



Optimization of transmission capacity of energy water pipeline networks with a tree-shaped configuration and multiple sources

Dmitry V. Sokolov*, Evgeny A. Barakhtenko**

Melentiev Energy Systems Institute of Siberian Branch of the Russian Academy of Sciences (ESI SB RAS), Lermontov Str., 130, Irkutsk, 664033, Russia



ARTICLE INFO

Article history:

Received 21 January 2020

Received in revised form

28 June 2020

Accepted 24 July 2020

Available online 12 August 2020

Keywords:

Pipeline networks

Network optimization

Pipeline diameters

Pumps

Algorithm

Dynamic programming

ABSTRACT

The problem of pipeline network transmission capacity optimization is solved during network design, optimization, and development. It consists in determining the optimal pipeline diameters, installation sites, and parameters of pumps and valves. This study proposes a methodology for optimizing the transmission capacity of tree-shaped water pipeline networks of energy systems that serve various purposes and have multiple sources. A method for constructing a network model that ensures the versatility of its representation regardless of the purpose of the network and the composition of its equipment is proposed. A method for modeling of the branch that enables one to flexibly alter the configuration of the branch in the process of running the optimization algorithm without changing the network topology is presented. The mathematical statement of the problem is formulated as the discrete-continuous optimization problem. This statement is based on the proposed methods of representing the network model and its elements. An algorithm based on dynamic programming is proposed, which implements a new approach to setting up the computational procedure and is versatile enough for calculating networks that serve various purposes. The results obtained during real network optimization are presented.

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1. Introduction

Water pipeline networks of energy systems are increasingly attracting interest as an object of study, which can be explained by the growing relevance of meeting the following requirements:

- Improvement of network efficiency, which implies reducing energy consumption by lowering losses during water transportation.
- Making available the necessary hydraulic modes of network operation when adopting intelligent systems technologies.
- Ensuring the operation of the network in the presence of active consumers.
- The coordinated operation of multiple sources within a single network.
- Identification and implementation of the appropriate redundancy level to ensure required network reliability.

In countries with a high level of technological and economic development, they aim at developing the concepts for the construction of energy pipeline networks, within the framework of which the above requirements will be implemented. For example, in the field of heat supply, the problem of transition to the concept of the 4th Generation District Heating [1] has been stated. Networks as a subject of study are known to be of high complexity, which is due to a large number of pipelines, multiple sources, the presence of active elements (pumps, pressure and flow regulators), and heterogeneous consumers. For this reason, to design networks that meet the above requirements, it is necessary to employ mathematical models, methods, and optimization algorithms that enable technologically and economically sound solutions.

Among the design problems of particular importance is the problem of optimizing the network transmission capacity, which consists in optimizing the diameter of pipelines, determining sites and parameters of pumps and valves. In this paper, the subject of study is the problem of ensuring the transmission capacity of tree-shaped networks that cover the following:

- Supply and return networks of the district heating system (DHS);

* Corresponding author.

** Corresponding author.

E-mail addresses: sokolov_dv@isem.irk.ru (D.V. Sokolov), barakhtenko@isem.irk.ru (E.A. Barakhtenko).

- Supply and return networks of the cooling system;
- Water distribution networks;
- Process networks of large enterprises.

The optimization problem can be reduced to various mathematical statements, for which various optimization algorithms can be used. To solve the problem, either universal mathematical software or developed specialized software can be used. The first case suggests using a standardized language for describing the problem, which eliminates the need to reprogram the optimizer but limits the ability to describe complex equipment and design procedures. In the second case, it becomes possible to flexibly configure the software for the problem but the optimizer must be reprogrammed. The time-consuming operation of reprogramming can be avoided by developing new approaches to solving the problem, which will allow standardizing algorithms and mathematical models and providing a unified interface for their integration.

In the 1960s, the Soviet scientist Khasilev [2,3] laid the groundwork for the theory of hydraulic circuits, which is a scientific discipline that integrates the achievements of various branches of science to build a common physical and mathematical basis for the analysis, design, and control of pipeline systems. One of the main principles of the discipline is that the methods and algorithms developed within its framework should be versatile and lend themselves to application to the calculation of pipeline networks of various types and purposes. The possibility of creating such methods and algorithms is underpinned by the commonality of topological properties of pipeline networks and the commonality of network laws of mass and energy conservation. In this study, a new methodology of optimization of transmission capacity of tree-shaped pipeline water networks of energy systems is presented; the methodology fulfills this principle of the theory of hydraulic circuits.

This paper presents the key ideas and components of the methodology. The results of applying the proposed methodology to a real DHS are presented.

2. Review of approaches to solving the problem

There are three main cases of network transmission capacity optimization.

- 1 Determining the optimal parameters for a new network when designing it.
- 2 Making recommendations for optimizing the existing network.
- 3 Determining the optimal parameters of the existing network as it undergoes development.

Due to the commonality of network laws, this literature review covers the most important approaches to optimizing the transmission capacity of networks that serve various purposes. Optimizing network transmission capacity is treated as a problem of deciding on the optimal parameters of network elements.

The application of linear programming methods is one of the approaches to determining optimal parameters of network elements. Alperovits and Shamir [4] proposed a computational procedure based on hierarchical decomposition of the optimization problem that employs the linear programming gradient method. This method was later used in Ref. [5], where a two-step decomposition model was proposed. The proposed computational procedure arrives at the lower boundary in the global optimal solution using the generalized duality theory. Linear programming has been used to construct iterative algorithms [6,7].

Some researchers [8] try to use combinatorial algorithms to

solve the problem of determining optimal network parameters. Some works [9,10] use the Cross-Entropy method for combinatorial optimization.

Other authors [11,12] reduced the problem of determining optimal parameters of pipeline network elements to nonlinear programming problems where diameters are treated as continuous variables.

The advent of dynamic programming (DP) [13] led to a considerable progress in the methods for determining optimal parameters of pipeline networks [14–16]. The studies to be highlighted among those devoted to the application of DP in determining optimal parameters of heating networks are [17,18].

Kalinci et al. [19] determined the optimal pipe diameter for a geothermal plant by solving the derivative of the formulated objective function. Dobersek and Goricanec [20] performed optimization work with nonlinear programming based on simplex method to find the optimal routing and pipe diameter in a tree-shaped heating network.

Recent studies have made attempts to apply various heuristic algorithms to solve the problem of optimal design of pipeline networks. The published research on the application of evolutionary algorithms to the problems of pipeline network optimization appeared at the end of the 1990s [21,22]. Genetic algorithms are widely used for design and optimization problems [23,24]. Cunha and Sousa [25] suggested the use of a simulated annealing to solve the optimization problem of a looped water distribution network. Li et al. [26] applied the genetic algorithm to find the optimal district heating and cooling network pipe diameter for a seawater-source heat pump plant. Ant colony optimization algorithms [27,28] also receive distribution for solving design and optimization problems.

