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# Comparison of evolutionary multi objective optimization algorithms in optimum design of water distribution network

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ABSTRACT

In this paper, the application of three well-known multi-objective optimization algorithms to water distribution network (WDN) optimum design has been considered. Non-dominated sorting genetic algorithm II (NSGA-II), Multi-objective differential evolution (MODE) and Multi-objective particle swarm optimization (MOPSO) algorithms are applied to benchmark mathematical test function problems for evaluating the performance of these algorithms. The Accuracy and computational runtime are the two indicators used for the comparison of these three algorithms. The optimization results of mathematical test functions show that all three algorithms were able to accurately produce Pareto Front, but the computational time of MODE algorithm to achieve the optimal solutions is lower than the two other algorithms. Then, the discussed algorithms have been used to optimize the WDN design problem. Comparison of the generated solutions on the Pareto Front for WDN design shows that the obtained Pareto Front of MODE is more accurate and faster.



## 1. Introduction

The Water Distribution Networks (WDN) is a critical municipal infrastructure. WDNs are designed to provide consumers with a minimum acceptable level of supply under operating conditions for the whole design period. WDNs today are very complex systems requiring a high investment for their construction and maintenance [1]. A suitable WDN should be able to provide water demand with the required pressure. Due to the varying amount of demand during the day, the pipe diameter should be selected so that the WDN would be able to give appropriate service to customers at all times.

According to computational and engineering complexity of WDNs optimal design, it has been thoroughly investigated over the past few decades [2, 3, and 4]. Because of nonlinearity between head loss and flow along pipes and also discrete design variables such as pipe diameter in WDNs design problems, this type of optimization is a highly challenging problem. The optimal design of WDNs is a combinatorial optimization problem included in the class of complex combinatorial problems known as non-deterministic polynomial-time Hard (NP-Hard) [5].

Early works on the optimization of WDN was based on single-objective optimization, i.e. least-cost design. One of the first WDN's optimization was presented by Alperovits and Shamir [6]. They used the linear programming gradient method. Savic and Walters [7] used EPANET hydraulic solver and integrated genetic algorithm to optimize three WDN benchmarks.

In the last decades, researchers have used multi-objective optimization instead of least-cost optimization for design of WDNs. One of the first multi-objective optimization of WDN design has been reported by Gessler [8]. He used partial enumeration method to minimize the network cost and maximize the minimum pressure. Because of unsatisfactory results of traditional deterministic optimization techniques, using the various evolutionary algorithms (EAs) developed to solve the WDN's design problems [9,10]. EAs are well-known methods which are used extensively for multi-objective optimization problems and they are well suited for solving complex optimization problems. As a result, the EAs

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were considered to solve the problem of optimal WDN's design and the researchers used other parameters such as hydraulic or mechanical reliability, water quality, operation cost and leakage as the second objective function along the network construction cost [11–13].

Halhal et al. in 1997 for the first time used a multi-objective genetic algorithm approach as an evolutionary algorithm for rehabilitation of WDN under a limited budget [14]. Farmani et al. compared four multi-objective evolutionary algorithms (MOEA) for WDN design optimization entitled Non-dominated Sorting Genetic Algorithm 2 (NSGA-II), Multi-Objective Genetic Algorithm (MOGA), Niched Pareto Genetic Algorithm (NPGA) and Pareto Archived Evolution Strategy (PAES) [11]. They found that the NSGA-II was the best of the tested algorithms. Also, Farmani et al. compared the NSGA-II to the Strength Pareto Evolutionary Algorithm 2 (SPEA-II) on a large WDN and concluded that SPEA-II produced better quality solutions [3]. Raad et al. in 2011 comparing numerous EAs in order to determine which method is adequate for WDN design optimization [15].They concluded that NSGA-II and some of its borrowed algorithms have the best performance for this purpose.

Zheng et al. investigated search behavior of the DE algorithm as a function of its parameter values [16]. They analyzed the influence of DE's parameters on measure run-time, search quality, convergence properties and solution generation statistics. Moosavian and Lence apply the non-dominated sorting algorithms on DE to obtain a multi-objective version of differential evolution algorithm for solving WDN's optimal design problem [17]. They show that presented MODE has an acceptable performance versus other multi-objective optimization algorithms. Shrivatava et al. used a Multi-objective particle swarm optimization for the design of WDN [18]. They also investigate the effect of swarm size and different inertia weights on the behavior of optimization algorithm.

In this paper, for a closer look at performance, three well known and usable multi-objective optimization methods, Non-dominated Sorting Genetic 2 (NSGA-II), Multi-Objective Differential Evolution (MODE) and Multi-Objective Particle Swarm Optimization (MOPSO), are compared for WDNs design optimization. Accuracy, convergence rapidity and solution's diversity are the parameters which used to assess the performance of these optimization algorithms.

## 2. Evolutionary multi-objective optimization algorithms

EAs are areas of multiple criteria decision making, where optimal decisions need to be taken in the presence of trade-offs between different objectives. EAs are very attractive for multi-objective analysis in relation to classical methods. EAs begin with a set of solutions which are randomly generated and called initial population. The offspring populations are generated by some operators such as the mutation, the crossover, and the selection. A brief description of three evolutionary algorithms, which have been used in this study, will be given in the following.

### 2.1. Non-dominated sorting Genetic algorithm II (NSGA-II)

Deb et al. in 2002 developed NSGA-II which is the integration of Genetic Algorithm and non-dominated sorting approach for multi-objective optimization [19]. NSGA-II algorithm contains three main parts for selection of the new generation's members: a non-dominated sorting, density estimation, and a crowded comparison.

