Banking Systems' Stability through Consumer Dynamics: An ABM analysis of Paper-money Adoption using Networks.

Term Project Paper - CSS 610

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Abstract

There are several aspects that a Banking System must achieve to reach stability. One of the many factors present in this system is consumer trust which is reflected in the adoption of its paper-money. A common consequence of a critical loss in trust are bank runs, which may lead to huge economic crisis. Traditional economical approach considers consumer money adoption as a top-down variable assuming a representative agent. The following model tries to analyze paper-money adoption using an ABM built from a theoretical framework of bank systems without central regulation. To study the behavior of adoption, the focus is set in how consumers interact given local trade or social influence. Results show that given certain conditions of social interaction it is possible to have definite paper-money adoption or even dynamic adoptions with stable competition in the bank system.

1. Introduction.

Historical development of monetary and banking systems have left evidence of particular behaviors of social complexity. Historical or theoretical scenarios referred as free-banking refer to the possibility of self-organization and regulation in a banking system with no central authority. These seemingly complex behaviors have been argued by economists considering aspects such as robustness and market efficiency. The particular behaviors implied in such a banking system are of interest to research beyond economics, and towards the studies of human organization, decision making and social complex behavior.

The following model attempts to analyze bank systems using an agent based model approach. This model will be developed from classical economic models of bank behavior and individual money adoption decision making. Throughout a heterogeneous population and a network implementation, the objective is to understand the dynamics given for a "self-organized" or stable bank system. This report provides result of a preliminary model with basic bank decision making. The results indicate how given specific individual preferences and interactions banks do have significantly different macro-behaviors (e.g. turn from stable competition to a monopoly).

The idea of working in this topic with the selected approach began with another PhD student from the Economics program. His contribution in this work has been guidance about the canonical theory and mathematical models of the banking system. My work consists in the design and implementation of a networked ABM of bank systems with consumers. Before beginning my work for this project the computational model already had banks but with limited behavior (issuing notes and receiving gold) and network dynamics. These implementations were simple and basic features that didn't behave as expected (the model converged to a single money type in 5 steps). In addition, my approach will only address theoretical aspects of the system's properties. For further works we will try to address historical issues. Given his background in economic history he will contribute with more theory and historical data.

On the actual model building I've coded the whole program from scratch using Python and popular libraries for agent based models.

2. Theoretical Framework.

Since Adam Smith economists have been close to the basic idea that markets generally follow the law of one price. This means that any market will tend to equilibrate around one unique price. The money market has always been problematic given that it is the product that serves as a unit of account or *numeraire*, as the general equilibrium model tends to describe. In these models money prices are generated exogenously.

According to this, the traditional literature that deals with money transmission mechanisms mostly works with a top – down approach. Considering that under certain conditions or thresholds, people will start adopting a specific kind of money. It also follows from this approach that people will tend to adopt a single monetary unit. Historically, we have seen that silver was adopted since Ancient Greece, passing through gold in the end of Medieval Era and getting to dollar adoption in the current days. However, there have always been monetary competition with the presence of several instruments to enable it (e.g. notes issuing with gold reserves).

Without the presence of a central monetary authority, banking systems have been witness of particular dynamics [1, 2]. These dynamics arise mainly due to the need of self-regulation and organization of the industry. An example of this dynamics is the stereotypical case of successful bank competition scheme: Scotland in the XIX century [1]. During this period the bank industry was divided into three distinctive market areas: Glasgow, Edinburgh and minor local banks. These banks co-existed issuing notes backed in gold through almost 50 years showing relative

signs of stability. These signs are mainly measured in low inflation, price stability (e.g. no excesses in monetary offers) and financial stability (e.g. bankruptcies of ill-managed banks or "healthy bankruptcies"). Examples like this show how under certain conditions banking systems may self-organize. The following work attempts to model a banking system in a networked agent based model to experiment on these conditions. Nevertheless, a further description of classic bank behavior model is useful before we address the model and the specific approach.

