Drawing a spiral

Stefan Kurtz

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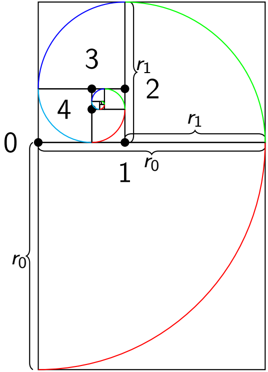
Drawing a spiral from quarter circles (1/4)

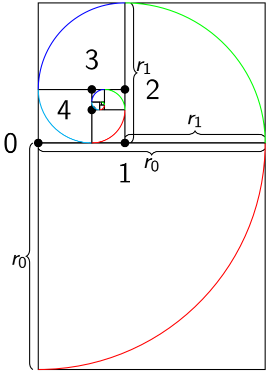
- a spiral can be split into many quarter circles with continuously reduced radii
- in the figure on the right start with a quarter circle in the south east quarter (quarter 0)
- the center of this red quarter circle is at the small dot marked by 0 (call it center 0)
- assumption:
  - radius of first quarter circle is r_0 and
  - there is a constant factor d, by which we reduce the radius in each step



- we continue the spiral in the north east quarter (quarter 1) by the green quarter circle with radius $r_1 = \frac{r_0}{d}$
- Y-coordinate of center 1 does not change relative to center 0
- but on the X-axes we place center 1 at distance $r_0 - r_1 = r_0 - \frac{r_0}{d} = r_0 \cdot \left(1 - \frac{1}{d}\right)$ to the right of center 0

Drawing a spiral from quarter circles (2/4)

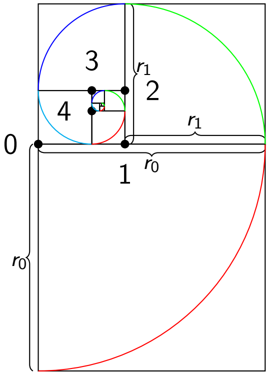
- the next quarter is the north west quarter (quarter 2)
- the corresponding blue quarter circle has center 2
- its radius is $r_2 = \frac{r_1}{d}$ and so place center 2 at distance $r_1 - r_2 = r_1 - \frac{r_1}{d} = r_1 \cdot \left(1 - \frac{1}{d}\right)$ above of center 1
- so the X-coordinate does not change relative to center 1



- the last quarter is the south west quarter (quarter 3) with center 3 and the cyan quarter circle
- the radius of this circle is $r_3 = \frac{r_2}{d}$, so we place it to the left of center 2 (same Y-coordinate) at distance $r_2 - r_3 = r_2 - \frac{r_2}{d} = r_2 \cdot \left(1 - \frac{1}{d}\right)$ to the right of center 2

Drawing a spiral from quarter circles (3/4)

- following quarter 3 we next consider quarter 0 again and draw a circle around center 4 with radius $r_4 = \frac{r_3}{d}$.
- center 4 is placed below center 3 at distance $r_3 - r_4 = r_3 - \frac{r_3}{d} = r_3 \cdot \left(1 - \frac{1}{d}\right)$.
- to summarize we move from quarter to quarter in counterclockwise order beginning with quarter 0
- in each step we draw a quarter circle of radius $r_{i+1} = \frac{r_i}{d}$ around a center which we obtain from the previous center ...



... by adding to or subtracting the X- or Y-coordinate the distance value $r_i - r_{i+1} = r_i \left(1 - \frac{1}{d}\right)$.

- the following table gives the detail where (x_i, y_i) is the center of the circle in quarter i

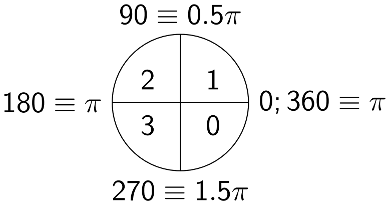
Drawing a spiral from quarter circles (4/4)

from quarter i	to quarter $(i+1) \bmod 4$	compute center for quarter circle in quarter $(i+1) \bmod 4$
0 (south east)	1 (north east)	$(x_{i+1}, y_{i+1}) = \left(x_i + r_i \left(1 - \frac{1}{d}\right), y_i\right)$
1 (north east)	2 (north west)	$(x_{i+1}, y_{i+1}) = \left(x_i, y_i + r_i \left(1 - \frac{1}{d}\right)\right)$
2 (north west)	3 (south west)	$(x_{i+1}, y_{i+1}) = \left(x_i - r_i \left(1 - \frac{1}{d}\right), y_i\right)$
3 (south west)	0 (south east)	$(x_{i+1}, y_{i+1}) = \left(x_i, y_i - \left(1 - \frac{1}{d}\right)\right)$

Drawing a quarter circle (1/2)

- now consider the remaining issue: how to draw a quarter circle
- we only have four different quarters, numbered from 0 to 3
- to draw a quarter circle, we need to specify a pair of angles:
  - a start angle where the quarter circle begins
  - an end angle, where the quarter circle ends
- these pairs are given in the following table

quarter	start angle		end angle	
	degree	radians	degree	radians
0	270	1.5π	360	2π
1	0	0	90	0.5π
2	90	0.5π	180	π
3	180	π	270	1.5π



Drawing a quarter circle (2/2)

- suppose we are given a center (c_x, c_y) and a radius r
- let α be the start angle and ω be the end angle of the quarter we have to draw the quarter circle in
- then the quarter circle consists of all points (x, y) with

$$
\begin{aligned}
x =& c_x + r \cdot \cos(a) \\
y =& c_y + r \cdot \sin(a)
\end{aligned}
$$

for all a, $\alpha \leq a \leq \omega$.