'Pricing under Rough Volatilty Models' Lab Report

Group 2

Artemy Sazonov   Andrei Petrov   Maxim Savostyanov

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Abstract

In the present paper we investigate the roughness of the Russian stock market in the context of the rough stochastic volatility model. We obtain the estimation of the Hurst parameter for the major Russian corporations. In the end we form the conclusion that the rough volatility model is a working model for the Russian stock market and formulate the future research horizon.

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Introduction

The first revolutionary model of Mathematical Finance was introduced by F. Black and M. Sholes in 1973's article [**BlackSholes1973**], and a similar model introduced in 1976 by F. Black in [**Black1976**] (the only fundamental difference was the use of the forward prices instead of spot prices, which is proven to be useful for some markets). Later there were created some local volatility models (like Dupire's LVM), and first stochastic volatility models (Heston SVM), but they still were not a perfect fit for pricing, even when first LSVMs were introduced.

Fractional Brownian motions were first employed in volatility modelling by F. Comte and E. Renault [**ComteRenault1998**]. Their model (called FSV) used a fractional Brownian motion with Hurst parameter $H > 0.5$ to model volatility as a long-memory process i.e. one where autocorrelation decays slowly, which was a widely accepted stylized fact. They thus introduced the class of fractional stochastic volatility models.

In 2014, J. Gatheral, T. Jaisson, and M. Rosenbaum showed in [**GatheralRosenbaum2014**] that for major American indices Hurst parameter estimations have the order of $1e-1$ (i.e. $H < 0.5$, therefore, there is no long-memory in the FSV-based models), and called the corresponding model (FSV + $H < 0.5$) a rough fractional stochastic volatility model (RFSV) to emphasise that the volatility is indeed rough.

However, their approach requires the use of a model, therefore, it is not perfect still. In 2022, R. Cont and P. Das [**Cont2022**] proposed a method of estimating the roughness of an asset without the need of a model, which can be used to find statistical evidence that volatility is rough even without RFSV.

In the present paper I show that the Hurst parameters of the major Russia-originated assets (stocks and depositary reciepts of Russian corporations) are less than 0.5 under RFSV, i.e. Comte and Renault's basic FSV model is not working well for the Russian stock markets, therefore, RFSV should be used instead.

Chapter 1

Rough Fractional Stochastic Volatilty Model Estimation

1.1 Realized volatility

Consider a stochastic volatility model

$$dS_t = \mu_t S_t dt + \sigma_t S_t dW_t, \tag{1.1.1}$$

where S_t is an asset price process, and σ_t is a stochastic volatility process representing a so-called *spot volatility.* Spot volatility, in fact, is not observable in the market, therefore, we should estimate it somehow.

Definition 1.1.1. The *realized variance* of a price process S over time interval $[t, t+\delta]$ sampled along the time partition π^n is defined as

$$RVar_{t,t+\delta}(\pi^n) = \sum_{\pi^n \cap [t,t+\delta]} \left(\log S_{t^n_{i+1}} - \log S_{t^n_i} \right)^2 \tag{1.1.2}$$

and *realized volatility* is defined as

$$RV_{t,t+\delta}(\pi^n) = \sqrt{\sum_{\pi^n \cap [t,t+\delta]} \left(\log S_{t^n_{i+1}} - \log S_{t^n_i} \right)^2} \tag{1.1.3}$$

Definition 1.1.2. Let S satisfy (**??**). Then the integrated variance is defined as

$$IVar_t = \int_0^t \sigma_s^2 ds \tag{1.1.4}$$

It has been shown many times (e.g. [**Barndorff-Nielsen2002**]) and mentioned in [**Cont2022**] that the realized variance converges in probability to the integrated variance as sampling frequency increases for all assets satistying the equation (**??**) (i.e. stochastic volatility models).

Proposition 1.1.1. *As time partition scale of π^n tends to 0, $RV_{t,t+\delta}(\pi^n) \approx \sqrt{\delta}\sigma_t$, i.e. $RV_{t,t+\delta}/\sqrt{\delta}$ could be considered as a consistent estimator of the spot volatility.*

Definition 1.1.3. The *fractional Brownian motion* $(W_t^H)_{t\in\mathbb{R}_+}$ with Hurst parameter $H \in (0,1)$ is a Gaussian process with the following properties:

1. $W_0^H = 0$,
2. $\mathbb{E}\left[W_t^H\right] \equiv 0$,
3. $\mathbb{E}\left[W_s^H W_t^H\right] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right)$

Definition 1.1.4. A stationary fOU process X_t is defined as the stationary solution of the stochastic differential equation

$$dX_t = \nu dW_t^H - \alpha(X_t - m)dt, \tag{1.1.5}$$

where $m \in \mathbb{R}$ and ν and α are positive parameters, see [**Cheridito2003**].

1.1.1 What is a long-memory process?

Definition 1.1.5. A process X_t is said to have a long memory, if

$$\sum_{k=0}^{\infty} Cov\left[X_1, X_k - X_{k-1}\right] = +\infty. \tag{1.1.6}$$

In particular, the fractional Brownian motion with $H > \frac{1}{2}$ is a long-memory process. Long-memory of the stochastic volatility process in stochastic volatilty models framework used to be a widely-accepted stylized fact [**Breidt1998**; **ComteRenault1998**; **Comte1996**; **Ding1993**].

1.2 Model description

In [**GatheralRosenbaum2014**] the authors considered the following model, as did I. Let there be a riskless asset $B_t \equiv 1$, and a risky asset, whose price S_t is defined by the following equations:

$$dS_t = \alpha S_t dt + \sigma_t S_t dW_t, \tag{1.2.1}$$

$$d\log\sigma_t = \alpha(m - \log\sigma_t)dt + \nu dW_t^H, \tag{1.2.2}$$

The risky asset is being traded in the market in numeraire prices (in our case, $B_t = 1$ RUB for stocks and GBP for depositary reciepts).

Definition 1.2.1. A model (**??**) – (**??**) is called a *Fractional Stochastic Volatility Model*[1] (FSV). For a special case $H < 0.5$ the model is called a *Rough Fractional Stochastic Volatility Model* (RFSV) to emphasise a so-called roughness of the trajectories of the fBm.

In [**Cheridito2003**] an exact formula for the autocovariance function of the log-volatility in the RFSV model was derived:

$$\begin{aligned} &\operatorname{cov}\left[\log\sigma_t, \log\sigma_{t+\Delta}\right] = \\ &= \frac{H(2H-1)\nu^2}{2\alpha^{2H}}\left(e^{-\alpha\Delta}\Gamma(2H-1) + e^{-\alpha\Delta}\int_0^{\alpha\Delta}\frac{e^u}{u^{2-2H}}du + e^{\alpha\Delta}\int_{\alpha\Delta}^{+\infty}\frac{e^u}{u^{2-2H}}du\right) \end{aligned} \tag{1.2.3}$$

Via the explicit formula for the covariance function of the log-volatility in the RFSV model (**??**), we can write a closed-form expression for a theoretical $m(q, \Delta)$:

$$m(q, \Delta) = 2\left(\operatorname{var}\log\sigma_t - \operatorname{cov}\left[\log\sigma_t, \log\sigma_{t+\Delta}\right]\right), {}^{2} \tag{1.2.4}$$

1.3 Normality Statistical Tests

In the following, x_i denotes a sample of n observations, g_1 and g_2 are the sample skewness and kurtosis, μ_j's are the j-th sample central moments, and $\overline{x}$ is the sample mean.

1.3.1 D'Agostino's K-squared test

The sample skewness and kurtosis are defined as

$$g_1 = \frac{m_3}{m_2^{3/2}} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2\right)^{3/2}}, \tag{1.3.1}$$

$$g_2 = \frac{m_4}{m_2^2} - 3 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2\right)^2} - 3. \tag{1.3.2}$$

[1] Despite the original [**ComteRenault1998**] naming (Comte and Renault used a model with long memory of volatility as a stylized fact), we shall call our model FSV for any $H \in (0, 1)$

[2] $m(q, \Delta) = \mathbb{E}\left[(\log\sigma_{t+\Delta} - \log\sigma_t)^2\right]$

Let

$$Z_1(g_1) = \delta \operatorname{asinh}\left(\frac{g_1}{\alpha\sqrt{\mu_2}}\right), \tag{1.3.3}$$

where constants α and δ are computed as

$$W^2 = \sqrt{2\gamma_2 + 4} - 1, \tag{1.3.4}$$
$$\delta = 1/\sqrt{\ln W}, \tag{1.3.5}$$
$$\alpha^2 = 2/(W^2 - 1), \tag{1.3.6}$$

and

$$Z_2(g_2) = \sqrt{\frac{9A}{2}}\left\{1 - \frac{2}{9A} - \left(\frac{1 - 2/A}{1 + \frac{g_2 - \mu_1}{\sqrt{\mu_2}}\sqrt{2/(A-4)}}\right)^{1/3}\right\}, \tag{1.3.7}$$

where

$$A = 6 + \frac{8}{\gamma_1}\left(\frac{2}{\gamma_1} + \sqrt{1 + 4/\gamma_1^2}\right), \tag{1.3.8}$$
$$\mu_1(g_2) = -\frac{6}{n+1}, \tag{1.3.9}$$
$$\mu_2(g_2) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}, \tag{1.3.10}$$
$$\gamma_1(g_2) \equiv \frac{\mu_3(g_2)}{\mu_2(g_2)^{3/2}} = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)}\sqrt{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}}, \tag{1.3.11}$$
$$\gamma_2(g_2) \equiv \frac{\mu_4(g_2)}{\mu_2(g_2)^2} - 3 = \frac{36(15n^6 - 36n^5 - 628n^4 + 982n^3 + 5777n^2 - 6402n + 900)}{n(n-3)(n-2)(n+7)(n+9)(n+11)(n+13)}. \tag{1.3.12}$$

The analytical expressions for skewness and kurtosis (**??**) - (**??**) were derived by E. Pearson in [**Pearson1931**].

Definition 1.3.1. The *D'Agostino-Pearson* statistic is defined as

$$K^2 = Z_1(g_1)^2 + Z_2(g_2)^2 \tag{1.3.13}$$

Null hypothesis: the sample is normally distributed.

NB. The K^2 statistic is able to detect deviations from both skewness and kurtosis. If the null hypothesis is true, then the test statistic has the χ^2 distribution with 2 degrees of freedom.

1.3.2 Shapiro-Wilk test

Definition 1.3.2. The Shapiro–Wilk test statistic is defined as

$$W = \frac{\left(\sum_{i=1}^n a_i x_{(i)}\right)^2}{\sum_{i=1}^n (x_i - \overline{x})^2}, \tag{1.3.14}$$

where

$$(a_1, \dots, a_n) = \frac{m^T V^{-1}}{C}, \quad C = \|V^{-1}m\| = (m^T V^{-1} V^{-1} m)^{1/2},$$

and $m = (m_1, \dots, m_n)^T$ is a mean of order statistic from a normally distributed sample, V is the covariance matrix of those normal order statistics. Null hypothesis: the sample is normally distributed.

NB. The W statistic has no distinguishable name, and the cutoff values are calculated numerically by Monte-Carlo simulation.

1.4 Statistical Analysis

1.4.1 Data Preprocessing and Realized Volatility Estimation

In the present paper we used high-frequency data for two types of assets:

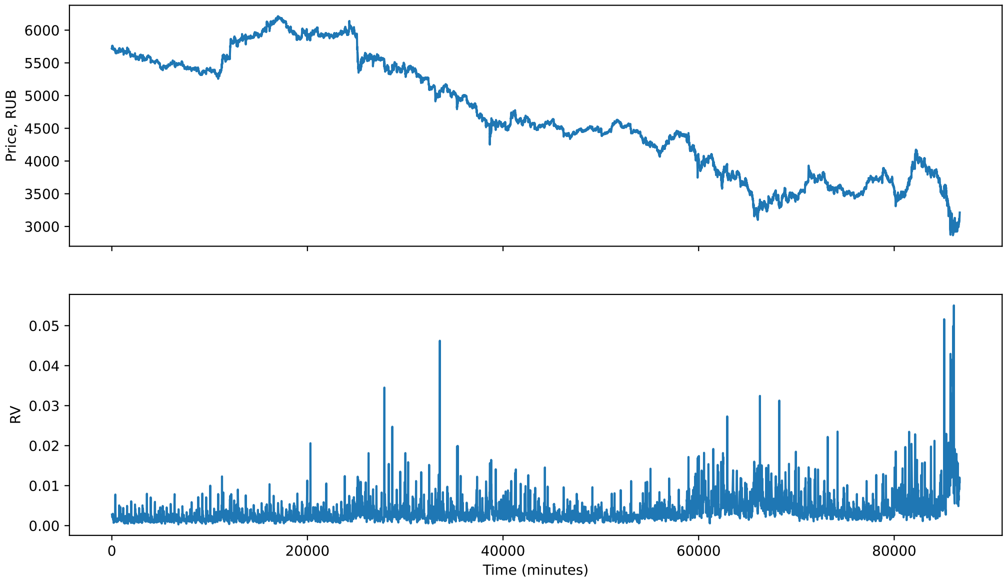

Figure 1.1: YNDX RX Equity. Price and Realized Volatility

1. **Stocks**: Yandex, Sberbank, Gazprom, VTB, Moscow Exchange, Lukoil, and X5 Group
2. **Depositary reciepts**: Sberbank, Gazprom, VTB, and Lukoil

Some text to be written

Using this approach to estimate the realized volatility we can be sure that our data is correlated in the least way possible.

1.4.2 Hurst Parameter Estimation

Let $m(q, \Delta, \pi^n)$ be a sample q-th absolute moment of $\log RV_{t+\Delta} - \log RV_t$ (under stationarity of increments assumptions):

$$m(q, \Delta, \pi^n) := \frac{1}{n} \sum_t \left| \log RV_{t+\Delta} - \log RV_t \right|^q, \tag{1.4.1}$$

i.e. $m(q, \Delta, \pi^n)$ is an empirical counterpart of $\mathbb{E}\left[|\log RV_\Delta - \log RV_0|^q \right]$. In my work we used the uniform partition with Δ-minutes increments, so we omit the π^n notation and use $m(q, \Delta)$.

Due to the similarities in the obtained results for all assets, we shall deeply analyze the Hurst parameter estimation only for the Yandex stocks (YNDX RX Equity). Plots for other equities could be found in the appendix, whereas the Hurst parameter estimations for them could be found in the table **??**. Estimation parameters:

- Realized volatility is estimated by 15 minute disjoint windows
- $\Delta \in [0, 40]$ for ζ_q estimation and other plots

NB. We did not manage to obtain more HF data (only 5 months of 1m-tick data), therefore my estimations are not precise and could not be used for further application.

In the figure **??** we can see that for $q = 0.6, 0.8, \text{and} 1.0$ the dots are very discrepant for $\log \Delta > 2.0$. However, we get a pretty decent linear fit for $q = 0.2$ and $q = 0.4$, therefore, the estimation on these two point would be the best one we can manage to extract. On the other hand, on ζ_q plot we observe a perfect linear fit for all q-s, therefore, H is its slope indeed.

We note that the graphs for ζ_q are slightly concave, which correlates with [**GatheralRosenbaum2014**] results. They conclude that this effect takes place due to the finite statistical population size.

Asset Type	Ticker	$\hat{H}$
Stock	YNDX	0.0521766
Stock	SBER	0.1551646
Stock	VTBR	0.0917236
Stock	MOEX	0.0853878
Stock	LKOH	0.0730521
Stock	GAZP	0.1309705
Stock	FIVE	0.0630289
Depositary reciept	OGZD	0.0523981
Depositary reciept	VTBR	0.0370185
Depositary reciept	SBER	0.0578053
Depositary reciept	LKOD	0.0352792

Table 1.1: Hurst parameter estimations for major Russian companies stocks and depositary reciepts

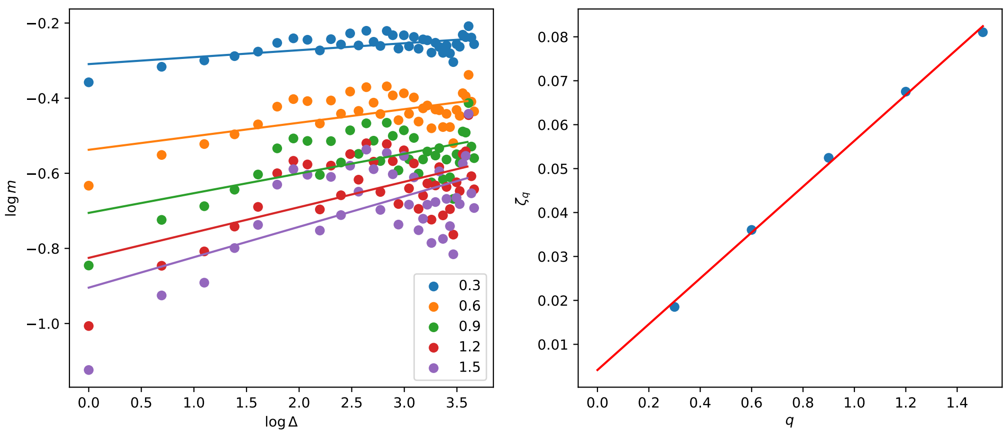

Figure 1.2: YNDX RX Equity. Plots for $\hat{H}$

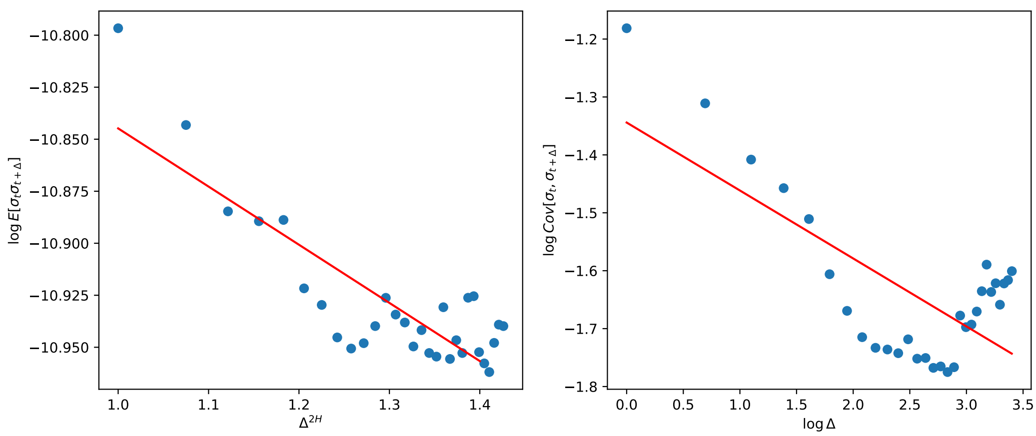

Figure 1.3: YNDX RX Equity. Empirical counterpart of $\log \mathbb{E}\left[\sigma_t \sigma_{t+\Delta}\right]$ as a function of Δ^{2H} (left) and Empirical counterpart of $\log \operatorname{cov}[\sigma_t, \sigma_{t+\Delta}]$ as a function of $\log \Delta$ (right)

1.4.3 Smoothing Effect Estimation

Smoothing effect is throroughly discussed in the appendix of [**GatheralRosenbaum2014**].

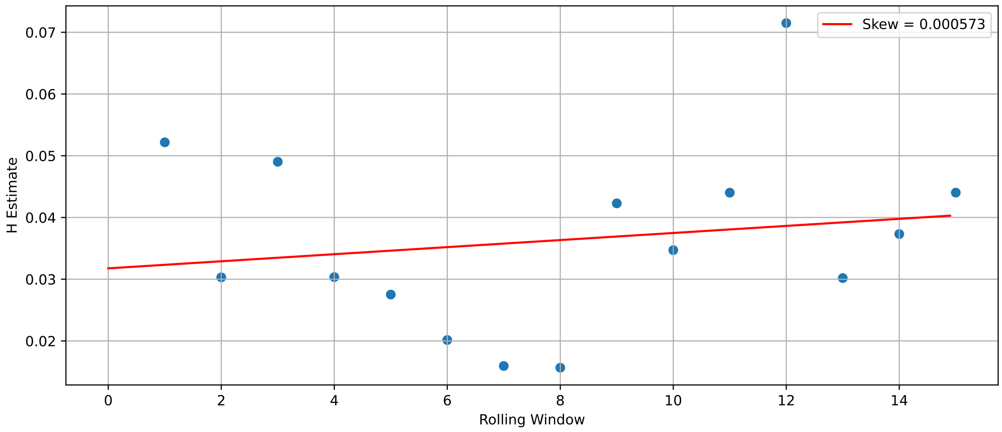

Figure 1.4: YNDX RX Equity. Smoothing Effect

We can clearly see that due to the positive slope of the plot **??**, the hypothesis about increasing $\hat{H}$ and decreasing $\hat{\alpha}$ as δ increases is to be accepted.

1.4.4 Tests for normality of volatility's log-increments

In order to test the normality of the log-increments of the realized volatility, we used the following tests:

1. Visual analysis of histograms: KDE vs normal fit vs empirical fit
2. Visual analysis of excessed kurtosis plot
3. D'Agostino's K Squared normality test
4. Shapiro-Wilk normality test

In [**GatheralRosenbaum2014**] the authors used **only** the visual analysis of the histograms, which, as we can now say, is not surprising due to the inadequacy of results for other numerical experiments.

Visual analysis of histograms and excessed kurtosis plot

1. KDE is the *kernel density estimator* of the data.
2. *Normal fit* $NF(\Delta)$ is the normal distribution fitted to the data with the same mean and variance.
3. *Empirical fit* $EF(\Delta)$ is the scaled normal distribution:
   - $EF(1)$ is said to be same as the $NF(1)$
   - $EF(\Delta)$ for $\Delta > 1$ is said to be a scaled $NF(1)$ by the factor of $\Delta^{\hat{H}}$ (by this we test the monofractal scaling property of normal distribution)

Looking at the figure **??**, we may form a conclusion: KDE and EF are a decent normality approximations for $\Delta = 10, 20$. For others, we don't get a fancy picture: $KDE(1)$ and $KDE(5)$ have a large kurtosis (they are too 'peaky' for them to be normally distributed). Excessed curtosis plot **??** confirms our visual conclusion for KDE and EF plots.

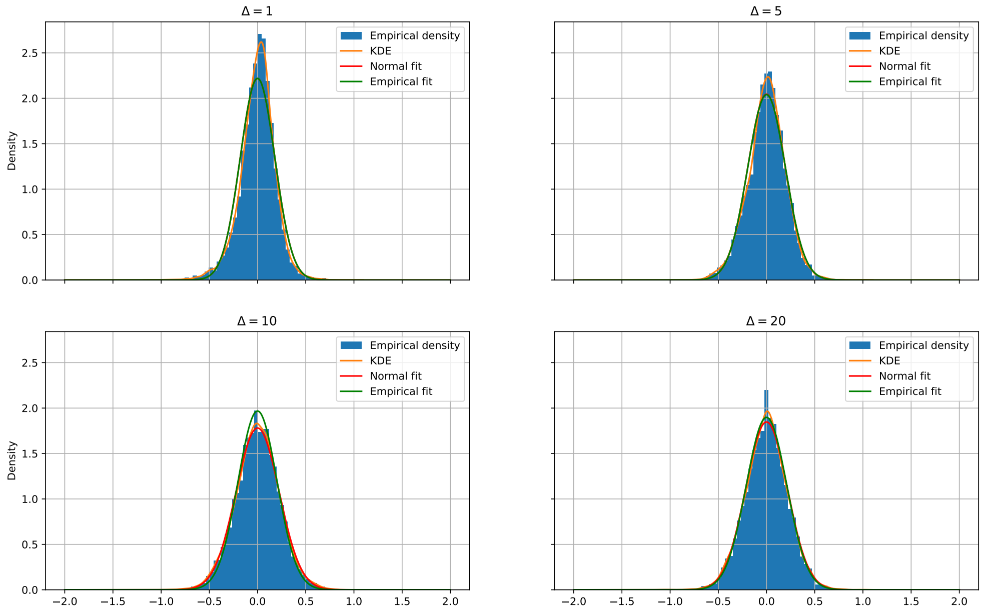

Figure 1.5: YNDX RX Equity. Empirical density of $\log \sigma_{t+\Delta} - \log \sigma_t$ for $\Delta = 1, 5, 10, 20$ days.

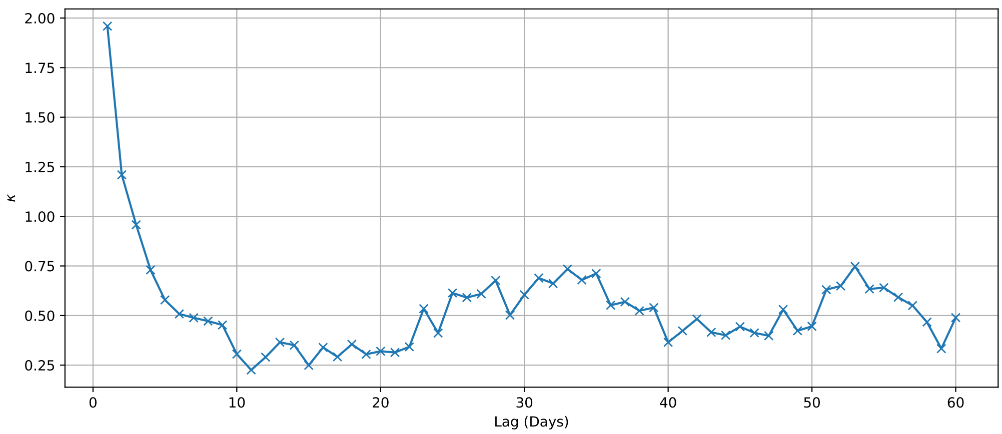

Figure 1.6: YNDX RX Equity. Excessed kurtosis κ as a function of Δ

Statistical tests for normality

We fix the confidence level to be $\alpha = 0.05$.