Methods of integer programming, which are implemented as optimizers distributed as part of mathematical software, have been developed. Zhang et al. [29] applied CPLEX solver for water network design problem. In Ref. [30,31] mixed integer nonlinear programming (MINLP) is used. In Ref. [32] the authors propose a hybrid evolutionary-MILP algorithm for optimization of multi-source complex DHSs.

The experience accumulated over years at the Melentiev Energy Systems Institute of Siberian branch of the Russian Academy of Sciences (ESI SB RAS) with respect to solving optimization problems proved the effectiveness of DP. The ESI SB RAS developed a method of multi-loop optimization [33,34] to optimize the parameters of looped pipeline networks that makes use of DP to optimize a network transformed into a tree [35]. These methods were applied at the ESI SB RAS [3,34] to determine optimal parameters of DHSs in the cities of Novosibirsk, Omsk, Khabarovsk, Saint-Petersburg [35], Bratsk [36], and others.

3. Constituents of the methodology. Modeling of the network and its elements

A new methodology is proposed to optimize the transmission capacity of tree-shaped water pipeline networks of energy systems that serve various purposes and have multiple sources. This methodology includes the following components:

- The principle of setting up the computational process when optimizing the transmission capacity of the network.
- A method for building a network model that ensures the universality of its representation regardless of the network purpose and the composition of the equipment.
- A method for modeling the branch as a network element that represents a model of one of the network elements, such as a

pipeline, a pump, a valve, or a model of their combination with the aggregated properties.

- The conceptual statement of the problem of optimization of the transmission capacity of tree-shaped pipeline networks, formulated on the basis of the proposed methods for modeling the network and its elements.
- The mathematical problem statement of the problem of optimization of the transmission capacity of tree-shaped pipeline networks, formulated as a problem of discrete-continuous optimization.
- A DP algorithm for optimizing the transmission capacity of tree-shaped pipeline networks.

The principle of the arrangement of the computational process when optimizing the transmission capacity of a pipeline network assumes the implementation of the following ideas:

- A single way of representing the model is used for networks that serve various purposes.
- Commercially available equipment is described by mathematical models that can be combined to obtain a model of a network element.
- The content of the optimization algorithm steps does not depend on the purpose of the network and the composition of its equipment.
- The integration of the algorithm, network model, and commercially available equipment models is carried out in the course of the computational process.

Fig. 1 shows an illustration of this principle.

Models of commercially available equipment are models of functionally independent components used to build a pipeline network. These are the models of pipelines, pumps, and valves. These models represent a set of the following components:

- Mathematical expressions describing the hydraulic laws as applied to the equipment;
- Mathematical expressions for calculating economic costs;
- Values of components of mathematical expressions;
- Limit values of hydraulic parameters of the equipment.

Subsections 3.1–3.5 will present the method for the network modeling and the method for the branch element modeling. Conceptual and mathematical problem statements are considered in **Section 4**. The optimization algorithm based on DP is presented in **Section 5**.

3.1. Method for building a network model

A distinguishing feature of the proposed method for building a

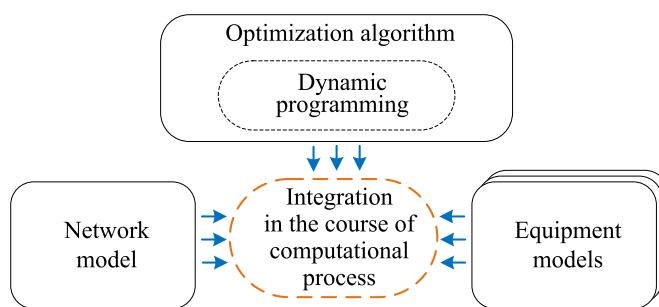


Fig. 1. An illustration of the principle of the arrangement of the computational process.

network model is that it ensures the universality of its representation regardless of the purpose of the network and the composition of equipment. According to this method, a network model is made up of the following components:

- Description of the structural configuration of the network.
- Values of the initial parameters of the network elements.
- Constraints on the values of variables.

The description of the structural configuration of the network is a description of the network topology and the types of its elements. Nodes and branches of the network are defined by multiple node and branch numbers. The network topology is defined by the directed graph consisting of these sets and incident functions. The sets of node and branch numbers consist of disjoint subsets, each of which corresponds to a certain type of elements.

The values of the initial parameters of the network elements reflect the properties of existing and new (to be designed) nodes and branches. For the existing elements, parameter values corresponding to the properties of the actual network elements are set, including the model numbers of the installed equipment. For new elements, initial parameters are set by determining their values on the basis of expert judgment.

Constraints on the values of the variables set the limits for the values of variables of interest in the optimal solution, and they include the following:

- Limit values of hydraulic parameters for network elements;
- Constraints on equipment permitted for installation at branches.

3.2. Method for branch modeling

Optimizing the transmission capacity of the pipeline network involves determining the optimal installation locations for pumps and valves. This operation implies changing the network topology, which proves a source of difficulties in the computational process. As part of the methodology, a method for branch modeling has been developed, which enables one to overcome these difficulties and flexibly alter the composition of the equipment in the course of the computational process. According to this method, the branch is treated as a mathematical model of the network element that connects two nodes and has a unified set of parameters. During the calculation, the branch models such elements as a pipeline, a pump, and a valve. Furthermore, the branch represents a model of a combination of the above elements. In this case, the branch has properties aggregated from the properties of the model components.

To build a unified network optimization model, the branch model should include the following components:

- The laws of changing the head at the branch;
- The law of the dependence of the flow velocity on the volume flow rate;
- Limit values of hydraulic parameters for equipment;
- Equations for calculating economic costs.

The change in the head at the j -th branch is defined by function

$$f_j(d_j, \gamma_j, \psi_j, \zeta_j, Q_j) = h_j^{\text{pl}}(d_j, Q_j) - H_j^P(\gamma_j, Q_j) + h_j^V(\psi_j, \zeta_j, Q_j), \quad (1)$$

where h_j^{pl} is the total loss of head at the pipeline section (m), H_j^P is pump head (m), h_j^V is head loss at the valve (m), d_j is internal

diameter of the pipeline (m), γ_j is pump model number, ψ_j is valve model number, ζ_j is local loss coefficient for the valve, Q_j is volume flow rate (m^3/s).

The total cost of the j -th branch is described by function

$$C_j^B(d_j, \gamma_j, \psi_j, Q_j) = C_j^{\text{pl}}(d_j) + C_j^p(\gamma_j, Q_j) + C_j^v(\psi_j), \quad (2)$$

where C_j^{pl} is pipeline costs, C_j^p is pump costs, C_j^v is valve costs.

Application of the proposed method for branch modeling allows obtaining the following results:

- To describe the topology of the pipeline network, the Kirchhoff's network laws and the laws of head changes in a unified matrix form.
- To change the branch configuration by changing the values of the corresponding variables without changing the network topology.

