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# Systemic vulnerability assessment of urban water distribution networks considering failure scenario uncertainty

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ABSTRACT

Water distribution networks (WDNs) are vital infrastructures in cities. However, reports about urban WDNs incidents that result in major system breakdowns and water outages are not uncommon, which repeatedly highlights the urgency of addressing vulnerability challenges of WDNs. This study aims to propose a new method for system-level, scenario-independent vulnerability assessment of WDNs, which considers the uncertainty in various failures that may happen in the system. The proposed method is developed based on the notion that the vulnerability of WDN is largely determined by its heterogeneity in node importance, which impacts how likely system malfunction or breakdown would happen should a small amount of nodes be attacked. Accordingly, the proposed method uses a set of indicators to measure the functional, structural and overall importance of each node in WDN, and introduces a novel network entropy model to measure the heterogeneity of importance of these nodes. The proposed method then assesses the systemic vulnerability of WDN, by benchmarking its current entropy against the entropies associated with the least and most vulnerable states of the system. The efficacy of the proposed method is demonstrated in a case study, in which the assessment yielded by the proposed method was found theoretically reasonable as well as consistent with actual conditions of the case WDN.



## 1. Introduction

The world has witnessed massive urban expansion and growth of urban population in the past decades. The global urbanization rate has increased from 37% in 1970 to 54% in 2016, and is projected to further increase to 67.2% by 2050 [1]. With the rapid expansion of urban areas, the demand for urban water resources is increasing. Water distribution networks (WDNs) are vital infrastructures for delivering water, which are expected to ensure safe and reliable water supplies even when disturbed by various natural and manmade hazards [2]. However, reports about urban WDN incidents that result in major water outages are not uncommon worldwide [3,4], which repeatedly highlights the criticality of addressing potential vulnerability issues of WDNs, as well as the challenges of doing so.

The vulnerability of a WDN refers to its inability to withstand the effects of instantaneous accidental or malicious events [5]. Assessing the vulnerability of WDNs is of fundamental importance to the evaluation and diagnosis of system reliability, prediction of system performance losses in relation to potential hazard scenarios, and design of disaster impact

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mitigation measures. Accordingly, a number of WND vulnerability assessment methods have been developed in the literature. The majority of these methods focus on component-level assessment [6–8], by investigating the failure probabilities of WDN components under given hazard scenarios, and assessing the severity of component failure consequences in terms of overall system performance losses. These methods, however, are inherently limited and potentially biased because of two reasons. First of all, the hazard scenarios are highly uncertain, owing to the fact that WDNs in many cases are faced with multiple hazards, the impacts of which may be largely coupled and difficult to predict with existing methods [9]; secondly, the WDNs are becoming more complex in their structure and function, which makes it increasingly challenging to predict the failure consequences under multiple hazard scenarios. Adding to such challenge is the interconnectedness of WDNs with other urban infrastructure systems, which may lead to significant cascading risks across systems [10] and cause damages to WDN components that are otherwise considered unlikely to fail. Existing methods for WDN vulnerability assessment cannot account for the above uncertainties in failure scenarios, and can only yield component-specific and scenario-dependent assessment results [11]. Therefore, stakeholders of water distribution systems have begun in recent years to seek for systemic vulnerability assessment methods for WDNs [12], which can provide comprehensive diagnosis of WDNs considering all uncertainties.

To address the above need, this study aims to develop a new method for WDNs vulnerability assessment. The authors posit that, given the significant uncertainties in hazard scenarios, the inherent attributes of a system can serve as the basis for assessing its systemic vulnerability, which can provide stakeholders of the system with an overall assessment of how vulnerable the system is when it is faced with various difficult-to-predict threats that could lead to highly uncertain failure consequences. The proposed method, which adopts a systemic perspective for assessing the vulnerability of WDN, is developed based on the notion that the vulnerability of WDN is largely determined by its heterogeneity in node importance [13]. This heterogeneity, reflecting how evenly the importance of nodes in ensuring both structural and functional integrity of the system is distributed among all nodes in the WDN, largely impacts how likely system malfunction or breakdown would happen should a small amount of nodes be attacked. Accordingly, the proposed method uses a set of indicators to measure the functional, structural and overall importance of each node in WDN, and introduces a novel entropy model to measure the heterogeneity of importance of these nodes. Then, the proposed method assesses the systemic vulnerability of WDN, by benchmarking its current entropy against the entropies associated with the least and most vulnerable states of the system.

The efficacy of the proposed method is demonstrated in a case study, based on real data from a middle-sized city in China. The validity of the results and their practical implications are discussed. This study contributes to the existing body of knowledge by enabling system-level, scenario-independent vulnerability assessment of WDNs, which considers the uncertainty in failures that may happen in the system. The remainder of this paper is organized as follows. Section 2 reviews prior studies and discusses current research gaps. Section 3 presents the details of the proposed vulnerability assessment method. Section 4 presents a case study, using the proposed method to assess the vulnerability of WDN in a case city. Section 5 concludes the paper.

## 2. Related work

### 2.1. Vulnerability assessment of WDNs

Vulnerability assessment is essential to the monitoring, prediction and intervention of the behavior of WDNs against various natural or manmade disruptions [5]. Existing literature is mostly concerned with component-level WDN vulnerability assessment, focusing on the estimation of failure probability of WDN components [6,7,14,15]. To achieve accurate failure probability estimation, various data pertaining to WDNs, such as records of pipe breakage and leakage incidents, water main break histories, pipe materials and soil type, are needed. However, access to these data is often limited and challenging in reality [16]. To overcome this issue, some probabilistic models or methods have been developed to estimate failure probabilities of components in WDNs [7,17,18].

The failure of a component in WDN affects not only the component itself but also other components that it relies on, causing cascading effects. This has motivated researchers to extend the investigation of the vulnerability of individual WDN components to that of the entire network, by analyzing failure consequences of components at the system level [19]. Along this line of research, indicators that are reflective of attributes of WDNs, particularly the networks structure and system operation state, are used to measure the severity of possible failure consequences and cascading effects. For instance, Pinto et al. [20] extended the concept of vulnerability in structural vulnerability theory [21] and proposed the theory of vulnerability of water pipe networks (TVWPN) using clustering criteria, such as minimum total head loss and maximum damage demand, to build a hierarchical model for evaluating the vulnerability of WDNs. Shuang et al. [22] developed a model to evaluate the nodal vulnerability of WDNs under cascading failures based on the connectivity loss of topological structure. Laucelli et al. [23] assessed the vulnerability of WDNs using repair rate (RR), which estimates the number of repairs after an earthquake for a unit length of pipeline, by considering that pipe breaks can reduce the hydraulic system capacity with respect to other network sections. Berardi et al. [24] proposed a model for analyzing WDN vulnerability under two types of failure scenarios, including segment failures and nodal failures. Using a betweenness centrality (BC) indicator, Agathokleous et al. [5] assessed the effect of the topology of WDN on its vulnerability.