- Non-dominated sorting retains members that are not dominated. If a descendant of a new generation is dominated, it would be immediately removed, otherwise, it becomes a member of the population and also if a member of parent generation is dominated by the descendants it will be removed too.
- The density of each particular member is measured as the distance of the considered point and two members of its neighbors.
- The crowded comparison operator aims to increase the diversity of Pareto Front. Population members are ranked taking into account seniority and local crowding distance. In this paper, a real-coded NSGA II was used to determine the optimum solutions in the search space.

### 2.2. Multi objective differential evolution (MODE)

Differential Evolution (DE) algorithm was proposed by Storn and Price in 1997 [20]. Because of its simplicity and excellent convergence characteristics, DE has been successfully applied to the wide range of engineering problems which are nonlinear, multi-criteria and multi-constrained [21–23]. Dong et al. reported that the DE algorithm is robust and converges fast compared to the GA algorithm [21]. In the DE optimization algorithm, for each parent set $x_i$, a different vector of $x_{i1}$ and $x_{i2}$ (randomly selected) is used to perturb another random vector $x_{i3}$ using the following mutation equation,

$$z_i = x_{i3} + F.(x_{i1} - x_{i2}) \tag{1}$$

where $x_{i1}$, $x_{i2}$, $x_{i3}$ are different random vectors from parent set, $z_i$ is the mutant vector and $F$ is a real constant factor between 0 and 2 called scaling factor. The suggested value for scaling factor is 0.4 to 0.6. To generate a child vector, crossover operator must be used as follows:

$$z'_{ji} = \begin{cases} z_{ji} & if \;\; rand(j) \le CR \\ x_{ji} & if \;\; rand(j) > CR \end{cases} \tag{2}$$

where $z'_{ji}$ is the value of the $j$th design variable of $i$th child vector, CR is crossover constant between 0 and 1 with the suggested value between 0.3 and 0.6, and rand($j$) is a randomly generated value for $j$th variable design between 0 and 1.

To decide whether the vector $z'_i$ should be a member of the next generation or not, it must be compared with the corresponding vector $x_i^{(G)}$ from generation G. If function $F$ denotes the objective function, the members of next-generation $x^{(G+1)}$ can be selected by relation (3):

$$x^{(G+1)}{}_i = \begin{cases} z'_i & if \;\; F(z'_i) < F\left(x^{(G)}{}_i\right) \\ x^{(G)}{}_i & if \;\; F(z'_i) \ge F\left(x^{(G)}{}_i\right) \end{cases} \tag{3}$$

In the last decade, researchers attempted to extend the DE algorithm to multi-objective optimization and they showed that DE can be an attractive alternative for multi-objective numerical optimization. In this study, the MODE is applied by integrating DE technique with non-dominated sorting, ranking, and crowding distance assignment procedures in [19]. In this process, Instead of using Eq. (3) to choose the members of the new generation, parent and new generated member are analyzed for dominance relation. If the parent dominates the new generated member, the new member is eliminated but if the new member dominates the parent, the parent is deleted. If the parent and new member are non-dominated, both of them are added to a temporary population. After repeating this progress for all the members, the non-dominated ranking and the crowding distance have been used to select the population of next generation from the temporary population. This procedure will continue to reach the Pareto Front

### 2.3. Multi-objective particle swarm optimization (MOPSO)

Particle swarm optimization (PSO) is originally presented by Kennedy and Eberhart in 1995 [24]. PSO is a stochastic, population-based evolutionary algorithm inspired by the bird herd behavior and uses swarm intelligence to find the optimal solutions. Particles (solutions) in PSO move toward the optimal solution with the regular velocity. The speed of any particle is composed of three components: the velocity of the same particle in the previous generations (inertia), the distance to the best position of the same particle in the past generations (personal guides) and the distance to the position of the leader particle (global guides). The leader is a particle with the best performance in the optimization procedure. The position of particles will be updated in each generation using the combination of these three components. The new position of each particle ($x_i$) in time t + 1 can be calculated using the following formula:

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (4)$$

where $v_i$ is the particle velocity and calculated as:

$$v_i(t+1) = w.v_i(t) + C_1.r_1(x_{Pbesti} - x_i) + C_2.r_2(x_{Gbesti} - x_i) \quad (5)$$

In above equation $w$ denotes the inertia weight with suggested values between 0.4 and 1.4, $C_1$ and $C_2$ are the non-negative constant coefficients with the proposed range between 1 and 2, $r_1$ and $r_2$ are random numbers between 0 and 1, $x_{Pbest}$ is the best position of the same particle in past generations and $x_{Gbest}$is the position of the leader particle.

**Table 1**
Settings of three optimization algorithms.

| Alrithms | NSGA II | MODE | MOPSO |
|---|---|---|---|
| Specific Parameters | Probability of crossover = 0.8<br>Probability of Mutation = 0.03 | Scaling Factor = 0.5<br>Probability of crossover = 0.3 | Size of repository = 50, 100, 150<br>Inertia weight (w) = 0.6<br>Personal Learning Coefficient ($C_1$) = 1<br>Global Learning Coefficient ($C_2$) = 2 |

Coello and Lechuga in 2002 extended PSO to deal with multi-objective optimization problems [24]. The developed method uses an external repository for storing the information of non-dominated particles. The leader is chosen from the repository members for calculating the particle's velocity. Padhye et al. in 2009 further developed the methodology to improve the performance of MOPSO.

In this paper, the presented MOPSO algorithm of Coello and Lechuga's [24] used with the external repository, global and local best positions. The global best position is chosen from the non-dominated particles stored in the external repository with roulette wheel selection on each generation. The density of points around each member of repository affects the probability of the member selects. The local best positions of each particle also refer to the non-dominated solution of the same particle in the past generation.