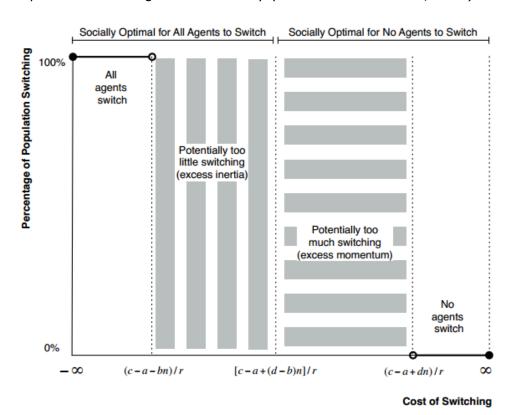
Classic theoretical approaches to monetary systems have as an assumption that people have gold, and –given certain needs that have been satisfied- they will exchange it for notes. Regarding the behavior of a bank we have the White-Selgin model [3]. The bank reserves' compositions are defined by Selgin and White as the stock of monetary base (e.g. gold) that equals the bank reserve. Monetary stock is defined by the sum of bank deposits and issued notes. According to this, a bank would find reserve equilibrium with the following equations [2]:

$$R = r(D+N)$$

with
$$r = \frac{R}{M}$$

On the other side, we would have individuals that evaluate which type of money they will use. Luther [4] defines the already mentioned thresholds for individual decision making. The model gives an inter-temporal approach of money switching within a group of individuals. As it is evident in Figure 1, the main parameters that provide an understanding of switching behavior are percentage of Population Switching and Cost of Switching. Though he talks about a network he just includes the "notion of a network" through a parameter (n) that measures the size of the population.

Figure 1. Network Effects, Switching Costs, and the Percentage of the Population Switching to a New Money (Extracted from Luther, 2013).



The model presented this far, as already noted, has a top-down perspective and considers the assumption of the representative agent.

This homogenous representation of individuals is critical for an analytical

approach with economic models but it doesn't enable a dynamic and systemic development. Given the nature of this system, it seems reasonable to consider different individuals and interactions between them and the banks. In other words, the implementation of heterogeneity and networks in the study of the organization of monetary markets could provide an adequate approach. The analytical model clearly describes banks behavior but it is not suitable to address industry dynamics, the effect of individuals' heterogeneity and its inter-connected behaviors.

The following work tries to implement the presented model in an agent based model that allocates its agents in a network. The social network would represent trading relationships between individuals (e.g. buying products) and note-gold exchange between individuals and banks (e.g. deposits, note issuing). Banks will try to get more deposits and individuals will attempt to maximize the benefit of using a particular type of money (e.g. gold, notes from bank A or bank B). To evaluate which type of money they would like to have they'll consider their trading relationships. The ideal condition would be that everybody shares the same type of money. The evaluation will follow from a particular threshold of "acceptable" amount of others with same type of money. A difference with Luther [4] is that the model considers discrete time evaluations. Simulations results show that the system may gain stability under certain conditions of bank's initial clients, average number of trading relationships and particular thresholds.

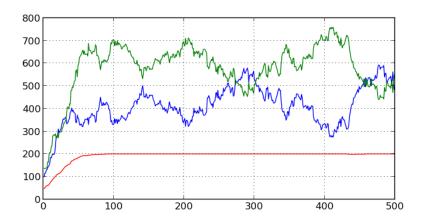
3. Agent Based Model of the Banking System in a Network.

As already stated the agent based model will try to represent a monetary system where a population of heterogeneous users will decide, considering their local trades, which type of money they want. From this particular decision, and the possibility to move their gold or notes from bank to bank or just keep the gold reserves, the model shows macrobehaviors of paper-money adoption. Thus, a typical run will show the specific amounts of bank reserves (equal to bank specific issued notes) and gold used as money.

For the purposes of this model competition is considered as the relatively stable presence of two or more banks within a significantly long period of time (Figure 2). Following this, whenever two or more banks share similar portions of reserves or issued notes within the population we will say they have a stable competition. The model also considers the possibility of bankruptcy (when reserves go to 0) without the possibility to regain clients. Finally, monopolies are possible, frequent and defined by the unique presence of one bank over the entire population (when every user decides it is better to adopt its paper-money).

According to this, the following model will try to answer "how does average network degrees and local trading affect stable competition in a banking system". Along with this fundamental question other significant interrogations appear: how does the micro level evaluation affect stability; how much impact does the network degree interconnection have on stable competition.

