15. Feedback

Titolo nota 4/21/2008

1) Concetto base dei sistemi con feedback

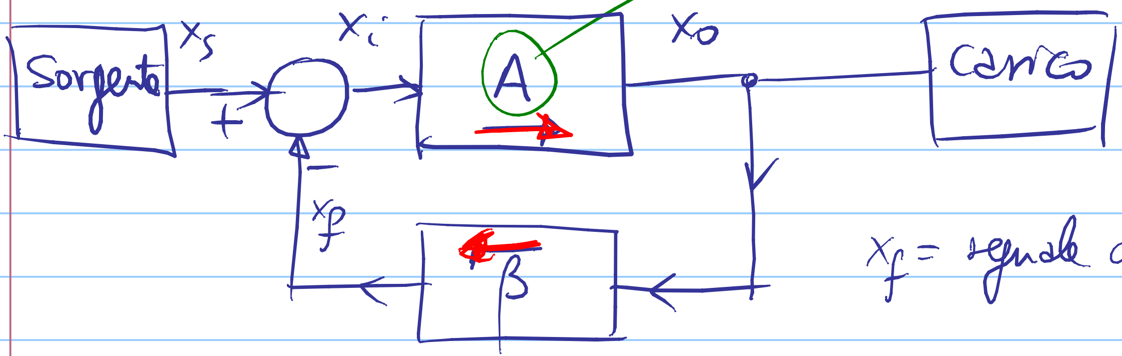
Uscita applicata all'ingresso

- Positivo → sistemi non lineari, oscillatori
- negativo → amplificatori, filtri
  1. Desensibilizzazione del guadagno
  2. Riduzione non linearità
  3. Riduzione rumore
  4. Controllo delle impedenze IN, OUT
  5. Aumento la larghezza di banda

  (tutto alle spese del guadagno)

Schema feedback

open loop gain



x_f = segnale di feedback

$$x_o = A x_i = A(x_s - x_f) = A(x_s - \beta x_o)$$

$$x_o = \frac{A}{1+\beta A} x_i \Rightarrow A_f = \frac{A}{1+\beta A}$$

loop gain

$$A_f \to \frac{1}{\beta} \quad \text{per } A \to \infty$$

Cioè il guadagno con feedback dipende solo dalla rete di feedback, e non più dal guadagno dell'amplificatore stesso.

NB: Segno delle frecce è un'ipotesi importante

Se $\beta < 0$ (feedback positivo) può essere $A_f \to \infty$

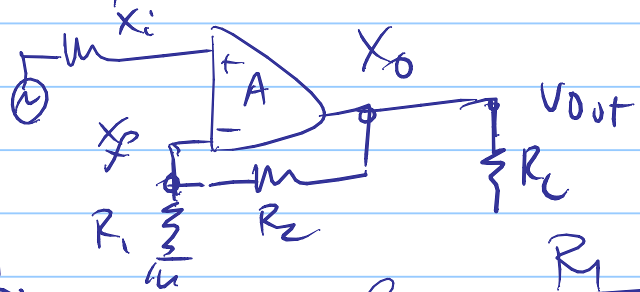
2. Proprietà generali

1. Desensibilizzazione del guadagno

$$dA_f = (1+\beta A - \beta A)/(1+\beta A)^2 \cdot dA = \frac{dA}{(1+\beta A)^2}$$

$$\frac{dA_f}{A_f} = \frac{dA}{A} \frac{1}{1+\beta A}$$ fattore di desensibilizzazione

Esempio



$$x_f = \frac{x_o R_1}{R_1+R_2} \qquad \beta = \frac{R_1}{R_1+R_2}$$

se $A = 10^4$ $\quad A_f = \frac{A}{1+\beta A}$ voglio $= 10$

$$\frac{1}{A} + \beta = \frac{1}{A_f} \rightarrow \beta = \frac{1}{A_f} - \frac{1}{A} = 10^{-1} - 10^{-4} = 9.99 \times 10^{-2} \simeq 1/10$$

$\beta A = \text{loop gain} = 10^3$

Effetto delle variazioni di guadagno

$$\frac{\Delta A}{A} = 20\% \rightarrow \frac{\Delta A_f}{A_f} = 0{,}02\%$$

(trascurando effetti sulle resistenze)