3.3. Pipeline modeling

The complete set of diameters is of the form $D = D_s \cup \{0\}$, where $D_s = \{d_1^s, \dots, d_k^s\}$ is a set of commercially available diameters, $0 \in D$ is the nominal diameter at the branches without a pipeline.

Flow velocity v_j (m/s) at the pipeline at the j -th branch is determined by the equation:

$$v_j(d_j, Q_j) = \frac{4Q_j}{\pi d_j^2}. \quad (3)$$

The total head loss at the pipeline at the j -th branch is modeled by function

$$h_j^{\text{pl}}(d_j, Q_j) = \begin{cases} 0, & d_j = 0, \\ \lambda_j \frac{L_j}{d_j} \frac{8Q_j^2 \cdot \text{sgn}(Q_j)}{g\pi^2 d_j^4}, & d_j > 0, \end{cases} \quad (4)$$

where λ_j is friction factor; L_j is length (m); g is gravitational acceleration (m/s^2). The equations proposed by Blasius [37], Colebrook [38], Nikuradse [39] and Altshul [40] are often used to calculate λ_j .

Pipeline costs C_j^{pl} at the j -th branch are calculated by the following formula:

$$C_j^{\text{pl}}(d_j) = \begin{cases} 0, & d_j = 0, \\ C_j^{\text{cpl}}(d_j) + C_j^o(d_j), & d_j > 0, \end{cases} \quad (5)$$

where C_j^{cpl} is capital investments for the pipeline determined by the formula:

$$C_j^{\text{cpl}}(d_j) = \begin{cases} f K_{\text{pl}}(d_j) L_j, & d_j = d_j^e, \\ (a + f) K_{\text{pl}}(d_j) L_j, & d_j \neq d_j^e, \end{cases} \quad (6)$$

f is coefficient of depreciation charges, K_{pl} is specific capital investments, a is discount rate, d_j^e is existing pipeline diameter, C_j^o is operating costs that depend on the purpose of the network, e.g. in the case of heating network, this is the cost of heat losses

$$C_j^o(d_j) = C_j^{\text{hl}}(d_j) = c_h q(d_j) L_j, \quad (7)$$

where c_h is specific cost of heat losses, $q(d_j)$ is specific heat losses.

3.4. Pump modeling

The complete set of pump model numbers is of the form $\Gamma = \Gamma_s \cup \{0\}$, where $\Gamma_s = \{1, \dots, \bar{\gamma}\}$ is a set of model numbers of commercially available pumps, $0 \in \Gamma$ is the model number at the branches with no active pumping equipment, $\bar{\gamma}$ is the number of models. For all pump models, the maximum branch volume flow rate limits are specified by the function $Q_{\text{max}}^p(\Gamma) = \{Q_{\text{max}}^p(\gamma) : \gamma \in \Gamma\}$.

In the proposed model, the pump head is represented as function of the increase in head across the pump

$$H_j^p(\gamma_j, Q_j) = \begin{cases} 0, & \gamma_j = 0, \\ \sum_{k=0}^{n_c} \alpha_k |Q_j|^{\beta_k} \cdot \text{sgn}(Q_j), & \gamma_j > 0, \end{cases} \quad (8)$$

$$\alpha_k = \alpha_k(\gamma_j), \quad \beta_k = \beta_k(\gamma_j),$$

$$n_c = n_c(\gamma_j), \quad k = 0, \dots, n_c,$$

where n_c is the number of polynomial components, α_k and β_k are coefficients and exponents that are functions of the model number of pump γ_j .

The costs of pump $C_j^p(\gamma_j, Q_j)$ at the j -th branch are calculated by the formula:

$$C_j^p(\gamma_j, Q_j) = \begin{cases} 0, & \gamma_j = 0, \\ C_j^{\text{cp}}(\gamma_j) + C_j^e(\gamma_j, Q_j), & \gamma_j > 0, \end{cases} \quad (9)$$

where C_j^{cp} is the pump equipment costs determined by the formula:

$$C_j^{\text{cp}}(\gamma_j) = \begin{cases} f K_p(\gamma_j), & \gamma_j = \gamma_j^e, \\ (a + f) K_p(\gamma_j), & \gamma_j \neq \gamma_j^e, \end{cases} \quad (10)$$

γ_j^e is the model number of existing pump, K_p is capital investments, C_j^e is electricity costs determined by the equation:

$$C_j^e(\gamma_j, Q_j) = 3600 \tau c_e \frac{\rho g Q_j H_j^p(\gamma_j, Q_j)}{\eta_p(\gamma_j)}, \quad (11)$$

ρ is water density (kg/m^3), τ is number of operation hours per year, c_e is electricity tariff, η_p is pump efficiency.

3.5. Valve modeling

The complete set of valve model numbers is of the form $\Psi = \Psi_s \cup \{0\}$, where $\Psi_s = \{1, \dots, \bar{\psi}\}$ is a set of model numbers of commercially available valves, $0 \in \Psi$ is the model number when there is no valve, $\bar{\psi}$ is the number of models. For all valve models, the constraints on the maximum volume flow rate at the branches are specified by the function $Q_{\text{max}}^v(\Psi) = \{Q_{\text{max}}^v(\psi) : \psi \in \Psi\}$.

The head loss h_j^v at the valve at the j -th branch is determined by the equation:

$$h_j^v = \zeta_j \frac{(v_j^v)^2}{2g}, \quad (12)$$

where v_j^v is flow velocity; ζ_j is local loss coefficient, the engineering

meaning of which follows from the equation below:

$$\zeta_j = \zeta_{\min}(\psi_j) + \Delta\zeta_j, \quad (13)$$

ψ_j is valve model number; $\zeta_{\min}(\psi_j)$ is local loss coefficient when the valve is in the open state; $\Delta\zeta_j$ is local loss coefficient increase when using the valve for reducing the pressure. Parameter ζ_j may be subject to upper limit $\zeta_{\max}(\psi_j)$.

Flow velocity v_j^v at the valve at the j -th branch is determined by the equation:

$$v_j^v = \frac{Q_j}{A(\psi_j)}, \quad (14)$$

where A is the cross-sectional area of the valve passageway (m^2).

Head loss at the valve is modeled by function

$$h_j^v(\psi_j, \zeta_j, Q_j) = \begin{cases} 0, & \psi_j = 0, \\ \zeta_j \frac{Q_j^2 \cdot \text{sgn}(Q_j)}{2gA(\psi_j)^2}, & \psi_j > 0. \end{cases} \quad (15)$$

The costs of the valve $C_j^v(\psi_j)$ equipment at the j -th branch are determined by the formula:

$$C_j^v(\psi_j) = \begin{cases} 0, & \psi_j = 0, \\ (a + f)K_v(\psi_j), & \psi_j > 0, \psi_j \neq \psi_j^e, \\ fK_v(\psi_j), & \psi_j > 0, \psi_j = \psi_j^e, \end{cases} \quad (16)$$

where ψ_j^e is the model number of existing valve, K_v is capital investments.