While existing literature has notably advanced the knowledge about the vulnerability of WDN, it is largely limited in that, instead of adopting a systemic perspective, most prior research that have examined WDN vulnerability start with component-level analysis, assessing failure probabilities of WDN components, and conclude with the severity of such failures consequences in the entire system. However, it is becoming increasingly challenging to estimate the failure probabilities and associated failure consequences, because failure scenarios could be highly uncertain involving multiple hazards, whose impacts may be highly coupled and complicated, and modern WDNs are complex in structure and function and heavily coupled with other infrastructure systems. Existing methods for WDN vulnerability assessment cannot account for the above uncertainties and complexities, and the assessment results which are component-specific and scenario-dependent could be inherently biased and misleading.

### 2.2. *Entropy-based assessment of node heterogeneity in complex networks*

Most natural or manmade complex system can be expressed by complex networks composed of a number of edges and nodes [25]. Nodes represent the internal elements or components of the system, and edges represent the relationships among the nodes. Using complex network, a complex system including its structure, feature and information can be properly modeled without losing important properties [26]. An important issue of a complex network is the analysis of attributes of its nodes. Node importance which is determined by a set of certain attributes of nodes in a network is a critical subject of research [27]. Heterogeneity of node importance has significant impact on the vulnerability of the network [28]. Specifically, large heterogeneity of node importance indicates that there are some nodes in the network that are substantially more important, either structurally or functionally, than others. In other words, the network is heavily dependent on these important nodes to function, therefore, attacking these nodes would lead to disproportionally severe consequences.

Entropy is a widely used metric for measuring the disorder or randomness of a system in the thermodynamics physics domain [29]. This concept has been applied and extended by a number of studies to analyze the heterogeneity of nodes in complex networks. These studies differ mainly in terms of the entropic indices used for calculating the entropy value. For instance, Sole et al. [30] defined the number of edges leaving a node (namely out-degree) as node remaining degree, and developed an entropy model for measuring the heterogeneity of nodes, using remaining degree as entropic index. Xiao et al. [31] proposed another entropy model for measuring the heterogeneity of nodes, using the ratio between the node degree and total number of node as entropic index. Similarly, Wu et al. [28] proposed to use the ratio between the node degree and the sum of all nodes degree as entropic index in their entropy model. They also analyzed the extremum condition of their model and calculated the maximum and minimum values of the entropy considering different network structures. Other typical entropy indices that have been used in the literature include critical coefficient of node [32] and degree distribution [33]. While the existing entropic indices are mainly based on node degree or its variances, an entropic index that can reflect both structural and functional characteristics of the nodes and the overall network is still lacking, which is needed in order to enable comprehensive assessment of the heterogeneity of node importance in WDN.

## 3. Proposed vulnerability assessment method

Motivated by the aforementioned gaps in the literature, this study proposes a new method for assessing the systemic vulnerability of WDNs. The proposed method first assesses the importance of each node in WDNs considering both its functional and structural characteristics. Then, by introducing a network entropy model, which uses a node importance-based entropic index, the proposed method assesses the heterogeneity of nodes importance in the WDNs. Based on the network entropy, the systemic vulnerability of WDNs under various uncertain failure scenarios can then be assessed. The method is explained in further details in the remainder of this section.

### 3.1. *Node importance*

The importance a node in WDN refers to the contribution of this node to function realization of the system and its controlling ability over structural connectivity of the network. Despite its several different definitions in prior research, node importance can be generally measured in two aspects [34], namely functional importance and structural importance. For a node in WDN, the functional importance mainly reflects its influence on water delivery capability, and the structural importance mainly reflects the criticality of its position in the network. Based on the above notions, the importance of nodes in WDN is assessed using a method improved upon the one reported in [35], as explained in detail below.

#### 3.1.1. *Functional importance of node*

WDN is built to ensure safe and reliable water supplies to its users, which depends on joint and coordinated functioning of all water distribution facilities in the WDN [36]. When a node in the WDN fails, all paths that contain this node would become invalid, which would result in flow reduction in the network.

In WDN, there are three types of nodes, namely source nodes (start nodes), transition nodes (middle nodes) and demand nodes (end nodes). Water flows from source nodes to demand nodes through transition nodes. In reality, the negative effects of node failure mainly involve a reduction of water flow. Specifically, for a WDN with $N$ nodes, denote its source nodes as $S = \{s_1, \cdots s_i, \cdots s_m\}$, its demand nodes as $D = \{d_1, \cdots d_j, \cdots d_z\}$, and its total amount of flow under normal situation as Q. For any given node $k$, denote the flow of any path that passes through node $k$ as $q_{k,\cdots,d_j}$, $q_{s_i,\cdots,k}$ or $q_{s_i,\cdots,k\cdots,d_j}$, when node $k$ is source node, demand node or transition node, respectively.

Assume that, when node $k$ fails, the failure would not cause flow reduction or redistribution in paths that do not contain node $k$. This assumption is based on the fact that in many cases proper technical or managerial measures can be taken in a timely manner to avoid the amplification of failure effects in the network. Then, the flow reduction that failure of node $k$ would cause, denotes as $\Delta Q(k)$, can be calculated as follows:

$$\Delta Q(k) = \begin{cases} \sum_{k=1}^{m}\sum_{j=1}^{z} q_{k,\cdots,d_j} & \text{when node } k \text{ is a source node} \\ \sum_{i=1}^{m}\sum_{k=1}^{z} q_{s_i,\cdots,k} & \text{when node } k \text{ is a demand node} \\ \sum_{i=1}^{m}\sum_{j=1}^{z} q_{s_i,\cdots,k,\cdots,d_j} & \text{otherwise} \end{cases} \tag{1}$$

Denote the amount of flow at node $k$ as $q(k)$, then obviously

$$\Delta Q(k) = q(k). \tag{2}$$

Then, the functional importance of node $k$, denoted as $\eta(k)$, can be defined as follows:

$$\eta(k) = \frac{\Delta Q(k)}{Q} = \frac{q(k)}{Q} \tag{3}$$

#### 3.1.2. *Structural importance of node*

The structural importance of a node depends on its controlling ability over the structural connectivity of the entire network. A few indicators have been proposed in prior research to measure the structural importance of node. Examples include degree centrality [37], betweenness centrality [38], and closeness centrality [39]. Generally speaking, a structural centrality measure assigns a real value to each node in the network, where the values produced are expected to provide a ranking of nodes subject to their structural importance [34]. For measuring nodal structural importance in WDN, flow betweenness [34], as an improvement of betweenness centrality, is widely used in prior research, which has an uncertain value range. In order to have a normalized value of structural importance, so as to be consistent with the value range of functional importance assessed above and hence allow for calculation of overall node importance, a variation of flow betweenness is introduced below.