## 3. Performance assessment of the multi-objective evolutionary algorithms

The three algorithms have been coded in mathematical software package MATLAB R2011b and run on a PC with1GB of RAM and Intel Core Due 2 GHz CPU. For evaluating the performance and strength of these three optimization algorithms, five well known mathematical test functions have been selected from [25]. These test functions have between 10 and 30 bounded design variables and 2 objective functions.

The quality of the obtained solutions is assessed using performance measures such as the distance between the generated Pareto Front and the known optimal Pareto Front solutions, and the diversity of the solutions on the Pareto Front. In this paper, the Inverted Generational Distance (IGD) measure is used for quantitative assessment of the three optimization algorithms [26]. This indicator measures the distance of elements in the true Pareto Front set of elements in the set of non-dominated vectors generated by the optimization algorithms. The IGD($A$,$P$) can be calculated as:

$$IGD(A,P) = \sum\nolimits_{\tau \epsilon P} d(\tau, A)/|P| \quad (6)$$

**Table 2**
Results of three optimization code for mathematical test functions for three parameter settings SD: Standard Deviation. The bolded texts in the table indicate the best results for each problem.

| | | Case I (Pop. size = 50) | | | Case II (Pop. size = 100) | | | Case III (Pop. size = 150) | | | [17] | | [18] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | MODE | NSGA II | MOPSO | MODE | NSGA II | MOPSO | MODE | NSGA II | MOPSO | MODEA | NSGA-II | MOPSO |
| ZDT1 | Best IGD | 0.009931 | 0.015038 | 0.013041 | 0.006207 | 0.009153 | 0.008509 | 0.003491 | 0.006008 | 0.005936 | – | – | 0.005414 |
| | Mean IGD | 0.013966 | 0.018975 | 0.016945 | 0.006838 | 0.010022 | 0.009982 | **0.004403** | 0.006732 | 0.006904 | 0.00445 | 0.005712 | 0.012500 |
| | SD | 0.001159 | 0.015038 | 0.002382 | 0.000323 | 0.009153 | 0.001412 | 0.00032 | 0.006008 | 0.000691 | 0.000107 | 0.000134 | – |
| | Time (s) | 3.27 | 4.97 | 72.11 | 5.74 | 9.75 | 271.39 | 9.07 | 16.35 | 602.8 | – | – | – |
| ZDT2 | Best IGD | 0.012704 | 0.015542 | 0.01548 | 0.005844 | 0.008437 | 0.008453 | 0.004007 | 0.005772 | 0.005963 | – | – | 0.004838 |
| | Mean IGD | 0.014378 | 0.018176 | 0.016499 | 0.006858 | 0.009364 | 0.009362 | **0.004638** | 0.006358 | 0.007083 | 0.004474 | 0.005424 | 0.013980 |
| | SD | 0.001042 | 0.015542 | 0.001028 | 0.00046 | 0.008437 | 0.000778 | 0.000223 | 0.005772 | 0.000641 | 0.000119 | 0.00019 | – |
| | Time (s) | 3.14 | 4.45 | 93.49 | 5.66 | 9.41 | 350.9 | 10.72 | 15.86 | 781 | – | – | – |
| ZDT3 | Best IGD | 0.013326 | 0.015452 | 0.019435 | 0.006754 | 0.008233 | 0.008528 | 0.004475 | 0.005353 | 0.00547 | – | – | 0.018860 |
| | Mean IGD | 0.015316 | 0.018047 | 0.043041 | 0.007454 | 0.009395 | 0.010878 | **0.004903** | 0.006358 | 0.007623 | 0.005202 | 0.005785 | 0.020050 |
| | SD | 0.00099 | 0.015452 | 0.024404 | 0.000379 | 0.008233 | 0.002414 | 0.000172 | 0.005353 | 0.001818 | 0.000089 | 0.000206 | – |
| | Time (s) | 3.33 | 5.09 | 58.42 | 5.83 | 9.80 | 205.63 | 9.23 | 16.39 | 420.28 | – | – | – |
| ZDT4 | Best IGD | 0.125242 | 0.009627 | 0.036622 | 0.004678 | 0.00722 | 0.009543 | 0.002978 | 0.003967 | 0.006804 | – | – | – |
| | Mean IGD | 0.638763 | 0.193089 | 0.384293 | 0.131584 | 0.16262 | 0.284069 | **0.012455** | 0.119508 | 0.106789 | 0.103042 | 0.005848 | – |
| | SD | 0.253382 | 0.009627 | 0.175655 | 0.081377 | 0.00722 | 0.134634 | 0.019329 | 0.003967 | 0.087308 | 0.093407 | 0.001032 | – |
| | Time (s) | 2.41 | 4.50 | 37.63 | 5.26 | 8.72 | 102.32 | 8.31 | 14.76 | 273.51 | – | – | – |
| ZDT6 | Best IGD | 0.010446 | 0.007996 | 0.01137 | 0.005428 | 0.003969 | 0.006924 | 0.003542 | 0.00279 | 0.004512 | – | – | 0.003508 |
| | Mean IGD | 0.011673 | 0.011467 | 0.012841 | 0.005764 | 0.005256 | 0.007801 | 0.003797 | **0.003739** | 0.005383 | 0.003585 | 0.012346 | 0.227100 |
| | SD | 0.000733 | 0.007996 | 0.000956 | 0.000182 | 0.003969 | 0.000803 | 0.00013 | 0.00279 | 0.000493 | 0.000451 | 0.001227 | – |
| | Time (s) | 2.77 | 4.42 | 63.4 | 5.08 | 8.91 | 267.8 | 8.64 | 14.9 | 592.33 | – | – | – |

where $P$ is the true Pareto Front vector set (the actual solutions), $A$ is non-dominated objective vector set (generated by optimization algorithm) and $d(\tau, A)$ is the Euclidean distance from the elements of $P$ to its nearest member in $A$. When the value of IGD is equal to zero, it means that all of the non-dominated solutions match the solutions on the true Pareto Front. Both diversity and convergence of solutions could be measured using IGD (A, P).

