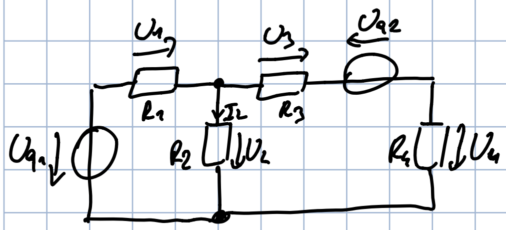
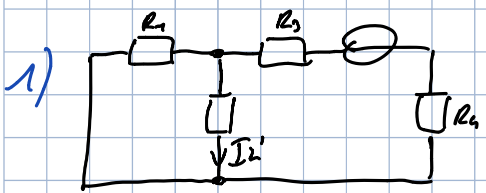
Wd.: Helmholz

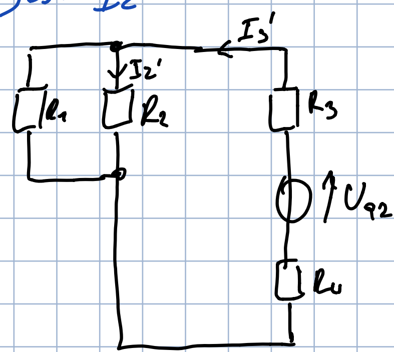


$U_{q1} = 10V$
$U_{q2} = 20V$
$R_1 = R_2 = 1k\Omega$
$R_3 = R_4 = 2k\Omega$

ges.: I_2

1)





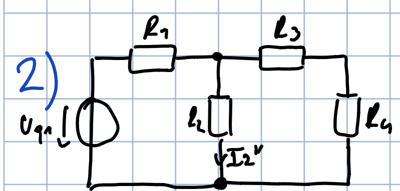
$$I_3' = \frac{U_{q2}}{\frac{R_1 \cdot R_2}{R_1 + R_2} + R_3 + R_4}$$

$$I_3' = -0{,}00\dot{4}\,A$$

$$I_3' \cdot \frac{R_1 \cdot R_2}{R_1 + R_2} = I_2' \cdot R_2 \qquad | : R_2$$

$$\underline{I_2' = -0{,}00\dot{2}\,A}$$

2)



$$I_1'' = \frac{U_{q1}}{R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}}$$

$$I_1'' = 0{,}00\dot{5}\,A$$

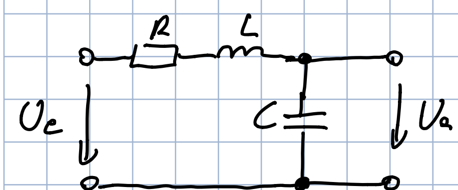
$$I_1'' \cdot \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} = I_2'' \cdot R_2 \qquad | : R_2$$

$$I_2'' = 0{,}00\dot{4}\,A$$

$$I_2 = I_2' + I_2'' = -0{,}00\dot{2} + 0{,}00\dot{4}$$

$$\underline{\underline{I_2 = 2{,}\dot{2}\,mA}}$$

Wd. Komplexe Rechnung



$f = 10^4\,Hz$
$U_e = 5V$

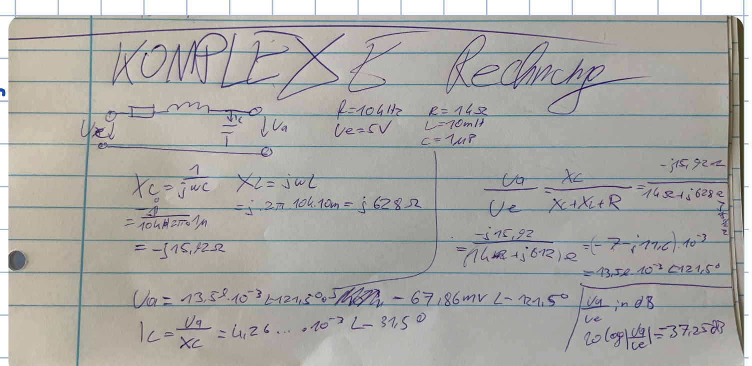
$R = 1k\Omega$
$L = 10mH$
$C = 1\mu F$

$$X_C = \frac{1}{j\omega C} = \frac{-j}{10^4 \cdot 2\pi \cdot 1\mu} = -j\,15{,}92\,\Omega$$

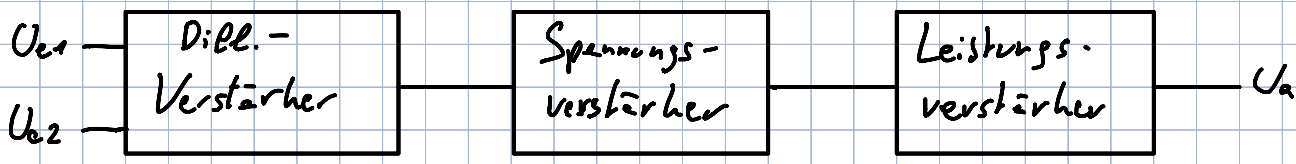
$$X_L = j\omega L = j \cdot 2\pi \cdot 10^4 \cdot 10m = j\,628\,\Omega$$

$$\frac{U_a}{U_e} = \frac{X_C}{X_C + X_L + R} = \frac{-j\,15{,}92}{1k\Omega + j\,628\Omega - j15{,}92\Omega}$$