NB. Both of these tests require the data to be independent, but we cannot guarantee this due to the dependence of fBm's increments. We do our best to analyse the population, but these two tests give us weak proof of normality due to possible correlations.

Looking at the tables with the results of Shapiro-Wilk and D'Agostino's K-Squared tests (tables **??** – **??**), we can see that for the majority of lags and for the majority of the considered assets, both tests showed the result "Not normal", i.e. both tests rejected the null hypothesis.

The three possible explanations are:

1. The tests are correct and the data is not normally distributed or is correlated strongly.
2. The visual analysis of the histograms show that for many lags the KDE plot, the normal fit and the empirical fit are very similar, therefore, the distribution is normal, but the data is correlated strongly. The excessed kurtosis plot shows that the data is distributed very close to the normal distribution for $\Delta > 5$, and at its closest distance for $\Delta \in [10, 22]$.
3. We get a population sampling error (not enough data).

Chapter 2

Zumbach Effect Estimation

2.1 Empirical Effect

2.2 Monte-Carlo Simulation of Zumbach Effect

Conclusion

Reproduced Hypotheses

We got aquainted with the fractional stochastic volatility models framework and studied the statistical properties of RFSV. We obtained roughness estimations for major Russian companies stocks and depositary reciepts, and reproduced some effects described in [**GatheralRosenbaum2014**].

1. The Hurst exponent of the considered assets has the order of $1e-1$ and is less than $\frac{1}{2}$.
2. The volatility of the considered assets **does not** have a property of long memory under fractional stochastic volatiltiy models.
3. Visual analysis and normality tests for the log-increments of volatility shows that for $\Delta \in [10, 25]$ the normality of log-inrements hypothesis holds.
4. The smoothing effect holds for the estimations of H and α (volatility of volatility under fOU). But **only** for VTBR LI Equity we got a negative slope of the smoothing effect. For other asset we got a nearly perfect linear fit and positive smoothing slopes.

Appendix A. Results for Additional Assets

Δ	Shapiro
1	9.16
2	9.44
3	9.52
4	9.49
5	9.53
6	9.65
7	9.62
8	9.59
9	9.57
10	9.61
11	9.57
12	9.61
13	9.56
14	9.59
15	9.61
16	9.64
17	9.63
18	9.63
19	9.69
20	9.67
21	9.66
22	9.69
23	9.67
24	9.71
25	9.68
26	9.70
27	9.73
28	9.74
29	9.71
30	9.72
31	9.73
32	9.73
33	9.77
34	9.73
35	9.75
36	9.73
37	9.72
38	9.71
39	9.72
40	9.73
41	9.73
42	9.68
43	9.71
44	9.71
45	9.68
46	9.73
47	9.66
48	9.68
49	9.65