2. Banda passante

Ampli con singolo polo $A = \dfrac{A_M}{1+s/\omega_H}$

$$A_f = \frac{A}{1+\beta A} = \frac{A_M/(1+s/\omega_H)}{1+\beta A_M/(1+s/\omega_H)} =$$

$$= \frac{A_M}{1+\frac{s}{\omega_H}+\beta A_M} = \frac{A_M/(1+\beta A_M)}{1+s/[\omega_H(1+\beta A_M)]}$$

$$= \frac{A_{Mf}}{1+s/\omega_{Hf}} \qquad A_{Mf} = \frac{A_M}{1+\beta A_M}$$

$$\omega_{Hf} = \omega_H(1+\beta A_M)$$

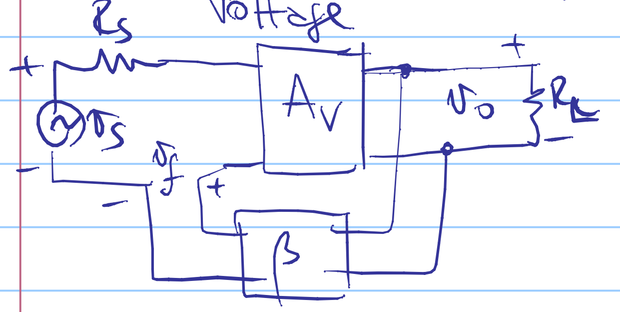
Gain-bandwidth product = $A_{Mf}\cdot\omega_{Hf} = A_M\omega_H$

→ Estende la banda a spese del guadagno.