$$= \frac{-j\,15{,}92\,\Omega}{(1k + j\,612)\,\Omega}$$

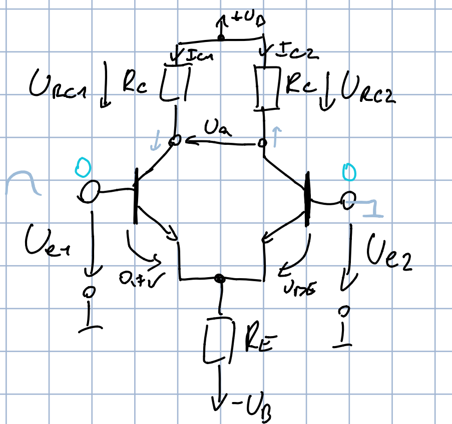


Aufbau realer OPV



Emitterschaltung

Gegentaktendstufe Kollektorschaltung



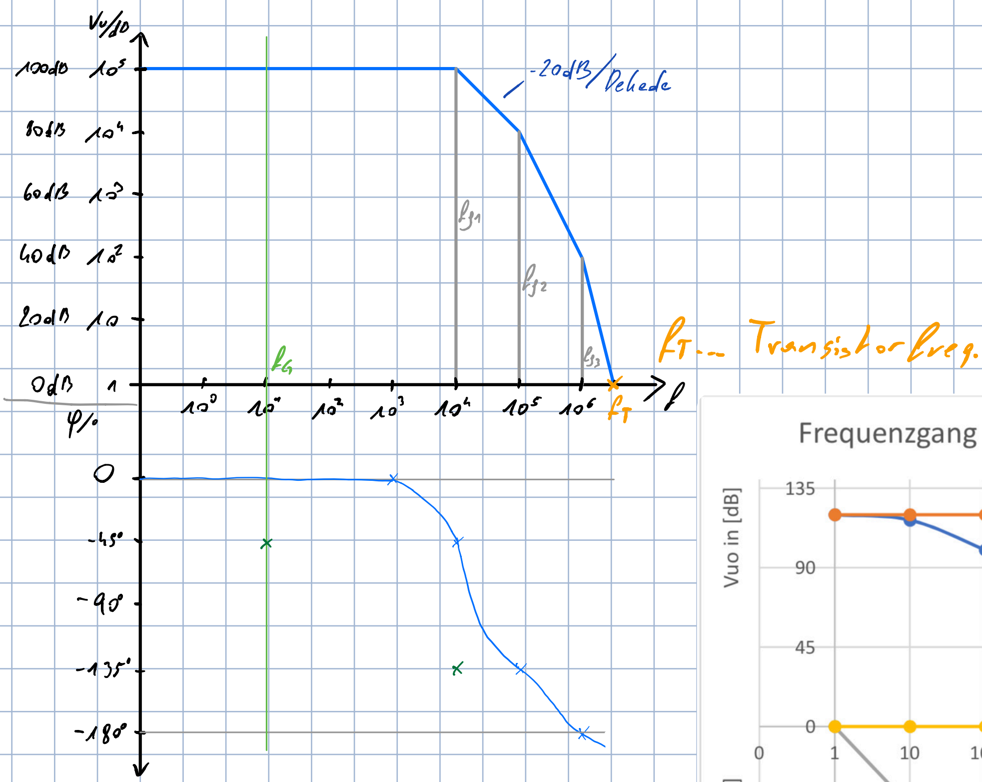
$I_{C1} > I_{C2}$ $\Rightarrow$ U_{BE2} wird kleiner $\Rightarrow$ U_{RC2} wird kleiner

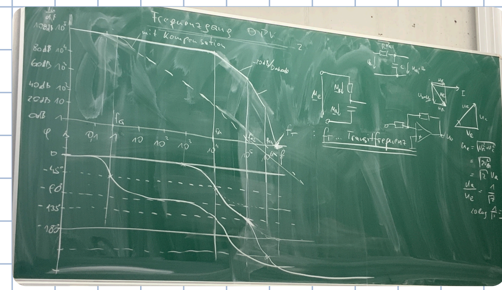
Common Mode Rejection Ratio
CMRR

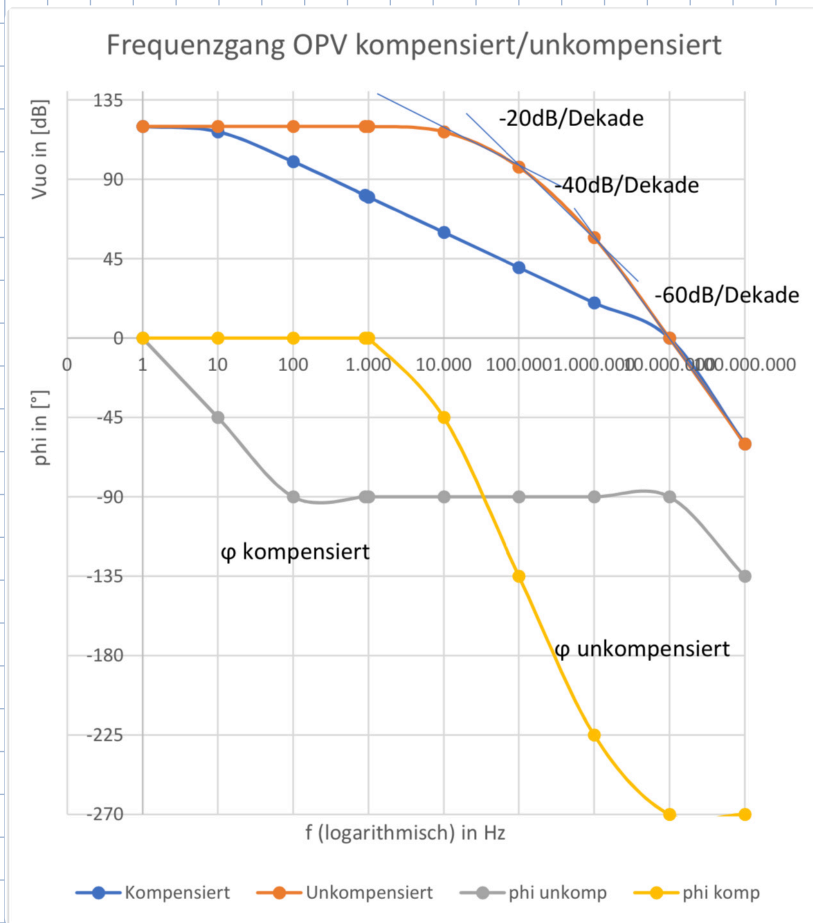
$U_a = 0$, da $U_{RC1} = U_{RC2}$
($\Rightarrow$ keine Spannungsdiff.)