Δ	Shapiro
1	9.13
2	9.37
3	9.52
4	9.50
5	9.54
6	9.59
7	9.59
8	9.57
9	9.56

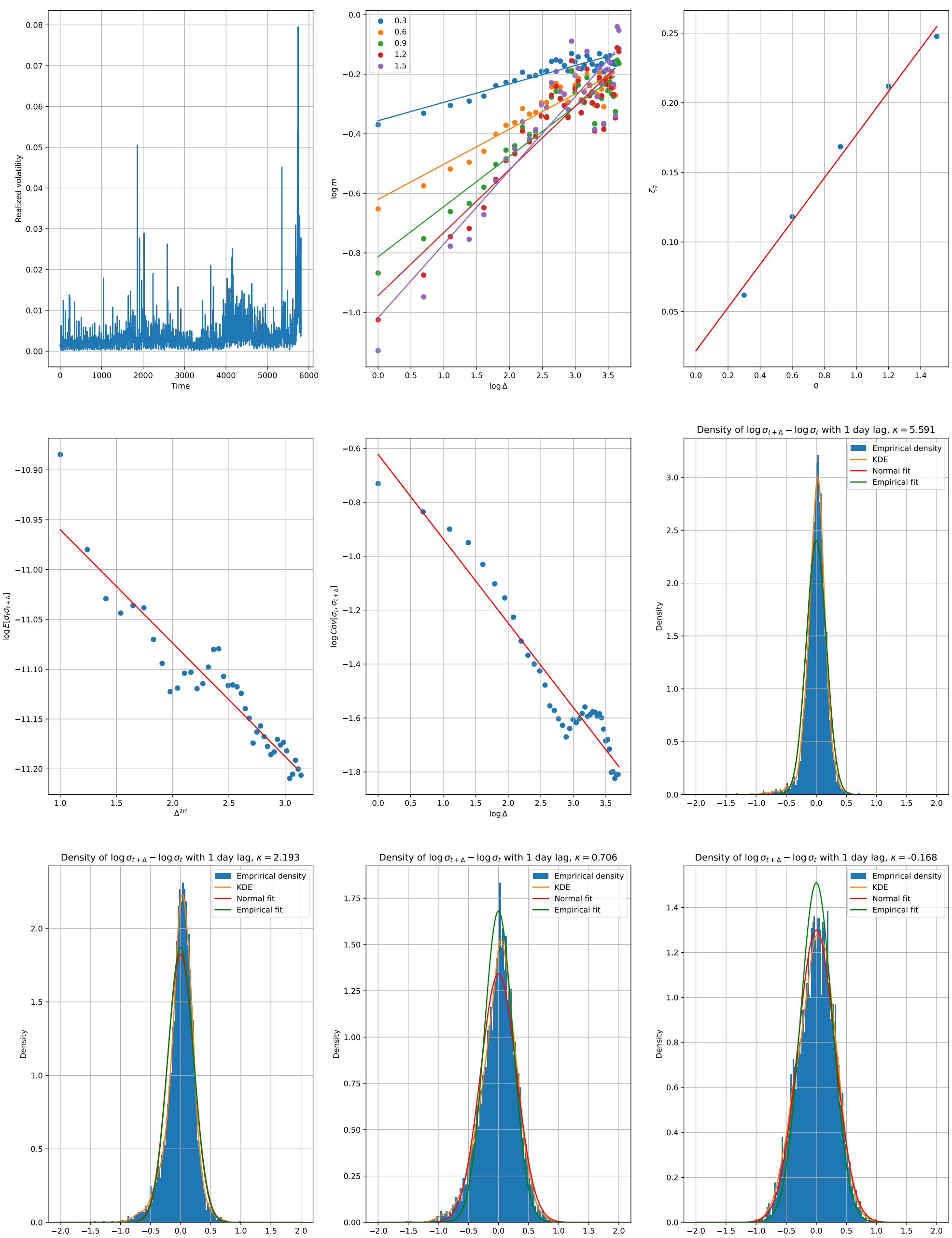

Figure 2.1: SBER RX Equity. $\hat{H}$ plots

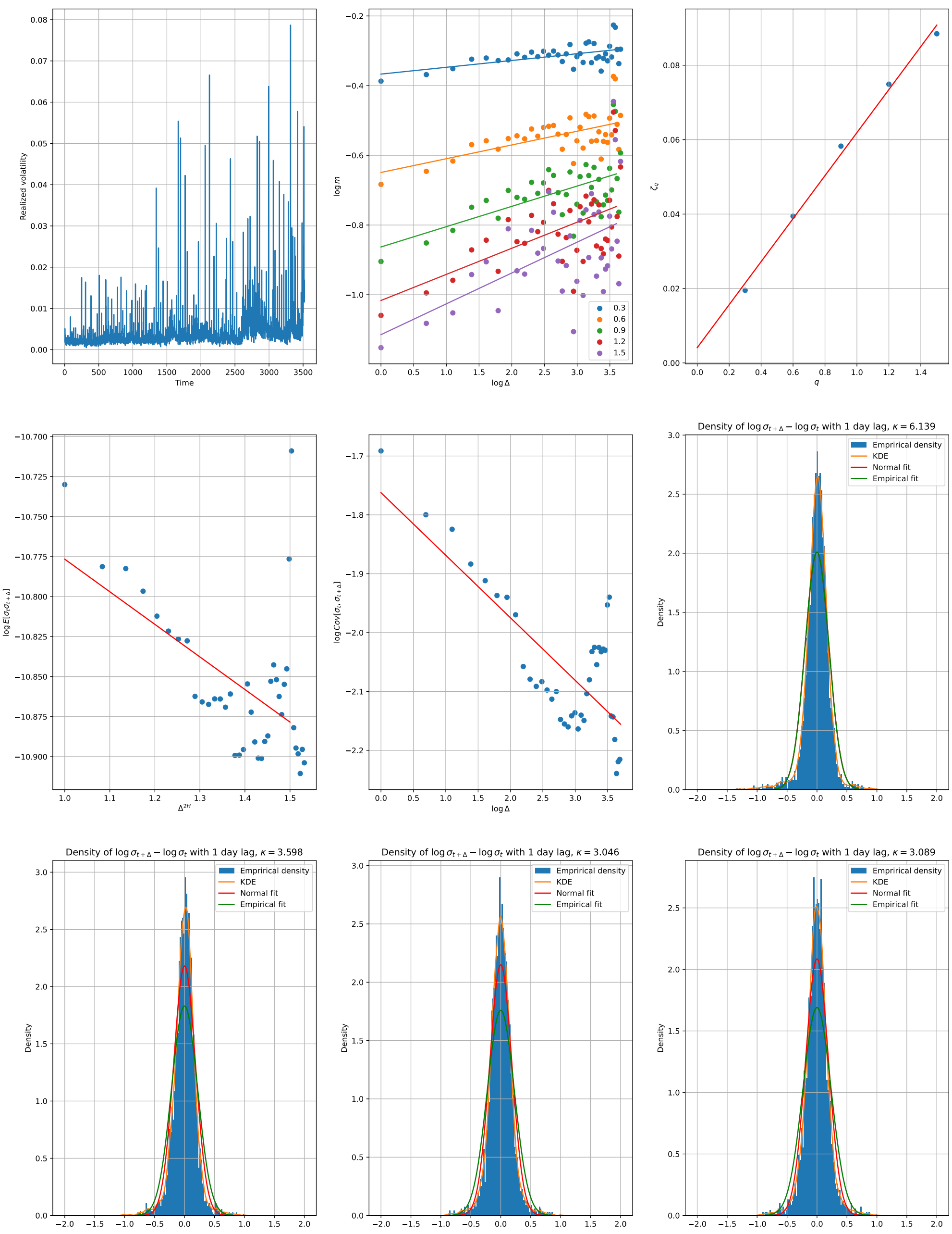

Figure 2.2: SBER LI Equity. $\hat{H}$ plots

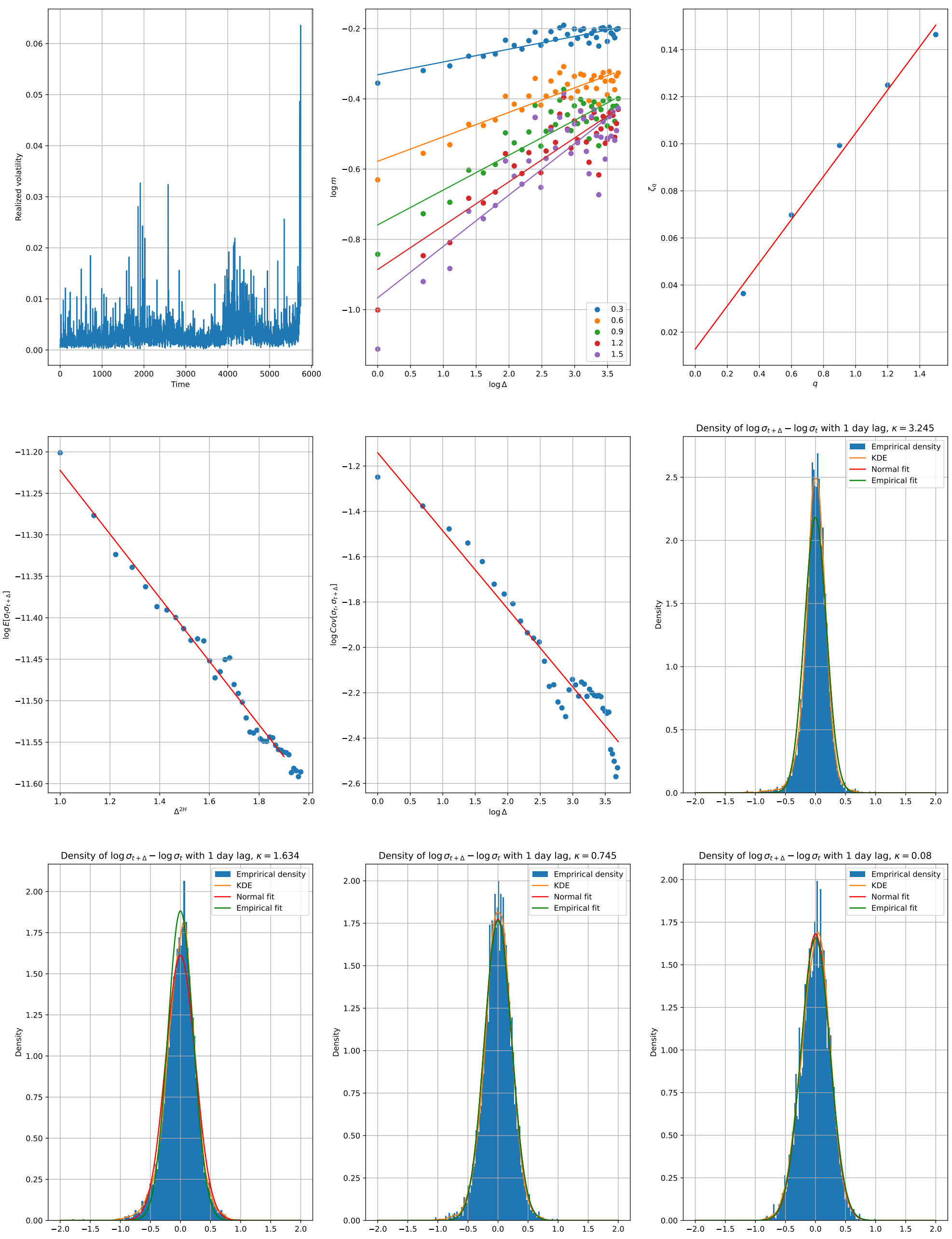

Figure 2.3: VTBR RX Equity. $\hat{H}$ plots

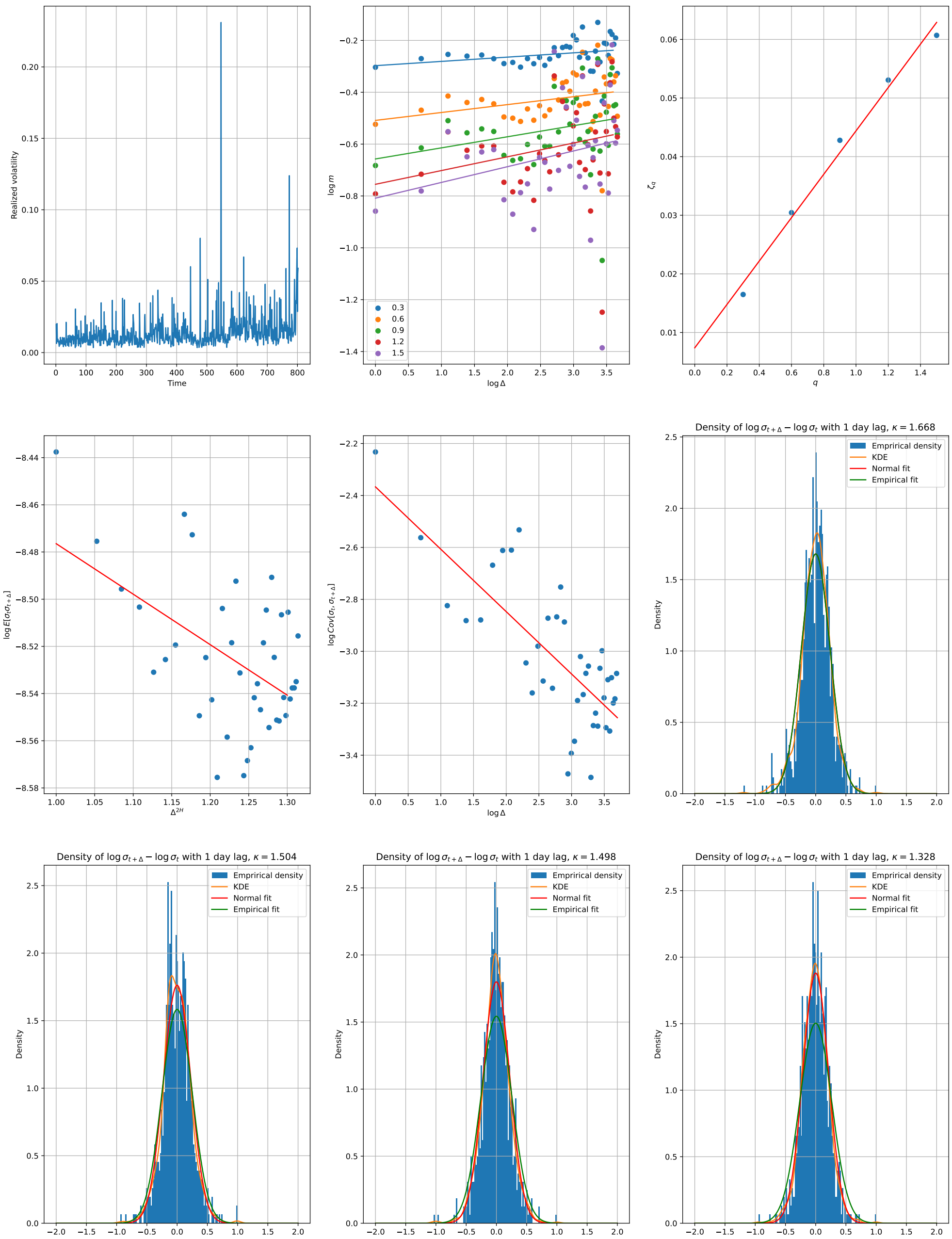

Figure 2.4: VTBR LI Equity. $\hat{H}$ plots

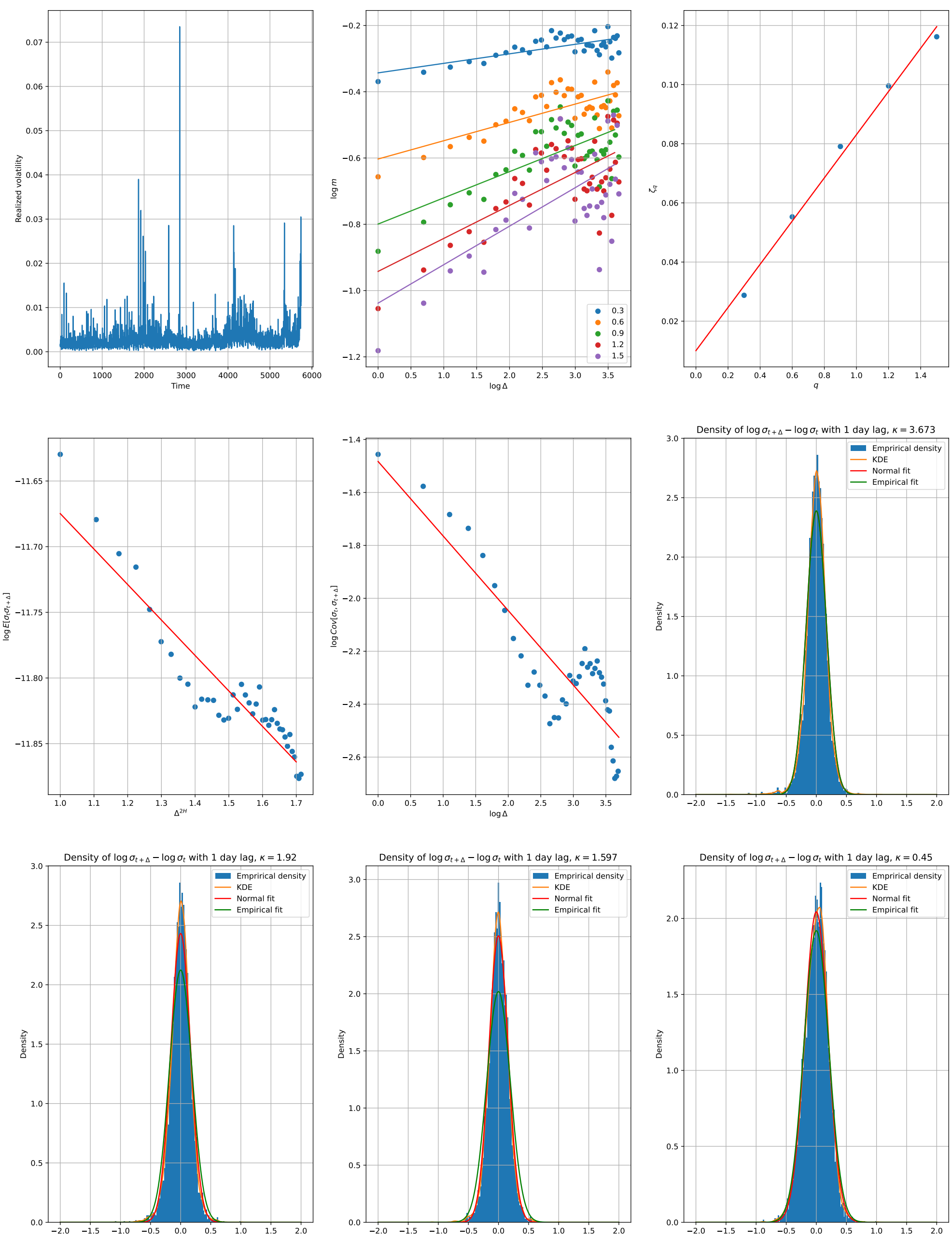

Figure 2.5: LKOH RX Equity. $\hat{H}$ plots

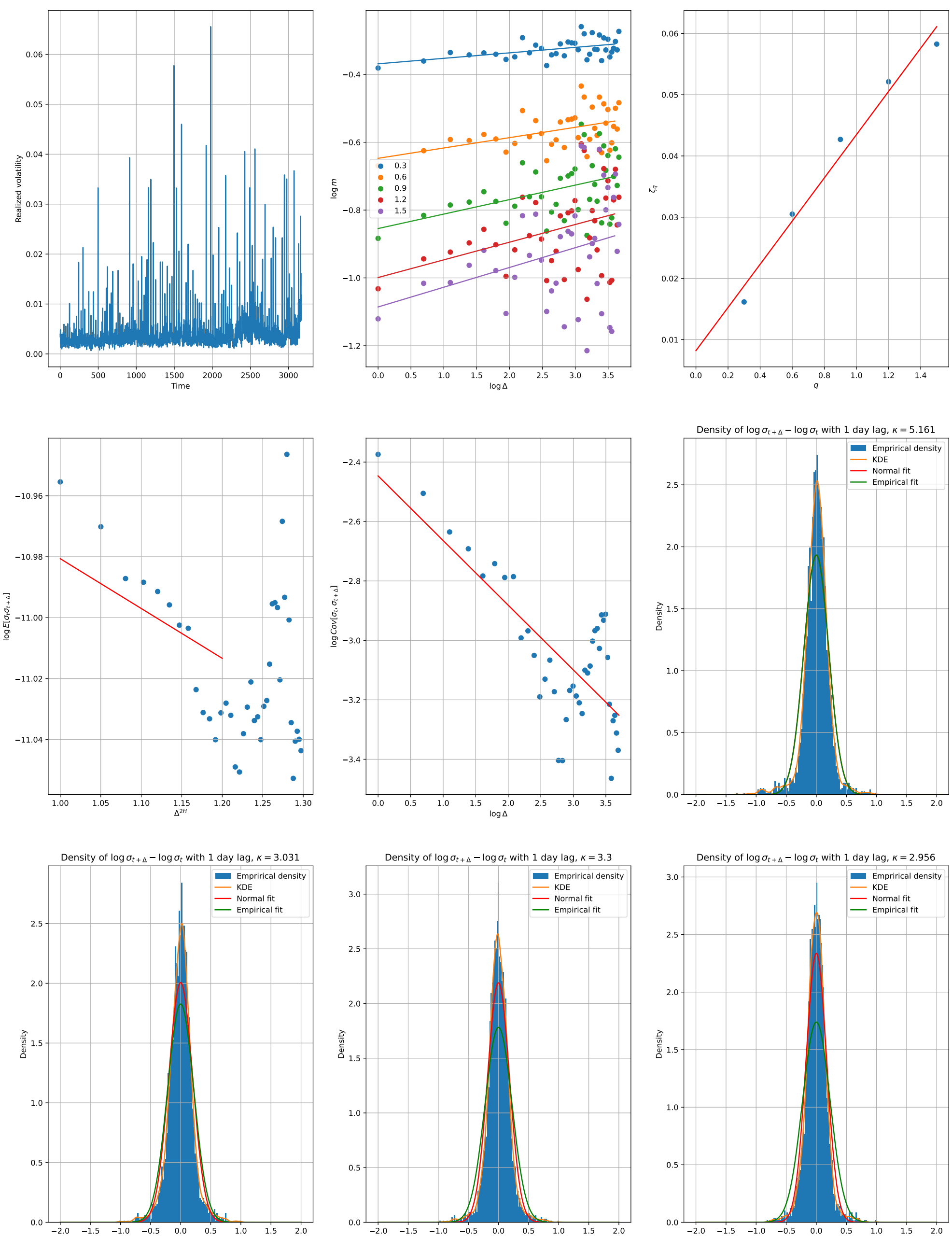

Figure 2.6: LKOD LI Equity. $\hat{H}$ plots

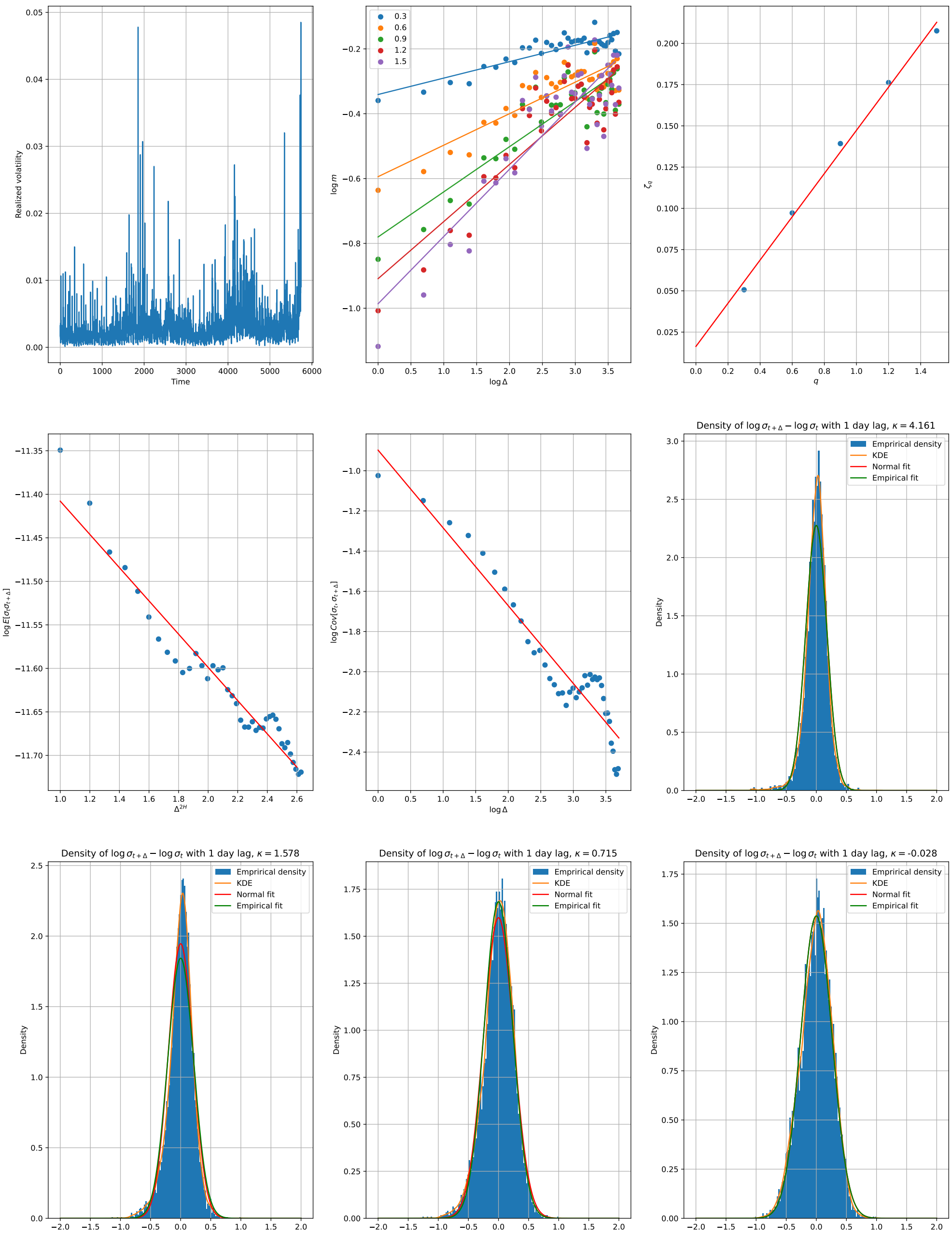

Figure 2.7: GAZP RX Equity. $\hat{H}$ plots

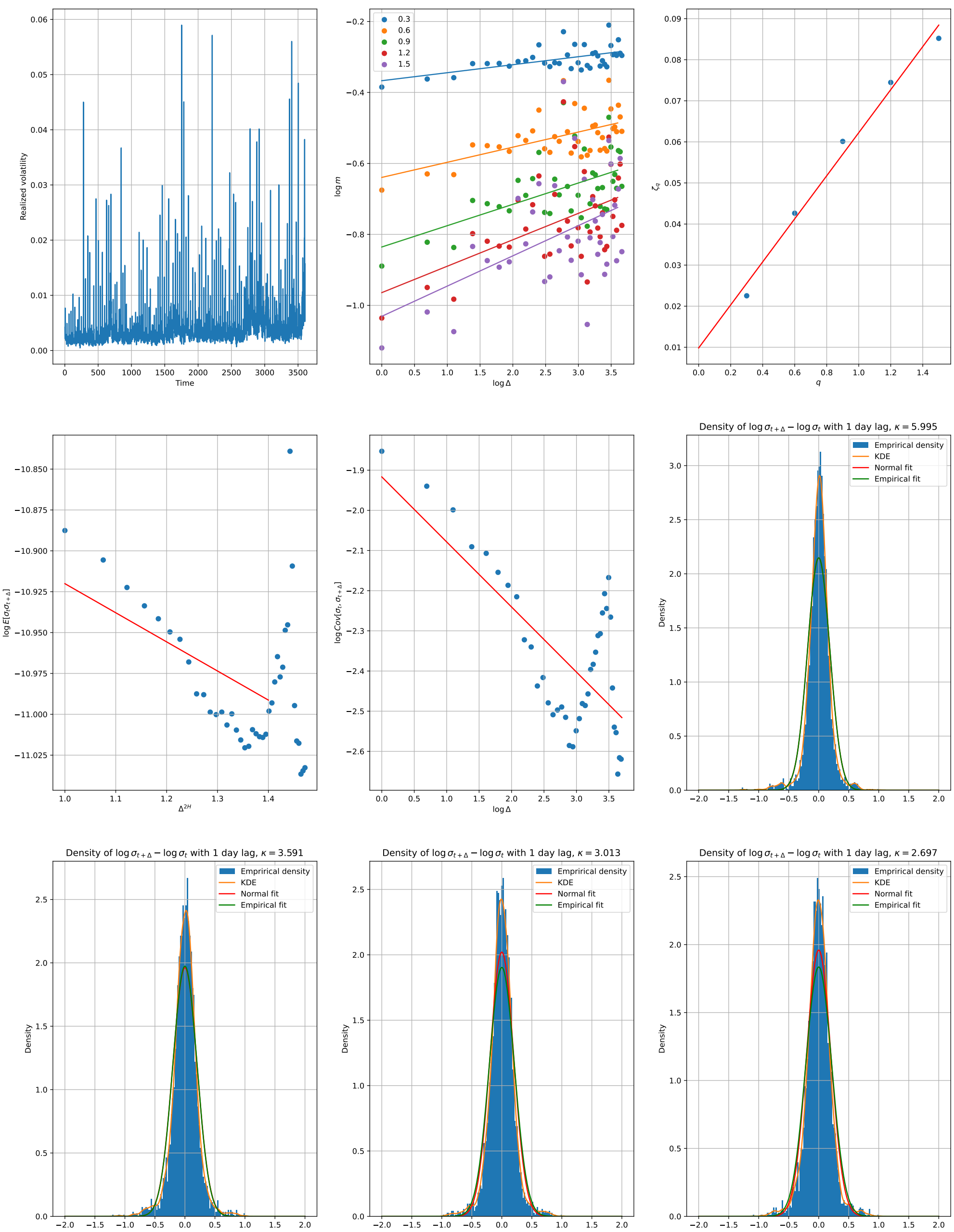

Figure 2.8: OGZD LI Equity. $\hat{H}$ plots

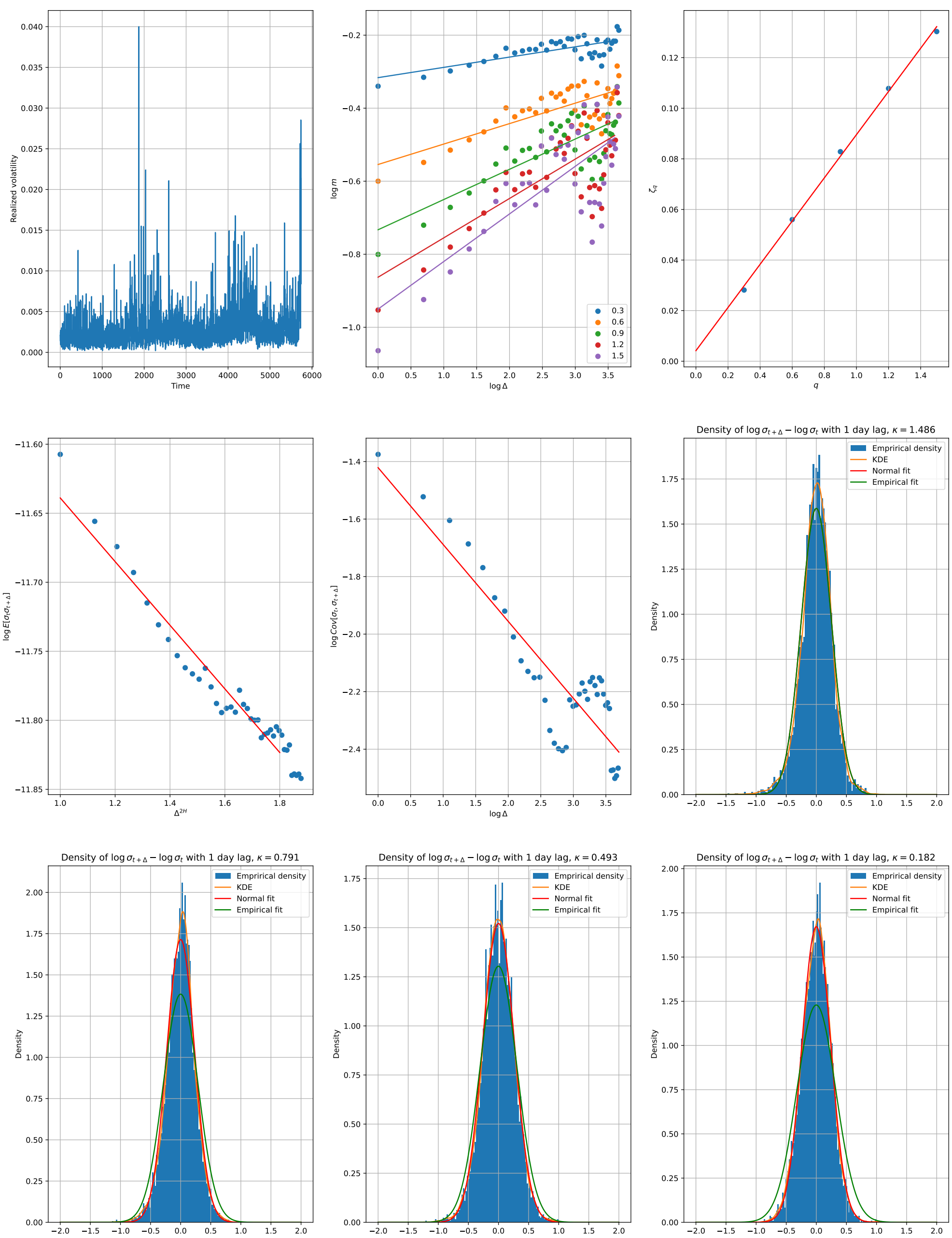

Figure 2.9: MOEX RX Equity. $\hat{H}$ plots

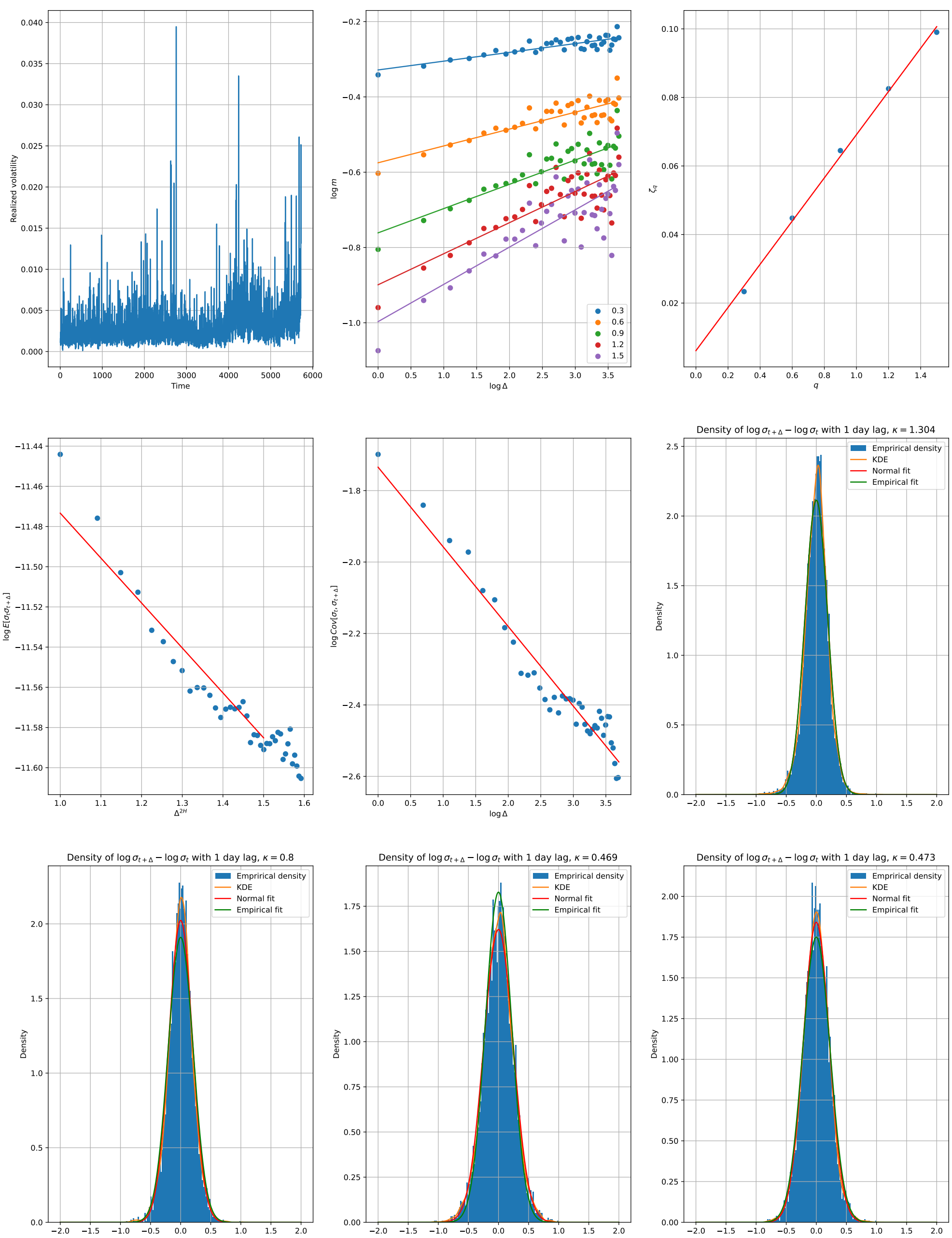

Figure 2.10: FIVE RX Equity. $\hat{H}$ plots

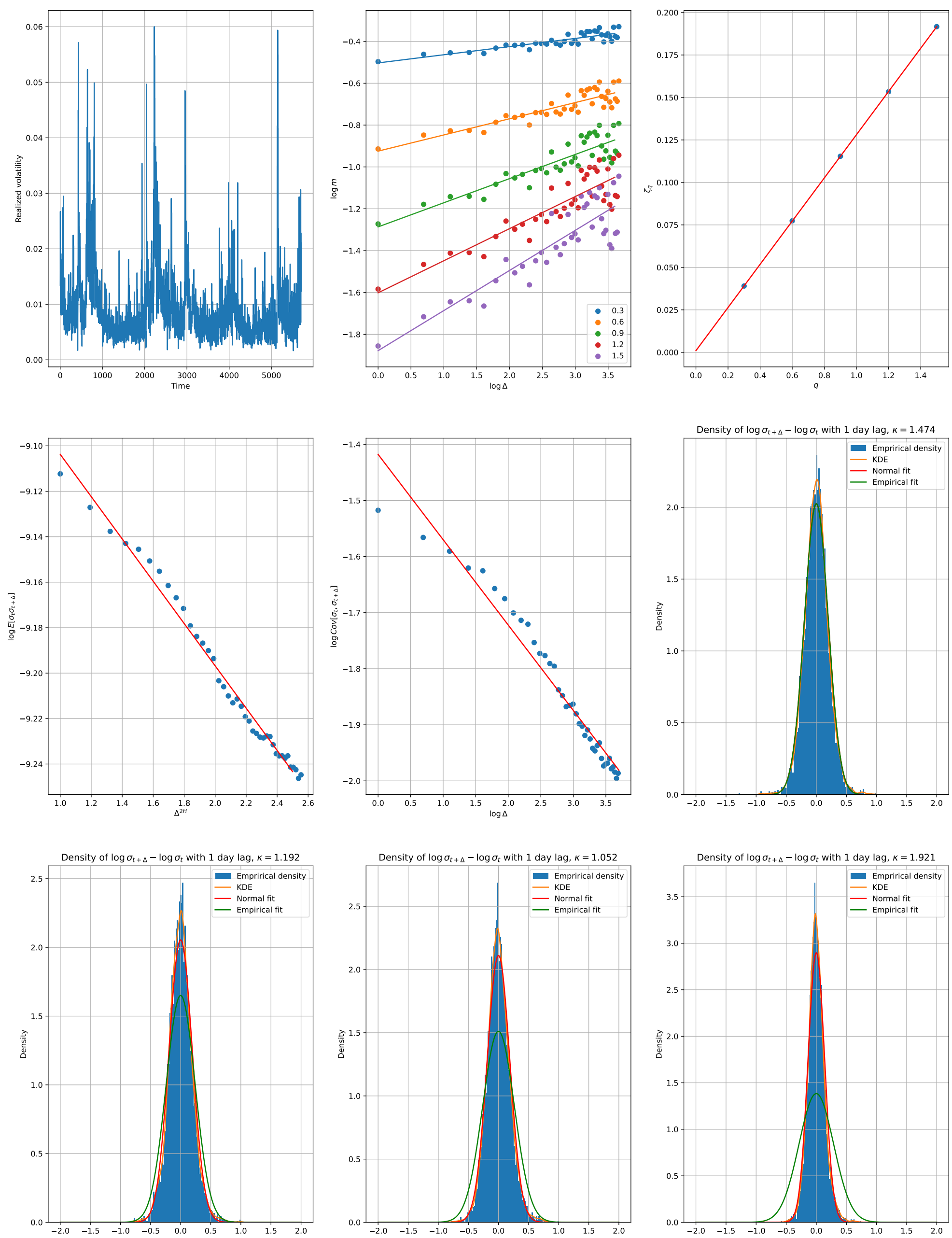

Figure 2.11: .AEX. $\hat{H}$ plots

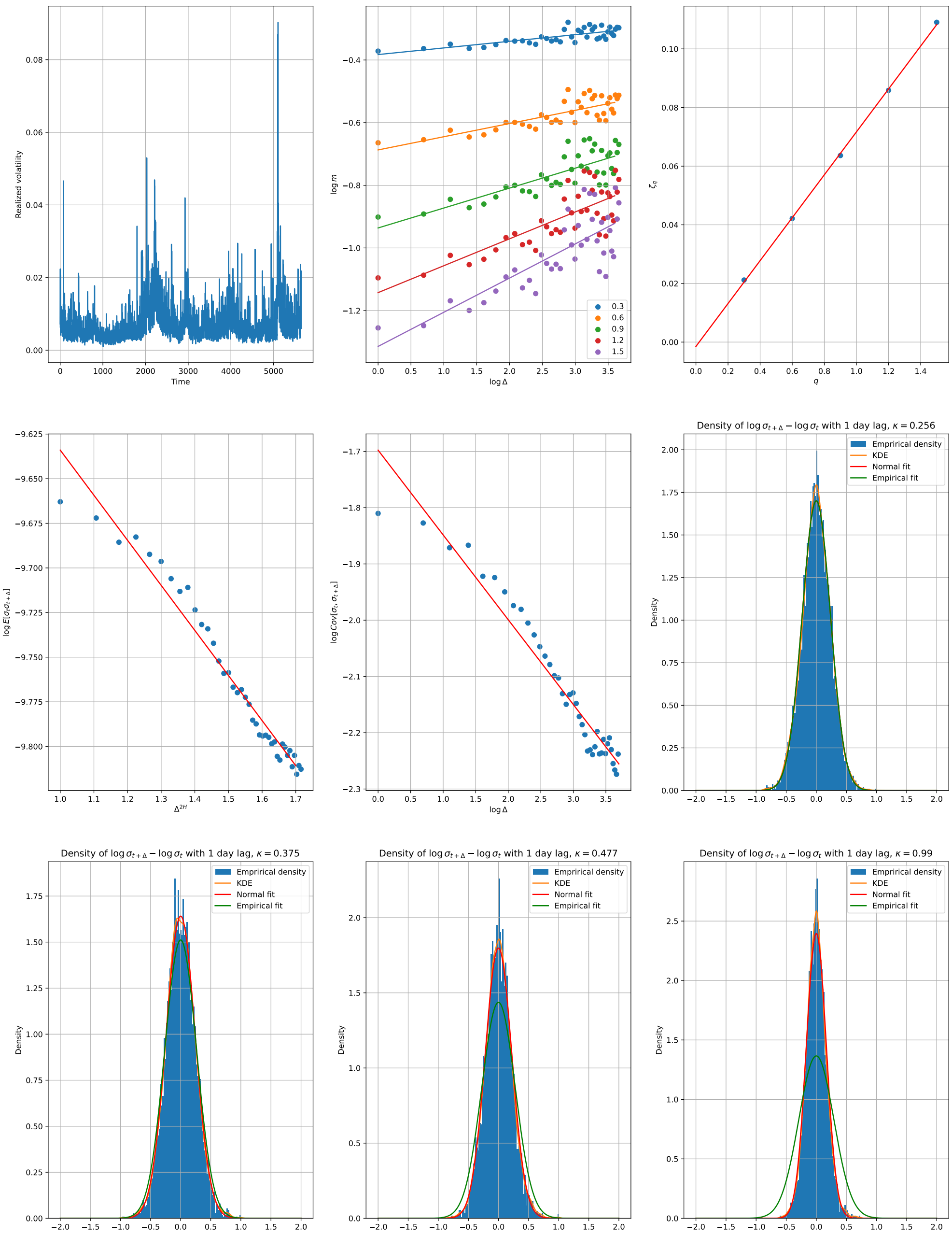

Figure 2.12: .AORD. $\hat{H}$ plots

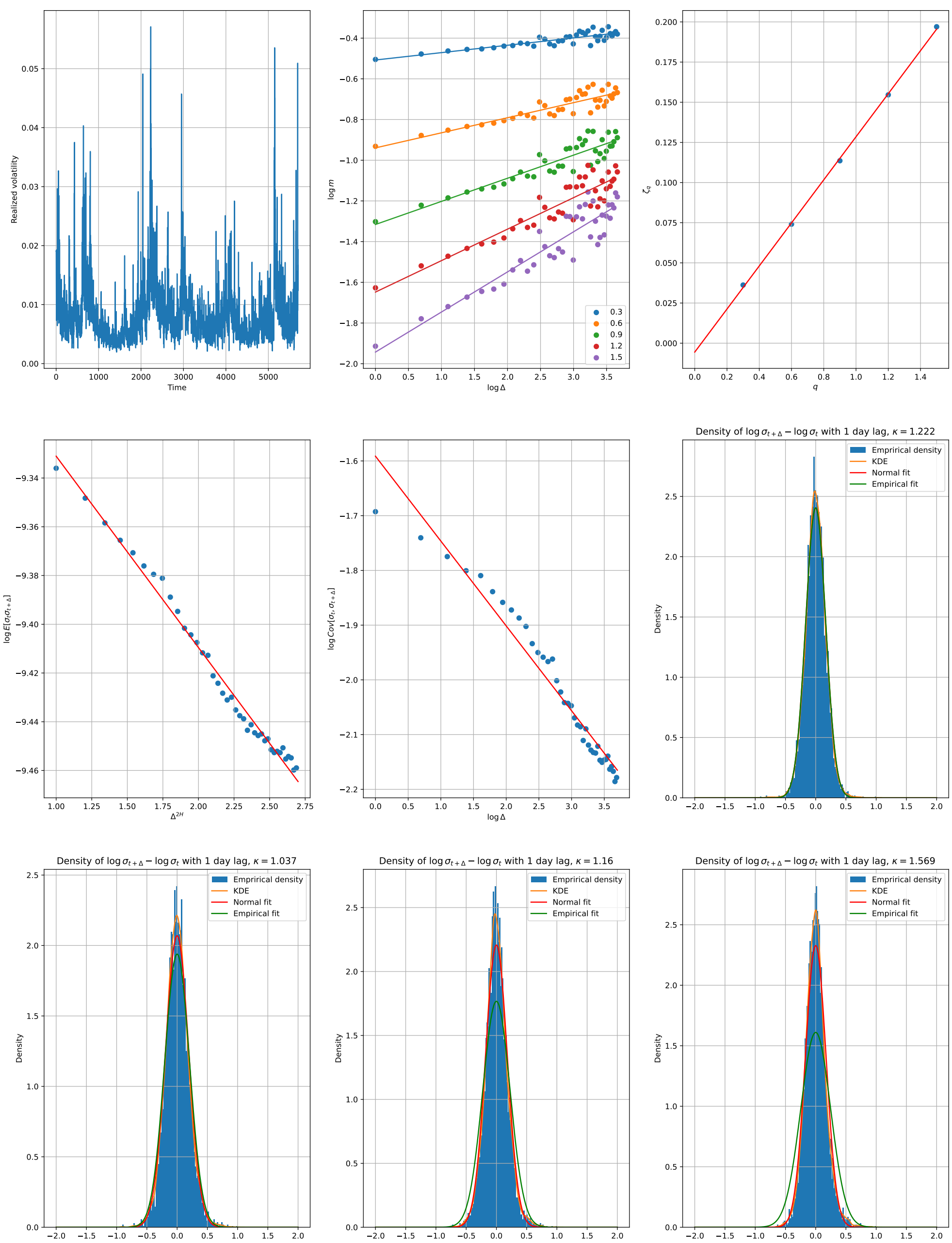

Figure 2.13: .BFX. $\hat{H}$ plots

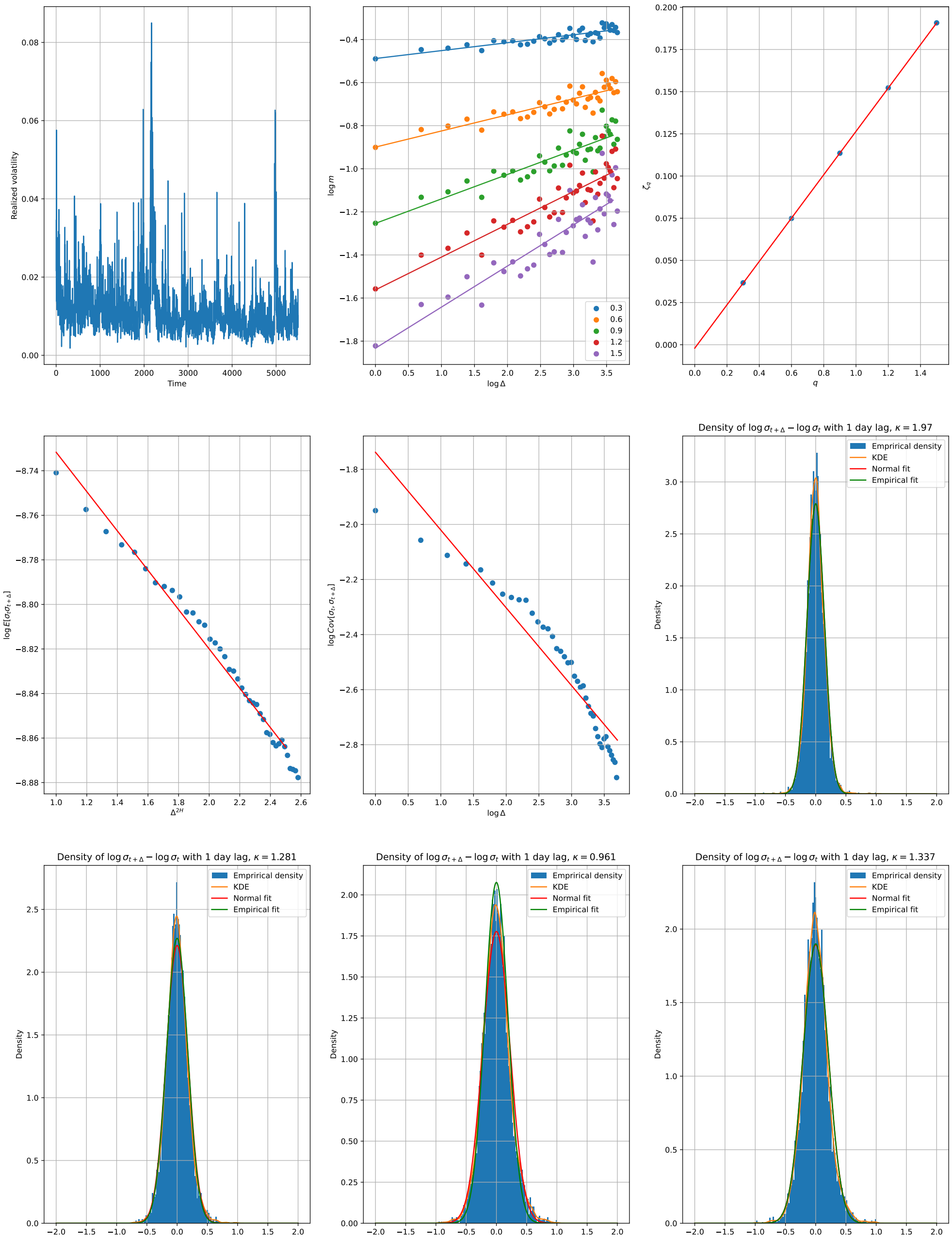

Figure 2.14: .BVSP. $\hat{H}$ plots

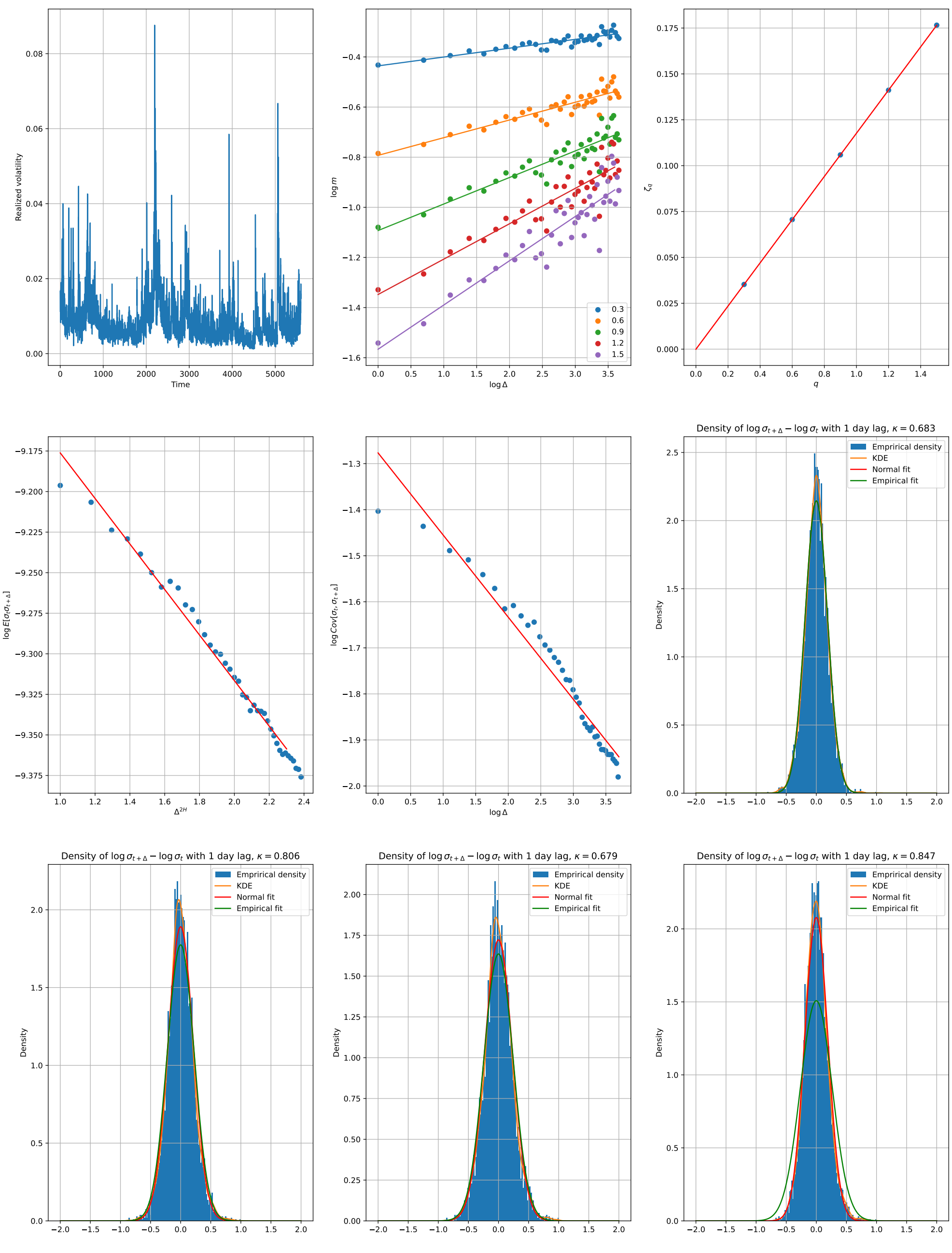

Figure 2.15: .DJI. $\hat{H}$ plots

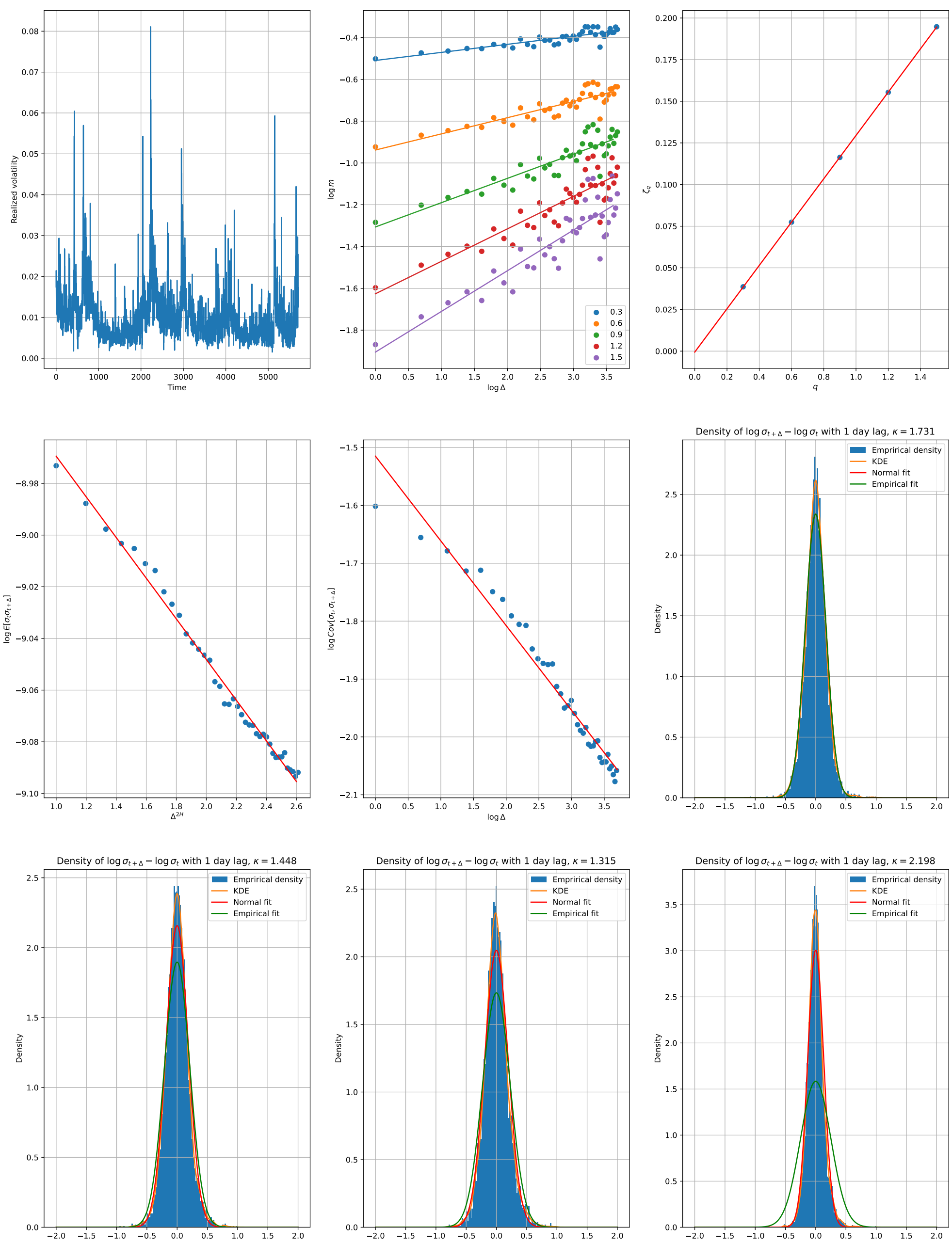

Figure 2.16: .FCHI. $\hat{H}$ plots

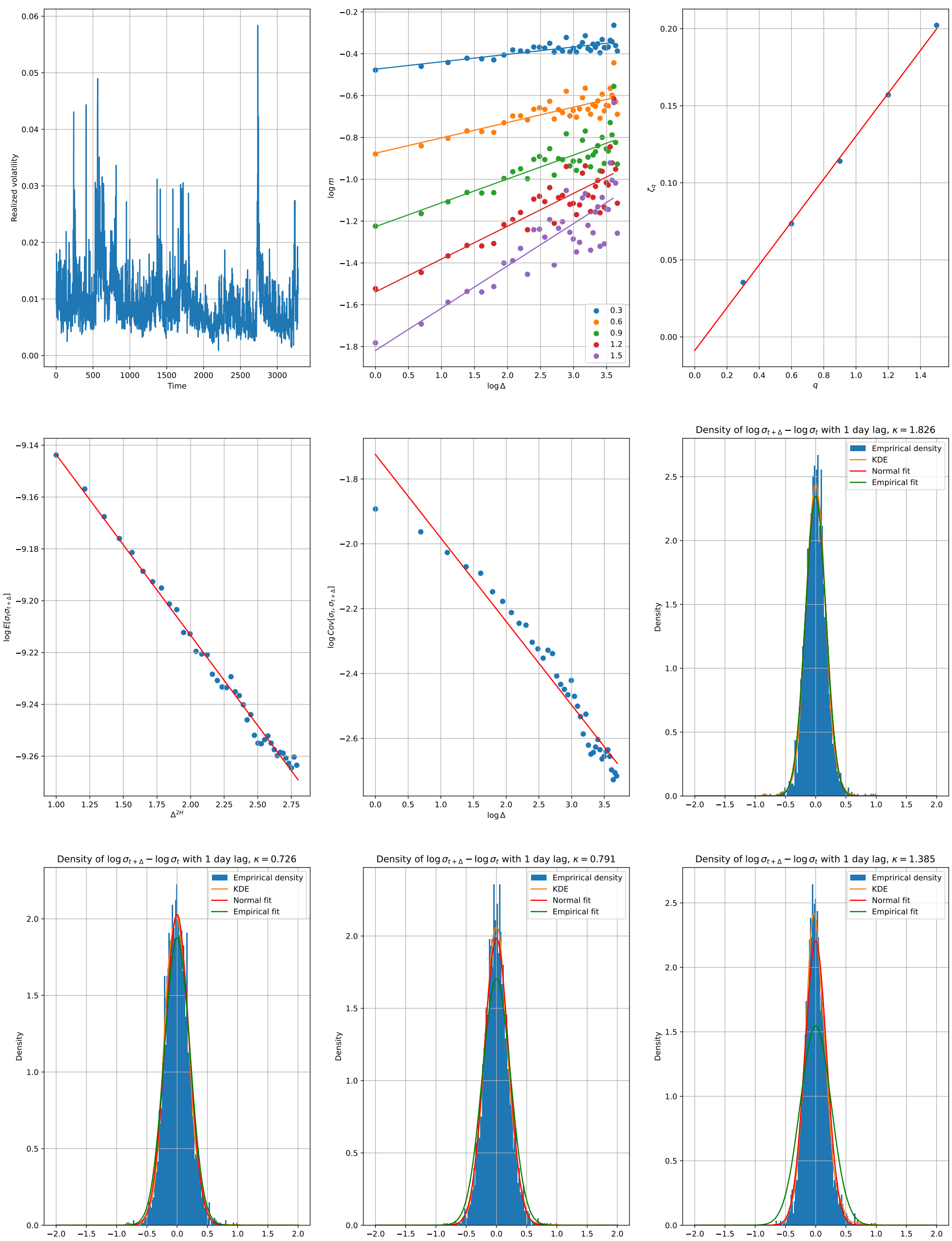

Figure 2.17: .FTMIB. $\hat{H}$ plots

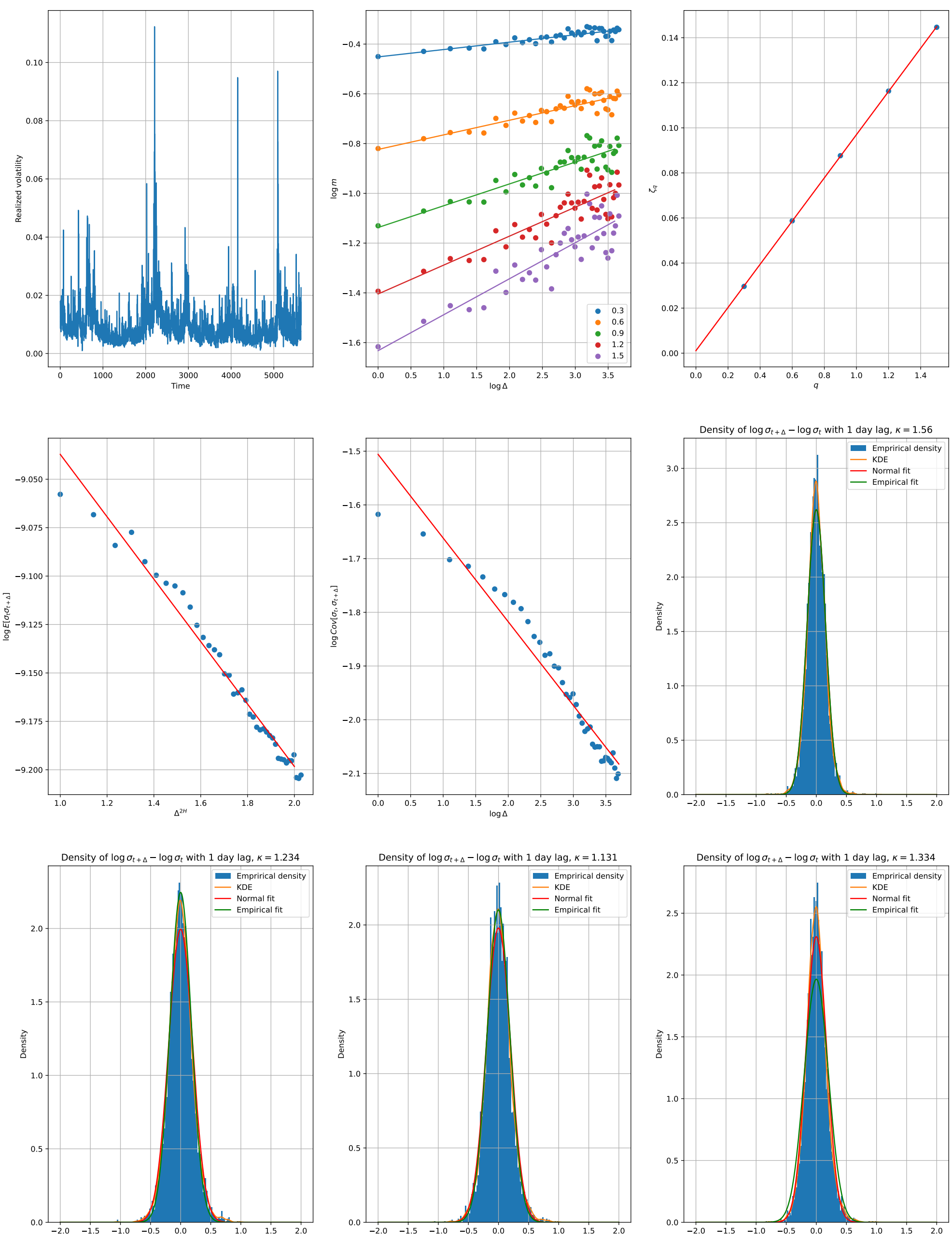

Figure 2.18: .FTSE. $\hat{H}$ plots

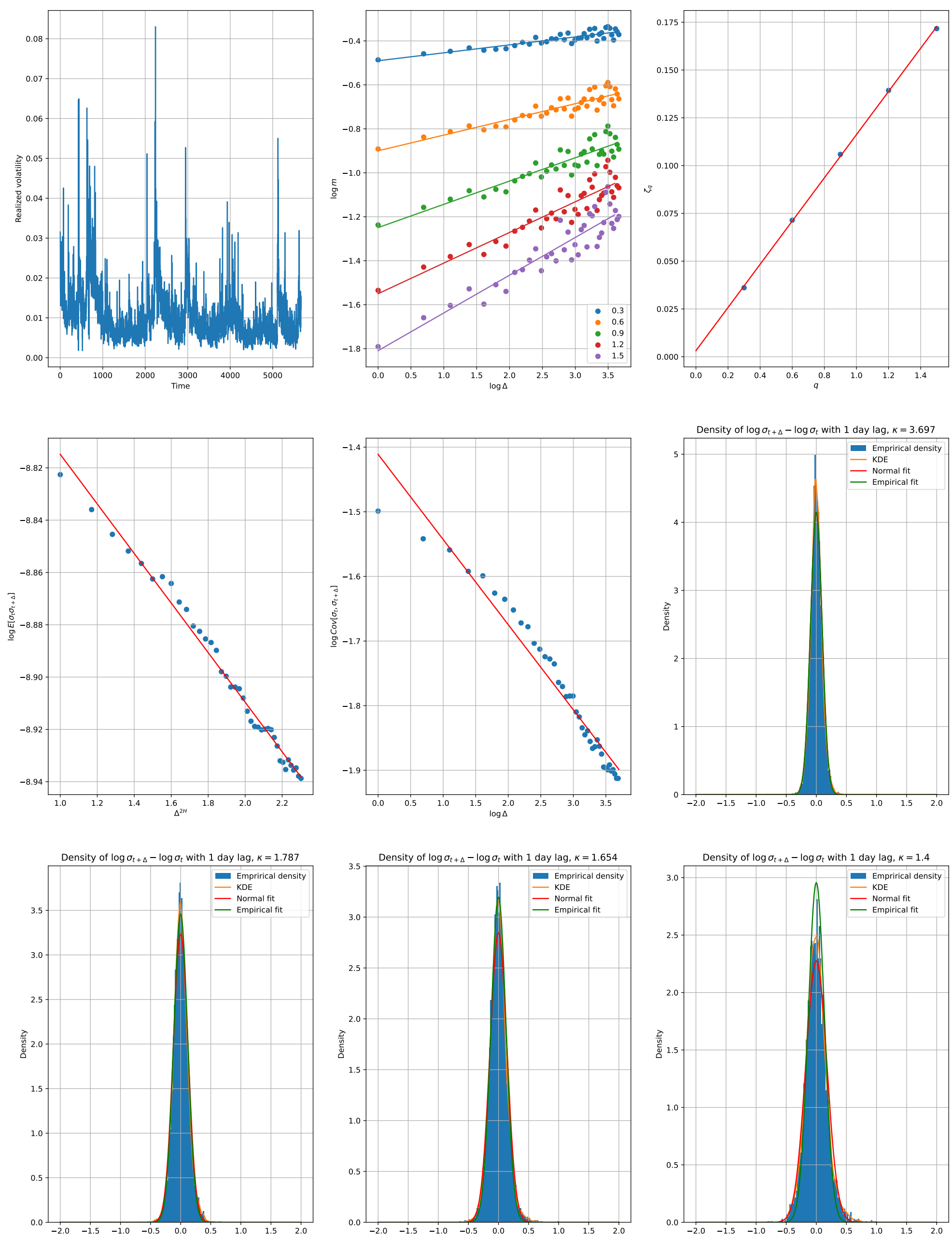

Figure 2.19: .GDAXI. $\hat{H}$ plots

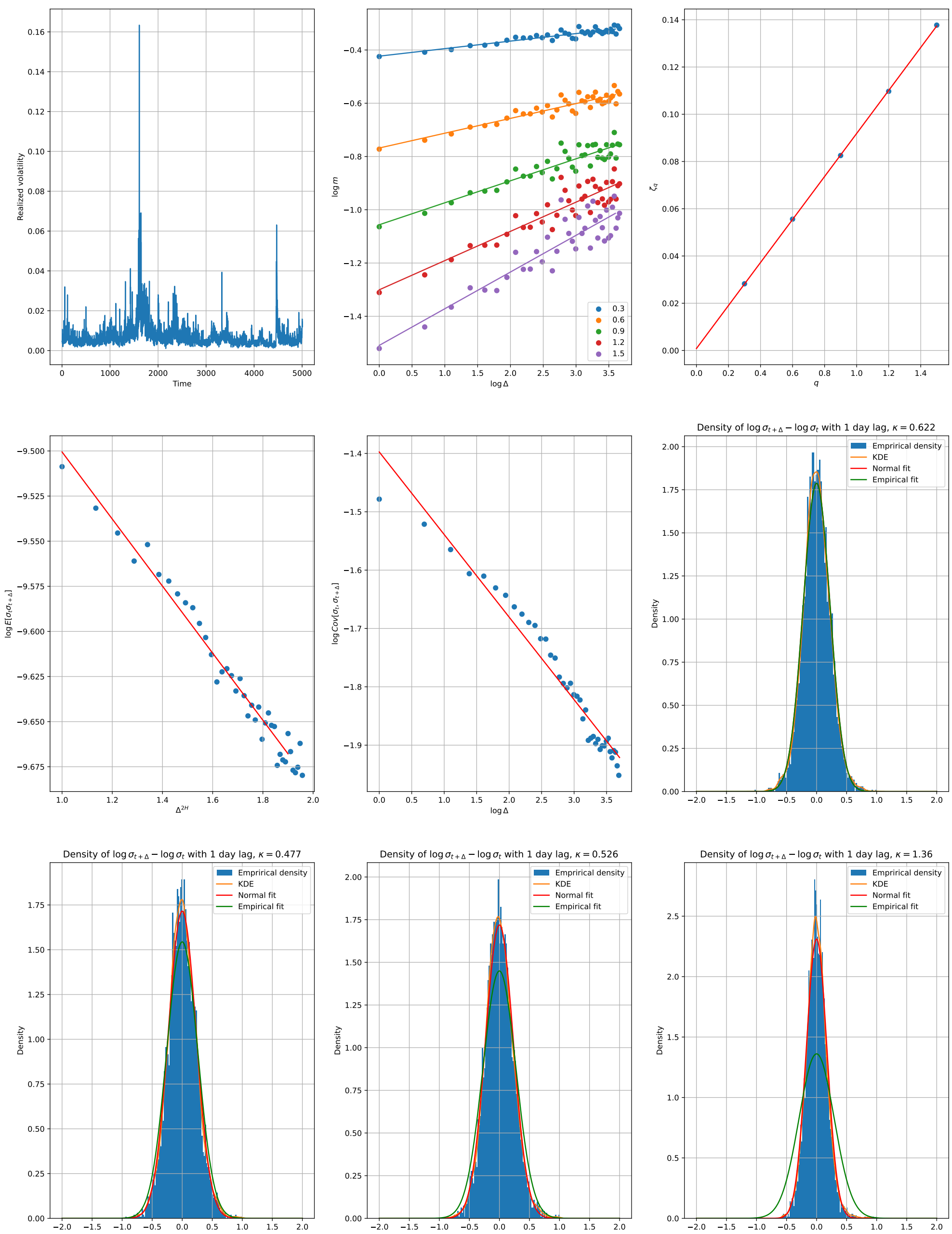

Figure 2.20: .GSPTSE. $\hat{H}$ plots

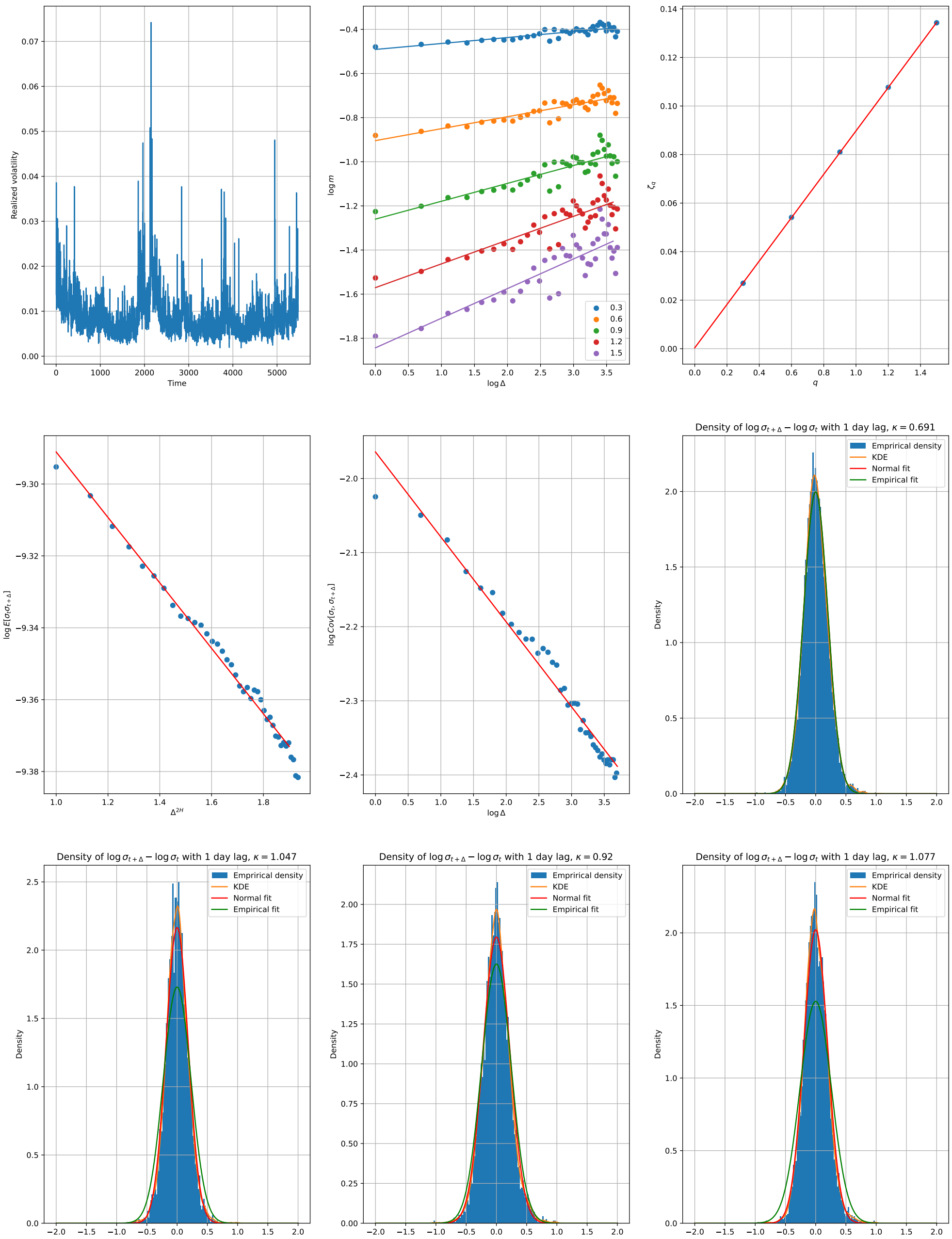

Figure 2.21: .HSI. $\hat{H}$ plots

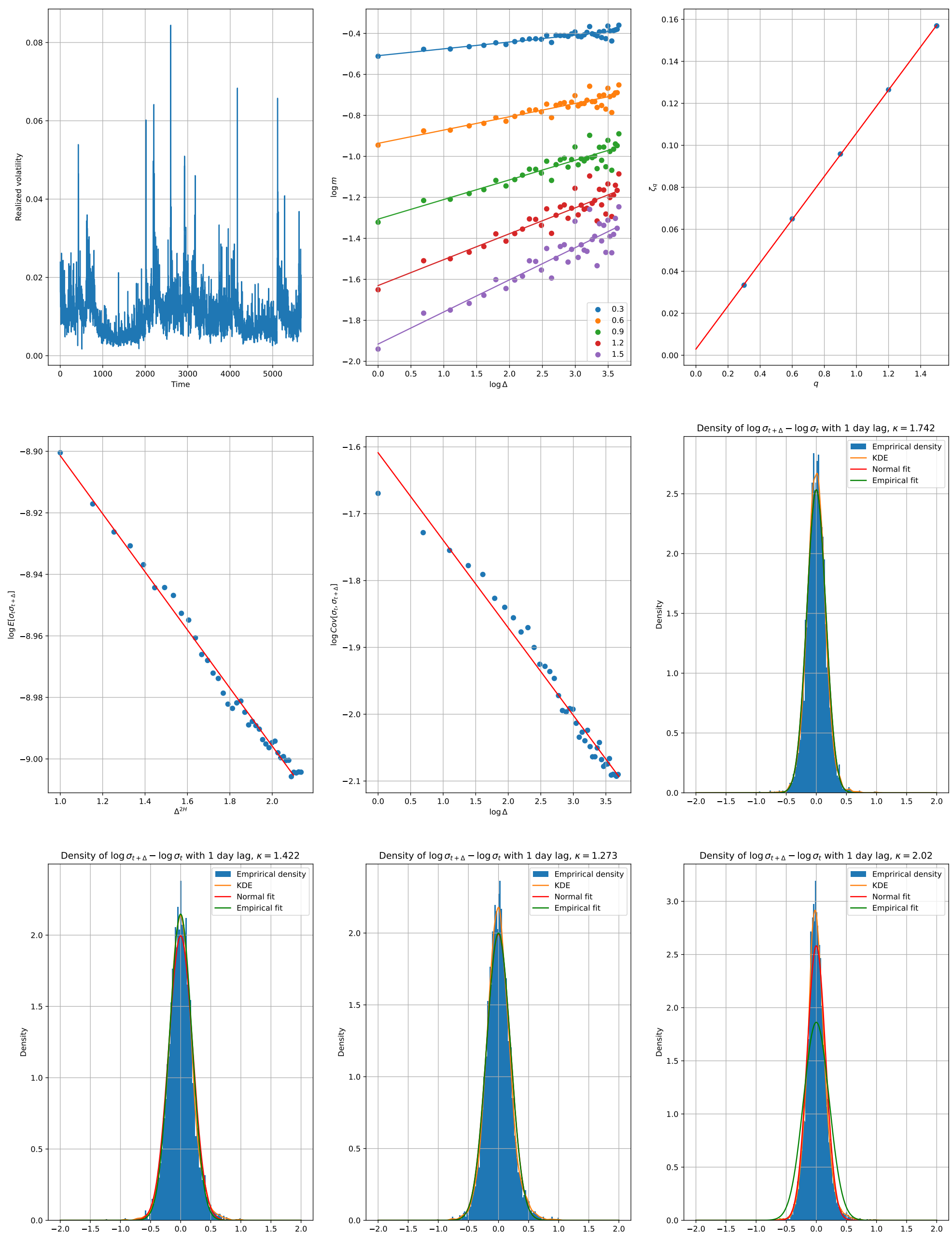

Figure 2.22: .IBEX. $\hat{H}$ plots

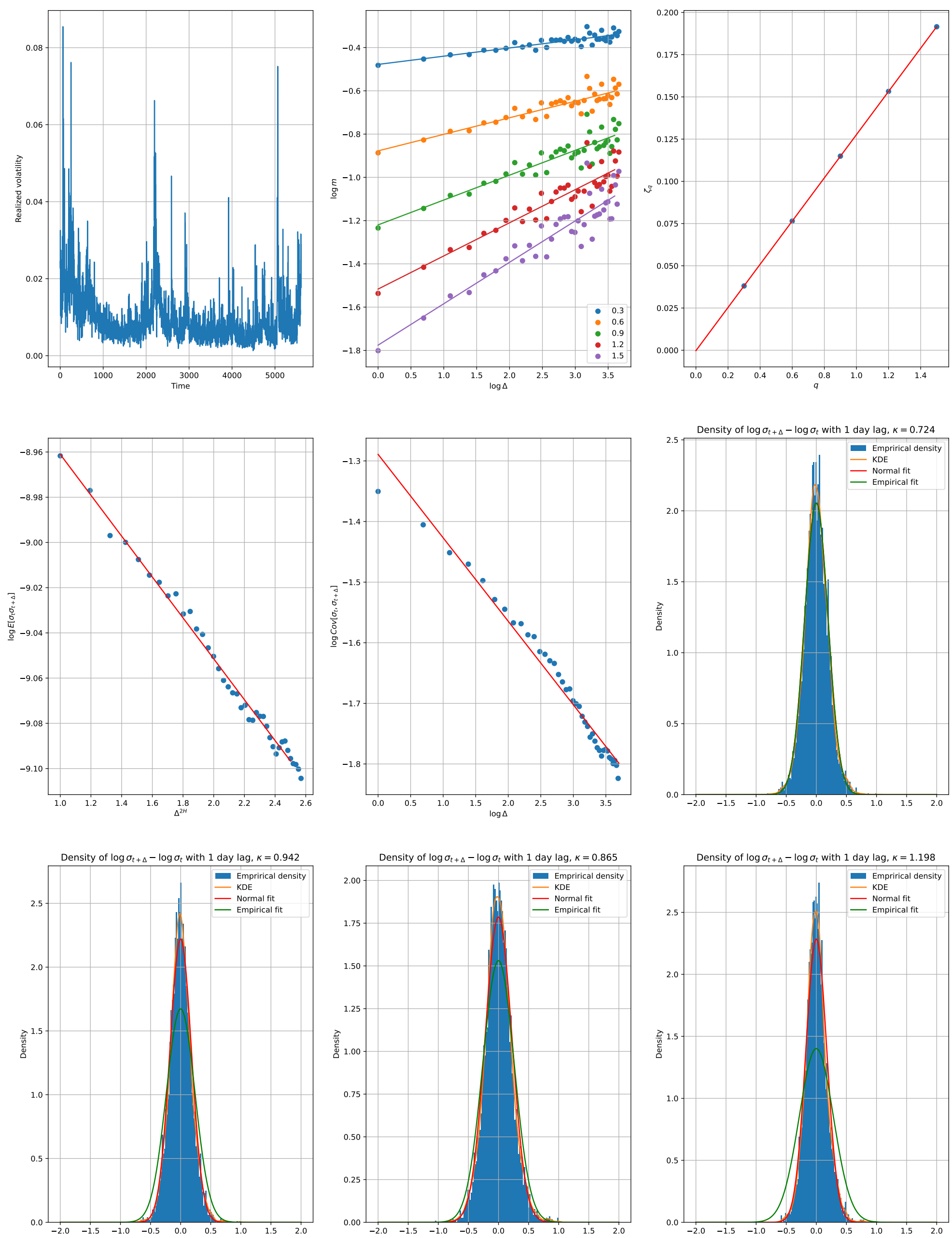