4. Formal description of the optimization problem

Subsection 4.1 presents a conceptual problem statement that has the network model and its elements presented on the basis of the proposed methods. **Subsection 4.2** covers the mathematical statement of the problem of network transmission capacity optimization, which is formulated as the problem of discrete-continuous optimization.

4.1. Conceptual problem statement

The problem of optimizing the transmission capacity of a pipeline network has the following form. The specified parameters include:

- A network model that consists of m nodes and n branches as defined by a directed graph $G = (N, A, \alpha(j), \omega(j))$, where $N = \{1, \dots, m\}$ is a set of nodes, $A = \{1, \dots, n\}$ is a set of branches, $\alpha(j)$ and $\omega(j)$ are incidence functions that respectively assign the initial and final nodes of j -th branch ($j \in A$); $N = N^+ \cup N^0 \cup N^-$ where N^+ is a set of nodes with inflows, N^0 is a set of simple points of connection of branches, N^- is a set of nodes with outflows;
- Lengths of pipelines L_j at branches, $j \in A$;
- Elevations of nodes z_i , $i \in N$;
- The upper p_i^{\max} and lower p_i^{\min} constraints on the manometer pressure at nodes, $i \in N^0 \cup N^-$;
- The upper v_j^{\max} and lower v_j^{\min} constraints on the flow velocity at branches, $j \in A$;

- Standard pipeline models, each of which corresponds to a diameter that belongs to set D ;
- Sets of pipeline diameters $D_j \subset D$, that are allowed for installation at specific branches, $j \in A$;
- Diameters of pipelines $d_j^e \in D$ at branches, $j \in A$;
- Pump models, each of which has its own number that belongs to set Γ ;
- Sets of pump model numbers $\Gamma_j \subset \Gamma$ that are allowed to be installed at specific branches, $j \in A$;
- Pump model numbers $\gamma_j^e \in \Gamma$ at branches, $j \in A$;
- Valves models each of which has its own number that belongs to set Ψ ;
- Sets of valve model numbers $\Psi_j \subset \Psi$ that are allowed to be installed at specific branches, $j \in A$;
- Valve model numbers $\psi_j^e \in \Psi$ at branches, $j \in A$;
- Hydraulic heads H_i at nodes with inflows to the network, $i \in N^+$;
- Volume flow rates q_i at nodes: $q_i > 0$ for inflows ($i \in N^+$), $q_i = 0$ for plain connections points ($i \in N^0$), $q_i < 0$ for outflows ($i \in N^-$).

The solution to the problem should be optimal parameters of the network:

- Diameters of pipelines d_j at branches, $j \in A$;
- Pump model numbers γ_j at branches, $j \in A$;
- Valve model numbers ψ_j at branches, $j \in A$;
- Local loss coefficients of valves ζ_j at branches, $j \in A$;
- Volume flow rates Q_j at branches, $j \in A$;
- Hydraulic heads H_i at nodes, $i \in N^0 \cup N^-$.

4.2. Mathematical problem statement

The problem of minimizing the function of the total cost of the network is solved, which has the following form:

$$C(\mathbf{d}, \mathbf{u}_\gamma, \mathbf{u}_\psi, \mathbf{Q}) = \sum_{j \in A} C_j^B(d_j, \gamma_j, \psi_j, Q_j), \quad (17)$$

subject to:

$$\mathbf{AQ} = \mathbf{q}, \quad (18)$$

$$\mathbf{AH} = \Delta \mathbf{H}, \quad (19)$$

$$\Delta \mathbf{H} = \mathbf{f}(\mathbf{d}, \mathbf{u}_\gamma, \mathbf{u}_\psi, \mathbf{u}_\zeta, \mathbf{Q}), \quad (20)$$

$$H_i = \frac{p_i}{\rho g} + z_i, \quad i \in N, \quad (21)$$

$$p_i^{\min} \leq p_i \leq p_i^{\max}, \quad i \in N^0 \cup N^-, \quad (22)$$

$$v_j^{\min} \leq |v_j(d_j, Q_j)| \leq v_j^{\max}, \quad j \in A_{pl}, \quad (23)$$

$$|Q_j| \leq Q_{\max}^p(\gamma_j), \quad j \in A, \quad (24)$$

$$|Q_j| \leq Q_{\max}^v(\psi_j), \quad j \in A, \quad (25)$$

$$d_j \in D_j \subset D, \quad j \in A, \quad (26)$$

$$\gamma_j \in \Gamma_j \subset \Gamma, \quad j \in A, \quad (27)$$

$$\psi_j \in \Psi_j \subset \Psi, \quad j \in A, \quad (28)$$

$$\zeta_{\min}(\psi_j) \leq \zeta_j \leq \zeta_{\max}(\psi_j), \quad j \in A, \quad (29)$$

where (18) and (19) are equivalents of the first and second Kirchhoff's laws; (20) is the law governing the change in the head at branches; (21) is relation between of hydraulic head and manometer pressure at the node; (22) is constraint on manometer pressure at the node; (23) is constrain on velocity at branches with the pipeline; (24)–(25) are constrains on maximum volume flow rate for the pump and valve; (26)–(28) are conditions of discreteness of the equipment models; (29) is constraint on local loss coefficient for the valve. The equations use the following notation: \mathbf{A} is an $m \times n$ incidence matrix with elements

$$a_{ij} = \begin{cases} 1, & \text{if } i = \alpha(j), \\ -1, & \text{if } i = \omega(j), \\ 0, & \text{otherwise,} \end{cases}$$

$\mathbf{d} = (d_1, \dots, d_n)^T$ is the vector of internal diameters of pipelines, $\mathbf{u}_\gamma = (\gamma_1, \dots, \gamma_n)^T$ and $\mathbf{u}_\psi = (\psi_1, \dots, \psi_n)^T$ are vectors of numbers of pump and valve models, $\mathbf{u}_\zeta = (\zeta_1, \dots, \zeta_n)^T$ is the vector of local loss coefficients for valves; $\mathbf{Q} = (Q_1, \dots, Q_n)^T$ and $\Delta\mathbf{H} = (\Delta H_1, \dots, \Delta H_n)^T$ are vectors of volume flow rates and head losses at branches, $\mathbf{q} = (q_1, \dots, q_m)^T$ and $\mathbf{H} = (H_1, \dots, H_m)^T$ are vectors of volume flow rates and hydraulic heads at nodes; \mathbf{f} is n -dimensional vector function with elements $f_j(d_j, \gamma_j, \psi_j, \zeta_j, Q_j)$ that reflect the laws of the change in the head at j -th branch, $j = 1, \dots, n$, $A_{\text{pl}} = \{j \in A : L_j > 0\}$ is a set of all branches with the pipeline.