### 3.1. Mathematical test functions

To analyze the computational time of each algorithm, the run-time was recorded for several parameter settings. The initial population size is set to 50, 100 and 150 and the number of generation for all cases is set as 250. The other parameter settings of three optimization algorithms are summarized in Table 1. These algorithms have been running for 30 times for each test function and the average results have been obtained. The results are shown in Table 2 and they are validated with the reported results of the recent articles [27,28].

In all cases, the results have been in a good agreement. Results show that the MODE has less elapsed time in contrast to the other methods in all cases. Time spent by MOPSO was very high compared to MODE and NSGA II. In most cases, the results of MODE are near to the real answers with smaller IGD.

To evaluate the convergence rate of these three algorithms, the mean amount of IGD for ZDT1 and ZDT6 test functions with their corresponding upper and lower bound were drawn versus to generation numbers in Fig. 1. These graphs are obtained from 30 times code execution for each test function. Diagrams show that MODE and MOPSO converge in less number of generations. But the variety of results in MODE and NSGAII is less than MOPSO. As a result, in the mathematical problems with a large number of design variables, the MODE has the best performance both in terms of convergence rate and the running time.

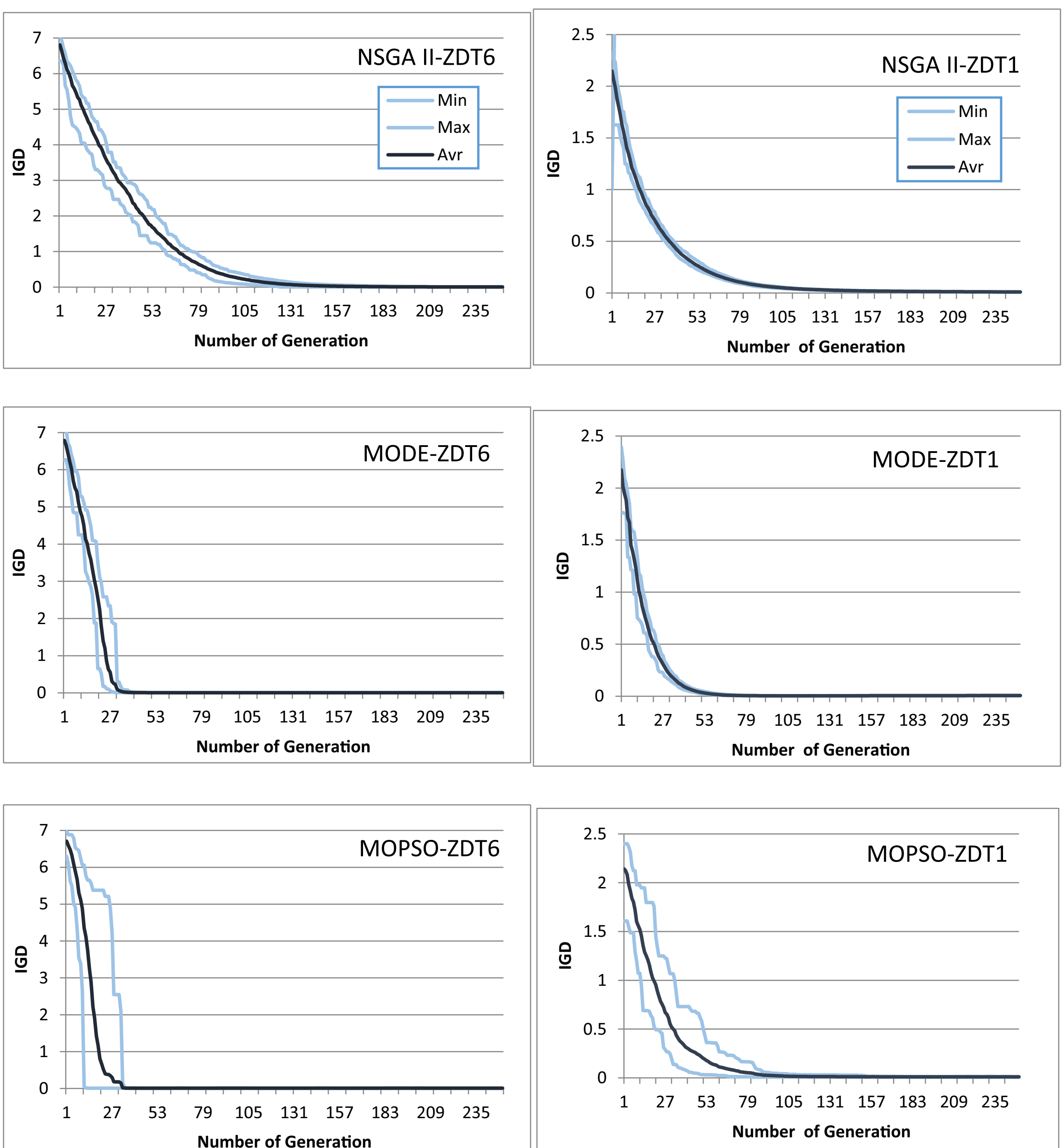


**Fig. 1.** Mean IGD values for ZDT1 and ZDT6 with corresponding upper and lower bounds.

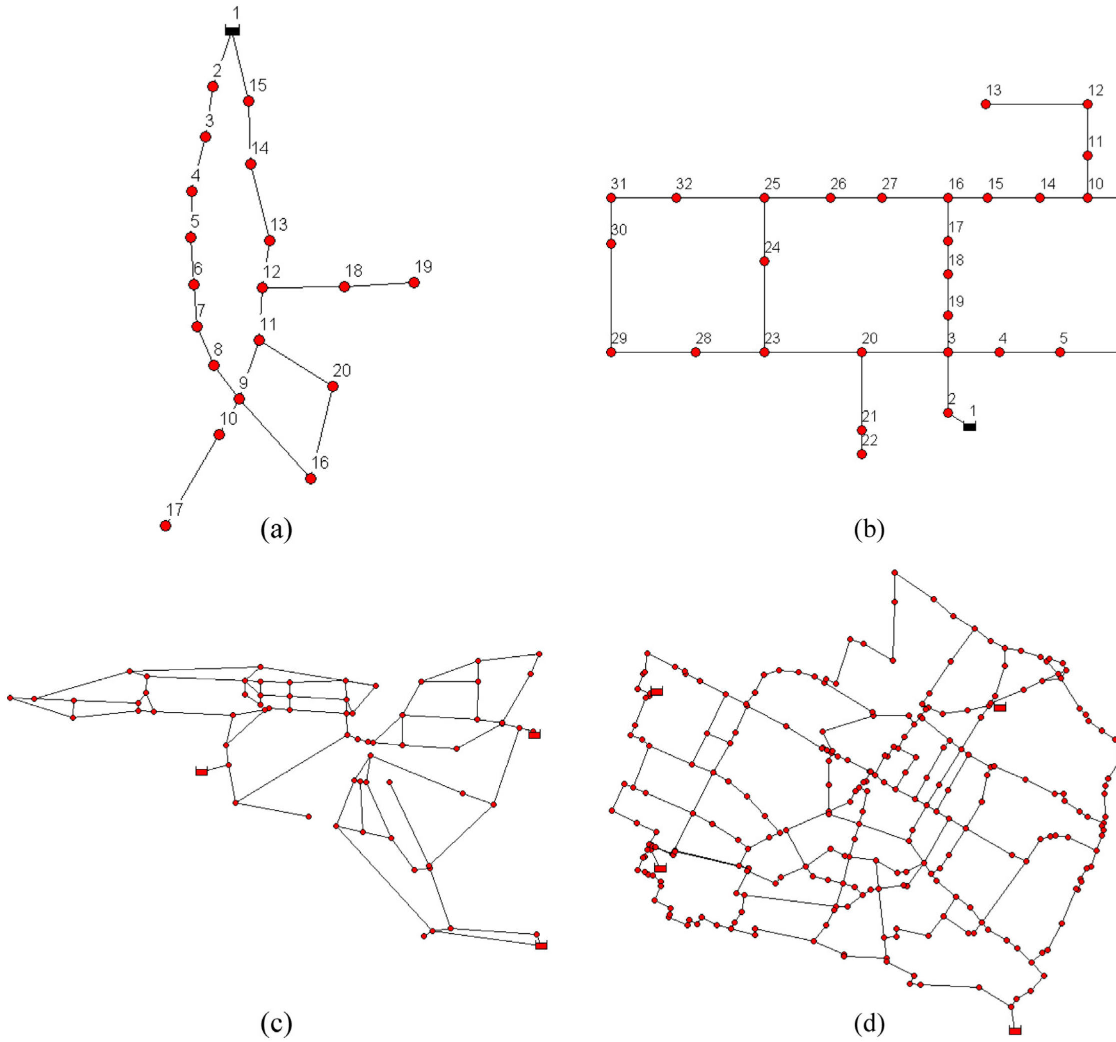


**Fig. 2.** (a) New York tunnel water network and (b) Hanoi water network (c) Pescara water network (d) Modena water network.

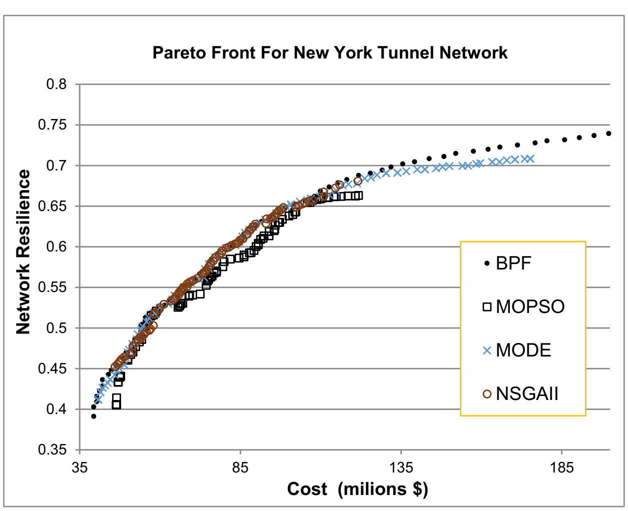


**Fig. 3.** The results of three optimization algorithm for New York tunnel network.

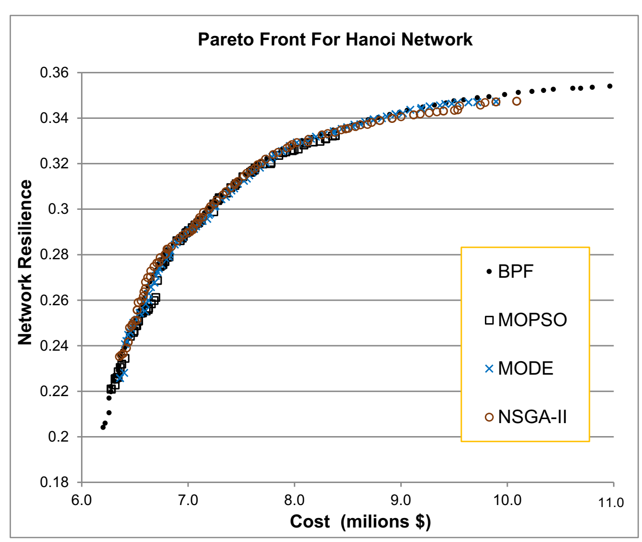


**Fig. 4.** The results of three optimization algorithm for Hanoi network.

### *3.2. Water distribution network optimal design*

The main objective function in WDN's design optimization is the capital cost which depends on the length and diameter of the pipes. The network reliability is considered as another objective which represents the ability of a WDN to satisfy the consumer's needs under normal or an abnormal condition [3]. Reliability in the context of WDNs is a somewhat nebulous concept, owing to the vast number of different interpretations over the years. The WDN's reliability has two main subcategories, the *hydraulic reliability* which reflects the network tolerance against operational change (e.g. demand change) and the *mechanical reliability* which reflects the network tolerance against physical changes such as pipe failure [29].

