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## Network Sectorisation Through Aggregation of Strong Connected Components

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### Abstract

This paper presents new algorithm for sectorisation of a Water Distribution Network (WDN) named Water Network Sectorisation (WNS) algorithm. Algorithm relies on Graph Theory to search for the Strong Connected Components (SCCs) in the graph, that are later on topologically sorted and aggregated in the sectors. Aggregation is driven by engineering criteria and heuristics such as sectors of approximately equal size with smallest number of links connecting them. Interventions in the network are not implemented in order to avoid negative effects on the networks' hydraulics. This is important especially for primary stages of sectorisation in which preserving hydraulic performance and minimal investment are the main objectives. Methodology is illustrated on a real size WDN. The results obtained show that the WNS algorithm can be used as an effective support tool in engineering practice.

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### 1. Introduction

Decomposition or sectorisation of a water distribution network (WDN) into zones (sectors, clusters or District Metered Areas - DMAs) has become one of the main strategies for efficient management of WDNs. Decomposition

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has been done traditionally to better control water losses by means of observing all inflows and outflows from the zone. In order to accomplish this, network interventions e.g. installation of isolation valves and flow metering devices are required. However, if not implemented carefully, such interventions can significantly worsen the network supply reliability, water quality, fire-flow supply and system response in the case of accidental bursts and other failures. This is due to the fact that historically WDNs are designed as extremely looped systems in order to provide aforementioned requirements, and decomposition into DMAs can considerably affect their topology.

Complexity of the real life WDN results in many different alternatives in which network decomposition can be done. Every WDN is unique in its topology and characteristics so there is no unique procedure for performing its' decomposition, but rather a series of guidelines provided by the different water authorities ([1], [2], [3], [4]) and used in this process by practice engineers. Decomposition in zones (sectors, clusters) is usually governed by the criteria of having zones of "manageable size" in terms of number of consumers, links or network length, often neglecting the network's topology. It can be also subjected to many other criteria and limitations leading to arbitrary solutions, usually obtained by the "trial and error" method done by a local expert. Generally, sectorisation process should be governed by general criteria in terms of zone size but also other criteria which should include evaluation of potential investments, energy consumption for pumping, increased water leakage, exceeded or insufficient pressures etc.

In recent years different algorithms for automated decomposition of the WDN into DMAs have been presented, as well as the tools that can be used to support this process [5, 6]. Majority of the presented methodologies are based on the Graph Theory algorithms [7, 8]. Others are using the modularity index [9] or community structure metrics [10] to perform the division of the WDN. Decomposition algorithm is usually coupled with an optimization algorithm [9, 11] in order to search within broader solution space and ensure that a suboptimal feasible solution is identified. It must be ensured that clustering interventions in the WDN, required to create sectors, do not worsen its operational performance and this can be evaluated using different performance indicators. In sectorisation process different objective functions are used. For example, in [7] reachability of every potential solution is minimized, in [9, 10] modularity metrics [12] is maximized and in [11] minimization of dissipated hydraulic power is adopted. Resilience index, that regards to the network post-segmentation reliability [13], is often used as a main performance indicator [3, 14] while in [10] fire-flow and water age metrics are added into consideration. The most comprehensive list of criteria and constraints used for decomposition, total of 13, can be found in [15]. However, despite all recent advancements made, scope exist to further improve existing water network sectorization algorithms, especially in terms of usability for practicing engineers.

The WNS methodology presented here is based on the heuristic aggregation of the Strong Connected Components (SCCs) determined using the Graph Theory algorithms [7,8]. The WNS procedure is conducted in three steps which will be explained in the following sections. Interventions in the network, such as closure of the valves or blockage of the pipes are not implemented at this point to avoid negative effects on networks' hydraulics. The main goal of the WNS is to search for the optimal decomposition suitable for the water balance control with minimal number of connecting links, implementing some engineering criteria and heuristics. Presented methodology can serve as a support tool for practice engineers when designing the sectorisation solution that will have the least effect on the hydraulics of the system and minimal investment. Furthermore, it can be used as a good starting point for narrowing down the solution space subjected to the optimization algorithm in search for the optimal solution. Methodology will be explained and demonstrated on a simple example and its' application illustrated on a real size network of Pozarevac town in Serbia.

## 2. Methodology

The proposed Water Network Sectorisation (WNS) is based on the Graph Theory and has three stages, as shown in Fig. 1. The algorithm requires a hydraulic model of water distribution network.