Figure 2. Example of stable competition (Green and Blue are Banks, Red is amount of users adopting paper-money).



ODD Overview: Agents' description and attributes:

The ABM has two types of agents: Banks and Users. The first ones have three important attributes: reserve, issued notes and clients. As the ABM follows the classic economic bank model the incoming reserve will always be equal to the issued notes. The amount of clients is fixed and is determined as a parameter of the simulation. Values of initial clients considering 10% and 20% of the population were used for the experiments presented in this report.

The other type of agent, Users, has the following attributes: reserve (gold), notes, relation with bank, relationships with users, adoption threshold and trades per step. As users decide which type of money they want they will only be able to keep gold (if chosen) or a specific bank notes (if paper-money is chosen). Users may only have one bank and thus only one type of paper-money. On the other hand, relationships with other users are based in a random value of a Gaussian distribution. The

mean value for this normal distribution will be determined by expected the average network degree. It is important to notice that relationships are different to trades; the first ones are all the possible trades an agent has, the second one are the actual trades executed in a time step. The threshold for adoption, different from Luther [4], is the percentage of neighbors needed to consider a majority. Similar to Schelling's [5] segregation model, the threshold would consider a significantly big enough group of people (in this case, a group of people using the same type of money). Luther [4] just considered the presence of a network, in this model we are effectively having micro-level interconnections with an actual network.

Agent attributes are displayed in Table 1. Attributes with their respective value type and initial conditions are described.

Table 1. Agent descriptions (ODD).

AGENTS	ATTRIBUTES	VALUES	INITIAL
BANK	Reserve	float[0,Total]	0
	Notes	float[0,Total]	0
	Clients	int[0,Population]	10%, 20%
USERS	Gold	float[0,Total]	Random Pareto
	Notes	float[0,Total]	0
	Banks	int[0,1]	0
	Relationships	Random Gauss	mean: 5,10,15
	Threshold	float[0,1]	.25,.5,.75
	Trades	float[0,1]	.2,.4,.6,.8

Agents' behaviors:

a) Banks:

The actual behavior of banks is simply a trade of reserve for its own notes. Further model development considers the implementation of banks investment. For now, the bank just receives reserves when a client wants its notes. If it receives reserves it gives the same amount back to the user, now its client, in notes. As people keep notes until they decide they want other type of money (e.g. gold or notes from another bank), whenever a client decides it wants to leave then the bank just returns the gold (reserve) and burns the returned notes.

b) Users:

As already described, initially the user will not get the money as an active procedure. On the contrary, it is a passive action of banks choosing their initial set of clients. Users, though, may decide whether to stay as a client or not doing so. To do this they just evaluate which type of money they would like to have after surveying the most popular type used in his immediate network of trades. As every user may only have one type of money (either having a Bank or just keeping his gold under the pillow) it has to decide which the best prospect is after some transactions with other users. When the decision is made the user may immediately change its currency in his bank.

Neither users nor banks may change their relationships (e.g. clients, trading relationships) during the simulation. The only changes enabled are those requested by clients when they decide to leave or enter a bank.

Random Distributions:

For the initialization of wealth and user relationships some random values are required. Specifically to acquire the wealth values a Pareto distribution is used. For the amount of relationships for each specific agent a random value is taken from a Gaussian distribution.

Network Properties:

As it should be evident so far the network is fundamental for the model. The implementation of the social network is as simple and straightforward as the agents already described. The network is built from a random selection of agents from the total population. For each agent the network generating procedure will count the amount of relationships (taken from the random Gaussian distribution) and select that quantity of random agents to be paired with. Thus, every agent will be connected with a random set of other agents with an overall normal distribution of connection throughout the population.

As the average degrees is a parameter of interest to the simulation this value will be able to be modified for the experiments. The modification is made through the mean value of the Gaussian distribution from where the amount of relationships is sampled.

Banks are also connected to the network but in a simpler and direct manner. At the beginning of the simulation banks will adhere a specific amount of clients to them. This means that initially the banks decides which are its connections in the network. As already stated, the relationships between bank and users will only be modified if users decide to. There are no bank to bank relationships.

Figure 3 shows a representation of the described network. It is possible to see the presence of three colored nodes meaning a distributed papermoney adoption (color node indicates type of money adopted). The three banks reside in the center of the network (bank node color is irrelevant).