U_a wird positiv

Frequenzgang OPV







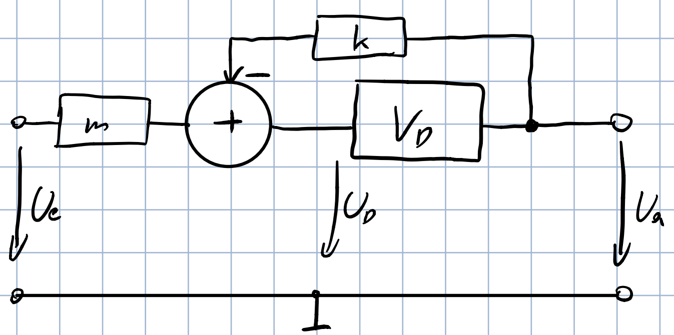
bei 180° Phasenverschiebung Problem

Verstärkungs-Bandbreiten-Produkt

$$V_{u,oc} \cdot f_o = f_T$$

Stabilitätsbetrachtung

Prinzip d. Gegenkopplung



k… Rückkopplungsvierpol
m… Einkopplungsvierpol
V_D… Differenzverstärkung

$$U_a = U_D \cdot V_D$$

$$U_D = m \cdot U_e - K \cdot U_a$$

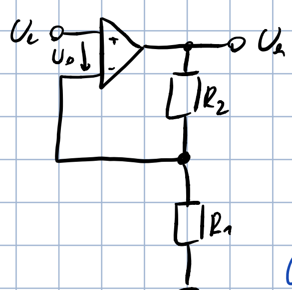
$$U_a = (m \cdot U_e - K \cdot U_a) \cdot V_D$$

$$U_a + K \cdot U_a \cdot V_D = m \cdot U_e \cdot V_D$$

$$U_a (1 + K \cdot V_D) = m \cdot U_e \cdot V_D$$

$$\frac{U_a}{U_e} = \frac{m \cdot V_D}{1 + K \cdot V_D} \qquad \left(= \frac{\text{Vorwärtsverst.}}{1 + \text{Schleifenverst.}} \right)$$

$$\frac{U_a}{U_e} = \frac{m}{\frac{1}{V_D} + K} \qquad V_D \gg \qquad \Rightarrow \frac{U_a}{U_e} = \frac{m}{K}$$



$$U_a = U_D \cdot V_D$$

$$U_D = U_e - U_a \cdot \frac{R_1}{R_1 + R_2}$$

$$U_a = U_e \cdot V_D - U_a \cdot V_D \cdot \frac{R_1}{R_1 + R_2}$$

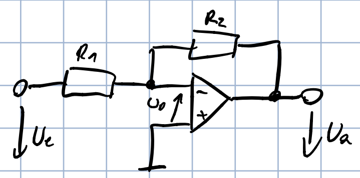
$$U_a + U_a \cdot V_D \cdot \frac{R_1}{R_1 + R_2} = U_e \cdot V_D$$

$$U_a \left(1 + V_D \frac{R_1}{R_1 + R_2}\right) = U_e \cdot V_D$$

$$\frac{U_a}{U_e} = \frac{V_D}{1 + V_D \frac{R_1}{R_1 + R_2}} = \frac{1}{\frac{1}{V_D} + \frac{R_1}{R_1 + R_2}}$$

$$V_D \gg : \quad \frac{U_a}{U_e} = \frac{\overbrace{1}^{m}}{\underbrace{\frac{R_1}{R_1 + R_2}}_{K}}$$

$$\frac{U_a}{U_e} = \frac{R_1 + R_2}{R_1} = \underline{\underline{1 + \frac{R_2}{R_1}}}$$



$V_v = \frac{-m}{K}$

Mitkopplung, Gegenkopplung

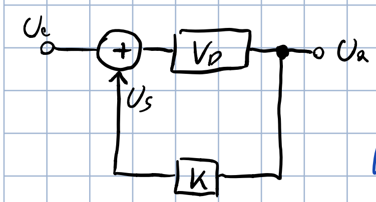
Für $U_a = 0$: $m = \frac{-U_D}{U_e}$ $\quad U_D = U_N = -U_e \frac{R_2}{R_1+R_2}$ $\quad m = -\frac{R_2}{R_1+R_2}$

