4.1 Case study

The Richmond WDN considers 6 water storage tanks, 7 booster pumps, and 11 water demand sectors. Moreover, there are 41 pressurized pipes and 41 non-storage nodes and connected in this network. The topology and organization of the Richmond WDN are shown in Fig 1. According to the modeling methodology presented in Section 2, the control-oriented model of the Richmond WDN can be formulated in the DAE form (11). The demand pattern is also given for a 24-h period, that is $T = 24$. In fact, by applying the mass balance at storage tanks, the system dynamics in (12) is obtained. The linear algebraic equation (13a) is determined by using the mass balance at non-storage nodes. Also, by using the Chezy-Manning head-flow formula (10), the algebraic equationn (13b) is obtained when $L_{i,j}$ and $D_{i,j}$ of (10) are given in the EPANET model of the Richmond WDN and the values of $C_{i,j}$ for each pipe in Chezy-Manny formula are obtained from Table 1 in Wang et al. (2018).

The EMPC-LPV formulation proposed in previous section (22) has been applied to a simulation model of Richmond WDN. In this simulation, the weights as $w_1 = 10$ and $w_2 = 0.1$ are considered, which they obtained from the tuning procedure presented in Toro et al. (2011). In the Richmond network, the minimal pressure at all the demand sectors is established to be 10m. Moreover, the period T is considered to be $T = 24$ h with the sampling time $\Delta t = 1$ h because of the periodicities of the water demand and electricity price respecting the variations in the daily tariff. Therefore, the prediction horizon of the proposed EMPC-LPV strategy is determined to be $T = 24$ h. To evaluate the effectiveness and economic efficiency and assess the performance of the presented approach in EMPC controller, another EMPC strategy based on the WDN inclining algebraic equations are tested in Richmond network such as:

- NEMPC: The optimization problem (18) is solved as a nonlinear optimization problem by using nonlinear periodic EMPC strategy and nonlinear solver.
- EMPC-Planner: By following Limon et al. (2014), it can be presented a finite-horizon optimization problem with the complete model of WDNs to find the optimal periodic steady trajectory, which it is introduced as the planner optimization problem. This optimization problem gives a similar solution if the time to be considered is any period that is $i \in [k, k + T], \forall k \in \mathbb{Z}_+$.
- EMPC-NCR: In this strategy, an iterative algorithm has been used to relax the nonlinear algebraic equations into a set of linear inequality constraints that will be implemented in the EMPC design (for more detail see Wang et al. (2018)).

All simulation and computations have been carried out using a commercial computer with i7 2.40-GHz Intel core processor with 12 GB of RAM running MATLAB R2016b. The optimization problems are solved by using the linear and nonlinear programming solvers available in YALMIP (Lofberg, 2004). All tests were done using the same prediction horizon, parameters and constraints.

In addition, the numeric assessment of the above-mentioned controllers is carried through different key performance indicators (KPIs), which are detailed below:

$$KPI_e := \frac{1}{n_s} \sum_{k=1}^{n_s} \left(\rho_j^T u(k) \right), \qquad j = mod(k, T), \qquad (23a)$$

$$KPI_s := \frac{1}{n_s} \sum_{i=1}^{n_x} \sum_{k=1}^{n_s} \max\{0, x_s(i,k) - x(i,k)\}, \qquad (23b)$$

$$KPI_m := \frac{1}{n_s} \sum_{i=1}^{n_x} \sum_{k=1}^{n_s} \left(x(i,k) - x_s(i,k) \right), \qquad (23c)$$

$$KPI_t := t_{opt}(k), \qquad (23d)$$

where KPI_e denotes the average economic performance of the pasteurization process, KPI_s is the safety KPI that measures the average differences of the water storage that are lower than safety hydraulic head $x_s(i,k)$ presented in Table 1. KPI_m is represented the additional water stored in storage tanks and KPI_t defines the difficulty to solve the optimization tasks associated with each approach accounting $t_{opt}(k)$ as the average time that gets to solve
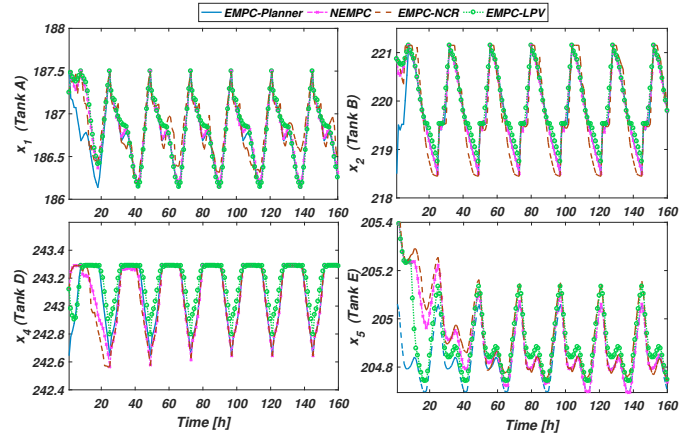