Assuming that $\delta_{ij}$ is the number of paths between $i$ and $j$, and $\delta$ is the total number of paths in the whole WDNs, where

$$\delta = \sum_{j \neq i} \delta_{ij} \tag{4}$$

$\delta_{ij}(k)$ ($k \neq i$ *and* $k \neq j$) denotes the total number of paths that link nodes $i$ and $j$ and pass through node $k$, and $\delta(k)$ denotes the number of all paths that pass through node $k$ in the whole network, where

$$\delta(k) = \sum_{j \neq i \neq k} \delta_{ij}(k) \tag{5}$$

Both $\delta_{ij}(k)$ and $\delta(k)$ can be calculated using existing methods [34]. Then, the structural importance of node $k$, denoted as $C(k)$, can be calculated as follows:

$$C(k) = \frac{\delta(k)}{\delta} \tag{6}$$

Eq. (5) indicates that nodes in central locations in the network where many paths pass through are generally considered more structurally important.

#### 3.1.3. *Overall importance of node*

For a given node, its overall importance should account for both the functional importance and the structural importance. As prior research has pointed out, there is no universally adopted notion about the respective contribution coefficients of the functional and structural importance of a node to its overall importance [34], partly because of the interdependencies of the structure and function of a network that make it difficult to differentiate their respective roles, as well as the varying managerial priorities in actual system operations [40]. That being said, however, prior studies have generally weighed the functional importance and structural importance equally [35,41], hence the overall importance of a node, denoted as $W(k)$, can be calculated as the average of its functional importance and structural importance:

$$W(k) = \frac{1}{2}(\eta(k) + C(k)) \tag{7}$$

the normalized node importance, denoted as $g(k)$, can be calculated as:

$$g(k) = \frac{W(k)}{\sum_{k=1}^{N} W(k)} \tag{8}$$

Obviously, the sum of $g(k)$ is 1, which matches the application requirement of entropy.

### 3.2. *Entropy model*

The heterogeneity of node importance in WDN can be measured by entropy. Entropy is a statistical measure of the amount of uncertainty associated with the probability distribution of any discrete random variable [42]. A network entropy model is proposed in this study for measuring the heterogeneity of node importance in WDNs. This subsection explains the calculation of the current, maximum and minimum network entropies in a given WDN. The results lay the foundation for the assessment of WDN vulnerability.

#### 3.2.1. *Network entropy*

Based on the analysis above, to measure the heterogeneity of node importance, by using $g(k)$ as an entropic index, the network entropy $E$ can be defined as:

$$E = -\sum_{k=1}^{N} g(k) \ln g(k) \tag{9}$$

where $N$ is the total number of nodes in the WDN. The above equation measures the level of heterogeneity of importance among all nodes. A larger value of $E$ indicates smaller heterogeneity of node importance in the network. It needs to be noted that, if a node is not connected with any other node in the network, it has zero functional, structural and overall importance, and hence should be excluded in the analysis in order to avoid invalidation of Eq. (9).

#### 3.2.2. *Maximum and minimum network entropies*

Based on Eq. (3), the value range of $\frac{q(k)}{Q}$ is [0, 1]. Similarly, based on Eq. (6), the value range of $\frac{\delta(k)}{\delta}$ is [0, 1]. Based Eq. (7), when $\frac{q(k)}{Q}$ or $\frac{\delta(k)}{\delta}$ increases, $g(k)$ will also increase.

To measure the heterogeneity of node importance, by using $g(k)$ as an entropic index, the network entropy $E$ can be defined as:

$$E = -\sum_{k=1}^{N} g(k)\ln g(k) = -\sum_{k=1}^{N-1} g(k)\ln g(k) - \left(1-\sum_{k}^{N-1} g(k)\right)\cdot \ln\left(1-\sum_{k}^{N-1} g(k)\right) \tag{10}$$

The partial derivative of $E$ with respect to variable $g(j)$ ($j = 1, 2, \ldots, N-1$) is:

$$\frac{\partial E}{\partial g(j)} = -\ln g(j) + \ln\left(1-\sum_{k}^{N-1} g(k)\right) \tag{11}$$

The second-order partial derivatives of $E$ with respect to variable $g(j)$ ($j = 1, 2, \ldots, N-1$) is:

$$\frac{\partial^2 E}{\partial^2 g(j)} = -\frac{1}{g(j)} - \frac{1}{1-\sum_{k}^{N-1} g(k)} \tag{12}$$

Obviously,

$$\frac{\partial^2 E}{\partial^2 g(j)} < 0 \tag{13}$$

Setting $\frac{\partial E}{\partial g(j)}$ to zero gives

$$-\frac{1}{g(j)} - \frac{1}{1-\sum_{k}^{N-1} g(k)} = 0 \tag{14}$$

Then

$$g(j) = 1-\sum_{k}^{N-1} g(k) = g(N) \tag{15}$$

Based on the analysis above, the network entropy E has its maximum value when $g(1) = \cdots = g(j) = \cdots = g(N) = \frac{1}{N}$, namely all nodes have the same normalized importance. The complete graph is an example of networks that meet this condition. Based on Eq. (9), the maximum network entropy has a value of $\ln N$.

As for the minimum network entropy, based on the above partial derivative analysis, $E$ has its minimum value when variables $g(k)$ ($k = 1, 2, \ldots, N-1$) are minimized, which in turn requires that all $\frac{\delta(k)}{\delta}$ ($k = 1, 2, \ldots, N-1$) are equal to zero, and that all $\frac{q(k)}{Q}$ ($k = 1, 2, \ldots, N-1$) are minimized.

Note that when all $\frac{\delta(k)}{\delta}$ ($k = 1, 2, \ldots, N-1$) are equal to zero, and assume the network does not have disconnected nodes, then none of these $N-1$ nodes is transition node. Consider such a WDN with $N$ nodes, denote its source nodes as $S = \{s_1, \ldots s_i, \ldots s_m\}$ and its demand nodes as $D = \{d_1, \ldots d_j, \ldots d_z\}$. If the $N$th node is a source nodes or demand nodes, there is

$$m + z = N \tag{16}$$

$$\sum_{i=1}^{m} q(s_i) = \sum_{j=1}^{z} q(d_j) = Q \tag{17}$$

hence, when $m > 1$ and $z > 1$, there is:

$$q(s_m) = Q - \sum_{i=1}^{m-1} q(s_i) \tag{18}$$

$$q(d_z) = Q - \sum_{i=1}^{z-1} q(d_j) \tag{19}$$

When a source node $s_i$ fails, the flow reduction is $q(s_i)$, hence the normalized importance of this node can be calculated as:

$$g(s_i) = \frac{\frac{q(s_i)}{2Q}}{\sum_{i=1}^{m} W(s_i) + \sum_{j=1}^{z} W(d_j)} \tag{20}$$