### 2.1. Stage 1: Determine the orientation of pipes

In the first stage, hydraulic simulation of the analysed WDN is performed to determine the orientation of pipes (based on water flow directions obtained in the simulation). As a result, directional graph (DIGRAPH) is defined with two sets  $G = \langle N, C \rangle$  (set of nodes  $N$  and set of links  $C$ , where each link is presented with ordered pair of nodes), as shown in Fig. 2. Network links with changing flow direction are identified as non-oriented (or links that can have both flow directions).

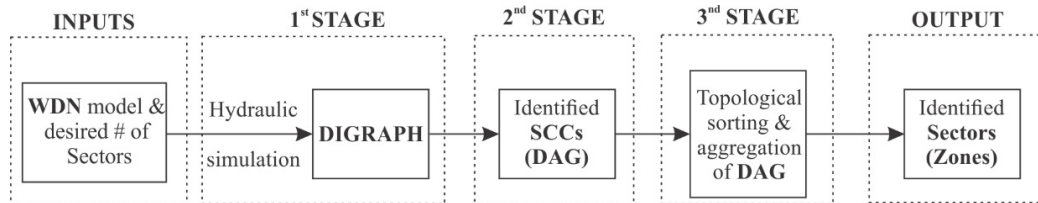


Fig. 1. WNS algorithm flowchart

In a simple example presented in Fig. 2. network consists of 13 nodes and 19 links, where two of those links are identified as not oriented. Putting that in the context of water networks, those are usually pipes (links) that are connecting tanks with the rest of the network. So in hypothetical simple network, nodes 8 and 2 could be tanks, and 1 and 10 are the source nodes. In a real size water networks parallel links often exist too. That is why a link should also have an identification number, because it cannot be uniquely defined with ordered pair of nodes.

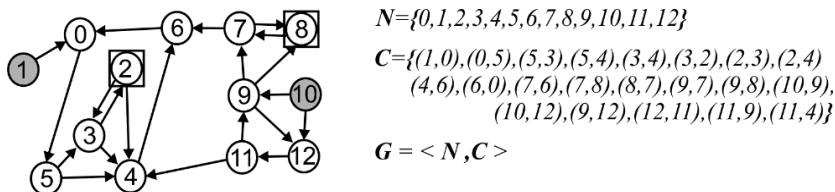


Fig. 2. Digraph presentation of a simple network with 2 sources and 2 undirected links

### 2.2. Stage 2: Identification of Strong Connected Components (SCCs)

In the second stage SCCs are identified within the DIGRAPH, resulting in the formation of the Directional Acyclic Graph (DAG). Strong connected component (SCC) is a term from Graph Theory, and it is defined as a subgraph in which each node can be reached from any other node within that subgraph. Without using the terminology from Graph Theory, SCCs are parts of network in which the water is circulating during the simulation period. Due to that fact, control of the water balance and/or water pressure regulation in SCC parts of the network could be difficult to achieve, so the idea is to detect SCCs and treat them as aggregated nodes in further network analysis and clustering. Algorithms for the extraction of SCCs from digraph are well known in the Graph Theory [16]. Here, the Gabow algorithm (explained in [17]) is used. It requires only one pass through the network (DIGRAPH) with recursive call of the Depth First Search (DFS) algorithm with arbitrary selection of the starting node.

For illustration purposes, a simple digraph shown in the Fig. 2. is used. Starting the DFS search from the node 2, nodes 3, 4, 6, 0 and 5 are visited (Fig. 3.a). During the DFS propagation, a check is made whether the selection of the next node forms a cyclic path or not. If yes, nodes forming the cyclic path are identified as a SCC. The algorithm continues until no further propagation is possible. In example shown in Fig. 3., the first SCC component identified is composed of nodes 2, 3, 4, 6, 5 and 0. No further propagation is possible, so the DFS starts again from randomly selected node, chosen from the set of nodes that were not visited during the first search. Assuming that the randomly selected node is node 9, and after nodes 12 and 11 are visited, the second SCC composed of these three nodes is

identified. DFS search is repeated again starting from node 8, and third SCC composed of nodes 8 and 7 is detected (Fig. 3.b). At the end, aggregated digraph is composed of three identified SCCs. The diagram can also be viewed as set of aggregated nodes and two remaining nodes 1 and 10 (Fig. 3.c). The most important property of new aggregated digraph is acyclicity, indicating it is a digraph without cycles. Such graph is referred to as Directed Acyclic Graph (DAG), and in terms of water network is very important, because it clearly separates source from the demand nodes and hence, makes the sectorisation of network easier.

