HIERARCHICAL METHODS AND APPLICATION TO WATER SYSTEMS

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Abstract. Hierarchical methods have been developed during the last two decades for solving problems not tractable by existing methods. Water systems seem to be ideal for application of hierarchical methods. Guided by characterization of water systems we shall survey some chosen hierarchical concepts and present their applications to these systems. The future trends are also indicated wherever possible.

Keywords. Water systems, hierarchical systems, aggregation, control and decision making, repetitive control, hierarchical optimization, multiobjective optimization, multilayer structure, multilevel structure, control correction, coordination, uncertainty factors, state constraints.

INTRODUCTION

The water systems are a subject of increasing interest to system researchers. This has been so since the demand on water has steadily grown and the unpolluted resource sources are getting scarce. Besides, the water system features imply that the system design and development or decision making and control in such systems constitutes a true challenge to research. The existing water systems are of a different kind starting from a single reservoir and progressing to a catchment area of a main river where there are storage reservoirs, urban and industrial areas affecting the system through their water demands and waste-water discharges, water surface and ground sources, water distribution networks and water treatment plants. The mentioned water system features are not common to each water system. However, if one of these features appears it creates serious problems for classical approaches and quite often the classical methods are simply useless. We will now list and briefly discuss the most important water system features.

There are a number of decision makers having a certain freedom within an existing legislation in the system, each of them may make decisions to realize their own, different than desired goals.

There are a large number of system objectives, even for one decision maker, and not all of them are commensurable or it is not possible to formalize some of them.

The human factor existing in the system cannot not completely be replaced by computers so there is a necessity of its appropriate incorporation into the "mechanical" part of a decision making and control structure.

There is a large uncertainty factor. This is, for example, due to hydrologic uncertainty or due to unknown future demands of the water users.

The dynamic characterisctics of various physical elements in the system are different. For example, a time constant of a regional retention reservoir and a time constant of a distribution network reservoir are quite different.

To initiate decision making or control actions the appropriately formulated problems mostly of an optimization type, have to be solved before to provide for decision values or control laws. The problems appear to be of a high dimension. There are two independent sources of the high dimensionality. A physical complexity of the system and a long time horizon on which the system behaviour is considered. For example, when an operating rule for a regional retention reservoir is designed, the time horizon of one year's length is usually chosen.

There exist information constraints of a different type. Some of them are directly connected with the water system features presented so far. For example, in a case of a complex and geographically distributed system it may appear to be impossible to ensure an on-line transmission of all information gathered currently in the system to one centre.

Among different constraints imposed on decions values there are so called state constraints. Now, taking into account an uncertainty factor pointed out before, we can see that some of the decision problems are under state constraints and in the presence of uncertainty. A mandatory water reserve and a mandatory flood capacity reserve for reservoirs are typical examples of the state constraints which have to be satisfied during the systems operation, in spite of an existing uncertainty i.e., unknown reservoir inflows and unknown water-users demands, in the considered case.

Hierarchical methods turned out to be very useful tools for solving problems arising in water systems. Guided by the above characterization of water systems we shall survey chosen hierarchical concepts, and present how they have been applied in the real water systems or during research performed on the water systems. The future trends are also indicated wherever possible.

AGGREGATION

The aggregation techniques were derived to cope with a problem dimensionality. The problem solution is

found by performing a multistage procedure. Each stage of the procedure consists of solving a problem of a lower dimension (aggregated problem) than the original one, providing for the data used in the next stage when another aggregated problem of a higher dimension than the previous one is solved. The original problem solution is obtained as a result of performing the last stage of the procedure. This concept is a particular case of something which is more general, that is a multi-layer concept (Findeisen and Lefkowitz, 1969). A proper formulation of an aggregated problem being solved at each stage (layer) is a crucial point of the technique. Although great research attempts have been made towards the development of general aggregation rules, this is still a part of an engineering art. In water systems the following two kinds of aggregations are in common use: spatial aggregation and time aggregation.