Similarly, when a demand node $d_j$ fails, the flow reduction is $q(d_j)$, hence the normalized importance of this node can be calculated as:

$$g(d_j) = \frac{\frac{q(d_j)}{2Q}}{\sum_{i=1}^{m} W(s_i) + \sum_{j=1}^{z} W(d_j)} \tag{21}$$

obviously,

$$\sum_{i=1}^{m} W(s_i) + \sum_{j=1}^{z} W(d_j) = \sum_{i=1}^{m} \frac{q(s_i)}{2Q} + \sum_{j=1}^{z} \frac{q(d_j)}{2Q} = 1 \tag{22}$$

hence

$$E = -\sum_{i=1}^{m} g(s_i)\ln g(s_i) - \sum_{j=1}^{z} g(d_j)\ln g(d_j) \tag{23}$$

namely

$$E = -\sum_{i=1}^{m-1} \frac{q(s_i)}{2Q}\ln\frac{q(s_i)}{2Q} - \frac{1}{2}\left(1-\sum_{i=1}^{m-1}\frac{q(s_i)}{Q}\right)\ln\frac{1}{2}\left(1-\sum_{i=1}^{m-1}\frac{q(s_i)}{Q}\right) - \sum_{j=1}^{z-1} \frac{q(d_j)}{2Q}\ln\frac{q(d_j)}{2Q} - \frac{1}{2}\left(1-\sum_{j=1}^{z-1}\frac{q(d_j)}{Q}\right)\ln\frac{1}{2}\left(1-\sum_{j=1}^{z-1}\frac{q(d_j)}{Q}\right) \tag{24}$$

For a water distribution system, there is flow between at least one pair of nodes (otherwise all nodes have zero flow and the entire network is not operational). Meanwhile:

$$\lim_{q(k)\to 0^+} \frac{q(k)}{2Q}\ln\frac{q(k)}{2Q} = \lim_{q(k)\to 0^+} \frac{\ln\frac{q(k)}{2Q}}{\frac{2Q}{q(k)}} = 0 \tag{25}$$

When the values of $\frac{q(s_i)}{Q}$ ($i = 1, 2, \ldots, m-1$) and $\frac{q(d_j)}{Q}$ ($j = 1, 2, \ldots, z-1$) are equal to zero and the values of $\frac{q(s_m)}{Q}$ and $\frac{q(d_z)}{Q}$ are equal to 1, the network has its minimum entropy with a value of $\ln 2$. It can be easily verified that this conclusion is still true when $m = 1$ or $z = 1$. The star graph is an example of networks that meet such condition.

On the other hand, when the $N$th node is a transition node, there is

$$m + z = N - 1 \tag{26}$$

Eqs. (17)–(19) and (22) are still true in this situation. The importance of node $s_i$, $d_j$ and $N$ can be calculated as:

$$g(s_i) = \frac{\frac{q(s_i)}{2Q}}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \tag{27}$$

$$g(d_j) = \frac{\frac{q(d_j)}{2Q}}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \tag{28}$$

$$g(N) = \frac{\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \tag{29}$$

hence

$$E = -\sum_{i=1}^{m} g(s_i)\ln g(s_i) - \sum_{j=1}^{z} g(d_j)\ln g(d_j) - g(N)\ln g(N) \tag{30}$$

namely

$$\begin{aligned} E = & -\sum_{i=1}^{m-1} \frac{\frac{q(s_i)}{2Q}}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \ln \frac{\frac{q(s_i)}{2Q}}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \\ & - \frac{\frac{1}{2}\left(1-\sum_{i=1}^{m-1}\frac{q(s_i)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \ln \frac{\frac{1}{2}\left(1-\sum_{i=1}^{m-1}\frac{q(s_i)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \\ & - \sum_{j=1}^{z-1} \frac{\frac{q(d_j)}{2Q}}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \ln \frac{\frac{q(d_j)}{2Q}}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \\ & - \frac{\frac{1}{2}\left(1-\sum_{j=1}^{z-1}\frac{q(d_j)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \ln \frac{\frac{1}{2}\left(1-\sum_{j=1}^{z-1}\frac{q(d_j)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \\ & - \frac{\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \ln \frac{\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)}{1+\frac{1}{2}\left(\frac{\delta(N)}{\delta}+\frac{q(N)}{Q}\right)} \end{aligned} \tag{31}$$

When the values of $\frac{q(s_i)}{Q}$ $(i = 1, 2, \ldots, m-1)$ and $\frac{q(d_j)}{Q}$ $(j = 1, 2, \ldots, z-1)$ are equal to zero, the values of $\frac{q(s_m)}{Q}$ and $\frac{q(d_z)}{Q}$ are equal to 1, and then $E$ will have its minimum value. if node $N$ is between nodes $s_m$ and $d_z$ in a three-node path, the network has its minimum entropy. In this situation, $E$ has its minimum value of 1.0889, which is larger than $\ln 2$; If not, namely node $N$ is between any other two nodes, $\frac{q(N)}{Q}$ is equal to zero, hence $E$ infinitely draw near to $\ln 2$ when $\frac{\delta(N)}{\delta}$ infinitely draw near to zero. It can be easily verified that this conclusion is still true when $m = 1$ or $z = 1$.

In summary, the maximum and minimum entropies of a network are $\ln N$ and $\ln 2$, respectively. It is noteworthy that these conclusions are consistent with suggestions in the prior studies [28,30,31], although those studies only considered network structure in the calculation of network entropy.

### 3.3. *Vulnerability assessment*

For a given WDN, its network entropy reflects the current vulnerability state of the system, and the maximum and minimum network entropy values represent, respectively, the states with the least and most vulnerability. When the node importance becomes more heterogeneous and the network entropy value approaches to the value of minimum network entropy, the WDNs become more vulnerable, and vice versa. Therefore, the systemic vulnerability assessment of the WDN can be carried out by benchmarking the current vulnerability state of the system against its least and most vulnerable states. Specifically, the vulnerability of WDNs, denoted as $\overline{E}$, can be calculated as:

$$\overline{E} = \frac{E_{\max} - E}{E_{\max} - E_{\min}} \tag{32}$$

It can be seen from Eq. (32) that the value range of $\overline{E}$ is [0, 1]. Higher $\overline{E}$ value indicates that the system has lower entropy, hence higher heterogeneity of node importance and therefore is more vulnerable to various possible failure scenarios, and vice versa.

## 4. Case study

### 4.1. *Case description*

To assess the efficacy of the proposed method, a case study was conducted in a middle-sized Chinese city. Located at an intersection of several major economic regions in Eastern China, the case city has an area of approximately 1000 $km^2$ and a population of approximately 300,000. To model the WDN in the city, reservoirs, pumps, plants and end users of the water distribution system were regarded as nodes, and the pipelines between them were regarded as edges. The location of the WDN nodes and edges between the nodes were determined based on the WDN design and operation documents obtained from local authorities and field visits. Based on these data, a WDN that consisted of 19 nodes and 20 edges was built. Its layout is shown in Fig. 1. Specifically, nodes 1–2 were both reservoirs, serving as water sources for two regions in the city; nodes 3–6 were water plants responsible for water treatment, and nodes 7–9 were pumps, which were used to increase path flow pressure. Nodes 9–19 were demand nodes.

The total amount of flow distributed by this system was $6.6 \times 10^4$ $m^3$ per day. The amount of flow between every pair of connected nodes, extracted from system monitoring data provided by the local authorities, is summarized in Table 1.

### 4.2. *Results*

#### 4.2.1. *Node importance assessment*

Based on Eq. (2) and the flow information in Table 1, the flow reduction that would result from for the failure of node $k$ was calculated. In addition, based on Eq. (4), the total number of paths in this WDN was 39. The consequences associated with the failure of each node in the network were calculated and summarized in Table 2. Then, based on Eqs. (3) (6) and (7), the functional, structural and overall importance of every node was calculated. The results are also summarized in Table 2.