Figure 2.23: .IXIC. $\hat{H}$ plots

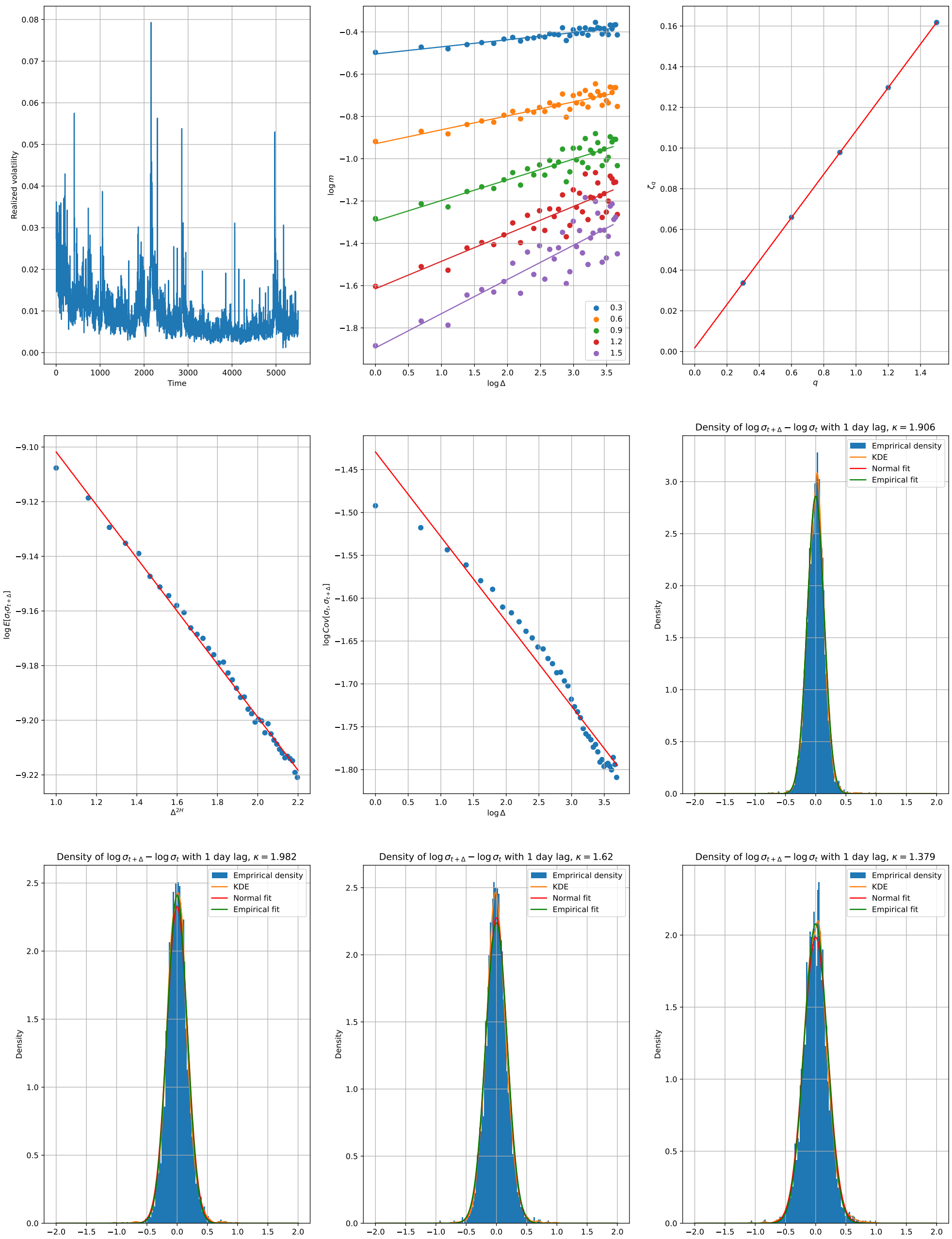

Figure 2.24: .KS11. $\hat{H}$ plots

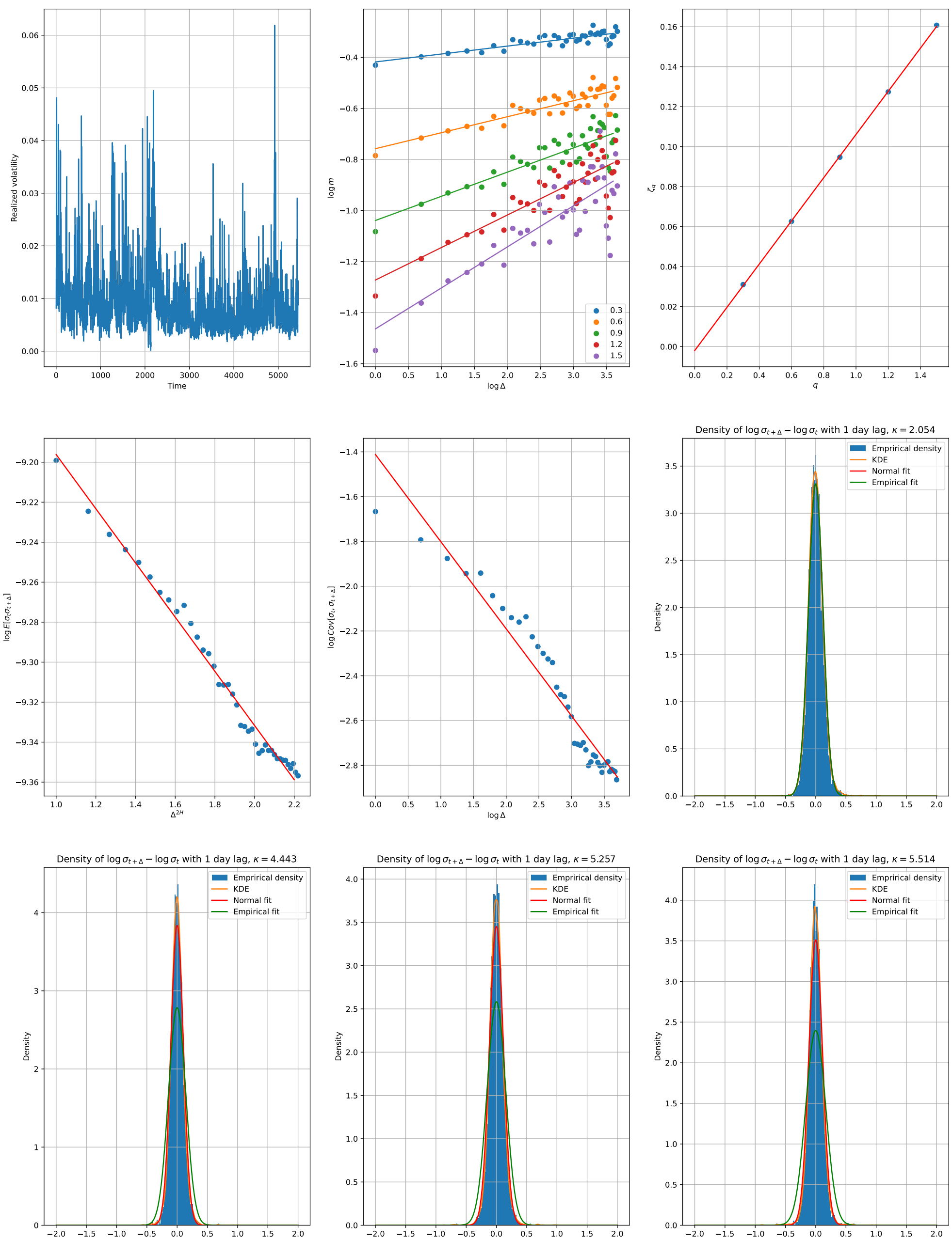

Figure 2.25: .KSE. $\hat{H}$ plots

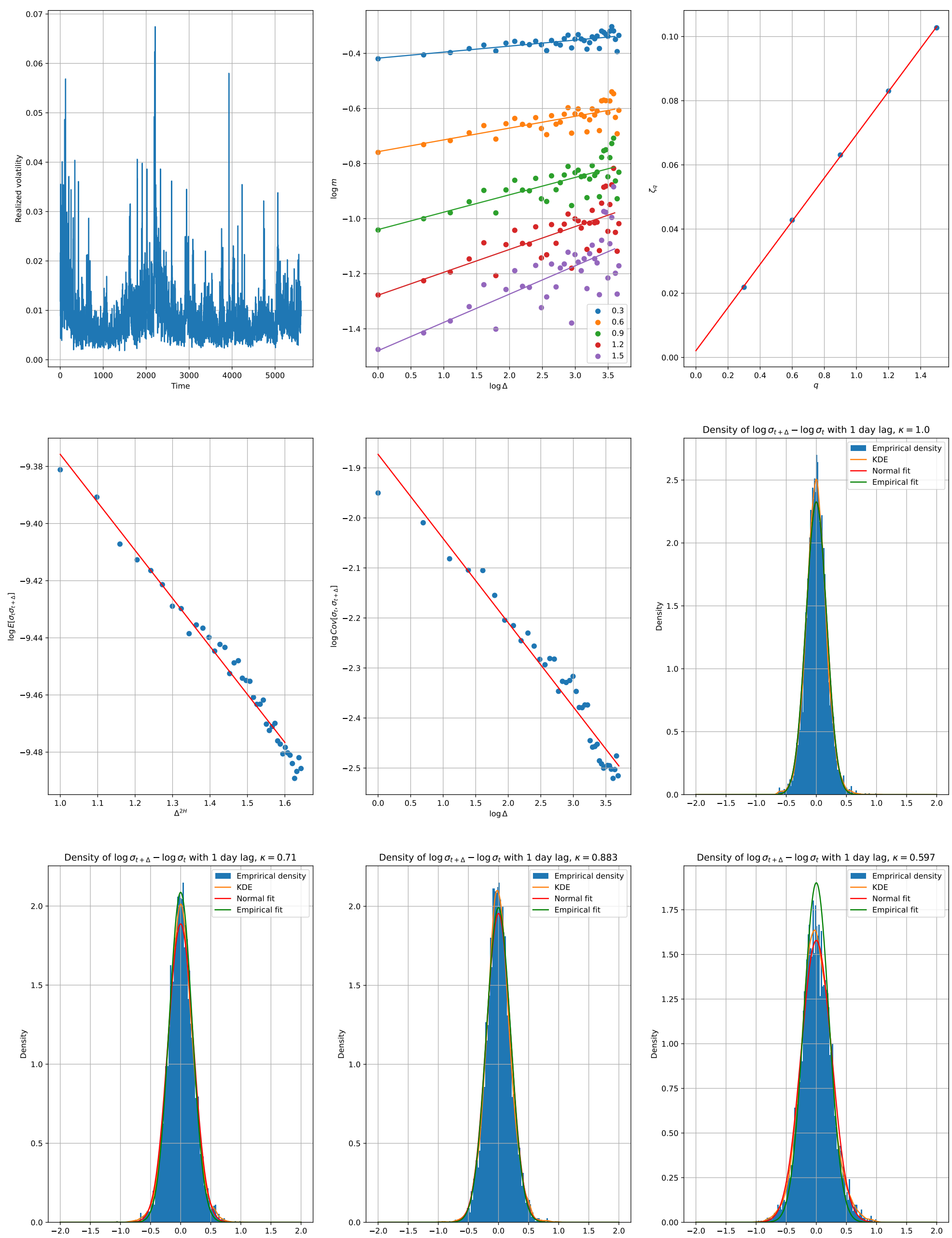

Figure 2.26: .MXX. $\hat{H}$ plots

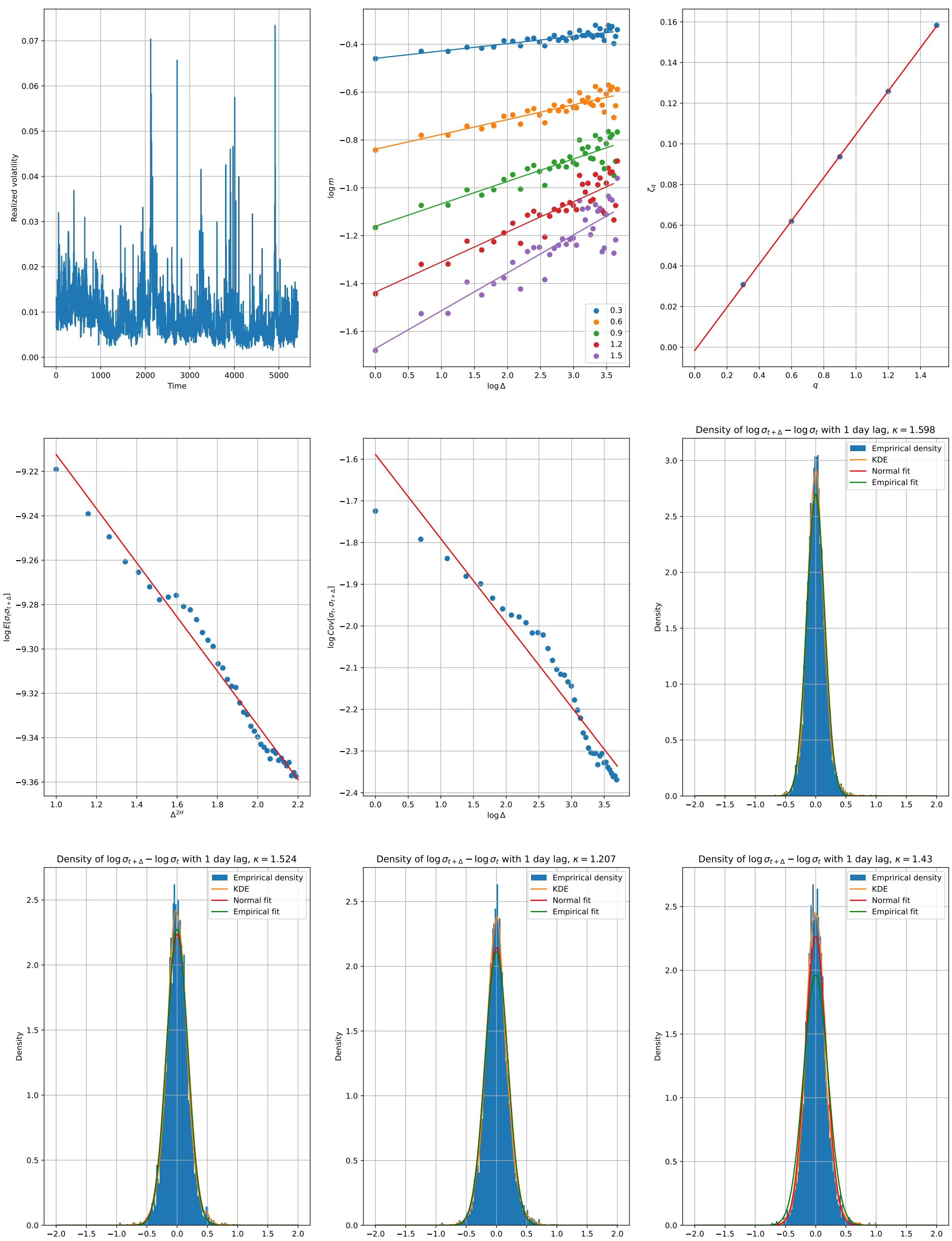

Figure 2.27: .N225. $\hat{H}$ plots

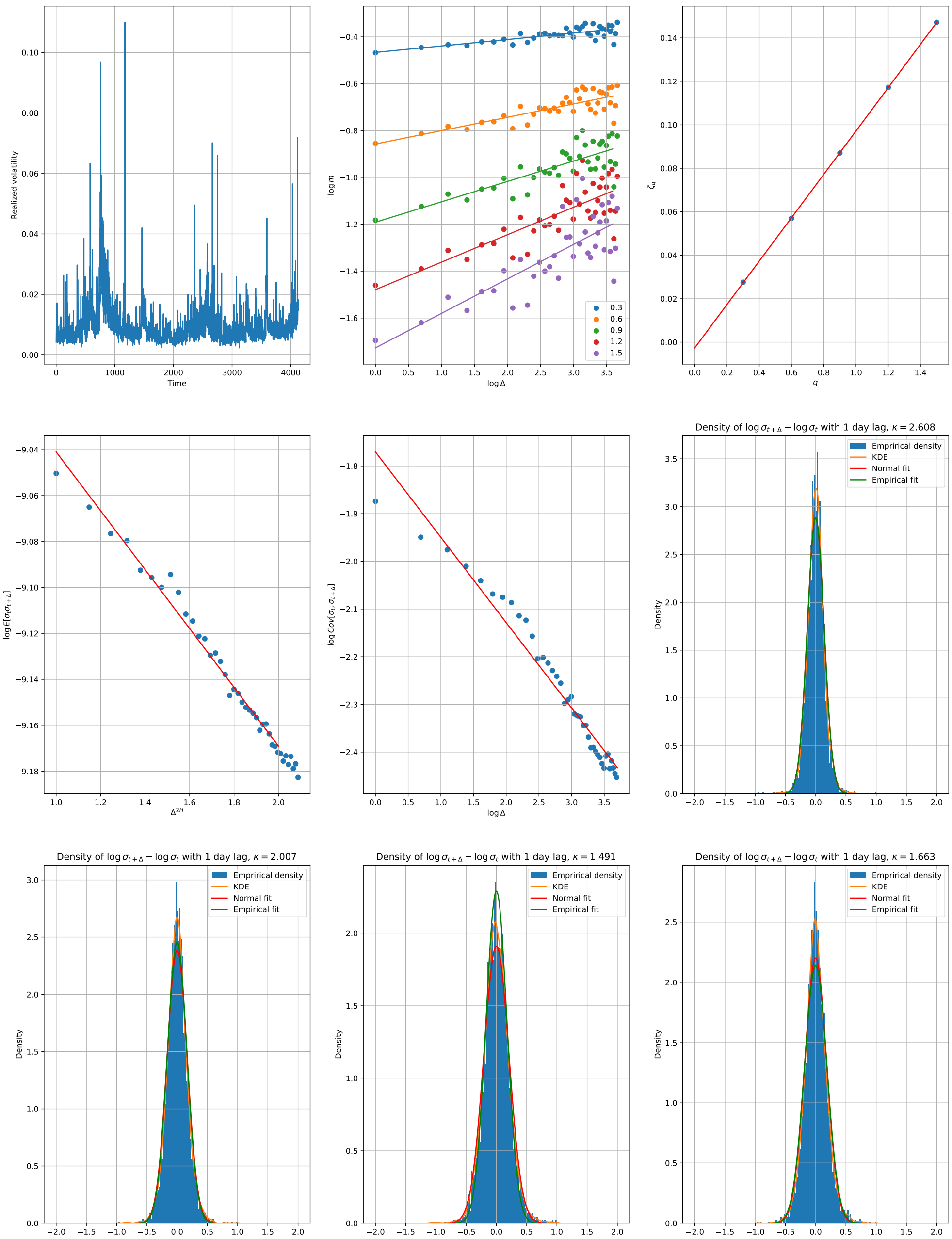

Figure 2.28: .OMXC20. $\hat{H}$ plots

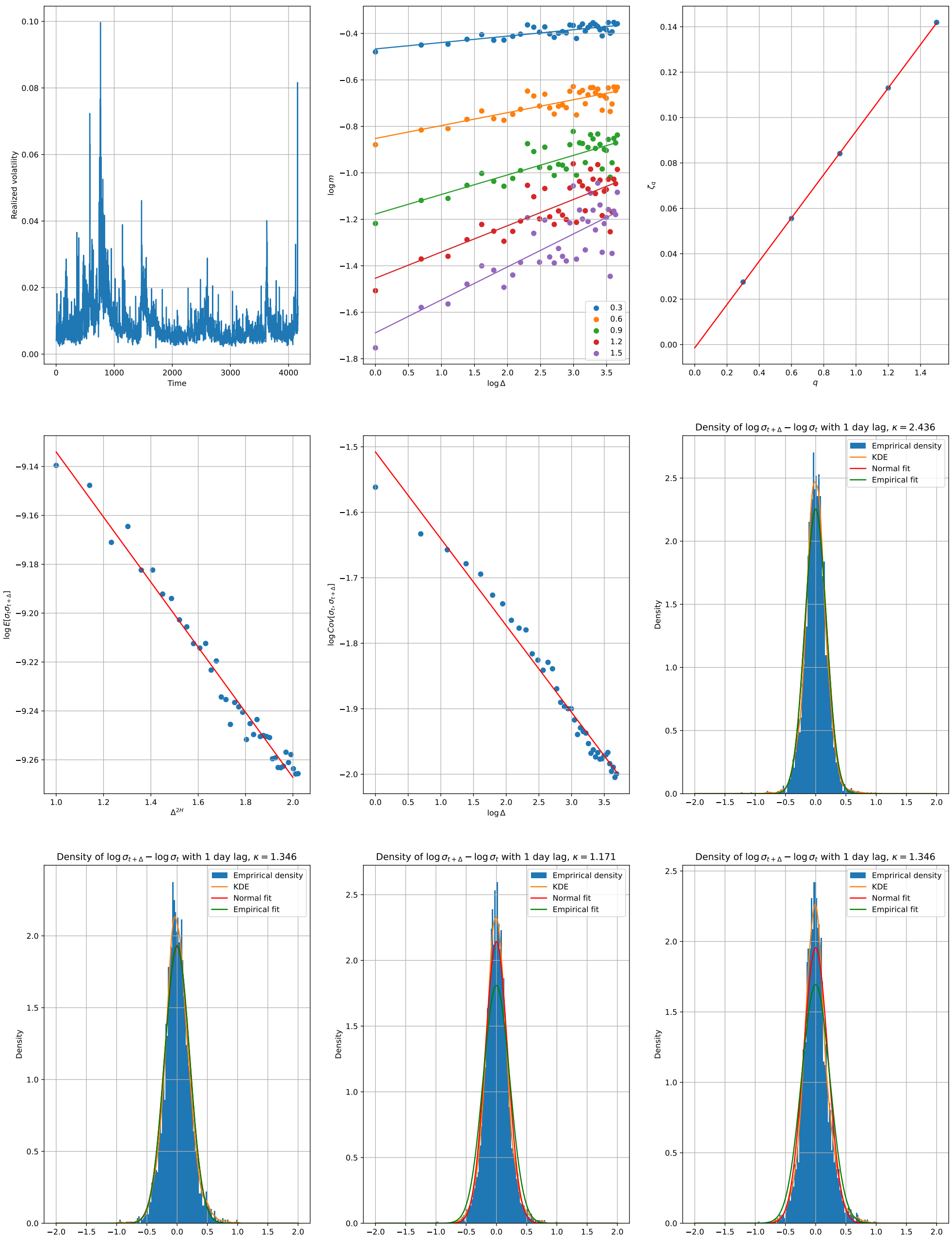

Figure 2.29: .OMXHPI. $\hat{H}$ plots

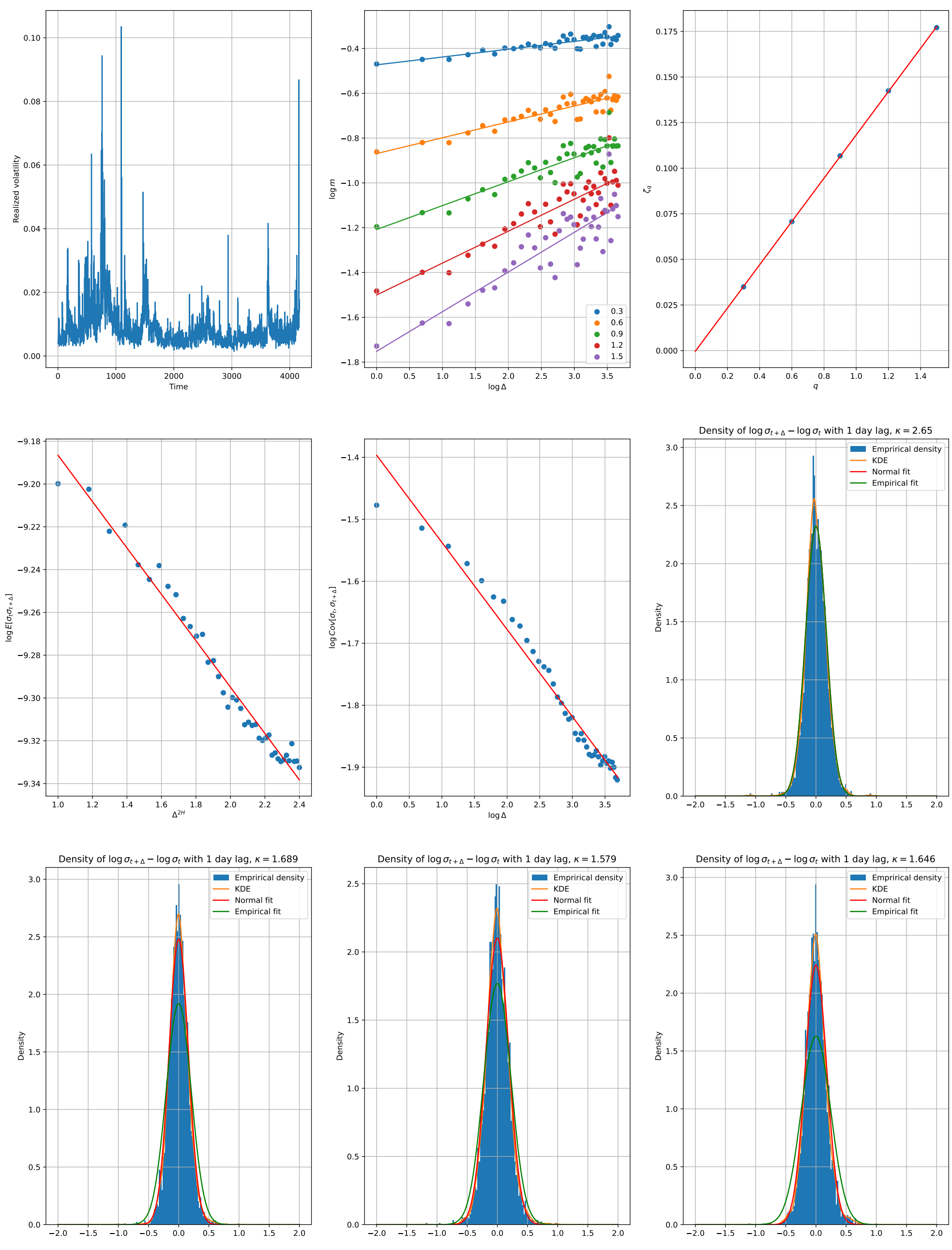

Figure 2.30: .OMXSPI. $\hat{H}$ plots

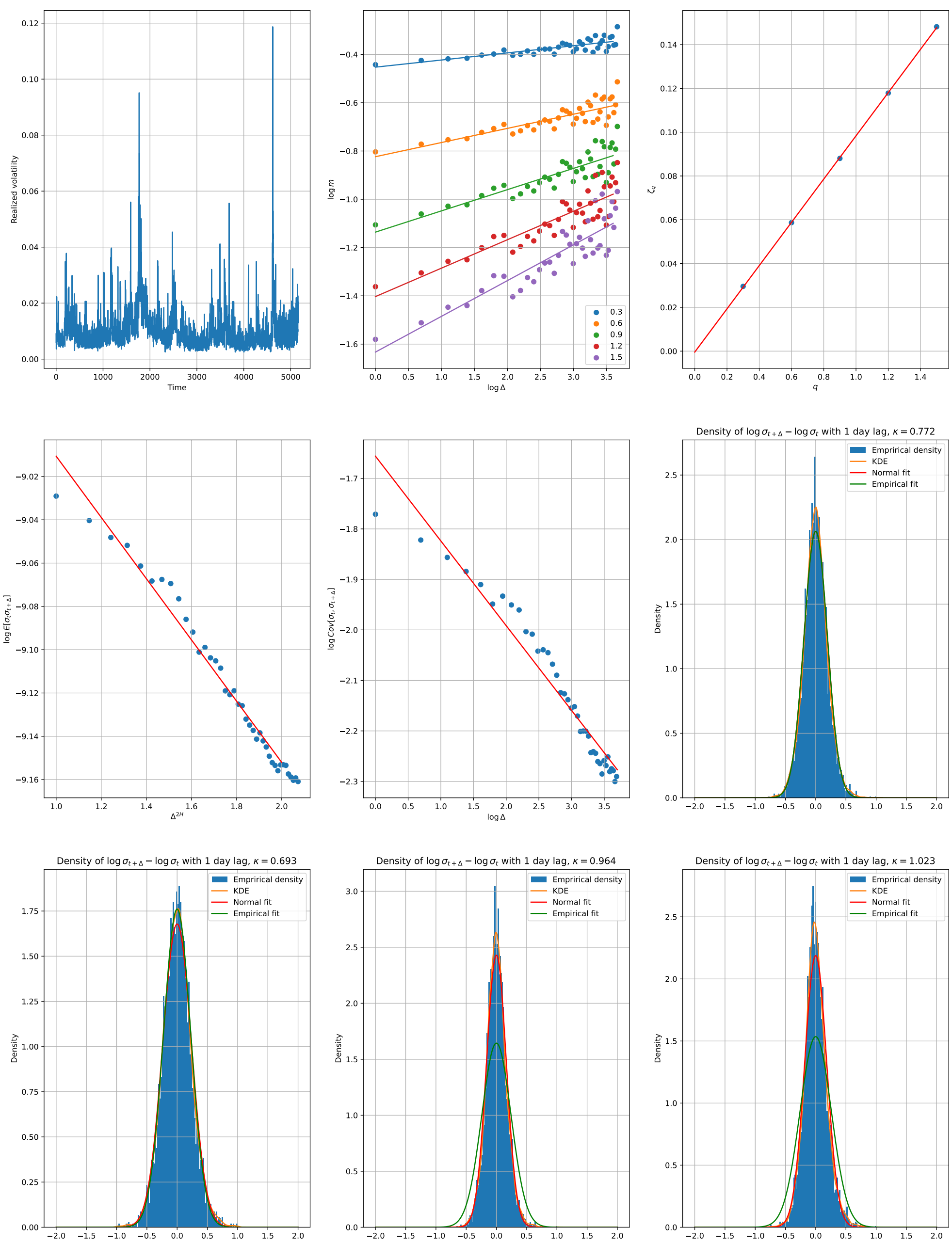

Figure 2.31: .OSEAX. $\hat{H}$ plots

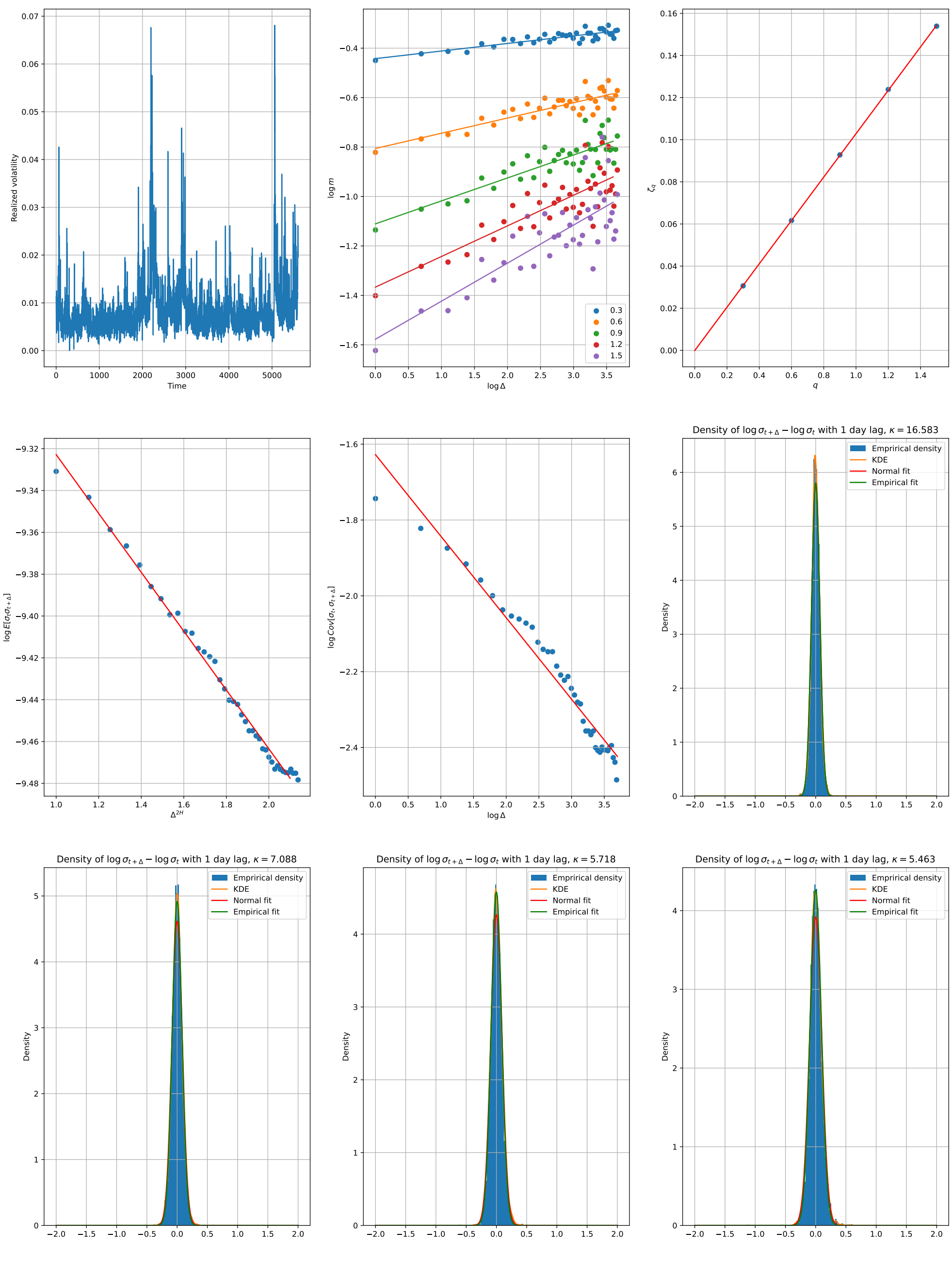

Figure 2.32: .RUT. $\hat{H}$ plots

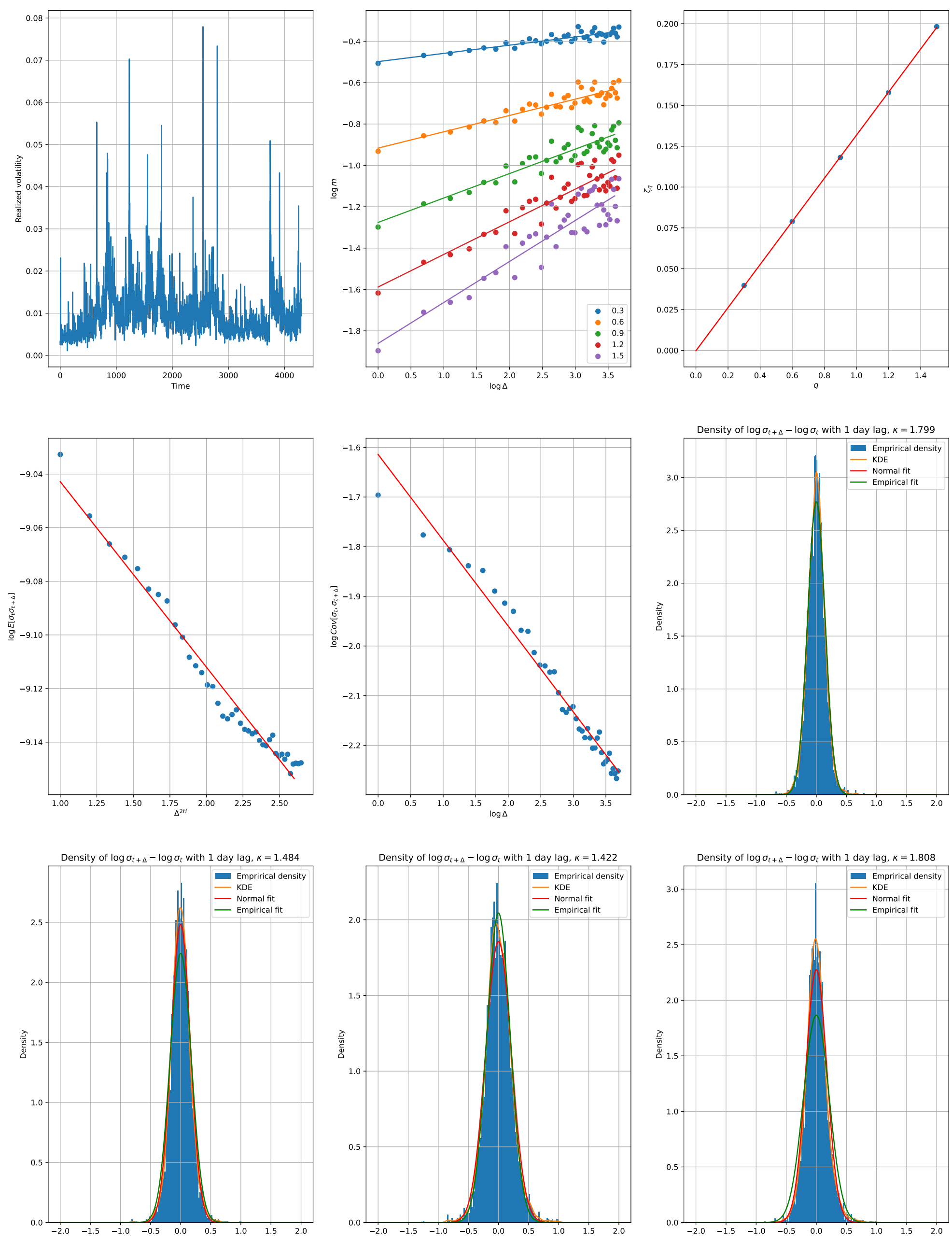

Figure 2.33: .SMSI. $\hat{H}$ plots

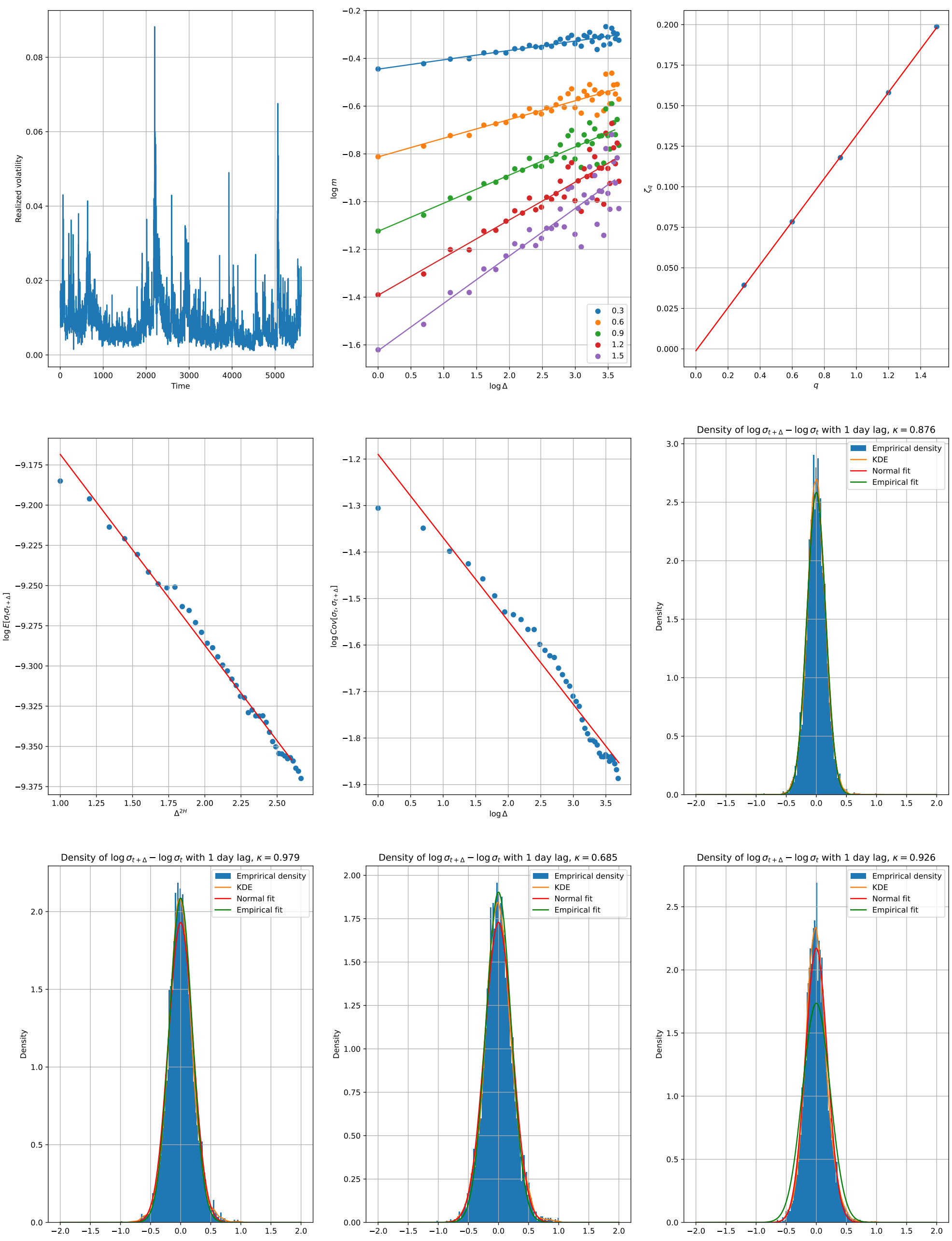

Figure 2.34: .SPX. $\hat{H}$ plots

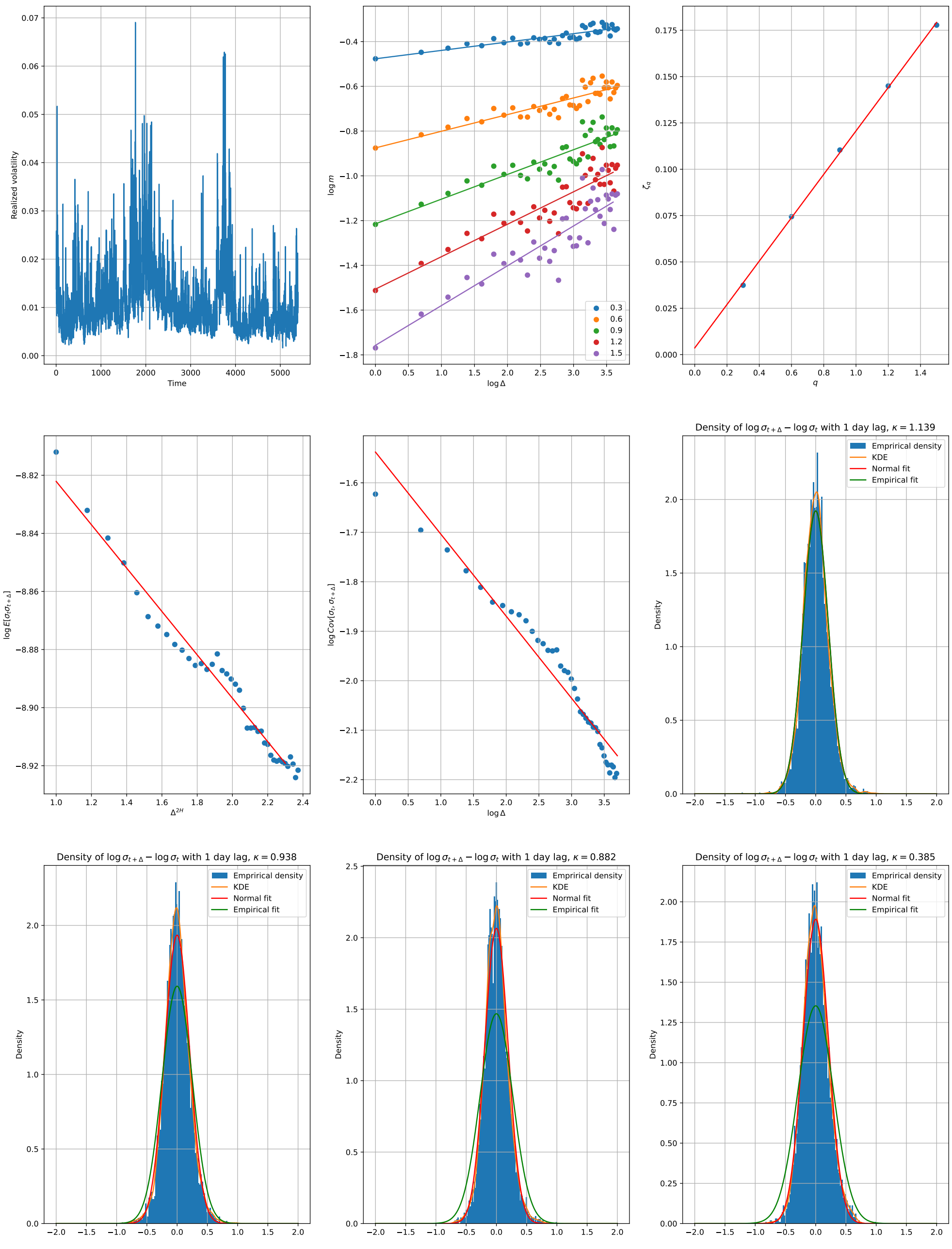

Figure 2.35: .SSEC. $\hat{H}$ plots

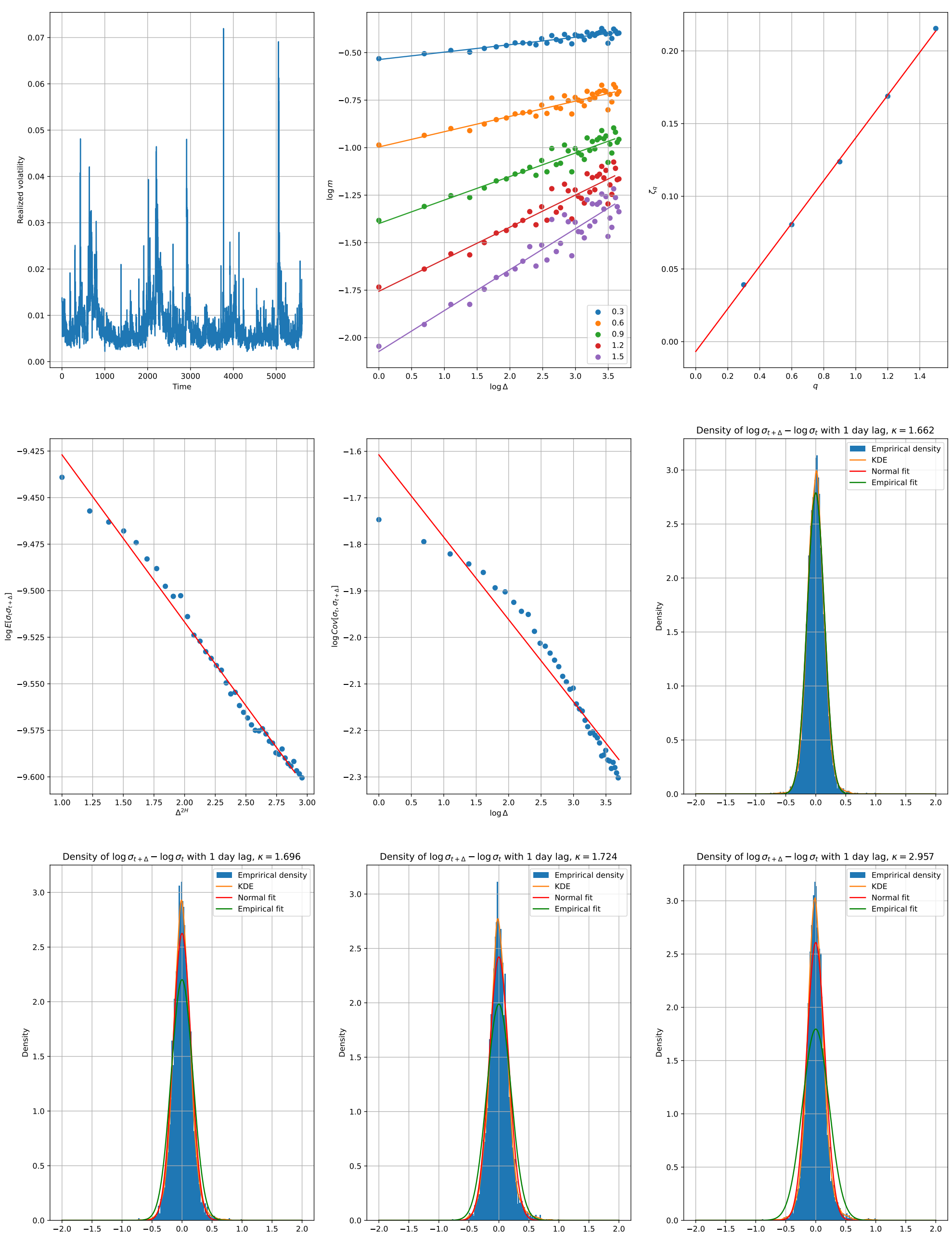

Figure 2.36: .SSMI. $\hat{H}$ plots

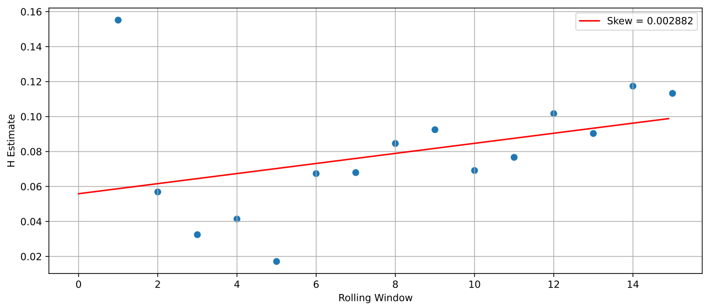

Figure 2.37: SBER RX Equity Smoothing Effect

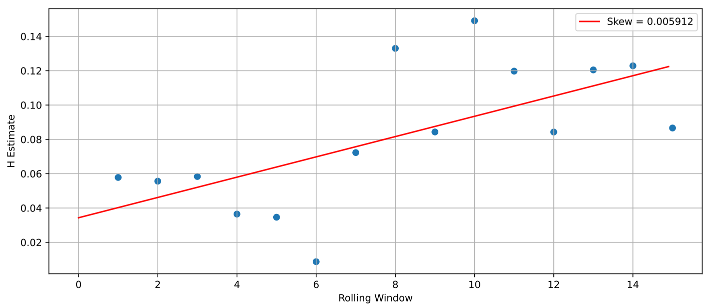

Figure 2.38: SBER LI Equity Smoothing Effect

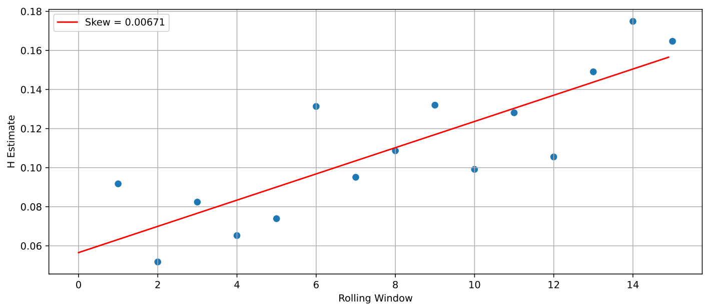

Figure 2.39: VTBR RX Equity Smoothing Effect

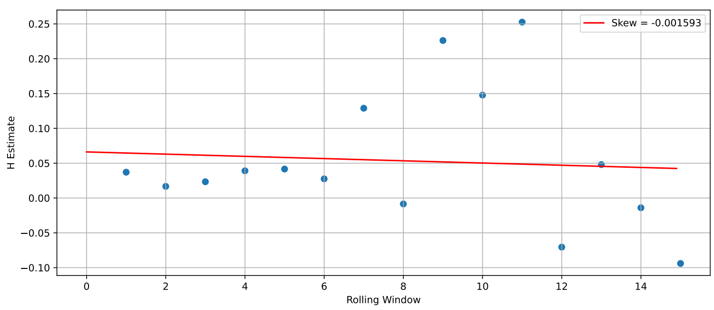

Figure 2.40: VTBR LI Equity Smoothing Effect

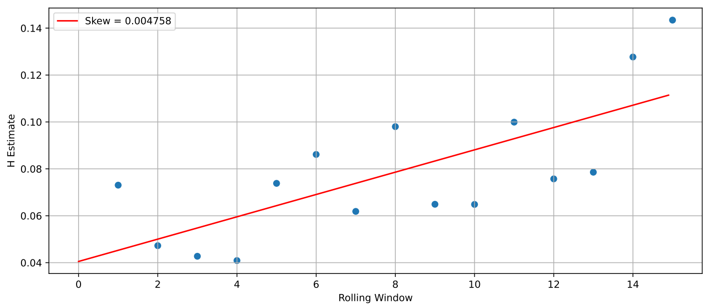

Figure 2.41: LKOH RX Equity Smoothing Effect

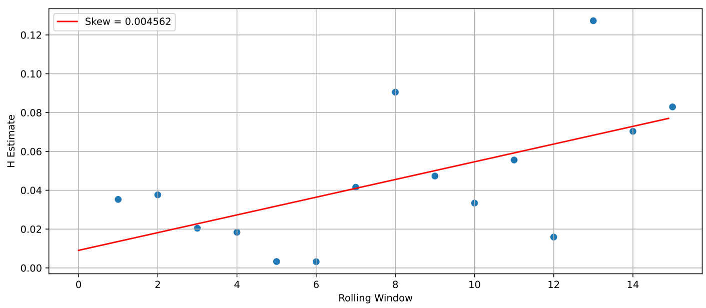

Figure 2.42: LKOD LI Equity Smoothing Effect

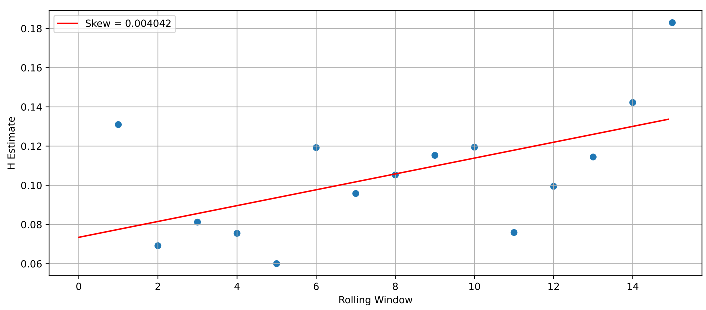

Figure 2.43: GAZP RX Equity Smoothing Effect

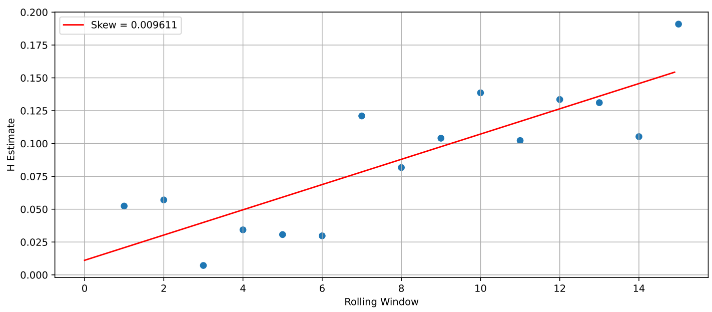

Figure 2.44: OGZD LI Equity Smoothing Effect

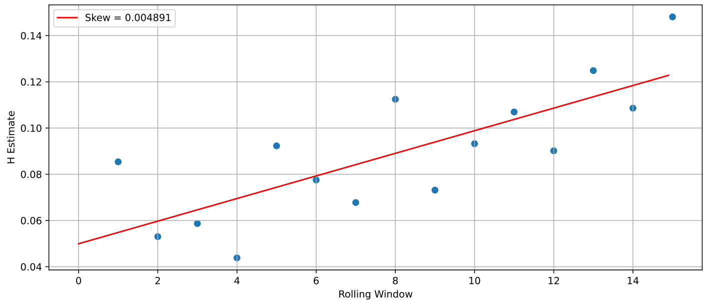

Figure 2.45: MOEX RX Equity Smoothing Effect

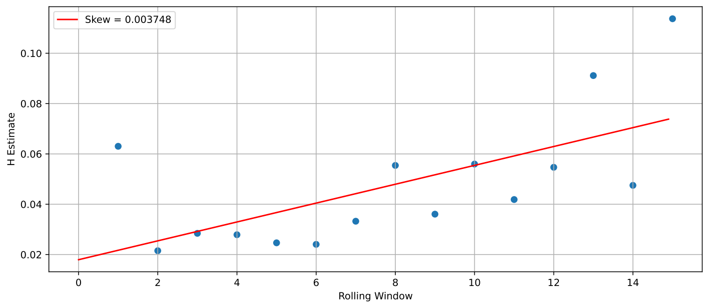

Figure 2.46: FIVE RX Equity Smoothing Effect

