3D − n_e-el. QD

$$H = \frac{1}{2m^*}\sum_{i=1}^{n_e}(\vec{p}_i - e\vec{A}_i)^2 + \frac{1}{2}m^*\omega_0^2\sum_{i=1}^{n_e}\rho_i^2 + \frac{1}{2}m^*\omega_z^2\sum_{i=1}^{n_e} z_i^2 + \frac{e^2}{4\pi\varepsilon_0\varepsilon_r}\sum_{i<j}^{1,n_e}\frac{1}{r_{ij}}$$

$$= \frac{1}{2m^*}\sum_{i=1}^{n_e}\vec{p}_i^{\,2} + \frac{1}{2}m^*\left(\Omega^2\sum_{i=1}^{n_e}\rho_i^2 + \omega_z^2\sum_{i=1}^{n_e} z_i^2\right) + \frac{e^2}{4\pi\varepsilon_0\varepsilon_r}\sum_{i<j}^{1,n_e}\frac{1}{r_{ij}} - \omega_L L_z$$

$$\Omega^2 = \omega_0^2 + \omega_L^2$$

If $\tilde{\vec{r}}_i = \vec{r}_i/l_0$, $\tilde{\vec{p}}_i = \vec{p}_i l_0/\hbar$, $l_0 = (\hbar/m^*\omega_0)^{1/2}$

$$\mathcal{H} \equiv \frac{H}{\hbar\omega_0} = \frac{1}{2}\sum_{i=1}^{n_e}\left(\tilde{p}_i^2 + \tilde{\Omega}^2\tilde{\rho}_i^2 + \tilde{\omega}_z^2\tilde{z}_i^2\right) + k\sum_{i<j}^{1,n_e}\frac{1}{\tilde{r}_{ij}} - \tilde{\omega}_L M$$

Noninteracting case: $k=0$

$$\mathcal{H}_0 = \sum_{i=1}^{n_e} h_i,$$

$$h_i = \frac{1}{2}\left(\tilde{p}_i^2 + \tilde{\Omega}^2\tilde{\rho}_i^2 + \tilde{\omega}_z^2\tilde{z}_i^2\right) - \tilde{\omega}_L m_i$$

$$\epsilon_{n_{\rho i}, m_i, n_{zi}} = \tilde{\Omega}(2n_{\rho i} + |m_i| + 1) + \tilde{\omega}_z(n_{zi} + \tfrac{1}{2}) - \tilde{\omega}_L m_i$$

$$\psi_{n_{\rho i}, m_i, n_{zi}}(\tilde{\vec{r}}_i) = f_{n_{\rho i}, m_i}(\tilde{\rho}_i)\, g_{n_{zi}}(\tilde{z}_i)\,\frac{e^{i m_i \varphi_i}}{\sqrt{2\pi}}$$

$$f_{n_{\rho i}, m_i}(\tilde{\rho}_i) = \sqrt{\frac{2\tilde{\Omega}\, n_{\rho i}!}{(n_{\rho i}+|m_i|)!}}\, x_i^{|m_i|}\, e^{-\frac{1}{2}x_i^2}\, L_{n_i}^{|m_i|}(x_i^2), \quad x_i = \tilde{\Omega}^{\frac{1}{2}}\tilde{\rho}_i,$$

$$g_{n_{zi}}(\tilde{z}_i) = \frac{(\tilde{\omega}_z/\pi)^{1/4}}{\sqrt{2^{n_{zi}}\, n_{zi}!}}\, e^{-\frac{1}{2}y_i^2} H_{n_{zi}}(y_i), \quad y_i = \tilde{\omega}_z^{\frac{1}{2}}\tilde{z}_i.$$

Then:

$$E_\alpha^{(0)} = \sum_{i=1}^{n_e}\epsilon_{n_{\rho i}, m_i}, \quad \Psi_\alpha^{(0)}(\tilde{\vec{r}}_1, \ldots, \tilde{\vec{r}}_{n_e}) = \prod_{i=1}^{n_e}\psi_{n_{\rho i}, m_i, n_{zi}}(\tilde{\vec{r}}_i),$$

$$\alpha = \{n_{\rho 1}, m_1, n_{z1}, \ldots, n_{\rho n_e}, m_{n_e}, n_{z n_e}\}.$$