Hydraulic reliability reflects how well the WDN can cope with changes over time, such as demand variations. It is an important

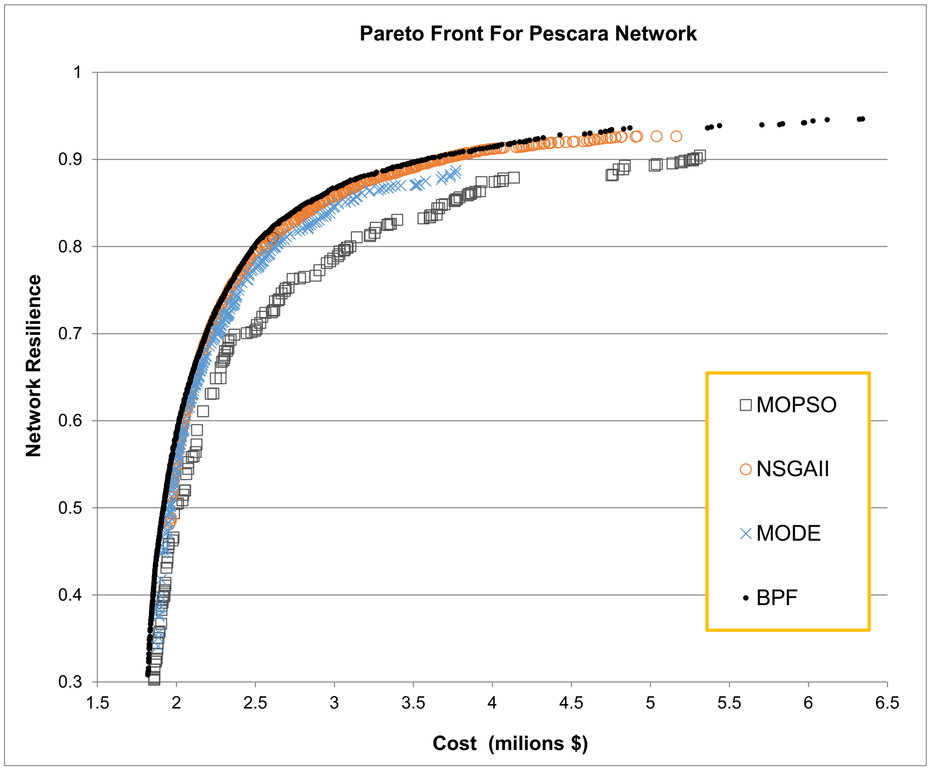


**Fig. 5.** The results of three optimization algorithm for Pescara network.

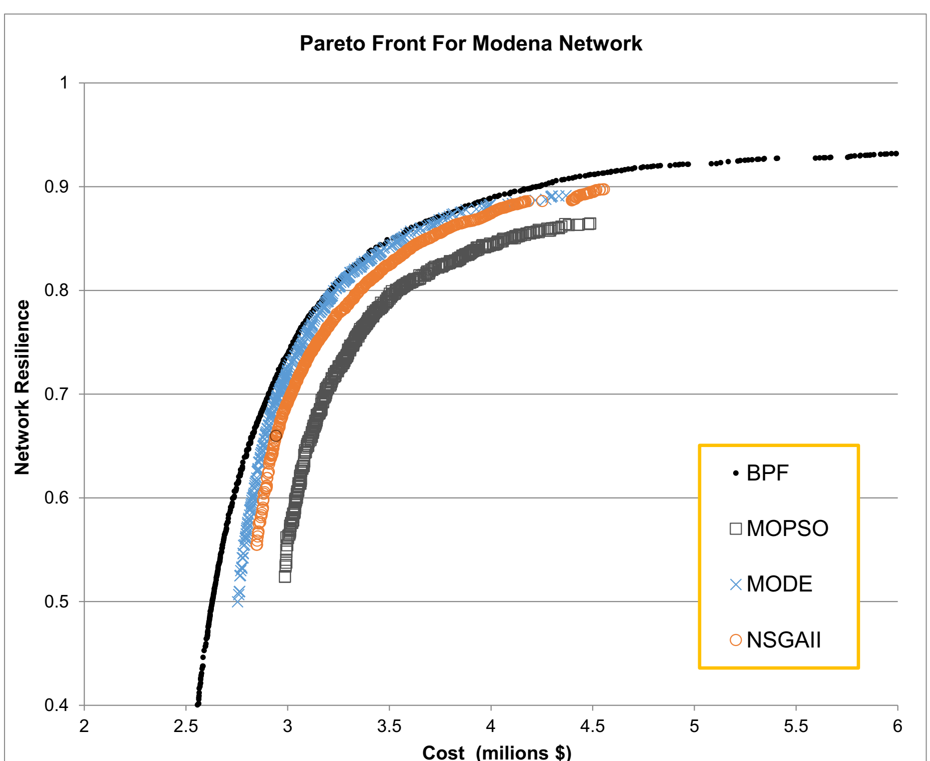


**Fig. 6.** The results of three optimization algorithm for Modena network.

performance measure of WDNs, as it refers directly to their basic function. It is, therefore, often considered as the ultimate goal of the WDN design. Network Resilience (NR) presented by Prasad and Park [2] and it is a surrogate measure of WDN's hydraulic reliability which considers surplus hydraulic power as a proportion of available hydraulic power, considering the number of inlet and outlet pipes in each demand node [29]. NR is strongly related to the intrinsic capability of the system to overcome failures while still satisfying demands and pressures in nodes. The value of NR is in the continuous range between 0 and 1 and it is defined by Eqs. (7) and (8) [2].