5. Optimization algorithm

In a tree-shaped network its configuration and the vector of nodal inflows and outflows \mathbf{q} unambiguously define a vector of volume flow rates at branches \mathbf{Q} . This property of flow distribution follows from the fact that in this case there is only one solution to equation (18). Given the tree-shaped network configuration and fixed vector \mathbf{Q} , objective function (17) is additive. This allows the use of DP to solve the problem (17)–(29).

In the present paper, a new algorithm for optimization of transmission capacity of tree-shaped pipeline networks of energy systems with multiple sources is proposed. This algorithm is based on DP and is a development of the algorithm proposed in Ref. [35]. The main difference and novelty of the algorithm consist in the following:

- versatility, which is ensured by independence from the properties and purpose of the network;
- the unified method is used to represent the network model and its elements;
- a new approach to the computational procedure;
- independence from mathematical models of equipment.

The algorithm has the following form.

Algorithm 1. A DP algorithm for optimizing the transmission capacity of tree-shaped pipeline networks. Input parameters: μ is the number of cells that determines the accuracy of the resulting solution; i_R is the node that is the root of the tree.

Step 1: Calculate the volume flow rates at the network branches and change the orientation of the branches in the direction of moving from the root of the tree to its end nodes.

Step 2: Determine the range of acceptable values of hydraulic heads and form a two-dimensional data structure for storing components of conditionally-optimal solutions to the problem.

Step 3: Determine conditionally-optimal solutions to the problem, each of which includes parameters of network nodes and branches and shows the head curves from the root of the tree to its end nodes.

Step 4: Identify the best (that with the minimum total costs) of the found conditionally-optimal solutions to the problem.

Step 5: Identify the components of the best solution out of the found conditionally-optimal solutions to the problem and restore the original orientation of the branches.

At Step 1 of **Algorithm 1**, the system of linear equation (18) to determine vector \mathbf{Q} of volume flow rates at the branches is solved. Then the orientation of the branches is changed to correspond to the direction of moving from the root of the tree i_R to its end nodes. When the branch orientation changes, the corresponding components of vector \mathbf{Q} have their signs changed to the opposite.

At Step 2 of **Algorithm 1**, data structures are formed to store the components of conditionally-optimal solutions to the problem, which will be determined during the DP computational process. To this end, the algorithm performs the following actions.

The upper and lower boundaries for hydraulic heads at the network nodes are calculated by formulas:

$$H_i^{\min} = \begin{cases} H_i, & i \in N^+, \\ \frac{p_i^{\min}}{\rho g} + z_i, & i \in N^0 \cup N^-, \end{cases} \quad (30)$$

$$H_i^{\max} = \begin{cases} H_i, & i \in N^+, \\ \frac{p_i^{\max}}{\rho g} + z_i, & i \in N^0 \cup N^-. \end{cases} \quad (31)$$

Based on the calculated constraints on hydraulic heads, cells are created to store parameters of network elements when constructing conditionally-optimal solutions of the problem. The principle of construction of these cells is illustrated in Fig. 2, which shows a fragment of the network model. Perpendiculars to the reference elevation plane are drawn through the connection points of the network elements. The values of the upper ($H_i^{\max}, i \in N$) and lower ($H_i^{\min}, i \in N$) limits on hydraulic heads are marked on the resulting lines. The obtained ranges of feasible values of hydraulic heads are divided into μ intervals the number of which determines the accuracy of the obtained solution to a problem. The obtained intervals belonging to one element are connected by segments. As a result, there are μ cells for each network element (nodes and branches). The total number of cells for the entire network model is $(m+n)\mu$. Each of the obtained cells is used in the construction of conditionally-optimal solutions to the problem to store a certain combination of network element parameters.

Below, the following functions are used to calculate boundaries of the k -th interval at the i -th node:

the lower boundary

$$H_i^{\min}(k) = H_i^{\min} + (k-1) \frac{H_i^{\max} - H_i^{\min}}{\mu}, \quad (32)$$

the upper boundary

$$H_i^{\max}(k) = H_i^{\min} + k \frac{H_i^{\max} - H_i^{\min}}{\mu}. \quad (33)$$

The solution at the j -th branch is in the k -th cell, if the following

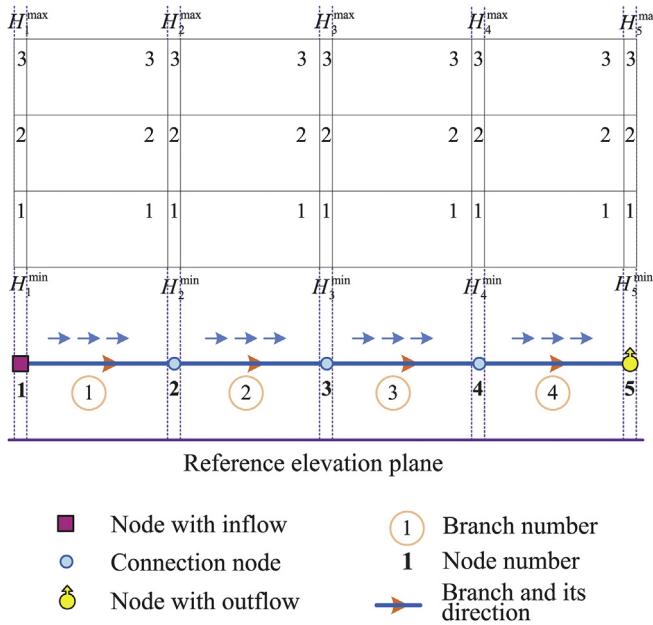


Fig. 2. Cells for nodes and branches.

condition is met for the head \tilde{H}_{jk} at the beginning of branch

$$H_i^{\min}(k) \leq \tilde{H}_{jk} \leq H_i^{\max}(k), \quad (34)$$

where i is the initial node of the j -th branch.

A two-dimensional data structure is formed for storing components of conditionally-optimal solutions. This data structure consists of the following two-dimensional arrays: $d[1\dots n, 1\dots \mu]$, $\gamma[1\dots n, 1\dots \mu]$, $\psi[1\dots n, 1\dots \mu]$ and $\zeta[1\dots n, 1\dots \mu]$ are arrays of pipeline diameters, model numbers of pumps and valves, local loss coefficients for valves, respectively; $H[1\dots m, 1\dots \mu]$ and $\tilde{H}[1\dots n, 1\dots \mu]$ are arrays of hydraulic heads at nodes and at the beginning of branches, respectively; $C[1\dots m, 1\dots \mu]$ and $\tilde{C}[1\dots n, 1\dots \mu]$ are arrays of cost sums in cells at nodes and branches, respectively; and $w[1\dots n, 1\dots \mu]$ is an array of relations between cells for adjacent network elements.

Step 3 of [Algorithm 1](#) constructs conditionally-optimal solutions to the problem.

