

The Fourier Transform and Its Applications

Third Edition

Ronald N. Bracewell

*Lewis M. Terman Professor of Electrical Engineering Emeritus
Stanford University*



Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis
Bangkok Bogota Caracas Lisbon London Madrid
Mexico City Milan New Delhi Seoul Singapore Sydney Taipei Toronto

C O N T E N T S

	Preface	xvii
1	Introduction	1
2	Groundwork	5
	The Fourier Transform and Fourier's Integral Theorem	5
	Conditions for the Existence of Fourier Transforms	8
	Transforms in the Limit	10
	Oddness and Evenness	11
	Significance of Oddness and Evenness	13
	Complex Conjugates	14
	Cosine and Sine Transforms	16
	Interpretation of the Formulas	18
3	Convolution	24
	Examples of Convolution	27
	Serial Products	30
	<i>Inversion of serial multiplication / The serial product in matrix notation I</i>	
	<i>Sequences as vectors</i>	
	Convolution by Computer	39
	The Autocorrelation Function and Pentagram Notation	40
	The Triple Correlation	45
	The Cross Correlation	46
	The Energy Spectrum	47
4	Notation for Some Useful Functions	55
	Rectangle Function of Unit Height and Base, $Il(x)$	55
	Triangle Function of Unit Height and Area, $A(x)$	57
	Various Exponentials and Gaussian and Rayleigh Curves	57
	Heaviside's Unit Step Function, $H(x)$	61
	The Sign Function, $\operatorname{sgn} x$	65
	The Filtering or Interpolating Function, $\operatorname{sine} x$	65
	Pictorial Representation	68
	Summary of Special Symbols	71

5	The Impulse Symbol	74
	The Sifting Property	78
	The Sampling or Replicating Symbol $\text{III}(x)$	81
	The Even and Odd Impulse Pairs $n(x)$ and $h(x)$	84
	Derivatives of the Impulse Symbol	85
	Null Functions	87
	Some Functions in Two or More Dimensions	89
	The Concept of Generalized Function	92
	<i>Particularly well-behaved functions / Regular sequences / Generalized functions /</i>	
	<i>Algebra of generalized functions I Differentiation of ordinary functions</i>	
6	The Basic Theorems	105
	A Few Transforms for Illustration	105
	Similarity Theorem	108
	Addition Theorem	110
	Shift Theorem	111
	Modulation Theorem	113
	Convolution Theorem	115
	Rayleigh's Theorem	119
	Power Theorem	120
	Autocorrelation Theorem	122
	Derivative Theorem	124
	Derivative of a Convolution Integral	126
	The Transform of a Generalized Function	127
	Proofs of Theorems	128
	<i>Similarity and shift theorems / Derivative theorem / Power theorem</i>	
	Summary of Theorems	129
7	Obtaining Transforms	136
	Integration in Closed Form	137
	Numerical Fourier Transformation	140
	The Slow Fourier Transform Program	142
	Generation of Transforms by Theorems	145
	Application of the Derivative Theorem to Segmented Functions	145
	Measurement of Spectra	147
	<i>Radiofrequency spectral analysis / Optical Fourier transform spectroscopy</i>	
8	The Two Domains	151
	Definite Integral	152
	The First Moment	153
	Centroid	155
	Moment of Inertia (Second Moment)	156
	Moments	157
	Mean-Square Abscissa	158
	Radius of Gyration	159

Variance	159
Smoothness and Compactness	160
Smoothness under Convolution	162
Asymptotic Behavior	163
Equivalent Width	164
Autocorrelation Width	170
Mean Square Widths	171
Sampling and Replication Commute	172
Some Inequalities	174
<i>Upper limits to ordinate and slope / Schwarz's inequality</i>	
The Uncertainty Relation	177
<i>Proof of uncertainty relation / Example of uncertainty relation</i>	
The Finite Difference	180
Running Means	184
Central Limit Theorem	186
Summary of Correspondences in the Two Domains	191
 9 Waveforms, Spectra, Filters, and Linearity	 198
Electrical Waveforms and Spectra	198
Filters	200
Generality of Linear Filter Theory	203
Digital Filtering	204
Interpretation of Theorems	205
<i>Similarity theorem / Addition theorem / Shift theorem / Modulation theorem /</i>	
<i>Converse of modulation theorem</i>	
Linearity and Time Invariance	209
Periodicity	211
 10 Sampling and Series	 219
Sampling Theorem	219
Interpolation	224
Rectangular Filtering in Frequency Domain	224
Smoothing by Running Means	226
Undersampling	229
Ordinate and Slope Sampling	230
Interlaced Sampling	232
Sampling in the Presence of Noise	234
Fourier Series	235
<i>Gibbs phenomenon / Finite Fourier transforms / Fourier coefficients</i>	
Impulse Trains That Are Periodic	245
The <i>Shah</i> Symbol Is Its Own Fourier Transform	246
 11 The Discrete Fourier Transform and the FFT	 258
The Discrete Transform Formula	258
Cyclic Convolution	264
Examples of Discrete Fourier Transforms	265