As can be seen in Table 2, with respect to the functional importance, node 1 was the most important, with a functional importance value of 0.8485. This was reasonable as node 1 was the only water source node in one of the two subareas in the

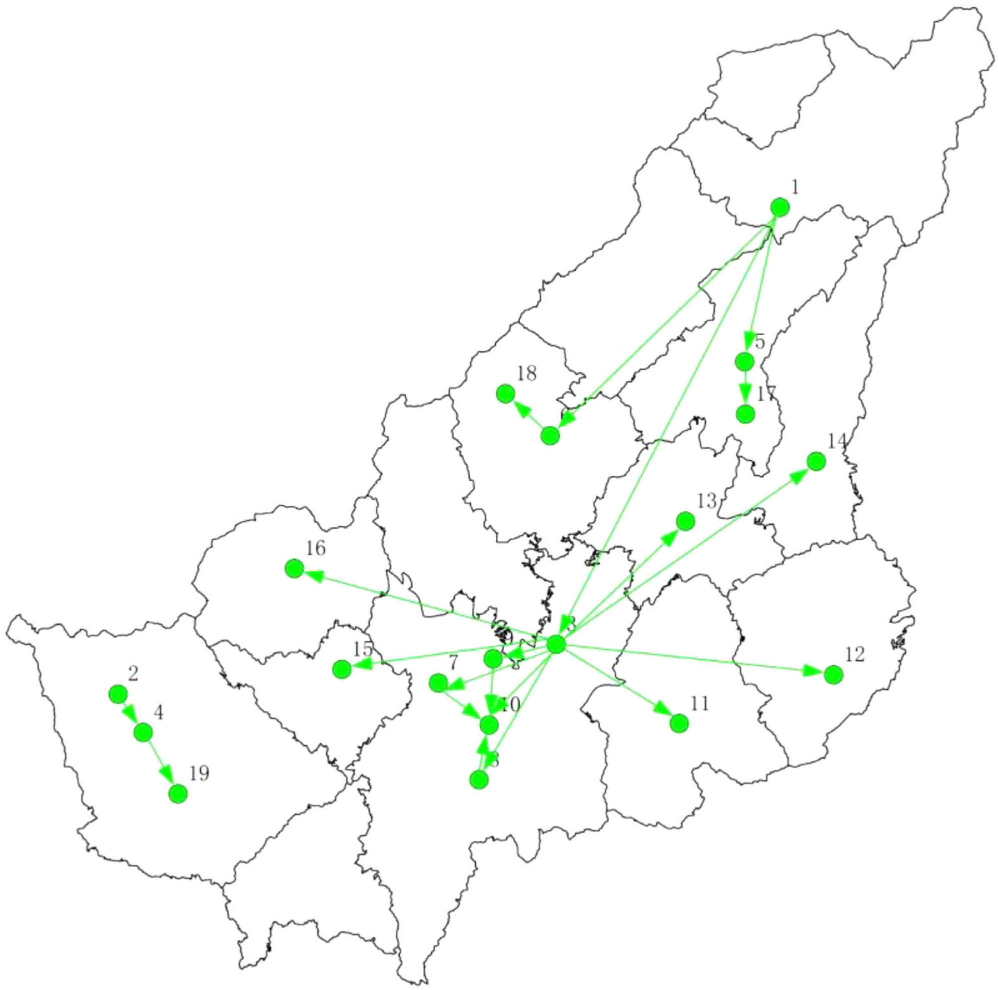



**Fig. 1 – Layout of WDN in the case city.**

**Table 1 – Flow information in the case WDN.**

| Pipeline | Flow of pipeline ($10^4$ $m^3$/d) | Pipeline | Flow of pipeline ($10^4$ $m^3$/d) |
|---|---|---|---|
| 1–3 | 5.0000 | 3–13 | 0.2700 |
| 1–5 | 0.3000 | 3–14 | 0.2500 |
| 1–6 | 0.3000 | 3–15 | 0.1100 |
| 2–4 | 1.0000 | 3–16 | 0.2000 |
| 3–7 | 0.8200 | 4–19 | 1.0000 |
| 3–8 | 1.3000 | 5–17 | 0.3000 |
| 3–9 | 0.2200 | 6–18 | 0.3000 |
| 3–10 | 0.7600 | 7–10 | 0.8200 |
| 3–11 | 0.2000 | 8–10 | 1.3000 |
| 3–12 | 0.4800 | 9–10 | 0.2200 |

Note: each pipeline is named after the two nodes at its two ends.

city within which all nodes except for nodes 2, 4, 19 were located. The flow of this subarea would become zero if node 1 failed. The lowest nodal functional importance in this subarea was 0.0167. The values of functional importance of all other nodes were generally equably distributed between the above two ends. With respect to the structural importance, the most important node was identified to be node 3, which was located at the center of the network and had a structural importance value of 0.3333. Ten nodes were identified to be the least structurally important, with structural importance value of 0, as they were located at either the beginning or the end of paths in the network. Moreover, for the overall node importance, node 3 was identified to be the most important, with an overall importance value of 0.5455. The second most important node was the water source node 1, which had the highest functional importance but the lowest structural importance. The node importance assessment results are depicted in Fig. 2, in which the size of each circle is proportional to the importance value of the corresponding node. The layout of the result also indicated that the structural importance of nodes would be larger when they were in more central locations.

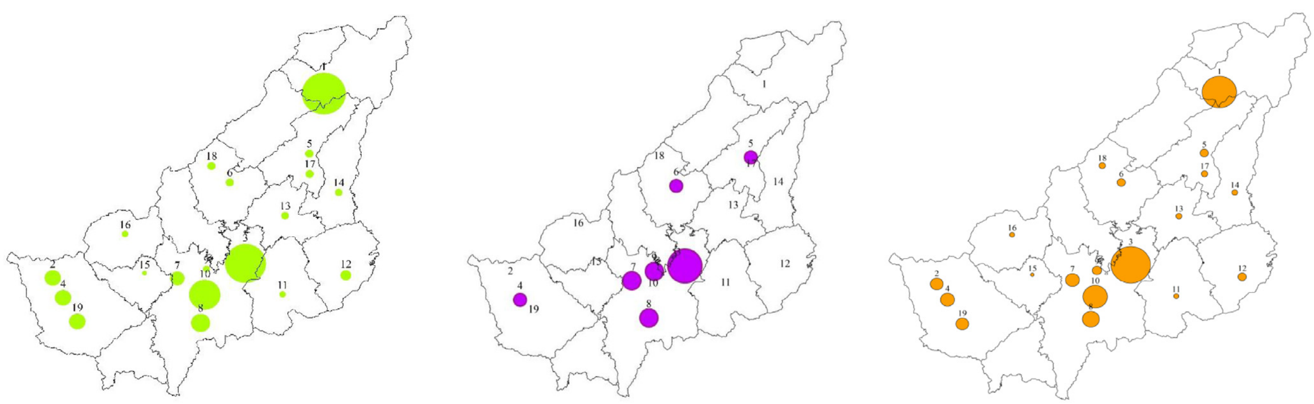
### 4.2.2. *Vulnerability assessment*

Based on the node importance assessment result, the network entropy $E$ of the WDN in the case city was calculated according to Eq. (9):