$$NR = \frac{\sum_{i=1}^{nn} c_i . q_i . (h_{a,i} - h_{r,i})}{\left(\sum_{i=1}^{nn} Q_i H_i + \sum_{j=1}^{np} \frac{P_j}{\gamma}\right) - \left(\sum_{i=1}^{nn} q_i h_{r,i}\right)} \tag{7}$$

$$c_i = \left(\sum_{j}^{npi} D_j\right) / (npi \times max\{D_j\}) \tag{8}$$

where nn is the number of demand and supply nodes, np is the number of pumps, $c_i$ is the uniformity of connected pipe to node i,$h_{a,i}$ is the available head at the supply node i in (kPa), $h_{r,i}$ is required head at supply node i in (kPa), $q_i$ is demand at node i in ($m^3$/s), $Q_i$ is supply at input node i in ($m^3$/s), $H_i$ is head of input node i in (kPa), $P_i$ is power from pump j in (kw), $\gamma$ is specific weight of water in (N/$m^3$), $npi$ is the number of pipes connected to node i and $D_j$ is the diameter of pipe $j$ connected to demand node $i$. Tanks act as a demand node when they are filling and they act as a reservoir when they are emptying.

Three optimization algorithms are compared for optimum design of four benchmark water distribution networks considering minimum capital cost and maximum network reliability as two objective functions. Epanet 2 software is used as a hydraulic solver. The design variables in all case studies are pipe diameters and the main constraint of the optimization problem is the acceptable pressure range in demand nodes and acceptable flow velocity in pipes. The initial population size and the number of generation of two first water networks considered as 100 and 250 respectively. Due to the more complex and larger search space in two other water networks optimization problem, the initial population size of Pescara and Modena are considered as 300 and 500, and the number of the generation of these problems considered 500 and 1000 respectively. For similarity of solving conditions, the same initial population has been used for all the optimization procedures.

**Table 3**
IGD and Spend time of WDN optimization in two benchmark network. The bolded texts in the table indicate the best results in each row.

| WDN | Measure | MODE | NSGA II | MOPSO |
|---|---|---|---|---|
| Hanoi | Mean IGD | **0.016333** | 0.016896 | 0.260512 |
| | SD | 0.001643 | 0.001237 | 0.259736 |
| | Best IGD | **0.014** | 0.014825 | 0.058007 |
| | Time (s) | **550.2** | 1083.5 | 1208 |
| New York Tunnel | Mean IGD | **0.469061** | 0.47375 | 0.590792 |
| | SD | 0.019095 | 0.012581 | 0.02615 |
| | Best IGD | **0.451543** | 0.462173 | 0.552421 |
| | Time (s) | **478.4** | 792.5 | 1231.4 |
| Pescara | Mean IGD | 0/035056 | **0/027438** | 0/0388 |
| | SD | 0/003623 | 0/011457 | 0/002003 |
| | Best IGD | 0/0302 | **0/0155** | 0/035 |
| | Time (s) | **3672/0** | 5057/2 | 6284/8 |
| Modena | Mean IGD | **0.021538** | 0.032372 | 0.085219 |
| | SD | 0.001878 | 0.002419 | 0.007641 |
| | Best IGD | **0.020931** | 0.031868 | 0.079915 |
| | Time (s) | **24725.7** | 43842.6 | 54256.1 |

**Table 4**
Pipe diameters of an optimal solution in the Pareto front for three optimization algorithms with the same cost.

| Network | Opt algorithm | Pipe No. and diameter (mm) | | | | | | | | | | | | | | | | | | | | | Objective function | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Hanoi** | | 1–7 | 8 | 9–11 | 12 | 13 | 14 | 15 | 16–20 | 21–22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | Cost (m $) | NR |
| | MOPSO | 1016 | 762 | 762 | 610 | 610 | 610 | 762 | 1016 | 610 | 1016 | 762 | 610 | 610 | 610 | 762 | 406 | 406 | 205 | 406 | 508 | 610 | 7.514 | 0.314 |
| | NSGA-II | 1016 | 762 | 762 | 610 | 610 | 610 | 762 | 1016 | 610 | 1016 | 762 | 610 | 508 | 610 | 762 | 508 | 406 | 205 | 205 | 508 | 610 | 7.523 | 0.314 |
| | MODE | 1016 | 1016 | 762 | 610 | 406 | 508 | 610 | 1016 | 610 | 1016 | 762 | 610 | 610 | 610 | 762 | 508 | 406 | 205 | 406 | 406 | 610 | 7.524 | 0.312 |

| Network | Opt algorithm | Pipe No. and diameter (mm) | | | | | | | | | | | Objective function | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **NYT** | | 1 | 2 | 3–10 | 11 | 12 | 13–15 | 16 | 17 | 18 | 19 | 20 | 21 | Cost (m $) | NR |
| | MOPSO | – | 156 | – | 72 | 108 | – | 120 | 72 | 192 | 204 | 168 | – | 84.575 | 0.5860 |
| | NSGA-II | – | – | – | 72 | 96 | – | 120 | 84 | 204 | 204 | 180 | 192 | 84.925 | 0.6084 |
| | MODE | – | – | – | 84 | 84 | – | 96 | 72 | 204 | 204 | 204 | 204 | 84.698 | 0.6062 |