The spatial aggregation

This kind of aggregation consists in combining several physical elements of a system into an artifical one which is from some standpoint equivalent. Examples of such aggregation used in water supply networks, water sewer networks, water distribution networks and water resource alloca-tion planning problem can be found e.g., in Fall-side and Perry (1974), Joalland and Cohen (1978), Klempous, Kotowski and Ułasiewicz (1983), Papageo-rgiou (1983), Nowosad (1983). The two-layer aggr-egation technique derived by Nowosad (1983) for solving water allocation problems yields an exact solution of the original oroblem, oppositely to commonly met aggregation methods. However, its applicability is somewhat limited in cases of multi-reservoir systems by the system structure. The water system consists of several retention reser-voirs and water users. The reservoir inflows and the users needs constitute the system uncertainty and only their forecasts are known. The planning is done under water deficit conditions. The aim of plaming is to minimize the users total losses under the conditions that water amounts in each reservoir dose not violate the given constraints. The planning structure is made-up of two layers. An upper layer problem is achieved by appropriate agrregations of the reservoirs, the users needs, the constraints, the reservoirs inflows and the decision variables. As a result of such aggregation the water allocation problem with one reservoir and the global water user is obtained. This is a simple dynamical problem with one state variable and it has an analytical solution. A lower layer problem is a static problem which consists of current opti-mal allocation to the users the global amount of water elaborated by the upper layer.

The time aggregation

This kind of aggregation is used for solving dynamic optimization problems on long time horizon. It is based upon relating the problem variables to different time scales associated with each layer. The time horizons corresponding to each layer are also different. A lower layer has a smaller time scale and a shorter time horizon than an upper layer. The upper layer provides the lower layer with a final state values (state targets). Clearly, the time scales and the time horizons are chosen so that the layer problems are consistent. Each layer problem consists of solving a number of independent (since separated by state targets) dynamic optimization problems on a shorter time horizons associated with the layer. The lower the layer the more detailed solution is obtained. The layers short term problems can be solved in parallel. Due to the difficulties in formulation the aggregate performance functions for each layer the structure produces rather suboptimal than optimal, i.e., exact solution. Examples of the

time aggregation used in solving a water retention planning and control problems were reported, e.g., Becker and colleagues (1976), Bechard and colleagues (1981),Findeisen and colleagues (1983). The reten-tion control task considered in Findeisen and colleagues (1983) consists of determining on-line a daily release for each of the three reservoirs in such a way that during control time horizon the losses caused by not meeting the water demands are minimised and the physical system constraints are satisfied. Due to slow reservoir dynamics a cho-sen time horizon is of one year's length. Hence the dimensionality problem arises here obviously. A three time scale aggregation is used and conse-quently three smaller subproblems are formulated. The long-term problem covers a one-year time hori-zon and is made up of the variables relating to one month, i.e., instead of the daily releases the monthly releases appear in the long-term pro-blem. The target storages resulting from this problem solving are used in the mid-term problem which covers a one-month time horizon and is form-ulated similarly to long-term problem by using variables relating to one decade. Constraining mid-term problems to meet at the end of its time horizon the target storages provided by long-term problems assures consistency between the two pro-blems. The resulting target storages are used in the short-term problem which covers a one-decade time horizon and is made up of variables relating to one day. The short-time problem solving pro-vides guidlelines for daily operation of the reservoirs.

HIERARCHICAL OPTIMIZATION

Hierarchical optimization methods were originally developed to solve single objective static optimi-zation problems of a truly large dimension (see e.g., Lasdon, 1970), by problem decomposition on a number of subproblems of smaller dimensions and their coordination. Thus, a computational struc-ture corresponding to hierarchical optimization method is organized in a form of multilevel hier-archy. These methods are used for solving such problems arising in complex water systems as model building, identification, system design, system development, system analysis (see e.g. Haimes (1977), Shima, Tarvainen and Haimes (1981) Oi, Kawai and Muchi (1981), Coulbeck (1983)). They are also useful in solving some static control tasks met in on-line control of static parts of water distribution networks (see e.g. Findeisen and colleagues (1983), Klempous and colleagues (1983)). Since the water systems as a whole must be usually considered as dynamical systems then single objec-tives but dynamic hierachical optimization (see e.g., Mesarović, Macko and Takahara (1970), Singh and Titli (1978), Findeisen and colleagues (1980)) methods are more often employed in these systems than the static ones. The Tamura's three level method of Goal Coordination type turned out extre-mely efficient in solving river pollution determin-istic open-loop control problem (Tamura 1974, 1975) and in optimization of a water supply network (Fallside and Perry (1975), Żelezik (1983)) as well. Thümler and Reinisch (1981) combined Tamura's approach with a repetitive control and derived hierarchical controller which was applied to a regional drinking water supply system. The dynamic hierarchical optimization methods were also used in solving deterministic closed-loop control pro-blems, i.e., in finding deterministic optimal control laws (see e.g., Singh and Titli (1978). Application of an Interaction Prediction Approach to river pollution deterministic feedback control was reported by Singh and Hassan (1976).