Δ	Shapiro-Wilk (stat)	Shapiro-Wilk (p-value)	K^2 (stat)	K^2 (p-value)	Conclusion
1	9.78331E-01	8.521E-29	4.21946E+02	2.374E-92	Not normal
2	9.87221E-01	1.827E-22	2.44050E+02	1.012E-53	Not normal
3	9.91074E-01	1.227E-18	1.66138E+02	8.388E-37	Not normal
4	9.93357E-01	8.959E-16	1.15158E+02	9.857E-26	Not normal
5	9.95637E-01	3.781E-12	7.34235E+01	1.138E-16	Not normal
6	9.97077E-01	3.510E-09	6.01998E+01	8.468E-14	Not normal
7	9.96985E-01	2.150E-09	4.91772E+01	2.096E-11	Not normal
8	9.97603E-01	6.943E-08	4.51977E+01	1.533E-10	Not normal
9	9.97847E-01	3.123E-07	3.43590E+01	3.460E-08	Not normal
10	9.98919E-01	7.683E-04	1.79663E+01	1.255E-04	Not normal
11	9.99302E-01	2.198E-02	1.02615E+01	5.912E-03	Not normal
12	9.98813E-01	3.220E-04	1.98233E+01	4.959E-05	Not normal
13	9.98313E-01	7.146E-06	2.80911E+01	7.945E-07	Not normal
14	9.98520E-01	3.258E-05	2.51641E+01	3.433E-06	Not normal
15	9.99092E-01	3.407E-03	1.24075E+01	2.022E-03	Not normal
16	9.98578E-01	5.054E-05	2.20390E+01	1.638E-05	Not normal
17	9.98762E-01	2.138E-04	1.73885E+01	1.675E-04	Not normal
18	9.98328E-01	8.030E-06	2.25733E+01	1.254E-05	Not normal
19	9.98844E-01	4.177E-04	1.91251E+01	7.031E-05	Not normal
20	9.98932E-01	8.695E-04	1.94969E+01	5.838E-05	Not normal
21	9.98787E-01	2.632E-04	2.03521E+01	3.807E-05	Not normal
22	9.98509E-01	3.033E-05	2.39327E+01	6.355E-06	Not normal
23	9.97368E-01	1.828E-08	4.51613E+01	1.561E-10	Not normal
24	9.98072E-01	1.391E-06	3.02773E+01	2.663E-07	Not normal
25	9.96688E-01	4.922E-10	5.77549E+01	2.875E-13	Not normal
26	9.96639E-01	3.875E-10	6.13147E+01	4.849E-14	Not normal
27	9.96502E-01	1.986E-10	6.18355E+01	3.738E-14	Not normal
28	9.95934E-01	1.454E-11	6.61228E+01	4.381E-15	Not normal
29	9.97702E-01	1.328E-07	4.04066E+01	1.682E-09	Not normal
30	9.96864E-01	1.220E-09	5.63125E+01	5.914E-13	Not normal
31	9.96448E-01	1.557E-10	6.58779E+01	4.952E-15	Not normal
32	9.95876E-01	1.144E-11	6.24651E+01	2.728E-14	Not normal
33	9.95495E-01	2.278E-12	7.60255E+01	3.099E-17	Not normal