$$E = -\sum_{k=1}^{19} g(k) ln g(k) = 2.2627 \tag{33}$$

**Table 2 – Failure consequence and importance of each node in the case WDN.**

| Node $k$ | Flow reduction $\Delta Q(k)$ | Number of broken paths $\delta(k)$ | Functional importance $\eta(k)$ | Structural importance $C(k)$ | Overall importance $W(k)$ |
|---|---|---|---|---|---|
| 1 | 5.6000 | 0 | 0.8485 | 0.0000 | 0.4242 |
| 2 | 1.0000 | 0 | 0.1515 | 0.0000 | 0.0758 |
| 3 | 5.0000 | 13 | 0.7576 | 0.3333 | 0.5455 |
| 4 | 1.0000 | 1 | 0.1515 | 0.0256 | 0.0886 |
| 5 | 0.3000 | 1 | 0.0455 | 0.0256 | 0.0355 |
| 6 | 0.3000 | 1 | 0.0455 | 0.0256 | 0.0355 |
| 7 | 0.8200 | 2 | 0.1242 | 0.0513 | 0.0878 |
| 8 | 1.3000 | 2 | 0.1970 | 0.0513 | 0.1241 |
| 9 | 0.2200 | 2 | 0.0333 | 0.0513 | 0.0423 |
| 10 | 3.1000 | 0 | 0.4697 | 0.0000 | 0.2348 |
| 11 | 0.2000 | 0 | 0.0303 | 0.0000 | 0.0152 |
| 12 | 0.4800 | 0 | 0.0727 | 0.0000 | 0.0364 |
| 13 | 0.2700 | 0 | 0.0409 | 0.0000 | 0.0205 |
| 14 | 0.2500 | 0 | 0.0379 | 0.0000 | 0.0189 |
| 15 | 0.1100 | 0 | 0.0167 | 0.0000 | 0.0083 |
| 16 | 0.2000 | 0 | 0.0303 | 0.0000 | 0.0152 |
| 17 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 18 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 19 | 1.0000 | 0 | 0.1515 | 0.0000 | 0.0758 |



(a) Node functional importance (b) Node structural importance (c) Node overall importance

**Fig. 2 – The functional, structural and overall importance of all nodes in the case WDN.**

In order to measure the vulnerability of the case WDN, the maximum network entropy and minimum network entropy were also calculated. Based on the analysis of Section 3.2.2, the maximum network entropy and minimum network entropy were determined to be ln19 and ln2, respectively.

Then, based on Eq. (32), the vulnerability level of the WDN in the case city was calculated below:

$$\overline{E} = \frac{E_{\max} - E}{E_{\max} - E_{\min}} = 0.3028 \tag{34}$$

### 4.3. *Discussions*

The results of the case study indicated that the case WDN had medium level vulnerability, which was assessed to be 0.3028 in a 0–1 scale. This medium level of vulnerability of the network could be mainly attributed to several factors: first of all, the nodal functional importance was equably distributed among the majority of the nodes, which in turn was because of the generally equable distribution of the water flows in the network. This significantly lowered the level of heterogeneity of the nodes and hence the vulnerability of the entire network; secondly, the network had an apparently heterogeneous structure, with nodes 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16 directly connected with node 3, forming a star-shaped component regionally, and nodes 2, 4 and 19 forming another star-shaped component with node 4 being at the center. Such heterogeneous structure was mainly responsible for the current vulnerability of the case network. In addition, the vulnerability also stemmed from the fact that the WDN served two subareas in the case city, which were disconnected with each other and each was served with one water source. The system would encounter severe functional losses should either of the two water source nodes break down. Such incidents actually had happened once in the case city. It experienced a serious and widespread system breakdown in 2014,

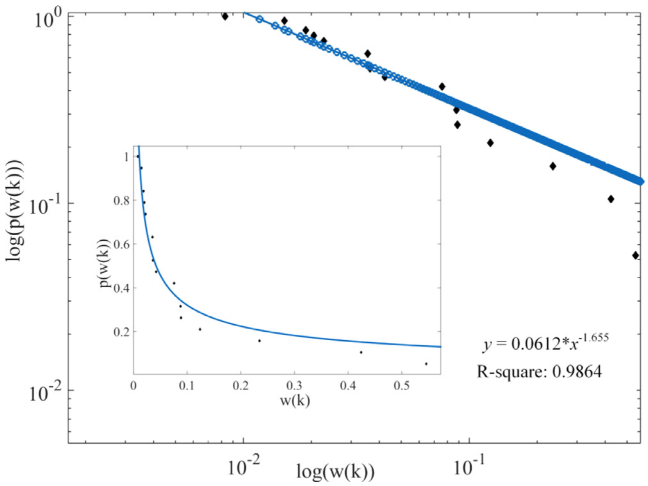



**Fig. 3 – The fitting curve of cumulative distribution function of node importance (in inside figure, the horizontal axis is node importance *W*(*k*), and vertical axis is cumulative probability p(*W*(*k*)); the curve is displayed in logarithmic coordinate system in outside figure).**

when node 1 failed due to unexpected technical issues, resulting in water outages in most part of the city for over 60 h, which remarkably demonstrated that the vulnerability of the case WDN against uncertain failure scenarios was yet to be reduced.

As the proposed method indicated, such medium level vulnerability of the case WDN was determined by the level of heterogeneity of node importance in the system. To further reveal the heterogeneous characteristic of the case WDN, it was assessed against power-law [43]. In a widely cited study, Barabási and Albert [44] showed that the distributions of node degree of many real-world networks are heterogeneous. Some nodes are highly connected to others while others have few links. If the degree distribution is found to follow a power-law distribution, this type of network is called scale-free network. The scale-free network is very heterogeneous when the power-law exponent varies from 2 to 3 [30,45,46]. Drawing upon this theory, the heterogeneity of the case WDN was tested based on the node importance distribution. Using a fitting method proposed by Mossa et al. [47] the cumulative distribution function of node importance in the case WDN was calculated as $y = 0.0612x^{-1.655}$ ($R^2$=0.9864), which had satisfactory fitting effect. The results are illustrated in Fig. 3. The fitting curve shown in the figure suggested a typical scale-free network with gentle gradient. However, the exponent of power-law was 1.655, which was smaller than the lower limiting value of 2. This result indicated that the case WDN was a scale-free network that did not have high heterogeneity.

Moreover, scale-free networks are known to be “invulnerable and fragile” [13]. These networks normally have high level of invulnerability against random failures, but could be broken into many isolated fragments when the most connected nodes are targeted. The proposed vulnerability assessment method considers the uncertainty of failure scenarios that could involve either random or targeted attacks, and the assessment results in the case study were reflective of such uncertainty and consistent with the known characteristics of scale-free networks in the literature.