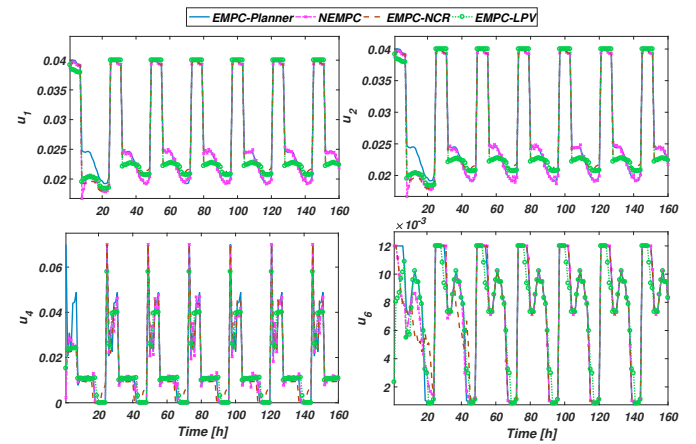
Fig. 2. Results of the evolutions of storage tanks.

Fig. 3. Evaluation of the control input results.

the corresponding FHOP. Moreover, $n_s \in \mathbb{I}_+$ is the number of seconds considered in the simulations and Δt is the sampling time in seconds. Note that in KPI_e, KPI_s, KPI_m and KPI_t lower values signify better performance results.

4.2 Results and Discussion

The closed-loop simulation results of some system control inputs and states for showing the system performance and comparison of EMP-LPV with NMPC, EMPC-NCR, and EMC-Planner are shown in Figures 2 and 3. Figure 2 presents the different evaluation of the system states which is the level of tanks, and they are obtained under the comparison of different mentioned EMPC approaches based on the dynamic model of Richmond network. Figure 2 shows the proper replenishment planning that the predictive controller dictates according to the cyclic behavior of demands. Notice that the net demand of each tank is properly satisfied along the simulation horizon. Figures 3 shows the evolution of the pumps commands obtained that are obtained under the comparison of different mentioned EMPC approaches based on the dynamic model of Richmond network. As shown in Figure 3, the behavior of the control input in EMPV-LPV, NMPC, EMPC-NCR and EMPC-Planner approaches are almost the same.

To evaluate the performance of different control strategies, the comparison is provided based on the KPI, which the

computation results are presented in Table 2. In general, the performances of different strategies of EMPC are similar. Particularly, from the KPI_e results, the economic cost of EMPC-LPV is slightly cheaper than the EMPC-NCR and NEMPC. Similar to KPI_e, the results of KPI_s and KPI_m in EMPC-LPV are better than other strategies. Based on the comparison of simulation time and average time that gets to solve the optimization problem, the time of EMPC-LPV is more decreased.

According to these results and reviewing the results in Table 2, it can be observed that the closed-loop trajectories obtained using the EMPV-LPV, EMPC-NCR, NEMPC and EMPC-Planner strategies are similar to those of the optimal planner trajectories (both states and control inputs). Forever, based on th the simulation time and KPI_t, it can be seen the EMPC-LPV approach has a significant improvement in the reduction of computational load and meanwhile based on the above comparison result, the performance of the EMPC-LPV strategy is similar to the EMPC-NCR and NEMPC strategy. This reduction in the computation time would be more relevant in larger networks.

Table 1. The hydraulic heads at storage tanks to evaluate the safety constraints.

Tank	Elevation (m)	Volume (m)	Hydraulic Head x_s (m)
A	184.13	1.02	185.15
B	216	2.03	2018.03
C	258.9	0.5	259.40
D	241.18	1.1	242.28
E	203.01	0.01	203.003
F	235.71	0.19	235.90

Table 2. Comparison of control performance.

Controller	KPI_e	KPI_s	KPI_m	KPI_t	Simulation Time
EMPC-LPV	0.9944	0.0118	5.2403	0.1499	25.1830 [s]
NMPC	1.0406	0.1966	7.0094	4.0961	688.1380 [s]
EMPC-NCR	1.0491	0.3302	6.8186	0.4856	81.5860 [s]

5. CONCLUSION

This paper focused on the design of an EMPC algorithm for the operational management of WDNs. In this approach for avoiding to use the nonlinear optimization problem because of the nonlinear algebraic equations, the nonlinear part of the model is embedded into the LPV model. Then the constrained optimization problem for an LPV model is solved iteratively by a series of QP problem while the scheduling parameters are calculated at each time instant. The proposed strategy is applied based on the Richmond WDN and the results have shown the performance is similar to a corresponding NEMPC, EMPC-Planner and EMPC-NCR. However, the proposed approach is the easiest one of the NEMPC approaches being compared and provides the best computational efficiency.

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