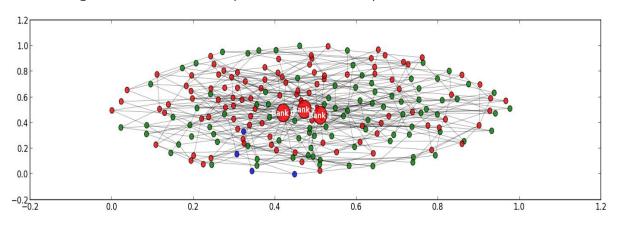


Figure 3. Network Example: Three bank competition final state.

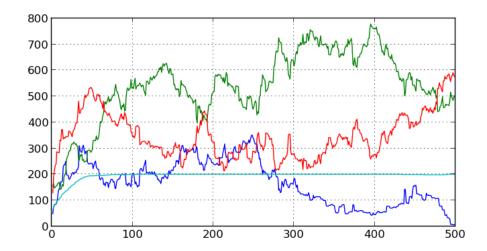
In summary, banks and user are nodes in the network. The social network edges consist of: clients (for bank to user relations) and trade relationships (for user to user relations).

Simulation Procedure:

Overall there are a set of relevant parameters that every simulation has to keep in mind: Average Degree (Relationships), Number of Initial Clients, Trade Threshold and actual ratio of Trades performed (percentage of trades from the whole set of relationships). The average degree will be given by our normal distribution for which we need to specify its mean and standard deviation. The number of initial clients is specified as a percentage of the total population; in the specific case of the experiments ran for this report the values are 10% and 20%. The trade threshold percentage is also a specified parameter which values are going to be alternated between "Low threshold" (25%) and "High threshold" (75%). Finally, the actual ratio of Trades performed is also a parameter that requires specification and will be quantified as "Low interaction" (33%) and "High interaction" (66%).

For each specific step in the simulation procedure only users will have an active role. Every step they interact with their effective trade relationships at a local level and evaluate the optimal money adoption. If they don't have the specified type of money then they will proceed to go to the bank and stop being clients. Banks, as reactive agents will only give back the gold and destroy the notes. If the new adoption is other bank then the agent will ask to be a client and give all its gold, the bank will answer issuing notes. The activation sequence used for this particular simulation is the uniform activation. The simulation runs for 500 steps in every simulation and demonstration provided in this report.

Figure 4. Three Bank Competition. Banks' reserves time series.



An example of the time series shown by a three bank competition simulation is shown in Figure 4. It is possible to appreciate how the green red banks managed to compete while the blue bank went down. The network shown in Figure 3 is the final state of this simulation.

Initial conditions:

Agents' populations consider a fixed set of 200 users for all simulation executions. On the other hand, bank presence will vary from 1 to 3 individuals. The interconnection between users is defined by the random Gaussian distribution and the wealth of each agents is dependent of a random Pareto distribution. Banks have a specific amount of initial clients that will stay the same throughout the simulation.

For a default model execution the initial conditions will proceed to build the network with the specified values and assign the clients to the banks. Besides the designated clients there will be no other money adoption rather than gold. Users will not evaluate their adoption decision during initialization.

Rumor implementation:

Banking system is always a potential victim of bank runs. These particular phenomena occurs when a significant amount of trust is loss regarding the banking system or a particular bank administration. The dynamic and behavior of these phenomena is easily modeled through the presence of rumors. Rumors have been implemented in the model as an exogenous random variable. It may trigger and when it does it spreads through the agents. Every time it spreads it may decay and, within several iterations, the rumor may finally die. Following 3 and 4, figure 5 displays the activation of rumors in throughout the same simulation.

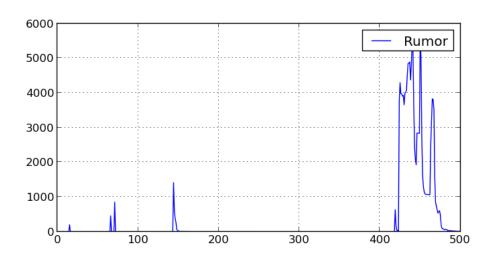


Figure 5. Three Bank Competition. Rumor presence.