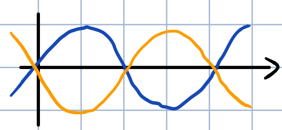
Für $U_e = 0$: $k = \frac{U_D}{U_a}$ $\quad U_D = U_N = -U_e \frac{R_1}{R_1+R_2}$ $\quad k = -\frac{R_1}{R_1+R_2}$

Minus kann entweder bei m oder bei K sein

$$V_v = \frac{m}{K} = \frac{+\frac{R_2}{\cancel{R_1+R_2}}}{-\frac{R_1}{\cancel{R_1+R_2}}} = -\frac{R_2}{R_1}$$

Schwingungsbedingung:



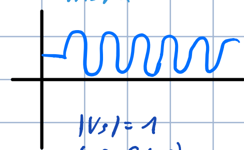


V_S ... Schleifenverstärkung

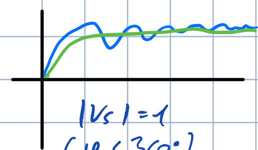
$U_S = k \cdot U_a$

$V_S = 1$ und 360° Phasendrehung => Schwingbedingung

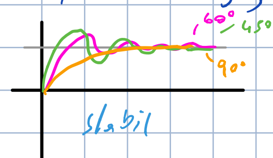
instabil



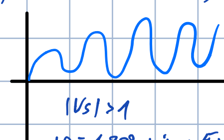
$|V_S| = 1$
($\varphi = 360°$)
$\varphi = 180°$ + inv. Eingang



$|V_S| = 1$
($\varphi < 360°$)
$\varphi < 180°$ + inv. Eingang



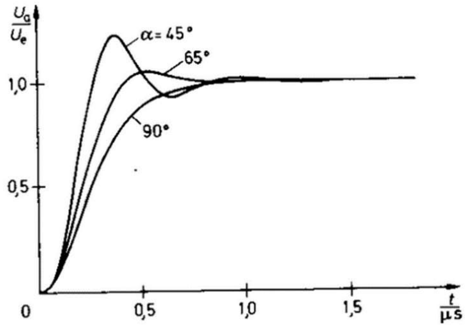
stabil



$|V_S| > 1$
$\varphi = 180°$ + inv. Eing.
anschwingendes System instabil

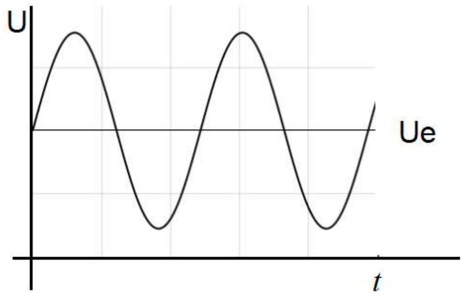
Beispiel: Sprungfunktion am Eingang:

1. φ = 180°, |Vs| < 1



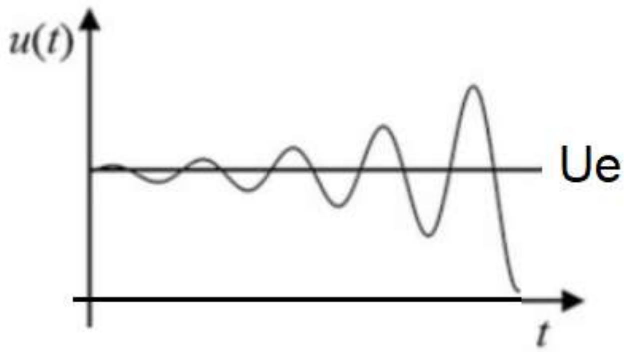
Schaltung ist stabil

2. φ = 180°, |Vs| = 1



Stabilitätsgrenze, Schaltung schwingt, instabil

3. φ = 180°, |Vs|>1

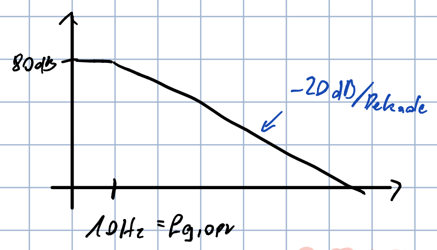


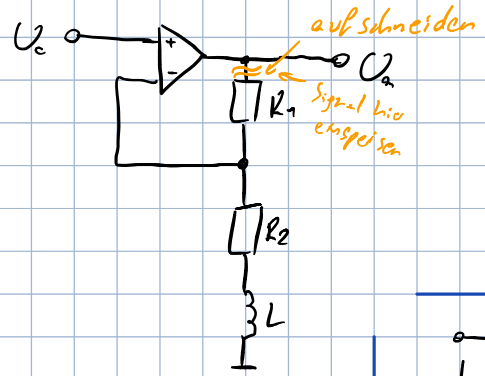
Schaltung instabil

Überprüfung d. Phasenreserve

Vorgehen:
- Der Einfluss von U_e wird vernachlässigt, U_a rückgekoppelt
- Rückkopplung wird am Ausgang aufgetrennt → U_a'
- Verhältnis $\frac{U_a}{U_a'}$ wird untersucht

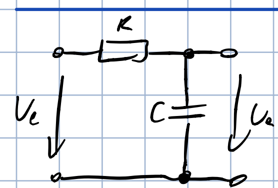
Bsp.: OPV mit $V_{u0,DC} = 80\,dB$
$f_{G,OPV} = 10\,Hz$