[Algorithm 2](#) implements the process of constructing conditionally-optimal solutions to the problem and is based on the depth-first search (DFS) algorithm [41]. The backward run of the DFS-algorithm is used to arrange the process of constructing conditionally-optimal solutions from the end nodes to the root of the network tree ([Fig. 3](#)). During the backward run along a branch, [Algorithm 3](#) is called to construct components of the solutions for that branch. When the algorithm returns to the node, [Algorithm 4](#) is called for the coupling of conditionally-optimal solutions and determining the components of solutions in this node. The tree-shaped network configuration guarantees that the path from the root to any other node is unique, thus ensuring the correct arrangement of the computational process of DP.

The operation of determining the components of conditionally-optimal solutions in a branch cell is performed on the basis of Bellman's principle of optimality [13,41]. At the j -th branch ($j \in A$) having i -th initial node and i' -th final node, for the k -th cell ($k = 1, \dots, \mu$) the following problem is solved:

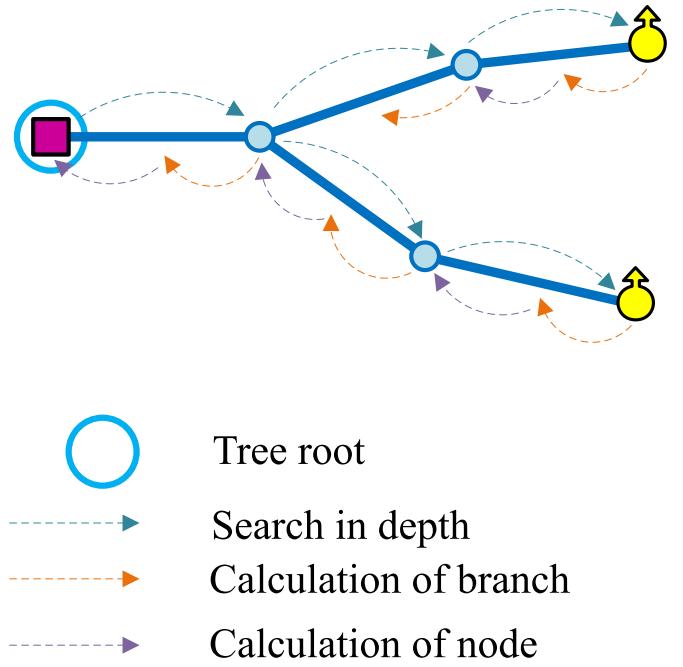


Fig. 3. DFS-algorithm is used to arrange the process of constructing conditionally-optimal solutions.

$$\left\{ \begin{array}{l} \tilde{C}_{jk} \rightarrow \min, \\ \tilde{C}_{jk} = C_j^B(d_{jk}, \gamma_{jk}, \psi_{jk}, Q_j) + C[i', w], \\ \tilde{H}_{jk} = H[i', w] + f_j(d_{jk}, \gamma_{jk}, \psi_{jk}, \zeta_{jk}, Q_j), \\ H_i^{\min}(k) \leq \tilde{H}_{jk} \leq H_i^{\max}(k), \\ |Q_j| \leq Q_{\max}^p(\psi_{jk}), \quad |Q_j| \leq Q_{\max}^v(\psi_{jk}), \\ \zeta_{\min}(\psi_{jk}) \leq \zeta_{jk} \leq \zeta_{\max}(\psi_{jk}), \\ v_j^{\min} \leq |v_j(d_{jk}, Q_j)| \leq v_j^{\max}, \quad j \in A_{pl}, \\ d_{jk} \in D_j, \quad \gamma_{jk} \in \Gamma_j, \quad \psi_{jk} \in \Psi_j, \quad w \in \{1, \dots, \mu\}, \end{array} \right. \quad (35)$$

where d_{jk} , γ_{jk} , ψ_{jk} , ζ_{jk} are components of the conditionally-optimal solution for the k -th cell, w is the number of a cell that gets the hydraulic head at i' -th node, $C[i', w]$ is costs in the w -th cell for a conditionally-optimal solution from the i' -th node to end nodes, $H[i', w]$ is hydraulic head at the w -th cell at i' -th node, \tilde{H}_{jk} is the hydraulic head at the beginning of j -th branch in the k -th cell, \tilde{C}_{jk} is costs in the k -th cell for a conditionally-optimal solution from the j -th branch to end nodes.

The problem (35) has the following engineering interpretation ([Fig. 4](#)). For the j -th branch in the k -th cell of all possible combinations of equipment the one for which the sum of costs (costs of the branch equipment and costs of previous steps) is minimal is stored. The hydraulic parameters for this combination should meet the specified constraints.

Determining the components of conditionally-optimal solutions for the node is performed by combining conditionally-optimal solutions at this node. Let A_i^- be a set of branches that have the i -th node as their initial node. When merging solutions at the k -th cell ($k = 1, \dots, \mu$), hydraulic head H_{ik} , and the sum of costs C_{ik} for the conditionally-optimal solution from the i -th node to end nodes of

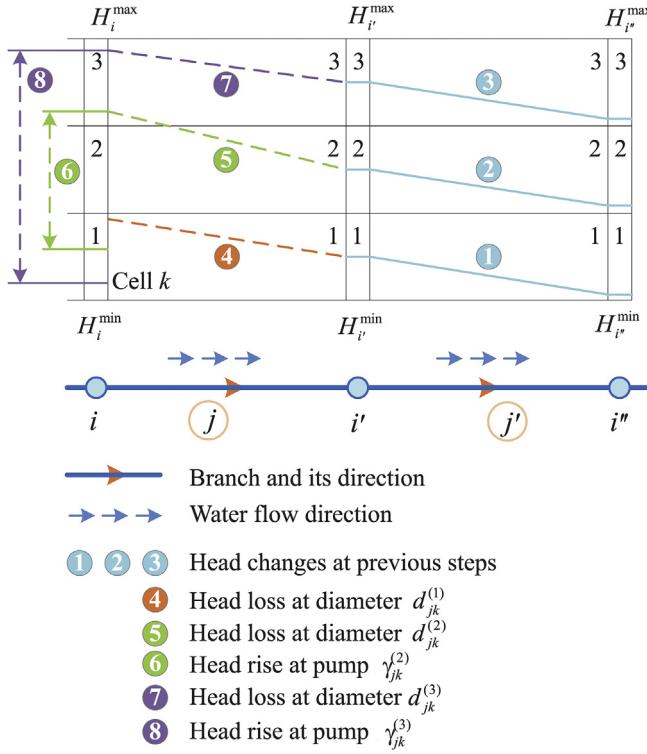


Fig. 4. The engineering interpretation of the problem (35).

the network are calculated. H_{ik} is calculated by the formula

$$H_{ik} = \begin{cases} H_i^{\min}(k), & \text{if } A_i^- = \emptyset, \\ \max_{j \in A_i^-} \tilde{H}[j, k], & \text{otherwise,} \end{cases} \quad (36)$$

C_{ik} is calculated by the formula

$$C_{ik} = \begin{cases} 0, & \text{if } A_i^- = \emptyset, \\ \sum_{j \in A_i^-} \tilde{C}[j, k], & \text{otherwise.} \end{cases} \quad (37)$$

Once Step 3 of **Algorithm 1** is completed, the components of conditionally-optimal solutions for all network elements are determined. As a result, μ conditionally-optimal solutions to the problem are constructed. Each of them (Fig. 5) corresponds to the head curve for the head change from the tree root i_R to end nodes of the network.