Following the aforementioned system breakdown incident, the local authority in the case city has decided to improve its WDN to present similar incidents in the future. Specifically, it has planned to add the following components to the existing WDN between year 2020 and 2030, as shown in Fig. 4, including a new water source (a reservoir, node 20), a water plant (node 21), and a few pipelines (blue links in the figure) that would link the new water source and water plant to the existing network and connect the two currently disconnected substructures of the network. Assume that the total water supply and total water demand of the WDN remain unchanged, and the total water supply is assigned to the three water source nodes proportionally to their planned capacity. Then, the importance of every node in the planned WDN can be calculated using the proposed method. The results are shown in Table 3. The values of $E_{max}$, $E$ and $E_{min}$ were 3.0445, 2.4408 and 0.6931, respectively. Accordingly, the level of vulnerability of the planned WDN was assessed to be 0.2568. Compared to the existing WDN, the level of vulnerability was reduced by 15.21%.

Moreover, in order to demonstrate the possibility of using the proposed method to guide the improvement of WDN, a hypothetical WDN was studied. The hypothetical WDN, as shown in Fig. 5, was developed by adding a few more pipelines (red links in the figure) to the planned WDN in the case city. It needs to be noted that these changes were not intended to optimize the case WDN. System optimization is beyond the scope of this study. Rather, these additional links were selected to apparently decrease the heterogeneity of the network and therefore, according to the prosed method, reduce the vulnerability of the network. Assume that the total amount of flow remains Q, and that the new pipelines share the water flows leaving the nodes they are connected to equally with existing pipelines that are connected to the same nodes, then the importance of every node in the hypothetical WDN can be calculated using the proposed method. The results are shown in Table 4. The values of $E_{max}$, $E$ and $E_{min}$ were 3.0445, 2.5843 and 0.6931, respectively. Accordingly, the level of vulnerability of the planned WDN was assessed to be 0.1943. Compared to the planned WDN, the level of vulnerability was reduced by 24.32%, which indicated the potential of WDN vulnerability reduction through measures that could possibly reduce the heterogeneity of node importance in the network.

Last but not least, the proposed systemic vulnerability assessment method has several important practical applications. It enables stakeholders of WDNs to face various possible failure scenarios, especially those that are low-probability but high-impact, when making planning, design and operation decisions. It would allow decision makers to evaluate the overall gains of system invulnerability from hazard-specific protective measures informed by existing vulnerability assessment methods. As demonstrated in the case study, the proposed method is promising in identifying potential vulnerability challenges in WDNs, and supporting the protection of WDNs in practice.

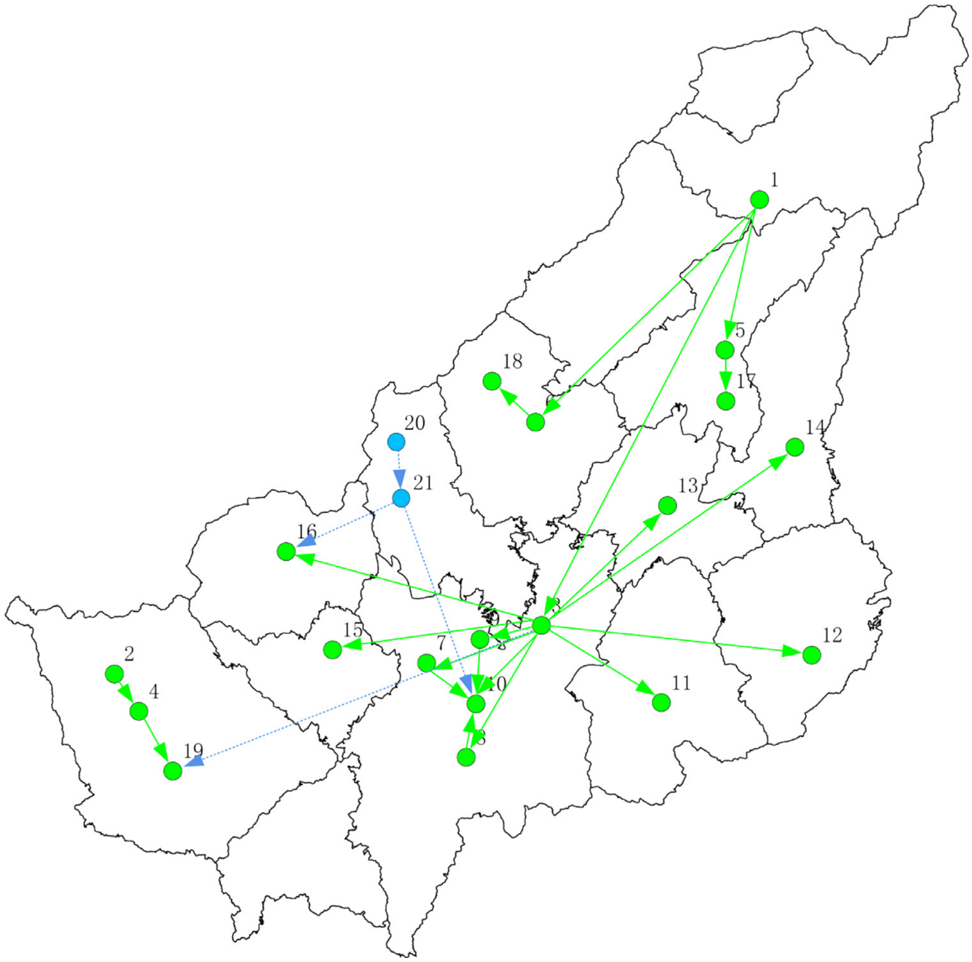



**Fig. 4 – Layout of the planned WDN in the case city.**

**Table 3 – Failure consequence and importance of each node in the planned WDN in the case city.**

| Node $k$ | Flow reduction $\Delta Q(k)$ | Number of broken paths $\delta(k)$ | Functional importance $\eta(k)$ | Structural importance $C(k)$ | Overall importance $W(k)$ |
|---|---|---|---|---|---|
| 1 | 3.7400 | 0 | 0.5667 | 0.0000 | 0.2833 |
| 2 | 0.3500 | 0 | 0.0530 | 0.0000 | 0.0265 |
| 3 | 3.3100 | 14 | 0.5015 | 0.3043 | 0.4029 |
| 4 | 0.3700 | 1 | 0.0561 | 0.0217 | 0.0389 |
| 5 | 0.2000 | 1 | 0.0303 | 0.0217 | 0.0260 |
| 6 | 0.2000 | 1 | 0.0303 | 0.0217 | 0.0260 |
| 7 | 0.2200 | 2 | 0.0333 | 0.0435 | 0.0384 |
| 8 | 0.2200 | 2 | 0.0333 | 0.0435 | 0.0384 |
| 9 | 0.2200 | 2 | 0.0333 | 0.0435 | 0.0384 |
| 10 | 3.2100 | 0 | 0.4864 | 0.0000 | 0.2432 |
| 11 | 0.2600 | 0 | 0.0394 | 0.0000 | 0.0197 |
| 12 | 0.5400 | 0 | 0.0818 | 0.0000 | 0.0409 |
| 13 | 0.3300 | 0 | 0.0500 | 0.0000 | 0.0250 |
| 14 | 0.3100 | 0 | 0.0470 | 0.0000 | 0.0235 |
| 15 | 0.1700 | 0 | 0.0258 | 0.0000 | 0.0129 |
| 16 | 0.2600 | 0 | 0.0394 | 0.0000 | 0.0197 |
| 17 | 0.2000 | 0 | 0.0303 | 0.0000 | 0.0152 |
| 18 | 0.2000 | 0 | 0.0303 | 0.0000 | 0.0152 |
| 19 | 1.0000 | 0 | 0.1515 | 0.0000 | 0.0758 |
| 20 | 2.4700 | 0 | 0.3742 | 0.0000 | 0.1871 |
| 21 | 2.4700 | 2 | 0.3742 | 0.0435 | 0.2089 |