Having in mind the behavior of the rumor it is possible to make an interpretation of the shift between Green and Red (Figure 4) after the

step 410. A critical rumor crisis may have produced a bank run for Red giving a chance for Green to regain territory.

Experiment Conditions:

The following report presents results for three experiment conditions or scenarios. As the model considers a relatively high number of parameters these three scenarios where built to describe the models' behavior and present highlights towards our questions.

First, an analysis is made with just one bank in the population. The purpose of this is to understand the behavior of gold or note convergence. In particular, our interest is set in how the bank is able to keep a stable presence with permanent or recurrent flows of clients. Results show mainly how robust is a one-bank under the conditions that we are going to utilize in the following experiments.

The following two experiments consider the presence of multiple banks: specifically two and three. For each of these experiments all the banks will have exactly the same initial parameters. The only effective difference will be the particular selection of clients and how they are connected in-between them. This random network composition and the particular dynamic it generates is what may allow one bank to overcome others or to compete in relative stability. An initial issue with bank choosing users led to a modification where banks chose by turns, one user at a time. A former implementation had one bank choosing all its clients and then allowing others to choose; this procedure had huge biases towards the bank that chose first.

4. Experiment Results.

As already stated, three major experimental conditions were performed; one bank performance and competitions with two and three banks. The experiments done with a single bank show quantitative results regarding its stability in the population. Results for experiments regarding the competition of two and three banks are delivered as a table in a qualitative manner. Divided by 4 different conditions: average network degree (5 and 15), and initial clients (10% and 20%). The tables show how changes in the parameters –and specifically individual parameters of paper-money adoption- indicate possibility of competition, dominance or monopoly.

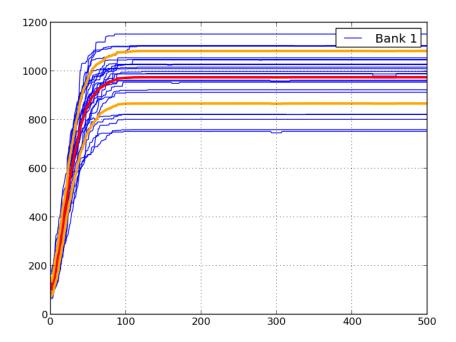
The experiments considers variation of the following parameters: average network degree (users' relationships), initial clients and local trade attributes (threshold, trades by step).

1) Single Bank scenario:

To begin with the single bank scenario an average degree of 5 is chosen with low trading requirements: low threshold (25% of neighbors) and low trade activity (33% of all possible trades). As it is possible to appreciate in Figure 6 these conditions enable a complete convergence to paper-money of the only bank present. With just an initial set of 20 clients the bank gets to monopolize the population with a unique money for all transactions. Blue lines indicate particular cases while the red line is the average time-series of 30 runs (orange lines show one standard

deviation up and down). The major differences between runs are based in the random sampling from the Pareto distribution for wealth. As different amount of wealth is provided the bank may get larger or smaller reserves from clients.

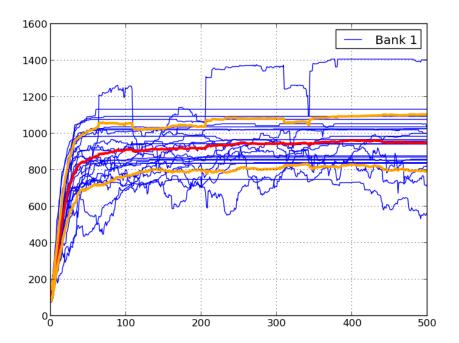
Figure 6. Single Bank with 10% Initial Clients and Low Requirements.



This particular stable and monopolistic behavior is clearly affected by the social requirements and thus dynamics through the network. As we can see in Figure 7, with higher requirements (threshold at 75% and trades with 66%) instability appears and in some cases the bank is not able to obtain a sovereign money distribution. In many cases the bank's reserve fluctuates given a significant decrease in its paper-money adoption. Average time series show how the effective monopolistic runs (those were the bank is able to take every user as a client) are relatively heavier

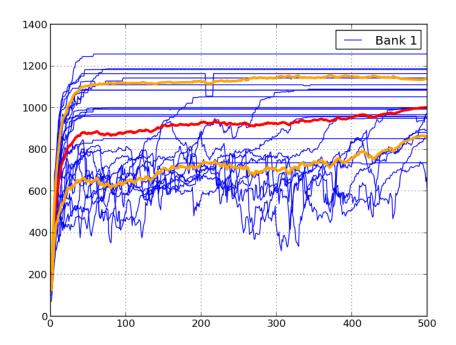
than the random processes where users adopt gold as their type of money.