Water systems are usually constructed to serve multiobjectives. Hence, decision problems are of multiobjective type. It is not always possible to replace those multiobjective problems by single

objective ones by emplying, e.g. a parameteriza-
tion by means of the weighingmatrices procedure
(see e.g. Haimes, Hall and Freedman, 1975). There
exists hierarchical multiobjective optimization
methods for solving such problems (see e.g. Haimes
(1977), Tarvainen and Haimes (1980), Haimes and
Tarvainen (1981), Simizu and Aiyoshi). New gener-
al algorithms have been recently presented by
Kiwiel (1983) and Tarvainen (1980). There is a
certain aspect of the hierarchical multiobjective
methods which make possible to extend their appli-
cation in decision making. Namely, a coordination
mechanisms of such methods can be used in so call-
ed programming phase of decision making. Hence,
the hierarchical algorithms for solving multi-
objective optimization problems can be utilized
in a synthesis of a programming decision making
structure. These and related problems amongst
which hierarchical games and hierarchical inter-
acting decision making schemes are of a great
importance (see for an excellent survey at
Findeisen, 1982) seem to be a main future area of
both research and application activities.

REPETITIVE CONTROL

The concept of repetitive control was developed
mainly to cope with the uncertainty factor in an
optimizing control. A repetitive control struct-
ure is multilayer hierarchical. Without loss of
generality we shall limit our considerations to
the two layer structure. A task of the upper
layer is to provide the lower layer with state
targets. The controls are elaborated by the lower
layer through solving optimal control problems
with state initial conditions taken from a real
system and with shorter time horizons than the
overall optimizing control time horizon. This
time horizon abridgement enables us to utilize
more efficiently current information about
uncertainty behaviour in the future. The state
targets produced by the upper layer are improved
from time to time depending on how much they dif-
fer from the real system state values. This is
done by solving again the upper layer problem
which state initial conditions are taken from the
real system. The upper layer problem and the low-
er level problem can be formulated in different
ways depending on how an uncertainty factor is
taking into account. The state constraints, if they
exist, play a fundamental role (see e.g. Brdyś
and colleagues (1981, Terlikowski and Malinowski
(1981)). When a system is of large scale then
the layer problems can be decomposed. The result-
ing structure is of multilayer-multilevel type.

A repetitive control structure with price decompo-
sition was first presented by Findeisen and
Malinowski (1976). The repetitive concept is
commonly used in operational control of multi-
reservoir systems (Findeisen, 1978). In Findeisen
and colleagues (1983) the repetitive and the time
aggregation concepts were combined to produce a
water retention management and control scheme
for Kamienna river basin. A further development
of the original repetitive structure with price
coordination was resulted in hierarchical
dispatching control structure for a multireservoir
system (Findeisen and Malinowski, 1981). The cont-
rol mechanism was based upon repetitive hierar-
chical optimization and upon more frequent and
simpler corrective actions. The short horizon
corrective control produced prices and consequently
current control corrections based upon suitable
prediction of a difference between future water
volumes stored in system reservoirs and those
obtained by solving long-term optimal planning
problems. The control structure was applied to
multireservoir Upper Vistula Water System
(Salewicz and Terlikowski, 1981).

An uncertainty, i.e., natural water inflow vol-
umes, water withdrawals and water users demands
was treated in the above methods in a determini-
stic way. The uncertain variables in the optimi-
zation problems were replaced by their forecasts
and a set membership uncertainty description was
used to guarantee the state constraints fulfilment.