34	9.95883E-01	1.185E-11	6.93021E+01	8.938E-16	Not normal
35	9.95461E-01	1.994E-12	7.52112E+01	4.657E-17	Not normal
36	9.95680E-01	4.986E-12	6.24106E+01	2.804E-14	Not normal
37	9.96425E-01	1.413E-10	5.74359E+01	3.372E-13	Not normal
38	9.97254E-01	1.001E-08	4.73976E+01	5.102E-11	Not normal
39	9.96900E-01	1.506E-09	5.13868E+01	6.942E-12	Not normal
40	9.98060E-01	1.332E-06	2.81409E+01	7.750E-07	Not normal
41	9.97998E-01	8.867E-07	3.33027E+01	5.867E-08	Not normal
42	9.97776E-01	2.150E-07	4.06532E+01	1.487E-09	Not normal
43	9.97970E-01	7.383E-07	3.31453E+01	6.347E-08	Not normal
44	9.98152E-01	2.478E-06	3.04860E+01	2.399E-07	Not normal
45	9.98049E-01	1.249E-06	3.81656E+01	5.158E-09	Not normal
46	9.98417E-01	1.614E-05	2.91195E+01	4.751E-07	Not normal
47	9.98447E-01	2.012E-05	2.85055E+01	6.458E-07	Not normal
48	9.97762E-01	2.007E-07	4.48911E+01	1.787E-10	Not normal
49	9.98279E-01	6.033E-06	3.10460E+01	1.813E-07	Not normal

Table 2.1: Normality tests for YNDX RX Equity

Δ	Shapiro-Wilk (stat)	Shapiro-Wilk (p-value)	K^2 (stat)	K^2 (p-value)	Conclusion
1	9.35813E-01	1.541E-44	1.49244E+03	0.000E+00	Not normal
2	9.46980E-01	1.658E-41	1.13069E+03	2.976E-246	Not normal
3	9.52602E-01	8.675E-40	9.65907E+02	1.803E-210	Not normal
4	9.57752E-01	4.594E-38	9.05537E+02	2.319E-197	Not normal
5	9.64613E-01	1.734E-35	7.31001E+02	1.842E-159	Not normal
6	9.69881E-01	3.170E-33	6.24109E+02	2.995E-136	Not normal
7	9.76307E-01	5.283E-30	4.83693E+02	9.276E-106	Not normal
8	9.79456E-01	3.603E-28	4.43458E+02	5.061E-97	Not normal
9	9.84334E-01	7.460E-25	3.27528E+02	7.555E-72	Not normal
10	9.88622E-01	3.169E-21	2.36699E+02	3.995E-52	Not normal
11	9.90766E-01	5.081E-19	1.93139E+02	1.149E-42	Not normal
12	9.91404E-01	2.708E-18	1.76024E+02	5.981E-39	Not normal
13	9.92096E-01	1.841E-17	1.39965E+02	4.046E-31	Not normal
14	9.94550E-01	4.942E-14	9.18491E+01	1.136E-20	Not normal
15	9.95707E-01	4.778E-12	7.06854E+01	4.476E-16	Not normal