Figure 7. Single Bank with 10% Initial Clients and High Requirements.



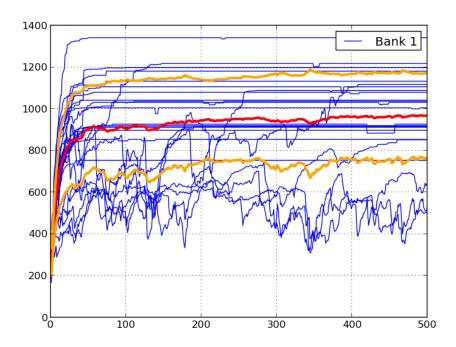
As already seen, the micro-level interaction may have a significant impact in the bank system stability, showing that its robustness is sensible to a differential change of 50% in threshold and 33% in effective trades. Even though requirements play an important role in bank's reserve time series, the average degree has also a significant impact. An average of 15 connections shows an even faster and noisier behavior for the bank's reserve (Figure 8).

Figure 8. Single Bank with 10% Initial Clients and High Requirements.



All the behavior we've seen so far in the single bank scenario with 10% of initial clients are also present in the scenario where the bank initializes with 20% of the population as its clients (40 individuals). This single difference of 10% more doesn't show a distinctive impact in the simulation in comparison with the same values in the 10% condition. As in the previous scenario the patterns show some cases with: complete convergence to paper-money; variability and late convergence; and persistent variability (Figure 9).

Figure 9. Single Bank with 20% Initial Clients and High Requirements.



Results regarding the single bank scenario show that under the specific parameters chosen the bank is able to behave with sensitivity towards the individuals parameters. The bank may turn from monopoly to unstable dynamics of a population with segmented adoptions.

2) Competition with Two Banks:

As in the single bank scenario the competition with two banks analyzes the effects of changing the mentioned parameters. In this case the results are given in a qualitative manner through Table 2, for competitions with 10% of initial clients, and Table 3, with 20% of initial clients.

Results shown in the following tables are presented by average degree (5, 15), effective trades (33%, 66%) and threshold (25%, 75%). The values show an ordinal scale from low to high providing a qualitative evaluation of the macro-behaviors of interest. The three major behavior involving bank stability (for our model purposes) are competition, upperhand and monopoly. As the others have already been defined we should clarify upper-hand. This classification is for the scenario where a particular bank has significantly higher participation of paper-money in the population but other banks are still competing (different from monopoly when the other banks go to bankruptcy).

Table 2. Competition with Two Banks and 10% of Initial Clients.

	Average Degree = 5				Average Degree = 15			
Trades	0.33		0.66		0.33		0.	66
Threshold	0.25	0.75	0.25	0.75	0.25	0.75	0.25	0.75
1. Competition	High	High	High	Low	Low	Low	Low	Low
2. Upper-hand	Mid	Mid	High	High	High	High	High	High
3. Monopoly	Low	Low	Mid	High	High	High	High	High
1. Convergence	High	High	High	High	High	High	High	High
2. Gold Presence	Low	Low	Low	Low	Low	Low	Low	Low
3. Speed	Low	L-M	L-M	L-M	М-Н	М-Н	H-	Mid

As we may see in Table 2, stable competition and unstable competition or monopoly appear to be distinguishable behavior that depend of average degree. With a degree of 5 most conditions allow a stable competition, only in the case of high trades and high thresholds is that

we see that unstable competition emerges. On the other hand, with an average degree of 15 relations all the individual conditions (trade and threshold) give low or none competition with a clear tendency towards monopolistic behavior. All the simulations show a convergence to papermoney and for obvious reason a lack of gold as a type of money.