An attempt to employ two stage stochastic program-
ming techniques to solve upper layer problems in
basic repetitive structure under prababilistic
model of uncertainty was presented by Ruszczyński
(1982), together with an example of application.

The repetitive concept seems to be quite powerful
in solving a class of problems under uncertainty
which arise in water systems. Existing structures
based upon this concept are, however, of open
loop with feedback type. Clearly, the resulting
solution quality can be improved by closing a loop
i.e., by taking into account a lower layer when an
upper layer problem is solved. At the present
stage, however, it leads to a highly complicated
numerical task. In spite of all difficulties,
at least a partial loop closing should constitute
a research task for the hear future. The results
obtained by Gessing, (1980), (1983) seem to con-
firm this statement.

CASE STUDY EXAMPLE

A case study presented by Findeisen and colleagues
(1983) constitutes a good illustration of the above
concepts. We shall recapitulate that case study.
The river basin under study extends over the catch-
ment area of a main river. There are three stor-
age reservoirs in the basin and three urban and
industrial areas affecting the system through
their water demands and wast water discharges.
Water is yielded both from surface and ground
sources, and is supplied to the users by municipal
water distribution networks. There exist water
authorities in the region. The detailed system
structure is illustrated in Fig. 1. The manage-
ment and control task is to accomplish the follow-
ing goals:

(I) supplying water for industrial, domestic and
municipal users in urban areas A,B and C to meet
their water demands;
(II) maintaining required water quality classes in
the river;
(III) satisfying qualitative and quantative requi-
rements on the water at the system output (node j
in Fig. 1.);
(IV) maintaining river flows on stretches c-d,
e-h and g-h within prescribed range;
(V) satisfying flood protection requirements, i.e.
keeping the instant and decade river flows on c-d,
e-h and g-h below prescribed values and not
exceeding the maximum resrvoir levels;
(VI) management of underground water resources by
satisfying prescribed instant and integral con-
straints on underground water charges;
(VII) minimizing the water yield and distribution
cost.
The goals of (IV), (V) and (VI) are assumed to be
of the highest priority. Goal (I) is considered
as more important than (II) and (III). Finally,
meeting the water demands is prior to minimiza-
tion of water yield and distribution costs.

An analysis of the water system goals leads to
splitting the control task into the following
interdependent control tasks: water retention
control task consists of determining on-line the
daily reservoir releases M5, M7 and M8 (see Fig.
2). The daily reservoir b-c releases are deter-
mined by control until RR1 while the daily releas-
es of the reservoirs d-e and f-g are determined
by control unit RR2 (see Fig. 2.). The action of
the units RR1 and RR2 as coordinated by the

retention authority (RA). The RA is given the aggregated forecasts of demands and discharges from water authority WA.

Water distribution control scheme consists of two control units: WDCAB unit whose aim is to control the distribution networks of the A and B areas and WDCC which is responsible for the distribution network control of the area C. Operating conditions of pumping stations (numbers of pumps switched on) and additionally in deficit situation, water supply limits for industrial users are WDC variables. The numbers of pumps switched on are kept constant within each one hour whilst the water supply limits can be adjusted once a day. The WDC unit is connected with other elements of the overall control structure. The connections are two-way. The WDC units gather information concerning water demands and send them to higher levels of the control structure, to obtain mid-term and long-term water demand forecasts necessary when deriving retention control. On the other hand, the higher level control units state conditions for WDC units to harmonize their operation with other elements. These conditions constitute operation goals and operation constraints. The operation goals are to cover water demands taking also into account water yield and distribution costs. The maximal water take-off from the retention reservoirs, from the river and from underground sources (instantaneous or integral constraints, respectively) are the time-varying operation constrains. Based on available information such as water demand forecasts, historical data, actual level in b-c reservoir (actual flow in the river) the WDCAB (WDCC) unit determines reservoir b-c level forecasts (river flow forecast) at the beginning of each day. The 7 days forecast for constraint activity on water take-off from the reservoir (from the river) is obtained. On the basis of this forecaste actual control situation is specified: normal or deficit situation. The control problem corresponding tc the normal situation has 24 hours time horizon while in the deficit situation seven days horizon is needed.