Reciprocal Property	266
Oddness and Evenness	266
Examples with Special Symmetry	267
Complex Conjugates	268
Reversal Property	268
Addition Theorem	268
Shift Theorem	268
Convolution Theorem	269
Product Theorem	269
Cross-Correlation	270
Autocorrelation	270
Sum of Sequence	270
First Value	270
Generalized Parseval-Rayleigh Theorem	271
Packing Theorem	271
Similarity Theorem	272
Examples Using MATLAB	272
The Fast Fourier Transform	275
Practical Considerations	278
Is the Discrete Fourier Transform Correct?	280
Applications of the FFT	281
Timing Diagrams	282
When N Is Not a Power of 2	283
Two-Dimensional Data	284
Power Spectra	285
 12 The Discrete Hartley Transform	 293
A Strictly Reciprocal Real Transform	293
Notation and Example	294
The Discrete Hartley Transform	295
Examples of DHT	297
Discussion	298
A Convolution of Algorithm in One and Two Dimensions	298
Two Dimensions	299
The Cas-Cas Transform	300
Theorems	300
The Discrete Sine and Cosine transforms	301
<i>Boundary value problems / Data compression application</i>	
Computing	305
Getting a Feel for Numerical Transforms	305
The Complex Hartley Transform	306
Physical Aspect of the Hartley Transformation	307
The Fast Hartley Transform	308
The Fast Algorithm	309
Running Time	314

Contents	xia
Timing via the Stripe Diagram	315
Matrix Formulation	317
Convolution	320
Permutation	321
A Fast Hartley Subroutine	322
13 Relatives of the Fourier Transform	329
The Two-Dimensional Fourier Transform	329
Two-Dimensional Convolution	331
The Hankel Transform	335
Fourier Kernels	339
The Three-Dimensional Fourier Transform	340
The Hankel Transform in n Dimensions	343
The Mellin Transform	343
The 2 Transform	347
The Abel Transform	351
The Radon Transform and Tomography	356
<i>The Abel-Fourier-Hankel ring of transforms /Projection-slice theorem I</i>	
<i>Reconstruction by modified back projection</i>	
The Hilbert Transform	359
<i>The analytic signal / Instantaneous frequency and envelope I Causality</i>	
Computing the Hilbert Transform	364
The Fractional Fourier Transform	367
<i>Shift theorem / Derivative theorems / Fractional convolution theorem /</i>	
<i>Examples of transforms</i>	
,14 The Laplace Transform	380
^Convergence of the Laplace Integral	382
place Transform	383
	385
	386
	389
	390

Modulation Transfer Function	416
Physical Aspects of the Angular Spectrum	417
Two-Dimensional Theory	417
Optical Diffraction	419
Fresnel Diffraction	420
Other Applications of Fourier Analysis	422
 16 Applications in Statistics	 428
Distribution of a Sum	429
Consequences of the Convolution Relation	434
The Characteristic Function	435
The Truncated Exponential Distribution	436
The Poisson Distribution	438
 17 Random Waveforms and Noise	 446
Discrete Representation by Random Digits	447
Filtering a Random Input: Effect on Amplitude Distribution	450
<i>Digression on independence / The convolution relation</i>	
Effect on Autocorrelation	455
Effect on Spectrum	458
<i>Spectrum of random input / The output spectrum</i>	
Some Noise Records	462
Envelope of Bandpass Noise	465
Detection of a Noise Waveform	466
Measurement of Noise Power	466
 18 Heat Conduction and Diffusion	 475
One-Dimensional Diffusion	475
Gaussian Diffusion from a Point	480
Diffusion of a Spatial Sinusoid	481
Sinusoidal Time Variation	485
 19 Dynamic Power Spectra	 489
The Concept of Dynamic Spectrum	489
The Dynamic Spectrograph	491
Computing the Dynamic Power Spectrum	494
<i>Frequency division / Time division / Presentation</i>	
Equivalence Theorem	497
Envelope and Phase	498
Using log / instead of /	499
The Wavelet Transform	500
Adaptive Cell Placement	502
Elementary Chirp Signals (Chirplets)	502
The Wigner Distribution	504

20	Tables of $\sin x$, $\sin^2 x$, and $\exp(-TTX^2)$	508
21	Solutions to Selected Problems	513
	Chapter 2 Groundwork	513
	Chapter 3 Convolution	514
	Chapter 4 Notation for Some Useful Functions	516
	Chapter 5 The Impulse Symbol	517
	Chapter 6 The Basic Theorems	522
	Chapter 7 Obtaining Transforms	524
	Chapter 8 The Two Domains	526
	Chapter 9 Waveforms, Spectra, Filters, and Linearity	530
	Chapter 10 Sampling and Series	532
	Chapter 11 The Discrete Fourier Transform and the FFT	534
	Chapter 12 The Hartley Transform	537
	Chapter 13 Relatives of the Fourier Transform	538
	Chapter 14 The Laplace Transform	539
	Chapter 15 Antennas and Optics	545
	Chapter 16 Applications in Statistics	555
	Chapter 17 Random Waveforms and Noise	557
	Chapter 18 Heat Conduction and Diffusion	565
	Chapter 19 Dynamic Spectra and Wavelets	571
22	Pictorial Dictionary of Fourier Transforms	573
	Hartley Transforms of Some Functions without Symmetry	592
23	The Life of Joseph Fourier	594
	Index	597