$R_1 = 9\,k\Omega$
$R_2 = 1\,k\Omega$
$L = \frac{1}{2\pi}\,H$

$X_L = \omega \cdot L$
$= 2\pi f \cdot L$
$X_L = f$

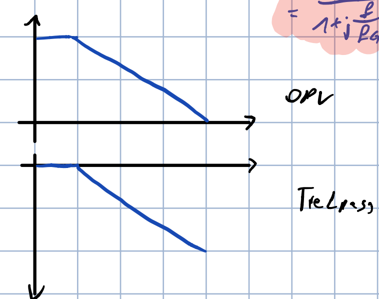


$$\frac{U_a}{\frac{1}{j\omega C}} = \frac{U_e}{R + \frac{1}{j\omega C}}$$

$$\frac{U_a}{U_e} = \frac{\frac{1}{j\omega C}}{\frac{j\omega CR + 1}{j\omega C}} = \frac{1}{j\omega RC + 1}$$

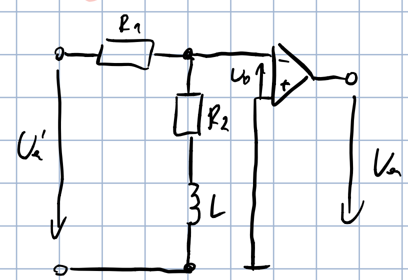
ω_g: $U_C = U_R$
$$\cancel{I} \frac{1}{\omega_g C} = \cancel{I} \cdot R$$
$$\omega_g = \frac{1}{RC}$$

$$\frac{U_a}{U_e} = \frac{1}{1 + j\frac{\omega}{\omega_g}} = \frac{1}{1 + j\frac{f}{f_g}}$$



Frequenzgang OPV: $\frac{U_a}{U_e} = \frac{V_{u0,DC}}{1 + j\frac{f}{f_{G,OPV}}}$

1) Aufschneiden:



2) $U_a = V_D(f) \cdot (\underset{\uparrow 0}{V_P} - V_N(f)) = -V_D(f) \cdot V_N(f)$

3) $\frac{U_N}{U_a'}$: $\frac{U_N}{R_2 + j\omega L} = \frac{U_a'}{R_1 + R_2 + j\omega L}$

$$\frac{U_N}{U_a'} = \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L} = \frac{R_2\left(1 + \frac{j\omega L}{R_2}\right)}{R_1 + R_2\left(1 + \frac{j\omega L}{R_1 + R_2}\right)} = \frac{R_2}{R_1 + R_2} \cdot \frac{1 + j\frac{\omega}{\omega_{G1}}}{1 + j\frac{\omega}{\omega_{G2}}}$$

4) $U_a = -V_D(f) \cdot U_N(f) = -V_D(f) \frac{R_2}{R_1 + R_2}\left(1 + j\frac{\omega}{\omega_{G1}}\right) \cdot \frac{1}{1 + j\frac{\omega}{\omega_{G2}}} \cdot U_a'$

5) $V_S(f) = \frac{U_a}{U_a'} = \frac{V_{u0,DC}}{1 + j\frac{f}{f_{G,OPV}}} \cdot \frac{R_2}{R_1 + R_2}\left(1 + j\frac{f}{f_{G1}}\right) \cdot \frac{1}{1 + \frac{f}{f_{G2}}}$

6) Grenzfrequenz ausrechnen: $\omega_{G1} = \frac{R_2}{L}$ $\quad f_{G1} = \frac{R_2}{2\pi L} = 1\,kHz$

$\omega_{G2} = \frac{R_1 + R_2}{2\pi L} = \frac{10\,k\Omega}{2\pi \cdot \frac{1}{2\pi}} = 10\,kHz$ $\qquad f_{G,OPV} = 10\,Hz$

7) $I \cdot j\omega L = I \cdot R$
$\omega_G = \frac{R}{L}$

8) $V_S = \frac{U_a}{U_a'} = (-1) \cdot 0{,}1 \cdot \frac{10^4}{1 + j\frac{f}{0{,}01}} \cdot \left(1 + j\frac{f}{1}\right) \cdot \frac{1}{1 + j\frac{f}{10}}$

WH: Betrag einer komplexen Zahl

$\left|\frac{U_a}{U_a'}\right| = 1$ $\quad z = a + jb$ $\quad |z| = \sqrt{a^2+b^2}$ $\quad |z|^2 = a^2 + b^2$

$z = \frac{1}{a+jb}$ $\quad |z| = \frac{1}{\sqrt{a^2+b^2}}$ $\quad |z|^2 = \frac{1}{a^2+b^2}$

---

9) $\left|\frac{U_a}{U_a'}\right| = 1 = \left|(-1)\cdot 0{,}1 \cdot \frac{10^4}{1+j\frac{f}{0{,}01}} \cdot \left(1 + j\frac{f}{1}\right) \cdot \frac{1}{1+j\frac{f}{10}}\right|$

10) $1^2 = 0{,}1^2 \left(1^2 + f^2\right) \frac{1}{2+\frac{f^2}{10^2}} \cdot \frac{10^4}{1+\frac{f^2}{0{,}01^2}}$

11) Nenner rüber multiplizieren: $\left(1 - \frac{f^2}{100}\right)\left(1 + \frac{f^2}{0{,}01^2}\right) = 0{,}01 \cdot 10^8 \left(1 + f^2\right)$

12) Ausmultiplizieren: $1 + \frac{f}{100} + \frac{f^2}{0{,}01^2} + \frac{f^4}{0{,}01} = 10^6 \left(1 + f^2\right)$

13) $f_s^4 \frac{1}{0{,}01} + f_s^2 \left(\frac{1}{100} + \frac{1}{0{,}01^2} - 10^6\right) = 0$