The river and its tributary are divided into a number of stretches (see Fig. 1.) and in each stretch BOD and DO are chosen as pollution indicators. Next a notion of a quality class is introduced and the water quality in the stretch is expressed in terms of the quality classes. The task of water quality control (WQC) is to keep water in the stretches in prescribed classes.

The stretch i-j placed below area C is highly polluted, so our requirements are restricted here to minimization so-called asymptotic concentration of oxygen deficit in this stretch. The WQC system consists of WQCC unit in area C, WQCB unit in area B and WQCA unit in area A (see Fig. 2). The main decision variables of the system are the volume, duration and initial time of effluent discharges from waste water treatment plants and rate of water releases from reservoirs (daily releases are already prescribed by the reservoir despatchers).

The structure of the overall management and control system is depicted in Fig. 2. The system is hierarchical and decomposition was done basing on differences in functions (functional decomposition) and frequency of actions (time decomposition). Water distribution control units WDCAB and WDCC, water quality control units WQAC, WQCB and WQCC, reservoir retention control units RR1 and RR2 compose lower level of the system. Element RA coordinates determination of local operating rules by RR1 and RR2 for the reservoir despatchers through solving appropriate long-term and mid-term problems. Unit WA the sole element of the highest level, represents the river water authority. In order to reduce an uncertainty

influence on attained performance the units work repeatedly.

CONCLUSIONS

Some features of water systems were listed and discussed from decision making and control standpoint. Guided by those features chosen hierarchical concepts, namely, aggregation, hierarchical optimization and repetitive contol, were surveyed and their application to water systems were presented. The future trends were also indicated.