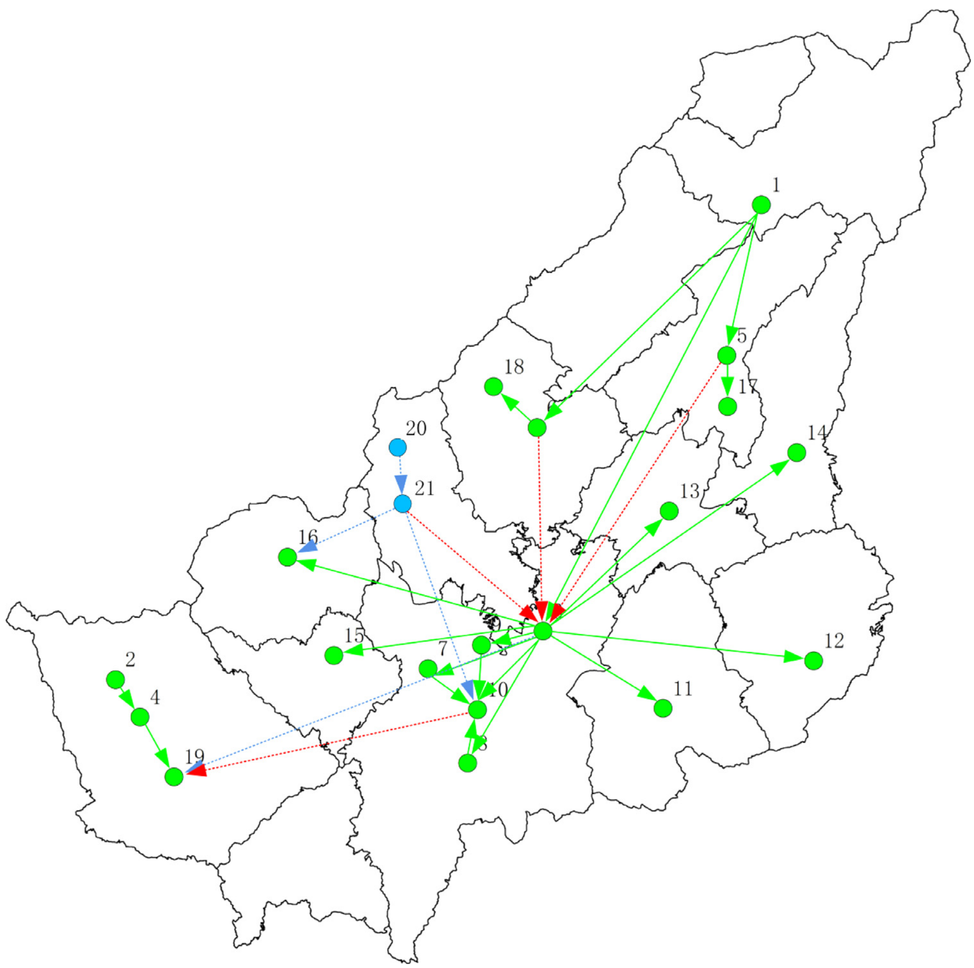



**Fig. 5 – Layout of the hypothetical WDN in the case city.**

**Table 4 – Failure consequence and importance of each node in the hypothetical WDN in the case city.**

| Node $k$ | Flow reduction $\Delta Q(k)$ | Number of broken paths $\delta(k)$ | Functional importance $\eta(k)$ | Structural importance $C(k)$ | Overall importance $W(k)$ |
|---|---|---|---|---|---|
| 1 | 3.4700 | 0 | 0.5258 | 0.0000 | 0.2629 |
| 2 | 0.3700 | 0 | 0.0561 | 0.0000 | 0.0280 |
| 3 | 3.3300 | 126 | 0.5045 | 0.7241 | 0.6143 |
| 4 | 0.3700 | 1 | 0.0561 | 0.0057 | 0.0309 |
| 5 | 1.2400 | 20 | 0.1879 | 0.1149 | 0.1514 |
| 6 | 1.2400 | 20 | 0.1879 | 0.1149 | 0.1514 |
| 7 | 0.3000 | 16 | 0.0455 | 0.0920 | 0.0687 |
| 8 | 0.3000 | 16 | 0.0455 | 0.0920 | 0.0687 |
| 9 | 0.3000 | 16 | 0.0455 | 0.0920 | 0.0687 |
| 10 | 2.0200 | 37 | 0.3061 | 0.2126 | 0.2594 |
| 11 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 12 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 13 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 14 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 15 | 0.3000 | 0 | 0.0455 | 0.0000 | 0.0227 |
| 16 | 1.1200 | 0 | 0.1697 | 0.0000 | 0.0848 |
| 17 | 1.2400 | 0 | 0.1879 | 0.0000 | 0.0939 |
| 18 | 1.2400 | 0 | 0.1879 | 0.0000 | 0.0939 |
| 19 | 0.7000 | 0 | 0.1061 | 0.0000 | 0.0530 |
| 20 | 2.4700 | 0 | 0.3742 | 0.0000 | 0.1871 |
| 21 | 2.4700 | 22 | 0.3742 | 0.1264 | 0.2503 |

## 5. Conclusions

With the continuous expansion of urban regions, WDNs in cities are becoming more complex in their structure and function. Meanwhile, they are exposed to increasing risks of various natural and manmade hazards, whose probabilities of occurrence and possible impacts are highly unpredictable. Systemic vulnerability assessment of WDNs considering the failure scenario uncertainty is of significant importance for properly managing WDNs to ensure the reliability of water supplies in cities. This paper described a new method for system-level, scenario-independent vulnerability assessment of WDNs, using a network entropy model whose entropic index is based on node importance. The efficacy of the proposed method was demonstrated in a case study, in which the assessment yielded by the proposed method was found theoretically reasonable as well as consistent with actual conditions of the case WDN.

That being said, there are several limitations in this study that should be noted. Specifically, the heterogeneity of edge importance was not considered in the vulnerability assessment, and the impacts by other infrastructure systems through their interdependencies with WDNs were not factored in. These limitations could be addressed in future studies to further advance this line of research.

## Conflict of interest

The authors declare no conflict of interest.

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## Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ijcip.2019.05.002.

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