Step 4 of **Algorithm 1** determines the best of the found conditionally-optimal solutions to the problem. Number k_{\min} of the best conditionally-optimal solution corresponding to the solution with the minimum total cost for the system is determined by the following formula:

$$k_{\min} = \arg \min_{k \in \{1, \dots, \mu\}} C[i_R, k]. \quad (38)$$

The optimal value of objective function (17) is equal to the value of the component with index k_{\min} and is determined by the formula:

$$C_{\min} = C[i_R, k_{\min}]. \quad (39)$$

At Step 5 of **Algorithm 1**, the vectors of the sought parameters \mathbf{d} , \mathbf{u}_γ , \mathbf{u}_ψ , \mathbf{u}_ζ , \mathbf{Q} and \mathbf{H} are formed, and their components are assigned

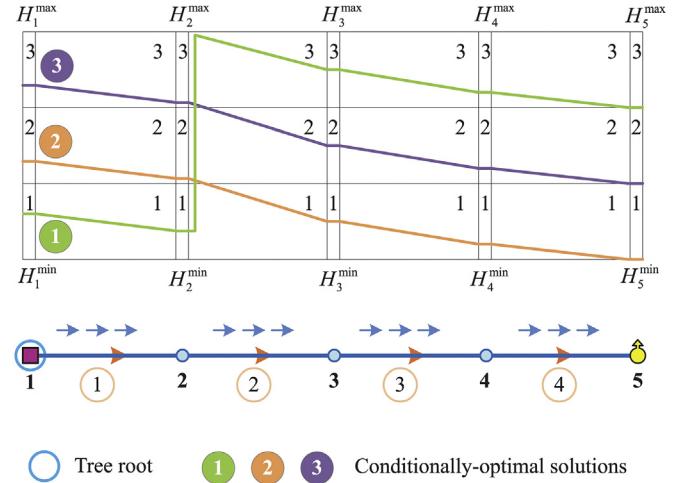


Fig. 5. Conditionally-optimal solutions to the problem.

the values corresponding to the optimal solution found.

Then the original orientation of the branches is restored. When the orientation of the branch changes, the corresponding components of vector \mathbf{Q} have their signs changed to the opposite.

The **Algorithm 2** has the following form.

Algorithm 2. A DFS-based algorithm for constructing conditionally-optimal solutions. Input parameter: i is the node number.

Step 1: Mark the i -th node as being in use.

Step 2: Identify the set of branches A_i^- for which the i -th node is the initial one.

Step 3: For all branches $j \in A_i^-$ do:

3.1: Determine the number of the final node of the j -th branch: $i' = \omega(j)$.

3.2: Recursively call **Algorithm 2** for the i' -th node.

3.2: Call **Algorithm 3** for the j -th branch.

Step 4: Call **Algorithm 4** for the i -th node.

Step 5: Mark the i -th node as already visited.

The **Algorithm 3** is as follows.

Algorithm 3. Algorithm for determining the components of conditionally-optimal solutions for the j -th branch.

Step 1: Determine numbers of initial and final nodes of the j -branch: $i = \alpha(j)$, $i' = \omega(j)$.

Step 2: For all cell numbers $k \in \{1, \dots, \mu\}$ do:

2.1: Solve problem (35) to determine the components of a conditionally-optimal solution in the k -th cell;

2.2: Store solution components in arrays: $d[j, k] \leftarrow d_{jk}$, $\gamma[j, k] \leftarrow \gamma_{jk}$, $\psi[j, k] \leftarrow \psi_{jk}$, $\zeta[j, k] \leftarrow \zeta_{jk}$, $w[j, k] \leftarrow w$, $\tilde{H}[j, k] \leftarrow \tilde{H}_{jk}$, $\tilde{C}[j, k] \leftarrow \tilde{C}_{jk}$.

The **Algorithm 4** is as follows.

Algorithm 4. The algorithm for determining the components of conditionally-optimal solutions for the i -th node.

Step 1: Identify the set of branches A_i^- for which the i -th node is the initial one.

Step 2: For all cell numbers $k \in \{1, \dots, \mu\}$ do:

2.1: Calculate hydraulic head H_{ik} by formula (36);

- 2.2: Calculate the costs of conditionally-optimal solution C_{ik} by formula (37);
 2.3: Store solution components in arrays: $H[i, k] \leftarrow H_{ik}$, $C[i, k] \leftarrow C_{ik}$.

The presented [Algorithm 1](#) is capable of finding a solution close to the global minimum of the objective function (17), which follows from the mathematical nature of DP.

6. Practical evaluation of the methodology

The proposed methodology is implemented as the IRNET software. IRNET was applied to solving the problem of development of the DHS network of the Magistralny urban locality. This is an urban-type settlement with a population of 7.1 thousand people, located in the Irkutsk region, Russia. The need to come up with recommendations on the development of the network is due to new consumers to be connected to the DHS and, accordingly, the growth of total heat loads of the consumers. The problem is thus reduced to the third case (see Section 2). The resulting solution should contain optimal parameters for new network elements and provide the required transmission capacity for the existing part of the network.

The diagram of the DHS of the Magistralny locality is shown in [Fig. 6](#). The district heating is supplied by a coal-fired boiler house at a temperature of 95/70 °C. The original models have been supplemented by 25 new consumers and 38 branches connecting these consumers to DHS. The same values of heat loads of 42.74 MJ/h are set for 25 consumers. Calculated tree-shaped models of supply and return networks contain 378 branches and 379 nodes each.