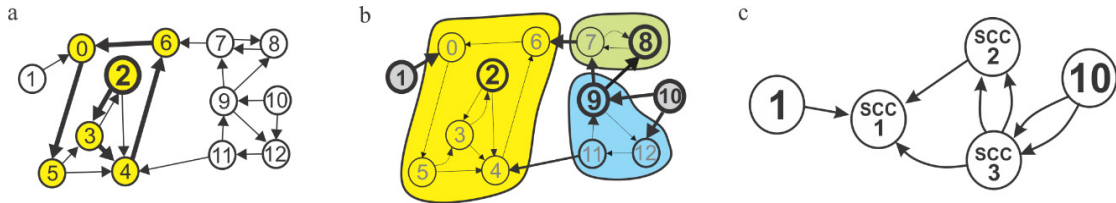


Fig. 3. DIGRAPH transformation to DAG: a) Start the DFS; b) Detected SCCs; c) Newly formed DAG

### 2.3 Stage 3: Topological sorting and aggregation

Finally, the topological sorting of DAG and its aggregation by using a number of pre-specified engineering criteria is conducted, as shown in Fig. 4. Although different algorithms from the Graph Theory could be used for topological sorting (for example algorithm for pre and post ordering of DAG nodes during recursive call of DFS), the customized algorithm with some heuristics that bias the process is used here, as explained below. Moreover, the DAG aggregation is done during the topological sorting by using pre-specified engineering criteria resulting in improved solutions identified (again, as explained below). The following engineering criteria are used here: size of the sector in terms of water demand, number of connections between the sectors, pipe diameters and pipe lengths.

Topological sorting of DAG starts from the most downstream (sink) nodes. Sink nodes are the one that have only inlet links, and propagation starts simultaneously from all these nodes. In the example discussed, SCC1 is the only sink node in the DAG, hence topological sorting will start from that node. Topological sorting continues in the direction opposite of the flow orientation thus adjacent nodes upstream of SCC1 are candidates for propagation. A precondition for visiting an adjacent upstream node, and putting it in the set of signed (sorted) nodes, is that a candidate node has all of its downstream nodes already signed. This means that only nodes that have sorted (signed) adjacent downstream nodes are valid candidates for further upstream propagation and possible aggregation to the downstream component. Applying this precondition on the example DAG, the only valid candidate for propagation from SCC1 is SCC2, so it is signed and moved to the set of sorted nodes (Fig. 4.a). Nodes 1 and 10 are source nodes (nodes without input links), so they are left for the final stage of algorithm.

The next step is to check if candidate node SCC2 could be aggregated with node SCC1. The primary criterion that is checked to determine if aggregation is feasible is the allowable size of the sector ( $Q_{\max}$ ) in terms of its demand. Maximum size for a sector is defined as  $Q_{\max} = Q_{\text{tot}}/N_{\text{sect}}$ , where  $Q_{\text{tot}}$  is total input in the WDN and  $N_{\text{sect}}$  is desired number of sectors, which has to be smaller than the number of identified SCCs. If the aggregated sector size  $Q_{\text{sect}}^{\text{agg}}$  is smaller than the  $Q_{\max}$  then upstream node is joined to the downstream one. Furthermore, even if this criteria is not satisfied, nodes are aggregated if change in the sector size is smaller than the 10%, i.e. if  $(Q_{\text{sect}}^{\text{agg}} - Q_{\text{sect}})/Q_{\text{sect}}^{\text{agg}} < 0.1$ . This is done to avoid the case where very small nodes are left as not aggregated. Following the simple example (Fig. 4.), if we assume that all demand nodes have the same water consumption, node SCC2 will not be aggregated to node SCC1.