14) substituieren

$f_s^2 = x$: $\frac{x^2}{0{,}01} - x \cdot 990 \cdot 10^3 - 10^6 = 0$

$x = 9{,}9 \cdot 10^3$

$f_s = \sqrt{x} = \sqrt{9900} \approx 100$ in kHz $\rightarrow \underline{\underline{99{,}5\,kHz}}$

15) Phase bei 99,5 kHz

in kHz

$V_s = \frac{U_a}{U_e} = \underset{-180°}{(-1)\cdot 0{,}1} \cdot \underset{89{,}42°}{\left(1+j\frac{99{,}5}{1}\right)} \cdot \underset{-84{,}26°}{\frac{1}{1+j\frac{99{,}5}{10}}} \cdot \underset{89{,}99°}{\frac{10^4}{1+j\cdot 9950}}$

$\varphi_{ges} = -264{,}8°$ $\Rightarrow$ Phasenreserve: $360° - 264{,}8° = 95{,}2° \rightarrow$ stabil

Vereinfachen und zusammenfassen

8) $(-1)\cdot 0{,}1 \cdot \left(\cancel{1} + j\frac{f}{1}\right) \frac{1}{\cancel{1}+j\frac{f}{10}} \cdot \frac{10^4}{\cancel{1}+j\frac{f}{0{,}01}} = 1$ $\quad$ 1er sind irrelevant

9) bis 16) $\quad -0{,}1 \cdot jf \cdot \frac{10}{jf} \cdot \frac{10^4}{j\cdot 100f} = 1$

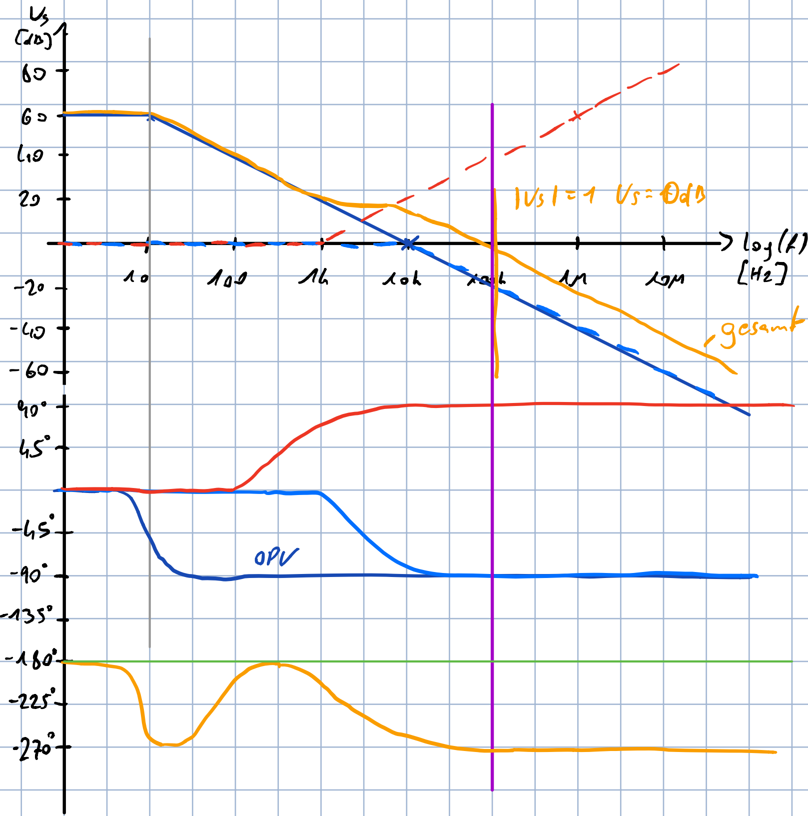
$\frac{f}{10} \cdot \frac{10}{f} \cdot \frac{10^4}{100f} = 1$

$100f = 10^4$

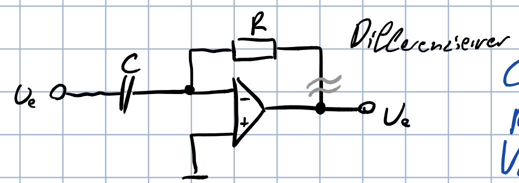
$\underline{f = 100\,kHz}$

$f_{g1} = 1\,kHz \quad f_{g2} = 10\,kHz \quad f_{g,opv} = 10\,Hz$

$V_s = \frac{U_a}{U_a'} \; \underline{-0{,}1} \cdot \underline{\left(1 + j\frac{f}{1}\right)} \; \underline{\frac{1}{1+j\frac{f}{10}}} \cdot \frac{10^4}{1+j\frac{f}{0{,}01}}$



Bsp.:

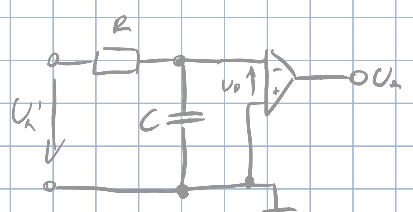


$C = 10\,\mu F$

$R = 10\,k\Omega$

$V_{U0,DC} = 120\,dB$

$f_{G,OPV} = 500\,Hz$



$$-\frac{U_D}{\frac{1}{j\omega C}} = \frac{U_a'}{R + \frac{1}{j\omega C}} \Rightarrow \frac{U_D}{U_a'} = \frac{\frac{1}{j\omega C}}{\frac{1+j\omega RC}{j\omega C}} = \frac{1}{1+j\omega RC} \quad f_g$$

$$= -\frac{1}{1+j\frac{f}{f_g}}$$

$$f_g = \frac{1}{2\pi RC} = 1591\,Hz$$

$$U_a = U_D \cdot V_{U0,DC} = -\frac{1}{1+j\frac{f}{f_g}} \cdot \frac{10^6}{1+j\frac{f}{f_{gOPV}}}$$

$$V_s = \frac{U_a}{U_a'} = -\frac{1}{1+j\frac{f}{1591}} \cdot \frac{10^6}{1+j\frac{f}{500}}$$

$$|V_s|^2 = 1^2 = \frac{1}{1+\frac{f^2}{1591^2}} \cdot \frac{10^{12}}{1+\frac{f^2}{500^2}} = 1$$

$$\left(1+\frac{f^2}{1591^2}\right)\left(1+\frac{f^2}{500^2}\right) = 10^{12}$$

$$1 + \frac{f^2}{1591^2} + \frac{f^2}{500^2} + \frac{f^4}{1591^2 \cdot 500^2} = 10^{12}$$

in kHz $$1 + \frac{f^2}{1{,}591^2} + \frac{f^2}{0{,}5^2} + \frac{f^4}{1{,}591^2 \cdot 0{,}5^2} = 10^{12}$$

$$f^4 \cdot 1{,}58 + f^2 \cdot 4{,}39^5 - 10^{12} = 0$$

$x = f^2$

$$x^2 \cdot 1{,}58 + x \cdot 4{,}39 - 10^{12} = 0$$

$${}_1x_2 = 795000 \qquad f = \sqrt{795000} = 891\,kHz$$

Phase:

$$\varphi_1 = -18^\circ$$

$$\varphi_2 = -\tan^{-1}\left(\frac{891\,kHz}{1{,}591\,kHz}\right)$$

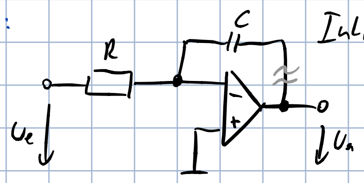
$$\varphi_3 = -99{,}88032312^\circ$$

Phase bei 891 kHz

$$V_s = \frac{U_a}{U_a'} = \underbrace{-}_{-180^\circ} \underbrace{\frac{1}{1+j\frac{891k}{1591}}}_{-89{,}89^\circ} \cdot \underbrace{\frac{10^5}{1+\frac{891k}{500}}}_{-89{,}86^\circ}$$

$$\varphi = -180^\circ - 89{,}89^\circ - 89{,}89^\circ = -358{,}87^\circ \Rightarrow \text{instabil}$$

Bsp.:



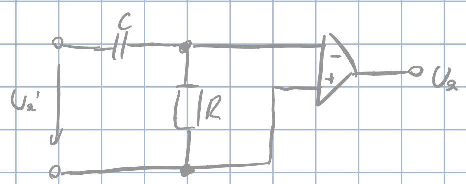
Integrator

$C = 10\,nF$

$R = 10\,k\Omega$

$V_{u0,DC} = 80\,dB$

$f_{g,OPV} = 500\,Hz$



$$\frac{U_a'}{R+\frac{1}{j\omega C}} = \frac{-U_P}{R}$$

$$\frac{-U_P}{U_a'} = \frac{R}{\frac{1+j\omega RC}{j\omega C}} = -\frac{j\omega RC}{1+j\omega RC}$$

$$= \frac{j\frac{\omega}{\omega_g}}{1+j\frac{\omega}{\omega_g}} = -\frac{j\frac{f}{f_g}}{1+j\frac{f}{f_g}}$$

$$f_g = \frac{1}{2\pi RC} = 1592\,Hz$$

$$U_a = -\frac{j\frac{f}{f_g}}{1+j\frac{f}{f_g}} \cdot \frac{10^4}{1+j\frac{f}{f_{g,OPV}}} \cdot U_a'$$

$$V_s = \frac{U_a}{U_a'} = -\frac{j\frac{f}{1592}}{1+j\frac{f}{1592}} \cdot \frac{10^4}{1+j\frac{f}{500}} = 1$$

$$|V_s|^2 = \left|(-1)^2 \cdot \frac{\frac{f^2}{1592^2}}{1+\frac{f^2}{1{,}592^2}} \cdot \frac{10^8}{1+\frac{f^2}{0{,}5^2}}\right| = 1$$

$$10^8\frac{f^2}{1{,}592^2} = (1+\frac{f^2}{1592^2}+\frac{f^2}{0{,}5^2}+\frac{f^4}{0{,}5^2\cdot 1{,}592^2}$$

$$0 = 1 - 10^8\frac{f^2}{1{,}592^2} + \underbrace{\frac{f^2}{1{,}592^2} + \frac{f^2}{0{,}5^2}}_{\text{Vernachlässigen}} + \frac{f^4}{0{,}5^2 \cdot 1{,}592^2}$$

$$0 = 1 - \frac{10^8 \cdot f^2}{1{,}592^2} + \frac{f^4}{1{,}592^2 \cdot 0{,}5^2}$$

$$0 = 1 - 39{,}5\cdot 10^6 \cdot f^2 + 1{,}578\,f^4$$

$$f^2 = 25\cdot 10^6$$

$$f = \sqrt{f^2} = 5\cdot 10^3\,kHz = \underline{\underline{5\,MHz}}$$

Vereinfacht

$$V_s = \frac{U_a}{U_a'} = -\frac{j\frac{f}{f_g}}{\cancel{1+j\frac{f}{f_g}}} \cdot \frac{10^4}{\cancel{1}+j\frac{f}{f_{OPV}}} = 1 \quad \text{Vernachlässigen}$$