Let us decompose the objective functions of costs of a branch (17) into the components: the sum of the costs of construction and operation of the pipeline

$$C_{pl} = \sum_{j \in A} C_j^{pl}(d_j), \quad (40)$$

the sum of the costs of electricity used to pump the heat transfer medium

$$C_e = \sum_{j \in A} C_j^e(\gamma_j, Q_j), \quad (41)$$

the sum of the costs of heat losses

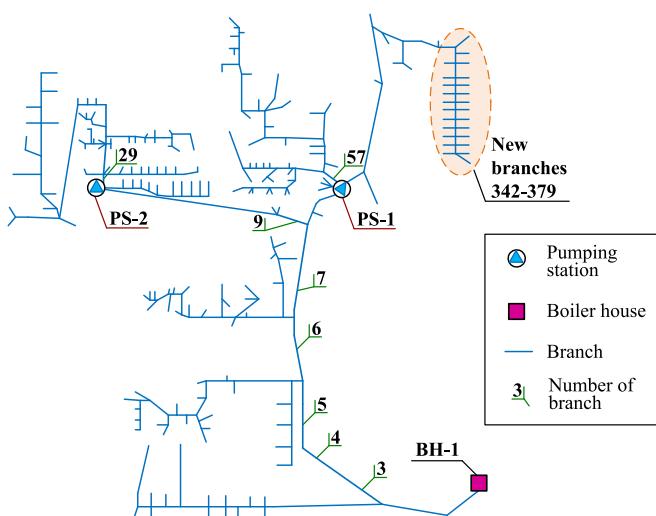


Fig. 6. District heating system diagram of the Magistralny urban locality.

$$C_{hl} = \sum_{j \in A} C_j^{hl}(d_j), \quad (42)$$

the sum of the costs of construction and operation of pumping units

$$C_p = \sum_{j \in A} C_j^p(\gamma_j). \quad (43)$$

Thus, the total cost of the network is calculated by the equation:

$$C_{total} = C_{pl} + C_e + C_{hl} + C_p. \quad (44)$$

The values of the total costs of the system and the values of its components are presented in [Table 1](#).

The calculations demonstrated that 16 branches required replacement of existing pipelines of supply and return pipelines. [Table 2](#) presents data for 8 pipelines of the supply network before and after its development. The following notation for pipeline parameters is used in the Table: length L , specific pressure loss Δp_{sp} , heat carrier flow rate Q , flow velocity v , inner diameter d . The head loss in the pipelines was calculated using the Nikuradse equation [39]. The values of Δp_{sp} are reduced, due to a decrease in pressure losses caused by friction. Capital investments in pipelines pay off by further reducing the cost of electricity used by pumping stations.

As a result of the performed calculations the diameters of pipelines for 76 new branches of supply and return networks were determined. [Table 3](#) presents data for 38 supply network pipelines. For branches with numbers 355 to 379 that connect new consumers to the network, identical values of pipeline diameters are assigned which is explained by identical volume flow rates at these branches.

The original network model contains four pumping units: one unit at the boiler house BH-1 and three pumping units at PS-1 and PS-2 pumping stations. PS-1 contains one pump operating in the return network. PS-2 contains two pumps operating in the supply and return networks. The calculations demonstrated that for the hydraulic heads to lie within a feasible range, a pumping unit should be installed at the PS-1 pumping station in the supply network. The parameters of the pumps before and after optimization are presented in [Table 4](#).

7. Conclusions

This study proposes a new methodology for optimizing the transmission capacity of tree-shaped water pipeline networks of energy systems that serve various purposes and have multiple sources. A unified method for modeling branches is devised. This method is designed to model a pipeline, a pump, a valve, and a combination of these elements. It can be used to develop mathematical models and statements of optimization problems for the

Table 1
The values of cost function and its components before and after optimization.

Network	Costs, million EUR/year				
	C_{pl}	C_e	C_{hl}	C_p	C_{total}
Before optimization					
Return	0.43	0.03	0.09	0.04	0.59
Supply	0.43	0.18	0.09	0.05	0.76
Total	0.87	0.22	0.18	0.09	1.36
After optimization					
Return	0.48	0.03	0.10	0.04	0.65
Supply	0.48	0.21	0.10	0.07	0.86
Total	0.95	0.24	0.20	0.11	1.50

Table 2

Parameters of branches before and after optimization.

Branch No.	L, m	Before optimization				After optimization			
		Δp_{sp} , Pa/m	Q, m^3/h	v, m/s	d, mm	Δp_{sp} , Pa/m	Q, m^3/h	v, m/s	d, mm
3	175	113.7	227.3	1.2	259	11.8	237.3	0.5	414
4	60	113.7	227.3	1.2	259	23.5	237.3	0.7	359
5	100	113.7	227.3	1.2	259	51.9	237.3	0.9	309
6	218	90.2	201.1	1.1	259	18.6	211.1	0.6	359
7	232	69.6	175.0	0.9	259	6.9	185.0	0.4	414
9	152	24.5	63.1	0.5	207	7.8	63.1	0.3	259
29	40	24.5	50.6	0.4	207	4.9	50.6	0.3	259
57	68	149.0	68.0	1.1	150	9.8	68.0	0.4	259

Table 3

Parameters of new branches.

Branch No.	Q, m^3/h	d, mm	v, m/s
342	10.0	100	0.4
343	9.6	100	0.4
344	9.2	80	0.5
345	8.4	80	0.5
346	7.6	80	0.4
347	6.8	80	0.4
348	6.0	50	0.9
349	5.2	50	0.8
350	4.4	50	0.6
351	3.6	50	0.5
352	2.8	50	0.4
353	2.0	50	0.3
354	1.2	50	0.2
355–379	0.4	25	0.2

Table 4

Parameters of pumping stations.

Name	Before optimization				After optimization			
	Supply network		Return network		Supply network		Return network	
	Q, m^3/h	H, m	Q, m^3/h	H, m	Q, m^3/h	H, m	Q, m^3/h	H, m
BH-1	239.7	110.9	—	—	249.7	110.3	—	—
PS-1	—	—	109.6	25.5	121.8	25.0	119.4	25.1
PS-2	62.3	33.0	61.1	33.1	62.3	33.0	61.1	33.1

design and expansion of pipeline systems. A method is also suggested to describe a network model. This method ensures the universality of the model presentation, regardless of the network purpose and composition of equipment. A unified mathematical statement of the optimization problem is given for different networks. Its versatility is provided by the application of the proposed methods for representing the network model and its elements. An algorithm is developed based on dynamic programming to implement a new approach to organizing a computational procedure and optimizing various networks.

The methodology is used as the basis for the development of the universal IRNET software. The proposed unified method for branch modeling was used to develop the universal interface for integrating software components that implement models of network elements (pipeline, pump, valve). The optimization algorithm does not depend on the equipment, which enabled the development of an optimizer that can be used repeatedly as a software component. These software components allow implementing the proposed principle of the computing process organization (see Fig. 1), according to which the optimization algorithm, network model, and equipment models are integrated during the computation process.

The IRNET software is widely used for optimization calculations. The results obtained on their basis are used to make recommendations for the design and expansion of the pipeline energy systems.

Credit author statement

Dmitry V. Sokolov Supervision, Conceptualization, Methodology, Software, Writing-Reviewing and Editing, Validation, Writing - original draft. Evgeny A. Barakhtenko Conceptualization, Methodology, Software, Writing-Reviewing and Editing, Visualization, Investigation, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The research was carried out under State Assignment, Project 17.4.1 (reg. no. AAAA-A17-117030310432-9) of the Fundamental Research of Siberian Branch of the Russian Academy of Sciences.

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