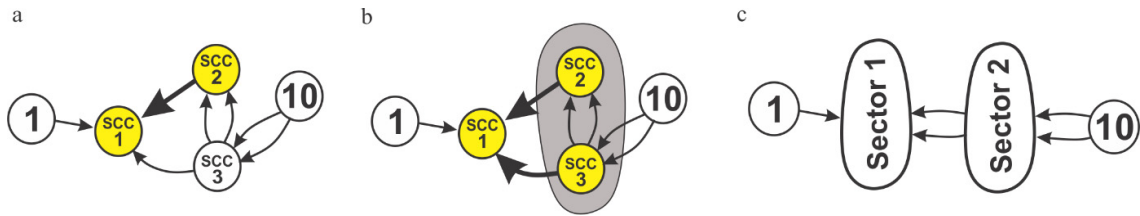


Fig. 4. DAG Aggregation: a) Upstream propagation and topological sorting; b) Components aggregation; c) Result of the algorithm

At this point, aggregation criteria have to be explained in more detail as the example in consideration does not illustrate complexity of real world networks. Unlike the Fig. 4. example, where only a single aggregation option exists (to aggregate SCC2 to SCC1 or not), in a real size network, a set of nodes that are valid candidates for propagation and aggregation according to the aforementioned primary criterion of maximum sector size is likely to exist. After checking that primary condition for aggregation, ranking of the remaining candidates is done according to the following additional criteria:

- Criterion 1: The number of links between the SCCs is checked. A candidate node whose aggregation would reduce the maximum number of links between the SCCs is chosen. Following this idea, aggregation of SCCs is done in such way that minimum number of links between SCCs remains at the end of algorithm run.
- Criterion 2: If multiple candidates exist after applying above criterion, a node is chosen by selecting the upstream link with the smallest diameter. This way the algorithm propagation through the main lines is postponed for the later stages. If several pipes with the same diameter exist, the pipe with minimum length is selected, in order to keep the sectors' pipeline length minimal.

Going back to the simple example shown above, the next node feasible for propagation is node SCC3, simply because its' adjacent downstream nodes SCC1 and SCC2 are already signed and topologically sorted. Next step is aggregation. Node SCC3 could be aggregated to SCC1 or SCC2. Given that SCC1 is already large enough and not feasible for further aggregation, node SCC3 is aggregated to SCC2 (Fig. 4.b). The algorithm stops after the source nodes have been reached. As a result of the algorithm (Fig. 4.c), there are 2 sectors – sector 1 made of nodes 0,2,3,4,5,6 (SCC1), and sector 2 made of nodes 7,8,9,11,12 (SCC2 and SCC3) – and 2 source nodes (1 and 10). There are five links connecting them.

### 3. Case Study

#### 3.1 Description

The WNS algorithm presented here is applied to a real size WDN of Pozarevac town in Serbia (Fig. 5.). This WDN supplies water to approximately 50,000 inhabitants, industry and public and commercial institutions. The water is pumped from wells into the reservoir **Kljuc** ( $Volume = 2 \times 2500 \text{ m}^3$ ), from where pumping station **PS Kljuc** delivers clean water to the network. The WDN is divided into 3 zones based on elevation: Zone I with ground elevation below 100 m.a.s.l, zone II with elevations between 100 and 150 m.a.s.l and zone III with elevation between 150 and 200 m.a.s.l. Pumping station **PS Kljuc** pumps water toward town and **Tulba** tank. It is equipped with 4 pumps and its' operating regime depends on the water consumption and water level in **Tulba** tank. Three pumps work normally 24 hours retaining pressure at 5 bars. Water is delivered through the pipes with diameters of 600/500 mm (main lines) to the **Tulba** tank. From the **Tulba** tank water is also pumped to the **Cacalica** tank, which serves for the consumers that are in the zone II, and for supply of small pumping station that delivers water for the consumers that are in the zone III. Total length of water distribution mains is 175 km, and summary length of the pipes with diameter below 100 mm is about 110 km (64.5 %). The average daily water consumption in the WDN is 203 l/s. The average pressure in the network is 42.5 m.