16	9.97007E-01	2.289E-09	4.37143E+01	3.218E-10	Not normal
17	9.96503E-01	1.787E-10	4.87933E+01	2.539E-11	Not normal
18	9.97407E-01	2.100E-08	3.22999E+01	9.687E-08	Not normal
19	9.97804E-01	2.276E-07	3.01283E+01	2.869E-07	Not normal
20	9.97415E-01	2.206E-08	3.40207E+01	4.097E-08	Not normal
21	9.97341E-01	1.446E-08	3.39454E+01	4.255E-08	Not normal
22	9.97780E-01	1.969E-07	2.82951E+01	7.175E-07	Not normal
23	9.97875E-01	3.600E-07	2.73240E+01	1.166E-06	Not normal
24	9.97805E-01	2.313E-07	2.97587E+01	3.451E-07	Not normal
25	9.97410E-01	2.166E-08	2.70730E+01	1.322E-06	Not normal
26	9.97560E-01	5.209E-08	1.46167E+01	6.699E-04	Not normal
27	9.97296E-01	1.140E-08	2.54806E+01	2.931E-06	Not normal
28	9.97741E-01	1.569E-07	2.50250E+01	3.680E-06	Not normal
29	9.97797E-01	2.221E-07	3.48181E+01	2.750E-08	Not normal
30	9.96929E-01	1.579E-09	5.98787E+01	9.943E-14	Not normal
31	9.96941E-01	1.683E-09	6.13445E+01	4.778E-14	Not normal
32	9.97385E-01	1.913E-08	5.86757E+01	1.814E-13	Not normal
33	9.97456E-01	2.877E-08	4.04238E+01	1.668E-09	Not normal
34	9.97256E-01	9.293E-09	4.68076E+01	6.853E-11	Not normal
35	9.97127E-01	4.582E-09	3.79309E+01	5.800E-09	Not normal
36	9.97779E-01	2.028E-07	2.57000E+01	2.626E-06	Not normal
37	9.96953E-01	1.821E-09	3.35454E+01	5.196E-08	Not normal
38	9.96201E-01	4.557E-11	3.44499E+01	3.306E-08	Not normal
39	9.95247E-01	7.724E-13	5.28028E+01	3.420E-12	Not normal
40	9.94196E-01	1.519E-14	6.50607E+01	7.451E-15	Not normal
41	9.93579E-01	1.882E-15	8.07111E+01	2.977E-18	Not normal
42	9.92698E-01	1.193E-16	1.11106E+02	7.477E-25	Not normal
43	9.92984E-01	2.862E-16	1.14048E+02	1.718E-25	Not normal
44	9.92021E-01	1.673E-17	1.18711E+02	1.668E-26	Not normal
45	9.91370E-01	2.811E-18	1.44601E+02	3.983E-32	Not normal
46	9.91957E-01	1.410E-17	1.33967E+02	8.116E-30	Not normal
47	9.92470E-01	6.180E-17	1.30649E+02	4.266E-29	Not normal
48	9.90678E-01	4.710E-19	1.66767E+02	6.125E-37	Not normal
49	9.91692E-01	6.837E-18	1.66856E+02	5.857E-37	Not normal

