

# Robust Scheduling of Water Distribution Networks

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**Abstract:** Optimal operation of water distribution networks can be posed as a scheduling problem where the objective is to meet the time varying demand while meeting constraints on supply, pressure etc. In the present work, we propose a robust optimization problem to address uncertainty in the parameters of the model used for optimization. The resulting problem is a second order cone program that can be solved efficiently. The formulation ensures a high probability of meeting the demands, adding to the practical significance. Further, we provide the results of applying this technique on a laboratory scale water distribution network.

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## 1. INTRODUCTION

The global demand for water is increasing rapidly and operators of Water Distribution Networks (WDNs) are finding it difficult to meet the demands. To cope with the situation, utility providers often choose to supply water intermittently, wherein each consumer would be provided water only for few hours in a day (Vairavamoorthy et al., 2007). It is common for these intermittent water supply systems (IWSSs) to have erratic operational schedules and inequitable water supply (Bhave and Gupta, 2013). Even for the IWSSs with a well planned operation, the system-model mismatch and variations in network parameters can cause the supply to be inequitable. Several mathematical programming techniques and meta-heuristics have been proposed in the past for developing these schedules for the operation of WDNs Mala-Jetmarova et al. (2017). However, most of these address the operation of continuous water supply systems (CWSs), which are less prevalent in developing countries as compared to the IWSSs.

Recently, we have presented a linear programming (LP) based technique for scheduling a class of IWSSs using mathematical models (Kurian et al., 2018) and experimentally generated data drive models (Chinnusamy et al., 2018). In the data driven modelling approach, it was assumed that flow measurements used to build the model are perfect and free of errors. However, in practice, flow measurements are corrupted with errors and hence, the model is subject to uncertainties. In the present work, we propose a robust optimization framework for the same scheduling problem to address the problem of model uncertainties. Further, we provide the results of applying this technique on a laboratory scale water distribution network.

## 2. PROBLEM DESCRIPTION

The system we consider consists of one source supplying water to multiple demand nodes located downstream. The flow may be pumped or gravity driven. The demand points are assumed to have storage tanks with sufficient buffer capacities. This allows water to be supplied at any time of the day. Demands to be met for these downstream tanks are known *a priori*. The flow in the network are controlled using ON/OFF valves located immediately upstream of the demand nodes. It is assumed that continuous control of flow cannot be realized using these ON/OFF valves. A schematic of one such system is shown in Figure 1. Networks of this type are common in IWSSs.

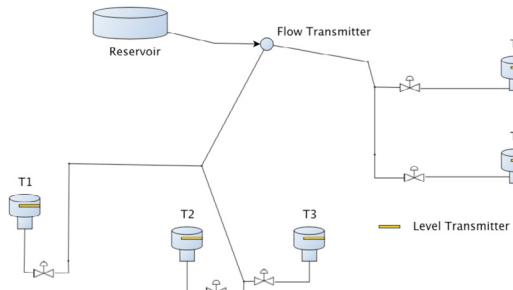


Fig. 1. Schematic of the system.

The objective is to meet the demands in minimal time by suitably operating the valves at different instants of the scheduling horizon. An optimization problem for identifying the sequence of valve operations has been shown to be a Mixed Integer Non-Linear Program (MINLP) (Kurian et al., 2018). The discrete states of the valves results in integer variables, while the non-linearities of the problem are induced by the head loss equation and pump curves.

Scheduling technique of less complexity is desirable considering that the problem has to be solved almost on a daily basis in several water networks in developing countries.

### 3. SCHEDULING FORMULATION

We propose to solve the problem in two steps. In the first step, we make use of the decomposition described by Chinnusamy et al. (2018) to decouple hydraulic simulations from the scheduling problem. For a given network with  $N$  tanks (demand nodes), each with a discrete ON/OFF valve, the total number of possible configurations (states) would be  $2^N$ . Hence, The achievable set of flow rates and pressures in a typical class of networks corresponding to all possible configurations of valve positions is finite. Flow measurements have to be performed to obtain the flow rates for each network configuration. These measurements are then used in a data driven model in place of a hydraulic model in the scheduling formulation.

With the available data, Chinnusamy et al. (2018) presents an LP to prepare the optimal schedule for supplying water. However, any errors in the measured flow rates can lead to deficit in supply while implementing the schedule. We show that the optimal solution of the nominal problem is biased towards network configurations with high measurement errors. To overcome this challenge, we propose a robust optimization framework for scheduling the WDN. Here it is assumed that the flow rates are corrupted with error and the uncertainty is described by a normal distribution function. Correspondingly, the realised supply of water is seen to be less than the predictions made using the data. The objective of our problem is to meet the demand in a minimum time under this assumption on measurement errors. We formulate a second order cone program to solve this problem efficiently (Lobo et al., 1998).

### 4. RESULTS

We tested the technique on the data obtained from a laboratory scale WDN. Assuming the deviation from the true flow rates is up to three standard deviations, the new framework could meet the demand in 709 out of 1000 different simulations. Contrary to this, the existing LP formulation could meet the demand only in 35 cases. This improvement was achieved with an increase in operational time of only about 7 %. The formulation also gives the flexibility to tune the parameters to arrive at a trade-off between operational time and probability of demand satisfaction.

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