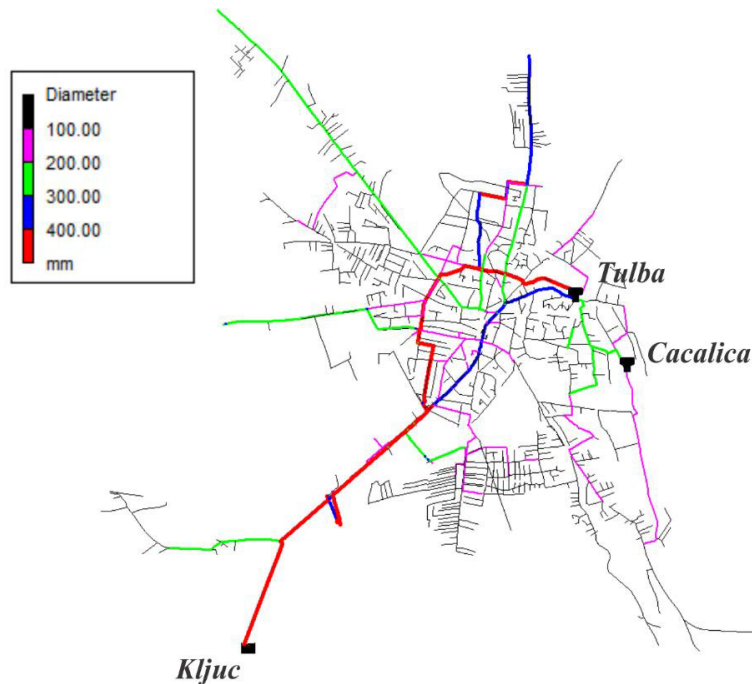


Fig. 5. Water Distribution Network of Pozarevac

### 3.2 Results and discussion

The implementation of WNS algorithm requires only a single hydraulic simulation and number of sectors to be specified. The results of hydraulic simulation were used for determining flow directions in pipes, in order to create the DIGRAPH. The final result of the WNS algorithm application is presented in Fig. 6. As it can be seen from this figure, the WDN is divided into 5 sectors (SCTs) with simulated peak consumption at 19h. A total of 14 links are identified as separation links between 5 SCTs, which is a very small number, bearing in mind that the WDN hydraulic model contains 2272 pipes, 568 valves and 12 pumps. This indicates that the WDN water balance could be controlled with relatively small number of measurement locations. The 2 mandatory discharge measuring locations are at the outlets from pumping stations *Kljuc* and *Tulba* (Fig. 6. – locations **A** and **B**). Water level in tanks *Tulba* and *Cacalica* have to be continuously measured as well. Moreover, as it will be explained below in more detail, 3 of 14 separation links between the SCTs can be safely closed without affecting the hydraulic performance of the network, hence leaving only 11 discharge measuring locations. It is worth mentioning that the algorithm is computationally very fast (execution lasts about 5 seconds on PC Intel i5 CPU), which makes it suitable for using it as a part of an optimization algorithm.

The centre of Pozarevac town is divided into 2 sectors (**SCT 1** and **SCT 2**), that are connected between themselves via 4 pipes. The most important connection is a pipe with a 500 mm diameter (location **C**), with maximum discharge that varies from about 100 to 153 l/s. Next two connection pipes have diameters of 200 and 80 mm (locations **D** and **E**, respectively), but with much smaller (almost negligible) discharges at the peak demand hour. In all three pipes flow direction is from **SCT 2** to **SCT 1**. Fourth pipe connecting these two sectors has the diameter of 200 mm (location **F**) and is in the vicinity of the aforementioned pipe with the 80 mm diameter. The closure of this pipe is suggested due to the fact that simulated discharge was below 0.1 l/s, so measuring would not make any sense and closing it would not affect hydraulic performance of the network. Finally, three flow measuring locations (**C**, **D** and **E**) are proposed to control the flow from **SCT 2** to **SCT 1**.