*3.2.1. Case 1 – New York city water distribution network*

The New York Tunnel network (Fig. 2a) was first proposed by Schaake and Lai in 1969 [30]. After them, several researchers have also investigated this problem for WDN optimization design [7]. This network has 20 nodes and 21 pipes, which is fed with an elevated reservoir. The total length of the network pipes is 223 Km with roughness coefficient of 100 and total nodal demand is 205,823 $m^3$ per hour. Schematic view of the New York Tunnel network has been shown in Fig. 2. The existing configuration of New York Tunnel network is unable to satisfy the expected demand in some nodes. To satisfy the minimum allowable pressure requirements in demand nodes, the network needs to be rehabilitated. The decision variables can duplicate some or all of the existing pipes. The minimum head requirement at all nodes is fixed at 77.72 m except for node 16, 17 and 1 which are 79.24, 83.14 and 91.44 m respectively. There are 16 commercially available pipe sizes for each duplicated pipe and the search space for this problem is $16^{21}$ ($1.9343 \times 10^{25}$) possible combinations.

*3.2.2. Case 2 – Hanoi water distribution network*

The second test problem is the Hanoi network in Vietnam (Fig. 2b) which was first presented by Fujiwara and Khang in 1990 [31]. The network has 32 nodes, 34 pipes and 3 loops, which is fed with a single elevated reservoir. The total length of the network pipes is 39.4 Km with roughness coefficient of 130 and the total nodal demand is 19940 $m^3$ per hour. The design of this network is restricted to selecting 6 commercially available pipes size. The minimum required pressure head for all the nodes is set at 30 m. The search space consists of $6^{34}$ ($2.865 \times 10^{26}$) possible combinations.

*3.2.3. Case 3 - Pescara water distribution network*

The third test problem is the Pescara network in Italy (Fig. 2c). This network is an intermediate problem and detailed in [32]. The network has 68 nodes and99 pipes, which is fed with three elevated reservoirs. The total length of the network pipes is 48.6 Km, the pipe's material is cast iron with roughness coefficient of 130 and the total nodal demand is 1794 $m^3$ per hour. The design of this network is restricted to selecting 13 commercially available pipes size. The minimum required pressure head for all the nodes is set at 20 m and the maximum total head of them is 57 m. The flow velocity of each pipe is enforced to be less than or equal to 2 m/s. The search space consists of $13^{99}$ ($1.91 \times 10^{110}$) possible combinations.

*3.2.4. Case 4 – Modena water distribution network*

The last case study is the Modena water network in Italy (Fig. 2d). This network is a large problem [32]. The network has 268 nodes and 317 pipes, which is fed with four elevated reservoirs. The total length of the network pipes is 71.8 Km, the material and roughness coefficient of all pipes are the same of Pescara water network and the total nodal demand is 1465 $m^3$ per hour. The design of this network is restricted to selecting 13 commercially available pipes size. The minimum required pressure head for all the nodes is set at 20 m and the maximum total head of them is 74.5 m. Also, the flow velocity of each pipe is enforced to be less than or equal to 2 m/s. The search space consists of $13^{317}$ ($1.32 \times 10^{353}$) possible combinations.

*3.2.5. Results of WDNs design optimization*

The results of employing the three algorithms (NSGA-II, MODE, and MOPSO) for WDN design have been shown in Figs. 3–6. These results compared with best-known Pareto Front (BPF) which was presented with [4]. The presented BPF in Ref. [4] is derived from the integration of the results of recent studies by other researchers. For numerical comparison of the obtained Pareto Fronts from the three algorithms, the IGD value is calculated for every four networks and has been shown in Table 3, also the mean elapsed times to reach the optimal solutions were measured and shown in Table 3. These data are obtained from 10 times code running for each algorithm and each WDNs. According to the obtained mean IGD, although the solutions of MODE and NSGA-II are close together the MODE provided best Pareto Front in all benchmark WDNs. Also in all cases, MODE was faster and also has covered a wider range of solutions. In all cases, the MOPSO running time is the biggest, but the difference between MOPSO and the other two algorithms execution time in WDN optimization has been lower in Comparison of the mathematical optimization problem in last part.

For closer examination, one solution of the Pareto Front of each optimization algorithm with the same construction cost has been selected for two case studies (New York and Hanoi network) and compared in Table 4. Results show that despite the similarity of the network structure, constraints and construction cost there will vary pipe diameter configurations that can lead to the networks with different reliability. This rule is the same for two other water networks.

## 4. Conclusions

The performance of three well known multi-objective optimization algorithms (NSGA-II, MODE, and MOPSO) have been assessed by applying a number of mathematical test functions and four WDN design under the same conditions, in which the results of the application to mathematical test functions show that in most cases MODE has the best performance, both in terms of IGD and converging speed. After that, the three algorithms applied to four WDNs design considering minimum cost and maximum network reliability as the two objective functions. Results show that MODE has the best Pareto Front in New York Tunnel design (expand existing network with duplicated parallel pipe). The Pareto Fronts generated by MODE and NSGA-II were almost identical and they were better than MOPSO in Hanoi network design (layout design). But the elapsed time in MODE was lower in comparison with the other two algorithms. In Pescara water network, the results of the NSGA-II were slightly better than the results of MODE and the results of the MOPSO were weaker than others. The MODE is still faster than the other two algorithms. In Modena water network (The most complicated network in this study), MODE has better results compared two other algorithms and MOPSO with the same initial population size and generation number could not fully approach the Pareto Front. As a result, the MODE is proposed as a fast and accurate algorithm to optimize water networks design as the multi-objective optimization problem.

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