$$1 = \frac{10^4}{\frac{f}{500}}$$

$$\frac{f}{500} = 10^4$$

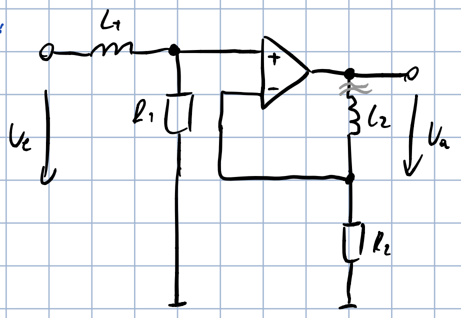
$$\underline{\underline{f = 5\,MHz}}$$

Phase:

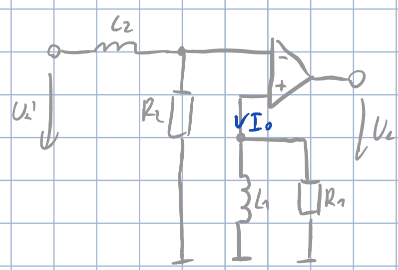
$$\underbrace{-}_{-180^\circ}\underbrace{j\frac{f}{f_g}}_{+90^\circ} \cdot \underbrace{\frac{1}{1+j\frac{f}{f_g}}}_{\tan^{-1}\left(\frac{5\,MHz}{1592}\right) = -90^\circ} \cdot \underbrace{\frac{10^4}{1+j\frac{f}{f_{OPV}}}}_{-89^\circ}$$

$\Rightarrow \varphi_r = 90^\circ \Rightarrow$ die Schaltung ist stabil

Bsp.:

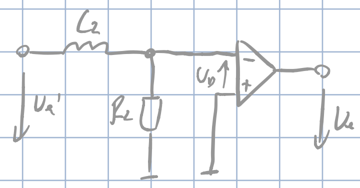


$L_1 = 15{,}9\,\text{mH}$
$L_2 = 159\,\text{mH}$
$R_1 = 10\,\text{k}\Omega$
$R_2 = 1\,\text{k}\Omega$
OPV: $V_{u0,DC} = 100\,\text{dB} \mathrel{\hat=} 10^5\,V$
$f_{g,OPV} = 100\,\text{Hz}$



$I_0 = 0\,A$

$\Rightarrow$



$$U_a = U_D \cdot V_D(f) = (V_P - V_N)\,V_D(f) = -V_N \cdot V_D(f) \qquad V_P = 0$$

$$\frac{U_N}{R_2} = \frac{U_a'}{R_2 + j\omega L_2}$$

$$\frac{U_N}{U_a'} = \frac{R_2}{R_2 + j\omega L_2} \qquad |:R_2$$

$$= \frac{1}{R_2 + j\omega L_2}$$

Grenzfreq.: $|U_L| = |U_R|$

$$I \cdot j\omega_g L_2 = I \cdot R_2$$
$$\omega_g = \frac{R_2}{L_2} = \frac{1\,\text{k}\Omega}{159\,\text{mH}} = 6{,}289 \cdot 10^{-3}$$
$$f_g = \frac{\omega_g}{2\pi} = 1\,\text{kHz}$$

$$V_s = \frac{U_a}{U_a'} = -\frac{1}{1 + j\frac{f}{f_g}} \cdot \frac{10^5}{1 + j\frac{f}{f_{g,OPV}}} = -\frac{1}{1 + j\frac{f}{1\,\text{kHz}}} \cdot \frac{10^5}{1 + j\frac{f}{100\,\text{Hz}}}$$

in kHz

$$|V_s|^2 = 1 = \frac{1}{1 + \frac{f^2}{1}} \cdot \frac{10^{10}}{1 + \frac{f^2}{0{,}1^2}}$$

$$(1 + f^2)\left(1 + \frac{f^2}{0{,}01}\right) = 10^{10}$$
$$1 + f^2\left(1 + \frac{1}{0{,}01}\right) + \frac{f^4}{0{,}01} = 10^{10}$$
$$\cancel{1} - 10^{10} + f^2 \cdot 101 + \frac{f^4}{0{,}01} = 0 \qquad x = f^2$$
$$100x^2 + 101x - 10^{10} = 0$$
$$x \approx 10000 \Rightarrow \underline{\underline{f}} = \sqrt{10000} = \underline{\underline{100\,\text{kHz}}}$$

Näherungsweise:

$$\left| -\frac{1}{1 + j\frac{f}{f_g}} \cdot \frac{10^5}{1 + j\frac{f}{f_{g,OPV}}} \right| = 1$$

$$\frac{1}{f} \cdot \frac{10^5 \cdot 0{,}1}{f} = 1$$

$$f^2 = 10^4$$
$$f = 10^2\,\text{kHz}$$
$$\underline{\underline{f = 100\,\text{kHz}}}$$

Phase: $V_s = -\frac{1}{1 + j\frac{100k}{1k}} \cdot \frac{10^5}{1 + j\frac{100k}{100}}$

$$\varphi = -180° - \tan^{-1}\left(\frac{100}{1}\right) - \tan^{-1}\left(\frac{1000}{1}\right) = -359{,}3697\ldots \Rightarrow \text{instabil}$$