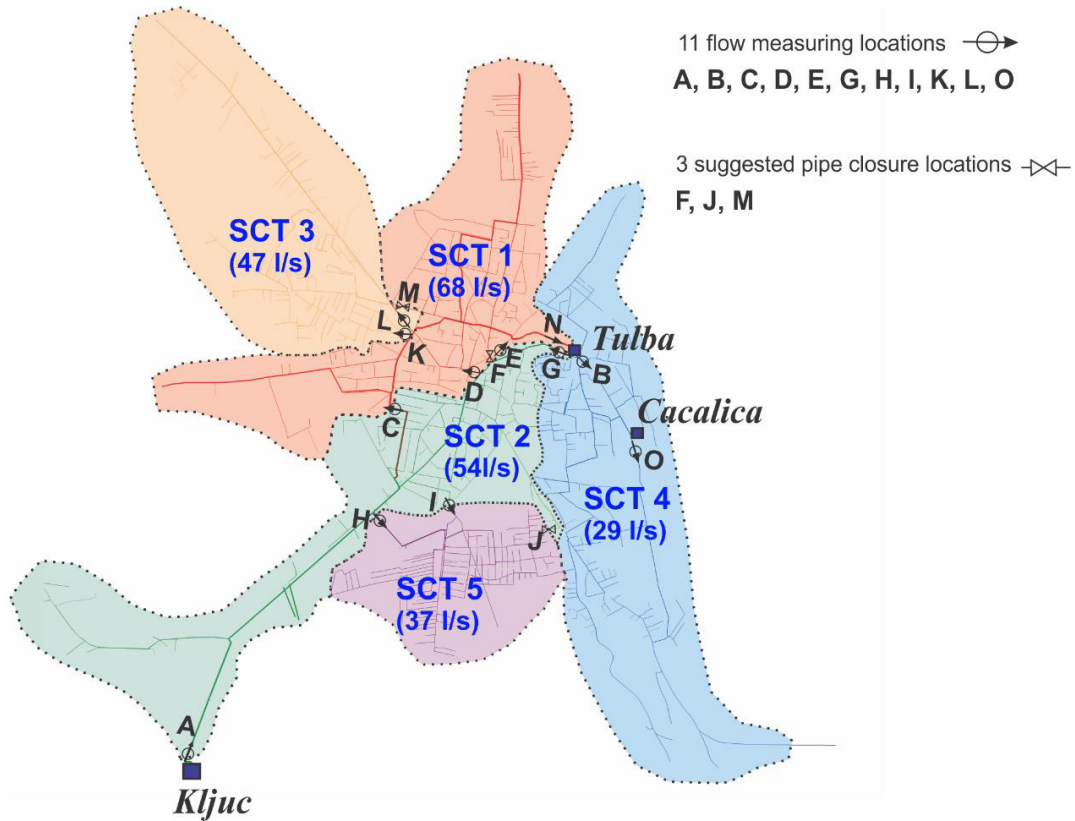


Fig. 6. Result of WNS algorithm applied to the WDN of Pozarevac town

In order to complete the water balance in **SCT 2**, it is necessary to measure the water flow supplied from the **Tulba** tank (pipe with diameter 350 mm – location **G**), and water that is delivered from **SCT 2** to **SCT 5** through the pipes with diameters 300 mm and 125 mm (locations **H** and **I**, respectively). It should be also noted that one pipe between these two sectors with the 80 mm diameter is to be closed as well, due to the negligible flow (location **J**). This completes the water balance in the sectors **SCT 2** and **SCT 5** with additional 3 measuring locations (**G**, **H** and **I**).

Sector **SCT 1** is supplied from **SCT 2**, and for the water balance control in **SCT 1**, it is necessary to measure water delivered to **SCT 3**. This is done through two pipes with the diameters of 200 and 80 mm (locations **L** and **K**, respectively), with one pipe with 100 mm diameter also being closed. Discharge that goes to **Tulba** tank from **SCT 1** has to be calculated as well (location **N**). There is no need to establish a new measurement point for discharge at this location, simply because the water level in the tank and discharges from the tank (locations **B** and **G**) are already measured. In this manner, the water balance is fully controlled for the sectors **SCT 1** and **SCT 3** with 2 additional measuring locations. To fully control water balance in the sector **SCT 4**, supplied from the **Tulba** tank, one more discharge measuring location is needed at the outlet of the small pumping station at **Cacalica** tank supplying consumers in zone III (location **O**).

In summary, to control the water balance in the whole WDN it is necessary to establish 11 discharge measuring locations (see Fig. 6.). Closure of 3 pipes with almost negligible discharges reduced the number of connecting links between the components to 11, from starting 14. It is obvious that some of the proposed measurement locations are suitable only for the water balance control, and can not be used for reducing the pressures in the network. Those are

the locations on water mains (600 and 500 mm) that deliver water to **Tulba** tank. Other measurement locations could possibly be equipped with valves, and used for reducing pressure in sectors and consequently, water leakage.

#### 4. Conclusion

Efficient and simple algorithm for water network sectorisation (WNS) is presented in the paper. Algorithm uses results of 24-hour hydraulic simulation to determine the orientation of the pipes and to identify non-oriented pipes in which water flows in both directions during the simulation period. Once the oriented graph (DIGRAPH) is defined, SCCs are identified and aggregated to sectors according to the number of sectors set as an input. Algorithm is tailored to the need of finding the sectorisation solution that will have the least effect on the hydraulics of the system with minimal investment. The WNS algorithm was tested and demonstrated on the real-life network of Pozarevac town. Based on the results obtained, it can be concluded that the algorithm can serve as a valuable tool for practicing engineers dealing with water network sectorisation. Future work should be aimed at improvement of the algorithm by adding the possibility of testing the effects of potential network interventions. In this manner, different sectorisation solutions could be investigated through an optimization method and evaluated using some performance indicators, allowing selection of a (sub)optimal one.

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