Maths

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R Markdown

GDP Prediction using Backpropagation

In this model, the GDP of a specific country is predicted based on the GDP of other countries, excluding the country's own GDP. The prediction for country i at time t is denoted as $\hat{GDP}_i(t)$, and the inputs consist of the GDPs of other countries.

The model computes the prediction as follows:

$$\hat{GDP}_i(t) = f\left(\sum_{j=1, j\neq i}^{n} w_{ij} \cdot GDP_j(t-1) + b_i\right)$$

Where: - w_{ij} represents the weight assigned to the GDP of country j for predicting country i's GDP. - b_i is the bias term for country i. - $f(\cdot)$ is the activation function, specifically the ReLU (Rectified Linear Unit), defined as:

$$f(x) = \max(0, x)$$

Backpropagation

To minimize the error in predictions, we update the weights w_{ij} and biases b_i using backpropagation. The error E at each iteration is calculated as:

$$E = \frac{1}{2}\sum_{i}\left(\hat{GDP}_i(t) - GDP_i(t)\right)^2$$

The weight update rule using gradient descent is:

$$w_{ij}^{(new)} = w_{ij}^{(old)} - \eta\frac{\partial E}{\partial w_{ij}}$$

Similarly, the bias update rule is:

$$b_i^{(new)} = b_i^{(old)} - \eta\frac{\partial E}{\partial b_i}$$

Where η is the learning rate.

By iterating through multiple training epochs, the model adjusts the weights and biases to minimize the prediction error.

The weight matrix W is a square matrix of dimension $n \times n$, where n is the number of countries. Each element w_{ij} in the matrix represents the weight assigned to the GDP of country j when predicting the GDP of country i. Since a country's own GDP is not used to predict itself, the diagonal elements of the matrix are set to zero:

$$W = \begin{pmatrix} 0 & w_{12} & w_{13} & \dots & w_{1n} \\ w_{21} & 0 & w_{23} & \dots & w_{2n} \\ w_{31} & w_{32} & 0 & \dots & w_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & w_{n3} & \dots & 0 \end{pmatrix}$$

This matrix W satisfies the following condition:

$$w_{ii} = 0 \quad \text{for all } i = 1, 2, \dots, n$$

Where: - w_{ij} denotes the weight connecting the GDP of country j to the prediction of country i's GDP. - n is the total number of countries.