In the condition of 20% of initial clients the competition of two banks shows different behaviors (Table 3). The low average degree does not provide the emergent competition behavior as clear as before. Though competition exists in some cases it is generally biased towards the "upper-hand" scenario (where one of the banks dominates but without complete monopoly). A particular case with high trade and high threshold appears again in the setting of 5 degrees of connection; the possibility of competition, upper-hand and monopoly appears to be somewhat similar. Following the same behavior as 10% initial clients, the simulations show a clear preference for paper-money and the macro-behaviors with an average degree of 15 tend to monopolies.

Table 3. Competition with Two Banks and 20% of Initial Clients.

	Average Degree = 5				Average Degree = 15			
Trades	0.33		0.66		0.33		0.66	
Threshold	0.25	0.75	0.25	0.75	0.25	0.75	0.25	0.75
1. Competition	High	Low	Low	Mid	Low	L-M	Low	Low
2. Upper-hand	Low	High	High	Mid	High	High	High	High
3. Monopoly	Low	Low	Low	Mid	High	H-	High	H-
1. Convergence	High	High	High	High	High	High	High	Low
2. Gold Presence	Low	Low	Low	Low	Low	Low	Low	High
3. Speed	Low	Low	Low	Low	Mid	L-M	М-Н	М-Н

3) Competition with Three Banks:

Similar to the two banks competition this scenario has a clear distinction between competitive and non-competitive behaviors sensible to the average degree. Although it has a tendency towards competition with 5 degrees, the behavior is different from the two bank scenario given that "upper-hand" situations are highly likely (Table 4). In the particular case of 15 degrees the behaviors are relatively the same. Considering the presence of a third actor, and the imperative condition of sharing the population as clients, it is reasonable to think that just the pressure of a new bank would make the 5 degree scenario a "difficult and competitive market" where one or more banks may go down and enable others to take the upper-hand.

Table 4. Competition with Three Banks and 10% of Initial Clients.

	Average Degree = 5				Average Degree = 15				
	0.33		0.66		0.33		0.0	66	
Inter-Bank	0.25	0.75	0.25	0.75	0.25	0.75	0.25	0.75	
1. Competition	High	High	High	High	Low	Low	Low	Low	
2. Upper-hand	High	H-	High	High	High	High	High	High	
3. Monopoly	Low	Low	M-H	L-M	High	High	High	High	
Money									
1. Convergence	High	High	High	H-	High	High	High	High	
2. Gold Presence	Low	Low	Low	High	Low	Low	Low	Low	
3. Speed	L-M	L-M	L-M	L-M	Mid	M-L	Mid	Mid	

When the conditions change to the initial clients of banks set at 20% (Table 5), the three banks simulations do not show significant differences with the results provided in the previous condition (clients at 10%).

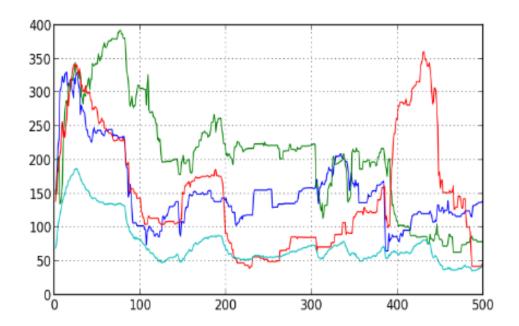
Table 5. Competition with Three Banks and 20% of Initial Clients.

	Average Degree = 5				Average Degree = 15			
	0.33		0.66		0.33		0.66	
Inter-Bank	0.25	0.75	0.25	0.75	0.25	0.75	0.25	0.75
1. Competition	High	High	High	High	H-	H-	Low	Low
2. Upper-hand	High	H-	High	High	High	High	High	High
3. Monopoly	Low	Low	L-M	Low	High	High	High	High
Money								
1. Convergence	High	H-	High	H-	High	High	High	High
2. Gold Presence	Low	Low+	Low	Low+	Low	Low	Low	Low
3. Speed	High	High	High	High	L-M	L-M	L-M	M-H

As it is possible to appreciate through the convergence factor, in some cases there is not a complete convergence to paper-money. There are some scenarios where is a probability of having user using gold as money.

To give more intuition about behaviors with three banks competing Figure 10 shows a single simulation run where it is possible to see a very "active" system that has competition, upper-hand positioning and user that still prefer to opt-out towards gold adoption.