Table 2.2: Normality tests for SBER RX Equity

Δ	Shapiro-Wilk (stat)	Shapiro-Wilk (p-value)	K^2 (stat)	K^2 (p-value)	Conclusion
1	9.65738E-01	7.232E-35	7.35050E+02	2.432E-160	Not normal
2	9.71389E-01	2.284E-32	6.49920E+02	7.440E-142	Not normal
3	9.79383E-01	4.481E-28	4.23467E+02	1.110E-92	Not normal
4	9.84988E-01	3.171E-24	3.30597E+02	1.628E-72	Not normal
5	9.85583E-01	9.374E-24	3.61521E+02	3.138E-79	Not normal
6	9.89094E-01	1.194E-20	2.07574E+02	8.430E-46	Not normal
7	9.88989E-01	9.445E-21	2.15389E+02	1.694E-47	Not normal
8	9.92174E-01	2.928E-17	1.53824E+02	3.960E-34	Not normal
9	9.93036E-01	3.810E-16	1.49945E+02	2.754E-33	Not normal
10	9.94618E-01	7.853E-14	1.16543E+02	4.932E-26	Not normal
11	9.97350E-01	1.758E-08	5.17100E+01	5.906E-12	Not normal
12	9.95967E-01	1.801E-11	8.29198E+01	9.867E-19	Not normal
13	9.96077E-01	2.950E-11	8.25761E+01	1.172E-18	Not normal
14	9.96054E-01	2.665E-11	7.90164E+01	6.947E-18	Not normal
15	9.97548E-01	5.559E-08	4.81896E+01	3.434E-11	Not normal
16	9.97226E-01	8.895E-09	5.08659E+01	9.007E-12	Not normal
17	9.98737E-01	1.866E-04	1.92402E+01	6.638E-05	Not normal
18	9.98414E-01	1.600E-05	2.86373E+01	6.046E-07	Not normal
19	9.99264E-01	1.642E-02	7.99361E+00	1.837E-02	Not normal
20	9.98622E-01	7.653E-05	1.74282E+01	1.643E-04	Not normal
21	9.98114E-01	1.984E-06	2.40289E+01	6.056E-06	Not normal
22	9.98424E-01	1.737E-05	1.73494E+01	1.709E-04	Not normal
23	9.99005E-01	1.703E-03	8.00492E+00	1.827E-02	Not normal
24	9.99050E-01	2.512E-03	9.11081E+00	1.051E-02	Not normal
25	9.98603E-01	6.657E-05	1.57325E+01	3.835E-04	Not normal
26	9.98893E-01	6.675E-04	8.89888E+00	1.169E-02	Not normal
27	9.99189E-01	8.530E-03	9.28383E+00	9.639E-03	Not normal
28	9.99264E-01	1.658E-02	5.41437E+00	6.672E-02	Normal
29	9.99255E-01	1.523E-02	5.62308E+00	6.011E-02	Normal
30	9.99415E-01	6.361E-02	5.14056E+00	7.651E-02	Normal
31	9.99631E-01	3.774E-01	2.60396E+00	2.720E-01	Normal
32	9.99790E-01	8.805E-01	6.81491E-01	7.112E-01	Normal
33	9.99692E-01	5.652E-01	3.60012E+00	1.653E-01	Normal
34	9.99200E-01	9.473E-03	4.29943E+00	1.165E-01	Normal
35	9.99333E-01	3.091E-02	2.46657E+00	2.913E-01	Normal
36	9.98552E-01	4.608E-05	1.67350E+01	2.323E-04	Not normal
37	9.98926E-01	8.949E-04	7.13522E+00	2.822E-02	Not normal
38	9.98399E-01	1.496E-05	1.43961E+01	7.481E-04	Not normal
39	9.98416E-01	1.696E-05	2.45176E+01	4.743E-06	Not normal
40	9.98278E-01	6.341E-06	2.15475E+01	2.094E-05	Not normal
41	9.97137E-01	5.840E-09	5.18449E+01	5.521E-12	Not normal
42	9.97576E-01	6.999E-08	4.07611E+01	1.409E-09	Not normal
43	9.98056E-01	1.409E-06	2.66872E+01	1.603E-06	Not normal
44	9.97798E-01	2.705E-07	3.80213E+01	5.543E-09	Not normal
45	9.95813E-01	1.015E-11	7.62067E+01	2.831E-17	Not normal
46	9.95736E-01	7.323E-12	8.19944E+01	1.567E-18	Not normal
47	9.95476E-01	2.481E-12	9.05709E+01	2.152E-20	Not normal
48	9.95270E-01	1.079E-12	8.31723E+01	8.697E-19	Not normal
49	9.95130E-01	6.226E-13	8.16565E+01	1.856E-18	Not normal

Table 2.3: Normality tests for VTBR RX Equity

Δ	Shapiro-Wilk (stat)	Shapiro-Wilk (p-value)	K^2 (stat)	K^2 (p-value)	Conclusion
1	9.86253E-01	3.280E-23	2.96230E+02	4.727E-65	Not normal
2	9.87673E-01	5.518E-22	2.53330E+02	9.773E-56	Not normal
3	9.89920E-01	8.135E-20	2.08311E+02	5.833E-46	Not normal
4	9.91729E-01	8.373E-18	1.65629E+02	1.082E-36	Not normal
5	9.94132E-01	1.375E-14	1.14501E+02	1.369E-25	Not normal
6	9.95203E-01	7.309E-13	9.26978E+01	7.429E-21	Not normal
7	9.95254E-01	8.981E-13	9.56711E+01	1.680E-21	Not normal
8	9.95090E-01	4.712E-13	1.09261E+02	1.880E-24	Not normal
9	9.96455E-01	1.692E-10	7.12182E+01	3.429E-16	Not normal
10	9.97602E-01	7.606E-08	4.66476E+01	7.423E-11	Not normal
11	9.97015E-01	2.845E-09	5.45817E+01	1.405E-12	Not normal
12	9.97146E-01	5.737E-09	6.12382E+01	5.038E-14	Not normal
13	9.97251E-01	1.025E-08	5.76060E+01	3.098E-13	Not normal
14	9.97845E-01	3.422E-07	4.35574E+01	3.480E-10	Not normal
15	9.97003E-01	2.699E-09	5.72794E+01	3.647E-13	Not normal
16	9.97793E-01	2.485E-07	4.14855E+01	9.807E-10	Not normal
17	9.97878E-01	4.243E-07	3.81333E+01	5.242E-09	Not normal
18	9.98191E-01	3.337E-06	3.21994E+01	1.019E-07	Not normal
19	9.98892E-01	6.580E-04	1.61855E+01	3.057E-04	Not normal
20	9.98923E-01	8.542E-04	1.16782E+01	2.911E-03	Not normal
21	9.98554E-01	4.585E-05	1.98113E+01	4.989E-05	Not normal
22	9.99001E-01	1.655E-03	1.57101E+01	3.878E-04	Not normal
23	9.98733E-01	1.840E-04	1.98965E+01	4.781E-05	Not normal
24	9.98817E-01	3.600E-04	1.92406E+01	6.637E-05	Not normal
25	9.98035E-01	1.186E-06	3.38480E+01	4.467E-08	Not normal
26	9.98353E-01	1.059E-05	2.56078E+01	2.750E-06	Not normal
27	9.98983E-01	1.435E-03	1.35009E+01	1.170E-03	Not normal
28	9.99120E-01	4.648E-03	1.53126E+01	4.731E-04	Not normal
29	9.98133E-01	2.303E-06	3.41161E+01	3.906E-08	Not normal
30	9.98871E-01	5.665E-04	1.91141E+01	7.070E-05	Not normal
31	9.98862E-01	5.234E-04	2.15337E+01	2.109E-05	Not normal
32	9.99436E-01	7.651E-02	9.07413E+00	1.070E-02	Normal
33	9.99371E-01	4.320E-02	1.00014E+01	6.733E-03	Not normal
34	9.99260E-01	1.615E-02	1.26737E+01	1.770E-03	Not normal
35	9.99227E-01	1.199E-02	7.20330E+00	2.728E-02	Not normal
36	9.98581E-01	5.747E-05	2.84615E+01	6.602E-07	Not normal
37	9.97641E-01	1.025E-07	4.68187E+01	6.815E-11	Not normal
38	9.96837E-01	1.209E-09	6.58118E+01	5.119E-15	Not normal
39	9.98318E-01	8.449E-06	2.93353E+01	4.265E-07	Not normal
40	9.97480E-01	4.005E-08	5.20385E+01	5.012E-12	Not normal
41	9.98181E-01	3.278E-06	3.50333E+01	2.470E-08	Not normal
42	9.98010E-01	1.050E-06	3.76342E+01	6.727E-09	Not normal
43	9.98705E-01	1.522E-04	2.39758E+01	6.219E-06	Not normal
44	9.98891E-01	6.788E-04	2.05627E+01	3.427E-05	Not normal
45	9.97689E-01	1.401E-07	4.19281E+01	7.860E-10	Not normal
46	9.98266E-01	5.947E-06	3.34811E+01	5.366E-08	Not normal
47	9.98035E-01	1.243E-06	3.95202E+01	2.620E-09	Not normal
48	9.97928E-01	6.234E-07	3.81473E+01	5.205E-09	Not normal
49	9.98907E-01	7.801E-04	1.76200E+01	1.492E-04	Not normal

Table 2.4: Normality tests for MOEX RX Equity

Δ	Shapiro-Wilk (stat)	Shapiro-Wilk (p-value)	K^2 (stat)	K^2 (p-value)	Conclusion
1	9.68911E-01	1.660E-33	7.20790E+02	3.037E-157	Not normal
2	9.71463E-01	2.492E-32	5.90572E+02	5.739E-129	Not normal
3	9.75711E-01	3.532E-30	4.26724E+02	2.178E-93	Not normal
4	9.79322E-01	4.153E-28	4.19047E+02	1.012E-91	Not normal
5	9.84734E-01	2.041E-24	3.13716E+02	7.542E-69	Not normal
6	9.86408E-01	4.474E-23	2.89088E+02	1.680E-63	Not normal
7	9.90080E-01	1.203E-19	2.01964E+02	1.393E-44	Not normal
8	9.91266E-01	2.421E-18	2.06180E+02	1.693E-45	Not normal
9	9.91447E-01	3.940E-18	1.84416E+02	9.009E-41	Not normal
10	9.89510E-01	3.160E-20	2.35492E+02	7.305E-52	Not normal
11	9.94163E-01	1.554E-14	1.24524E+02	9.121E-28	Not normal
12	9.93787E-01	4.326E-15	1.46109E+02	1.875E-32	Not normal
13	9.93911E-01	6.571E-15	1.38988E+02	6.595E-31	Not normal
14	9.93950E-01	7.543E-15	1.46965E+02	1.222E-32	Not normal
15	9.93708E-01	3.355E-15	1.37426E+02	1.440E-30	Not normal
16	9.95751E-01	7.134E-12	9.03277E+01	2.430E-20	Not normal
17	9.96952E-01	2.065E-09	5.74098E+01	3.417E-13	Not normal
18	9.96794E-01	9.177E-10	6.44851E+01	9.937E-15	Not normal
19	9.97277E-01	1.193E-08	4.13211E+01	1.065E-09	Not normal
20	9.96615E-01	3.774E-10	5.59601E+01	7.054E-13	Not normal
21	9.97329E-01	1.602E-08	3.70555E+01	8.985E-09	Not normal
22	9.97721E-01	1.600E-07	3.96039E+01	2.513E-09	Not normal
23	9.97719E-01	1.591E-07	3.68208E+01	1.010E-08	Not normal
24	9.97401E-01	2.431E-08	3.82352E+01	4.981E-09	Not normal
25	9.96982E-01	2.463E-09	5.46063E+01	1.388E-12	Not normal
26	9.96995E-01	2.649E-09	5.77449E+01	2.890E-13	Not normal
27	9.97392E-01	2.321E-08	4.11806E+01	1.142E-09	Not normal
28	9.97077E-01	4.101E-09	5.16604E+01	6.055E-12	Not normal
29	9.97912E-01	5.382E-07	3.21822E+01	1.027E-07	Not normal
30	9.97734E-01	1.764E-07	3.65232E+01	1.172E-08	Not normal
31	9.97191E-01	7.662E-09	4.69101E+01	6.510E-11	Not normal
32	9.98501E-01	3.134E-05	2.15089E+01	2.135E-05	Not normal
33	9.98792E-01	2.966E-04	1.40109E+01	9.069E-04	Not normal
34	9.98960E-01	1.192E-03	1.15583E+01	3.091E-03	Not normal
35	9.98922E-01	8.686E-04	1.34568E+01	1.196E-03	Not normal
36	9.98049E-01	1.331E-06	3.33640E+01	5.690E-08	Not normal
37	9.98014E-01	1.060E-06	3.78066E+01	6.172E-09	Not normal
38	9.98013E-01	1.054E-06	3.41280E+01	3.883E-08	Not normal
39	9.96742E-01	7.485E-10	6.64146E+01	3.787E-15	Not normal
40	9.96384E-01	1.313E-10	6.64622E+01	3.698E-15	Not normal
41	9.95934E-01	1.702E-11	8.70810E+01	1.232E-19	Not normal
42	9.96809E-01	1.053E-09	6.08614E+01	6.083E-14	Not normal
43	9.96112E-01	3.782E-11	7.46695E+01	6.106E-17	Not normal
44	9.96157E-01	4.657E-11	7.53412E+01	4.364E-17	Not normal
45	9.96278E-01	8.128E-11	7.58787E+01	3.335E-17	Not normal
46	9.95567E-01	3.604E-12	8.10987E+01	2.453E-18	Not normal
47	9.94378E-01	3.751E-14	1.08925E+02	2.224E-24	Not normal
48	9.94402E-01	4.100E-14	1.13494E+02	2.265E-25	Not normal
49	9.94118E-01	1.516E-14	1.08391E+02	2.905E-24	Not normal

Table 2.5: Normality tests for LKOH RX Equity

Δ	Shapiro-Wilk (stat)	Shapiro-Wilk (p-value)	K^2 (stat)	K^2 (p-value)	Conclusion
1	9.56049E-01	1.783E-38	1.06426E+03	7.904E-232	Not normal
2	9.61786E-01	2.017E-36	8.98275E+02	8.749E-196	Not normal
3	9.67511E-01	4.101E-34	7.02403E+02	2.986E-153	Not normal
4	9.73549E-01	2.647E-31	5.74303E+02	1.958E-125	Not normal
5	9.76946E-01	1.705E-29	4.72138E+02	2.996E-103	Not normal
6	9.81281E-01	7.299E-27	4.06361E+02	5.752E-89	Not normal
7	9.84266E-01	9.098E-25	3.31414E+02	1.082E-72	Not normal
8	9.86635E-01	7.001E-23	2.94586E+02	1.075E-64	Not normal
9	9.89172E-01	1.447E-20	2.45921E+02	3.970E-54	Not normal
10	9.91127E-01	1.692E-18	1.95091E+02	4.331E-43	Not normal
11	9.92274E-01	3.957E-17	1.63712E+02	2.820E-36	Not normal
12	9.92413E-01	5.935E-17	1.49801E+02	2.958E-33	Not normal
13	9.94956E-01	2.831E-13	9.53055E+01	2.017E-21	Not normal
14	9.95707E-01	5.883E-12	7.78854E+01	1.223E-17	Not normal
15	9.96084E-01	3.067E-11	7.48638E+01	5.540E-17	Not normal
16	9.97206E-01	8.010E-09	5.03216E+01	1.183E-11	Not normal
17	9.97038E-01	3.244E-09	4.70147E+01	6.179E-11	Not normal
18	9.97482E-01	3.829E-08	3.87159E+01	3.917E-09	Not normal
19	9.97719E-01	1.574E-07	3.03331E+01	2.590E-07	Not normal
20	9.97015E-01	2.888E-09	3.89996E+01	3.399E-09	Not normal
21	9.97399E-01	2.382E-08	3.65644E+01	1.149E-08	Not normal
22	9.96733E-01	6.830E-10	4.25579E+01	5.737E-10	Not normal
23	9.97563E-01	6.210E-08	3.22277E+01	1.004E-07	Not normal
24	9.97895E-01	4.771E-07	2.87768E+01	5.639E-07	Not normal
25	9.97505E-01	4.447E-08	2.47545E+01	4.213E-06	Not normal
26	9.97348E-01	1.804E-08	2.32894E+01	8.765E-06	Not normal
27	9.97913E-01	5.397E-07	2.41669E+01	5.652E-06	Not normal
28	9.98727E-01	1.754E-04	1.27265E+01	1.724E-03	Not normal
29	9.98934E-01	9.485E-04	1.29167E+01	1.567E-03	Not normal
30	9.98897E-01	6.994E-04	1.15337E+01	3.130E-03	Not normal
31	9.99183E-01	8.091E-03	1.26717E+01	1.772E-03	Not normal
32	9.99203E-01	9.687E-03	1.17127E+01	2.862E-03	Not normal
33	9.99607E-01	3.172E-01	4.10760E+00	1.282E-01	Normal
34	9.99541E-01	1.890E-01	2.43257E+00	2.963E-01	Normal
35	9.99309E-01	2.490E-02	2.24129E+00	3.261E-01	Normal
36	9.99159E-01	6.578E-03	5.40324E+00	6.710E-02	Normal
37	9.98308E-01	7.785E-06	1.57270E+01	3.845E-04	Not normal
38	9.97952E-01	7.093E-07	2.03002E+01	3.907E-05	Not normal
39	9.97223E-01	9.310E-09	2.57321E+01	2.584E-06	Not normal
40	9.95850E-01	1.177E-11	4.52170E+01	1.518E-10	Not normal
41	9.95429E-01	2.014E-12	6.20961E+01	3.281E-14	Not normal
42	9.95829E-01	1.081E-11	6.30434E+01	2.043E-14	Not normal
43	9.96428E-01	1.636E-10	5.93910E+01	1.269E-13	Not normal
44	9.95292E-01	1.168E-12	8.94902E+01	3.694E-20	Not normal
45	9.94243E-01	2.314E-14	1.19861E+02	9.386E-27	Not normal
46	9.94167E-01	1.776E-14	1.18459E+02	1.892E-26	Not normal
47	9.94912E-01	2.678E-13	9.61789E+01	1.303E-21	Not normal
48	9.93552E-01	2.264E-15	1.21170E+02	4.878E-27	Not normal
49	9.94539E-01	6.744E-14	9.76440E+01	6.264E-22	Not normal

Table 2.6: Normality tests for GAZP RX Equity

Appendix B. Estimation Code.

```
def rlz_vol_est(df: pd.DataFrame,
                count: int,
                rolling_window: int=1) -> np.ndarray:
    log_returns = np.zeros(int(df.shape[0]/rolling_window))
    for i in range(1, log_returns.size):
        log_returns[i] =
                math.log(df["Mean"][i*rolling_window]/
                                    df["Mean"][(i-1)*rolling_window])

    rlz_vol = np.zeros(int(log_returns.size/count))

    for i in range(rlz_vol.size):
        lr_n = np.zeros(count)
        for n in range(count):
            lr_n[n] = log_returns[i*count+n]

        tmp = 0.0
        for j in range(1, lr_n.size):
            tmp += (lr_n[j] - lr_n[j-1])**2

        rlz_vol[i] = math.sqrt(tmp)

    return rlz_vol

def hurst_estimation(name: str,
                     mode: str = 'yf',
                     rolling_window: int = 1,
                     show_pics = True,
                     save_pics = False):
    if mode == 'yf':
        count = days_count
        df = yf.download(name, '2000-01-01', '2019-01-01')
        df["Mean"] = 0.5*(df["Open"]+df["Close"])

    elif mode == 'bb':
        count = minutes_count
        df = pd.read_csv('data_bloomberg/'+name+'.csv', sep="\t")
        df["Mean"] = 0.5*(df["High"]+df["Low"])

    volatility_array = rlz_vol_est(df = df,
                                   count = count,
                                   rolling_window = rolling_window)
    zetaq            = np.zeros((2, num_of_q))

    for I in range(0, num_of_q):
        graph_data = np.zeros((2, pD-sD))
        q          = step_of_q*(1+I)
        line_start = math.log(sD)
        line_stop  = math.log(pD)

        for Delta in range(sD, pD):
            graph_data[0, Delta-sD] = math.log(Delta)
            graph_data[1, Delta-sD] = math.log(m(q, Delta, volatility_array))

        linear_model    = np.polyfit(graph_data[0],graph_data[1], 1)
        linear_model_fn = np.poly1d(linear_model)
        x_s             = np.arange(line_start, line_stop, 0.1)
```

```
        skew_of_linear_model = skew(line_start,
                                    line_stop,
                                    linear_model_fn(line_start),
                                    linear_model_fn(line_stop))

        zetaq[0, I] = q
        zetaq[1, I] = skew_of_linear_model

    linear_model_H    = np.polyfit(zetaq[0], zetaq[1], 1)
    linear_model_H_fn = np.poly1d(linear_model_H)
    x_s               = np.arange(0, step_of_q*(num_of_q+1), step_of_q)

    H_est = skew(0,
                 step_of_q*(num_of_q)+1,
                 linear_model_H_fn(0),
                 linear_model_H_fn(step_of_q*(num_of_q)+1))

    sz = 40
    graph_data = np.zeros((2, sz))

    for Delta in range(1, sz+1):
        graph_data[0, Delta-1] = Delta**(2*H_est)
        graph_data[1, Delta-1] = ACov(volatility_array, Delta)

    linear_model    = np.polyfit(graph_data[0],graph_data[1], 1)
    linear_model_fn = np.poly1d(linear_model)
    x_s             = np.arange(1, (sz+1)**(2*H_est), 0.1)

    for Delta in range(1, sz+1):
        graph_data[0, Delta-1] = math.log(Delta)

    linear_model    = np.polyfit(graph_data[0],graph_data[1], 1)
    linear_model_fn = np.poly1d(linear_model)
    x_s             = np.arange(0, math.log(sz+1), 0.1)

    def lag_array(Delta):
        retarr = np.zeros(volatility_array.size - Delta)
        if Delta >= 0:
            for i in range(0, volatility_array.size-Delta):
                retarr[i] = np.log(volatility_array[i+Delta]) -
                                        np.log(volatility_array[i])
        else:
            for i in range(0, volatility_array.size-math.abs(Delta)):
                retarr[i] = np.log(volatility_array[i]) -
                                        np.log(volatility_array[i-Delta])

        retarr = retarr/retarr.max()
        return retarr

    return H_est

def f(theta):
    return (1/((2*H+1)*(2*H+2)*theta**2)*((1+theta)**(2*H+2) - 2
                   - 2 * theta**(2*H+2) + (1-theta)**(2*H+2)))

def smoothing_theoretical(delta: float):
    num_of_Deltas = 200
    plot = np.zeros((2, num_of_Deltas))
```

```
    Delta = np.arange(1, num_of_Deltas+1, 1)
    plot[0] = np.log(Delta)
    plot[1] = np.log(Delta**(2*H) * f(delta/Delta))

    linear_model    = np.polyfit(plot[0],plot[1], 1)
    linear_model_fn = np.poly1d(linear_model)
    x_s             = np.arange(0, 5, 0.1)

    print(skew(0, 1, linear_model_fn(0), linear_model_fn(1))*0.5)
    print(skew(0, 1, linear_model_fn(0), linear_model_fn(1))*0.5/H - 1)

def smoothing_empirical(name: str, show_pics: bool=True):
    num_of_wind = 20
    graph_data = np.zeros((2, num_of_wind))
    for i in range(1, num_of_wind+1):
        graph_data[0, i-1] = i
        graph_data[1, i-1] = analyse_volatility(name=name,
                                                mode='bb',
                                                rolling_window=i,
                                                show_pics=False)

    return [np.mean(graph_data[1]),
            np.std(graph_data[1]),
            np.min(graph_data[1])]
```