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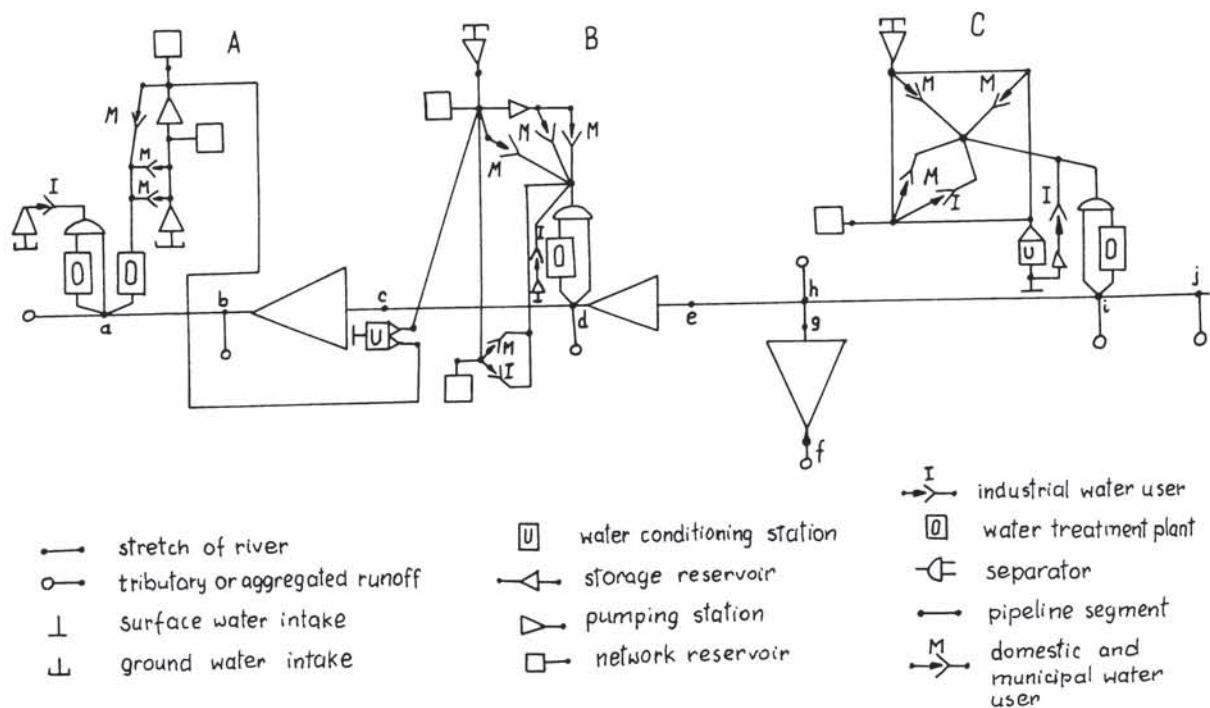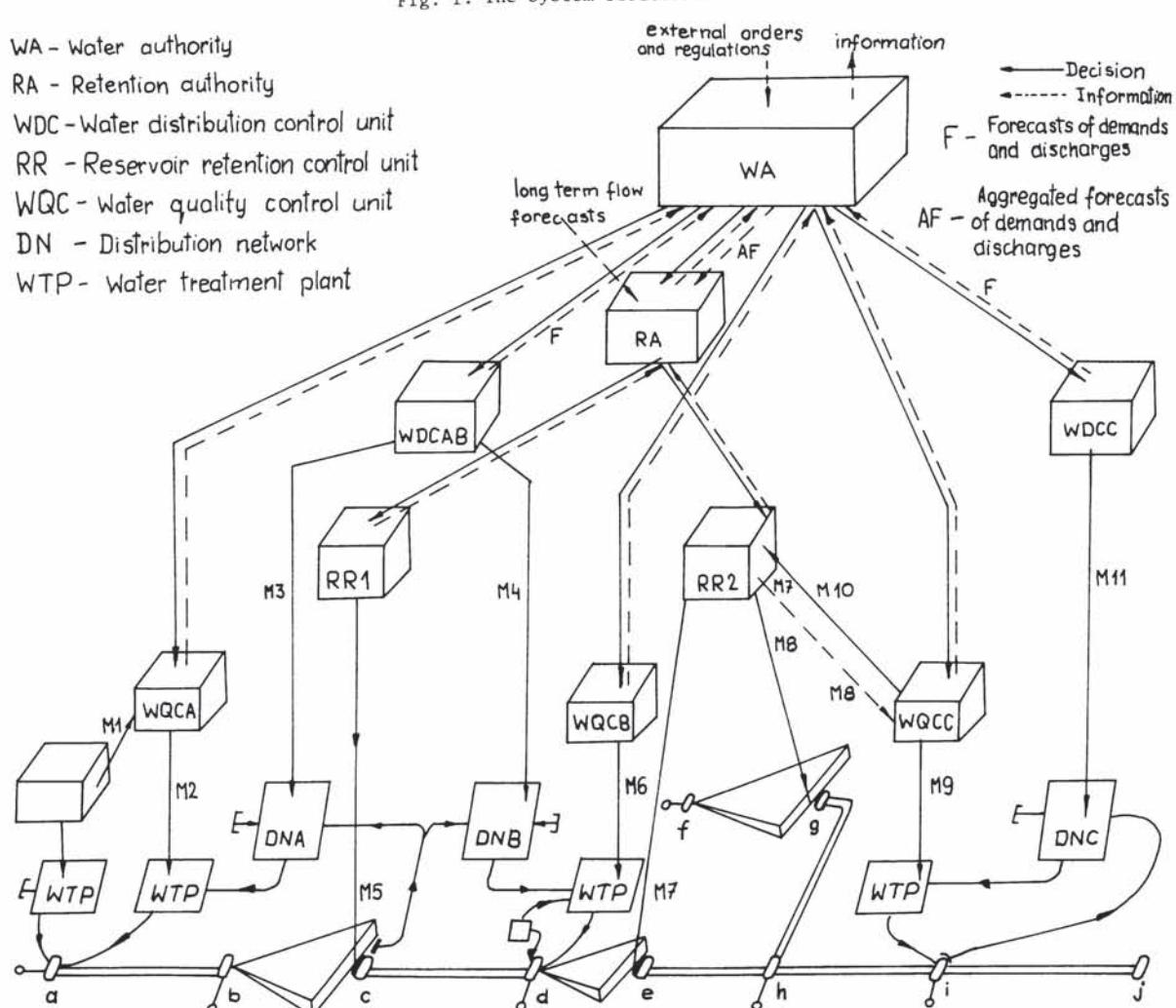 of Cliwice Press, Gliwice, Poland,
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Fig. 1. The System structure

Fig. 2. Control and management structure.