Figure 10. Single run with Three Banks [Clients=10%, Degree=5, Trade=0.66, Threshold=0.75]. Red, green and blue are banks. Light-blue represents people with paper-money.



In the whole simulation run we may see how all three banks began with high participation. Soon a decrease in paper-money (people going for gold adoption) made a significant impact in banks reserves. This impact and probably other significant inflection are not only produced by network properties but by rumors. In Figure 11 the different negative rumor activations appear. These rumor may explain some of the tipping points but others are just based in micro-level local interaction.

As a final remark, is important to consider that the amount of people with paper-money (Light-blue line) is measured in units of individuals and not the value of their possession. Figure 12 shows the final state of the simulation where it is evident the widespread distribution of users that preferred gold (yellow nodes).

Figure 11. Single run with Three Banks: Rumor activation.

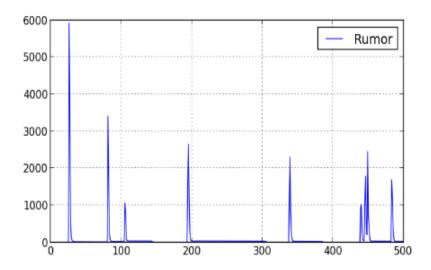
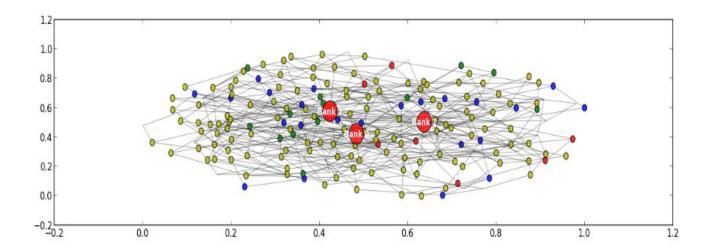


Figure 12. Single run with Three Banks: Final step network.



5. Conclusions and Discussion.

Different results regarding changes in the initial client degree show that the ratio of banks clients and population is critical. Just a small difference of 10% may provide a different scenario for the organization of the banking system. This gives an intuitive conclusion about the population size and stable competition. It would be true -for a big enough population- that two or more banks could compete and coexist without affecting each other. Further studies using this model should try to determine the value or interval in which the "big enough" population makes a transition from coexistence and unstable competition.

Considering that people have a limited amount of friendships, the average network degree would also be susceptible to the total population of agents. For a population with a sufficiently big size the natural limit of friendships of a normal individual would be too small. In other words, the average network degree would decrease in proportion to the population. This kind of behavior should be tested in further researches with the model. Given that the results show that smaller average has better competition, the intuition of this relatively small average trade relationship degree could prove to be a support for free-banking in big populations.

Another important conclusion is the possibility of gold adoption. Some scenarios of the simulation show that given an "active" banking system or even a monopolistic one, there is always a chance of certain clusters gaining preference for gold as type of money.

Further work would require to implement a more complex behavior of banks. Current economic mathematical models present some behaviors that should be added to this model. The additions would consider bank investment and active decision making about notes issuing and reserve management. Regarding user behaviors and attributes, trading relationships (provided by Gaussian distribution) should be changed from a normal to a Pareto distribution. This modification will be a better proxy fit to a traders interactions (some traders just buy products for their house, others go to the market and try to buy and resell). Another aspect of relevance is the calibration of the complete model with historical data. Present motivations are oriented to develop a calibrated model with the specific additions already mentioned.

6. References.

- [1] White, L. 1999. *The Theory of Monetary Institutions.* Blackwell Publishers, Malden, Massachusetts.
- [2] Arteaga, F. 2012. Repensando la banca central: Esquemas de Laissez-faire como alternativas para un orden monetario.
 Congreso Nacional de Investigación Económica México, DF.
- [3] Selgin, G. and White, L. 1994. *How would the invisible hand handle money?* Journal of Economic Literature. pp 32:1718–1749.
- [4] Luther, W. 2013. CryptoCurrencies, Network effects, and Switching Cost. Mercatus Center, George Mason University.
- [5] Schelling, T. 1978. *Micromotives and Macrobehaviors*. Norton.