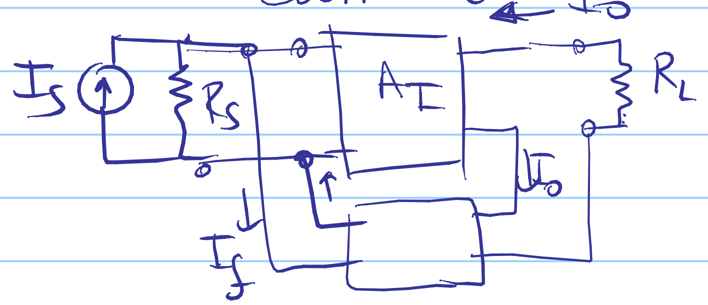
3. Topologie di feedback

Dipendono dal tipo di amplificatore

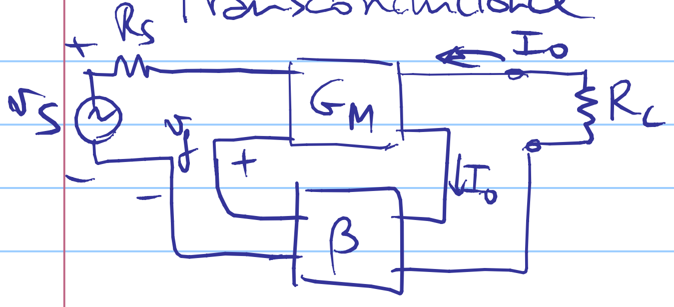
voltage



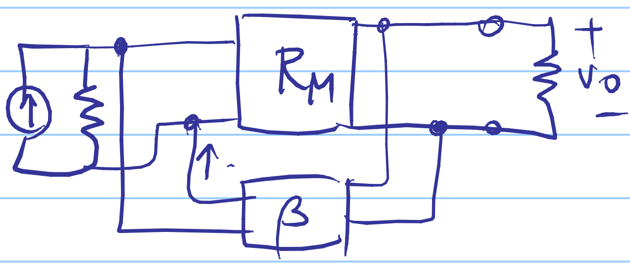
current



Transconductance



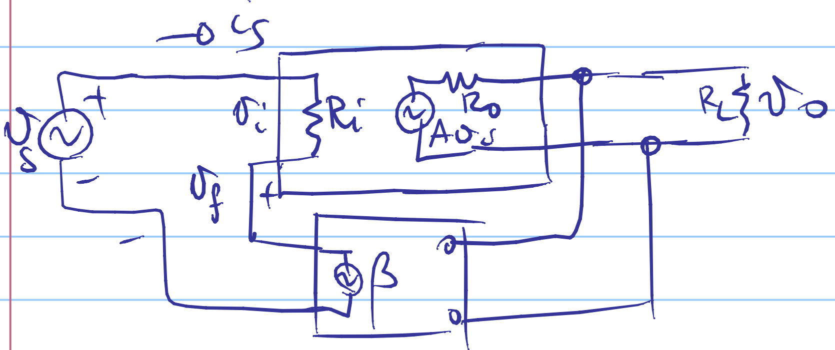
Transresistance



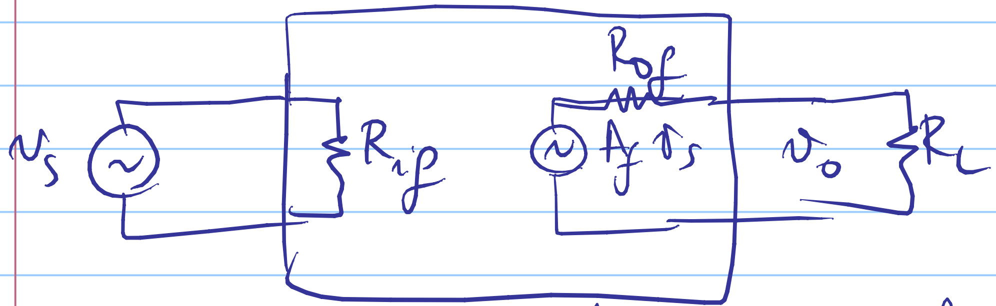
Ampli	Rin	Rout	Sampling	Summing
Voltage	∞	0	tensione	serie
Current	0	∞	corrente	parallelo
Transcond.	∞	∞	corrente	serie
Transres	0	0	tensione	parallelo

4. Effetto sulle impedenze di IN/OUT

1) tensione serie



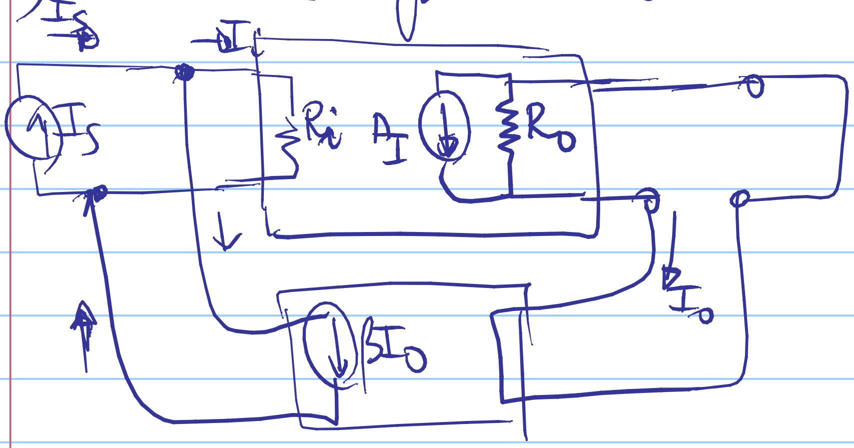
$$R_{if} = \frac{v_s}{i_s} = \frac{v_s}{v_i/R_i} = R_i \frac{v_i + v_f}{v_i} = R_i(1+\beta A)$$



$$R_{of} = \frac{v(\text{open})}{i(\text{short})} = \frac{A_f v_s}{A v_i / R_o} = R_o \frac{A_f v_s}{A\, v_s} = \frac{R_o}{1+\beta A}$$

Aumenta Rin, diminuisce Rout

2) Corrente parallelo



$$R_{if} = \frac{V_S}{I_S} = \frac{I_i R_i}{I_S} = R_i \frac{I_i}{I_i + I_f} = R_i \frac{\cancel{I_i}}{\cancel{I_i}(1+\beta A)} = \frac{R_i}{1+\beta A}$$

$$R_{of} = \frac{v(\text{open})}{i(\text{short})} = \frac{A \cancel{I_S} R_o}{A_f \cancel{I_S}} = R_o(1+\beta A)$$

Summing	RIN	Sampling	ROUT
serie	$R_i(1+\beta A)$	tensione-serie	$R_o/(1+\beta A)$
parallelo	$R_i/(1+\beta A)$	corrente-parall.	$R_o(1+\beta A)$

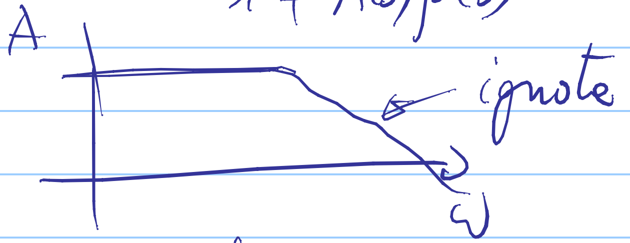
Il feedback negativo è virtuoso

5. Stabilità

In funzione delle frequenze $A = \frac{A(s)}{1 + A(s)\beta(s)} =$

$s = j\omega$

$= \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$



Ipotesi: A, β costanti a basse frequenze

Loop gain $L(j\omega) = A(j\omega)\beta(j\omega) \equiv |A||\beta| e^{j\varphi(\omega)}$

Per ω_{180} $\varphi(\omega_{180}) = 180°$ → $L = -|A||\beta|$

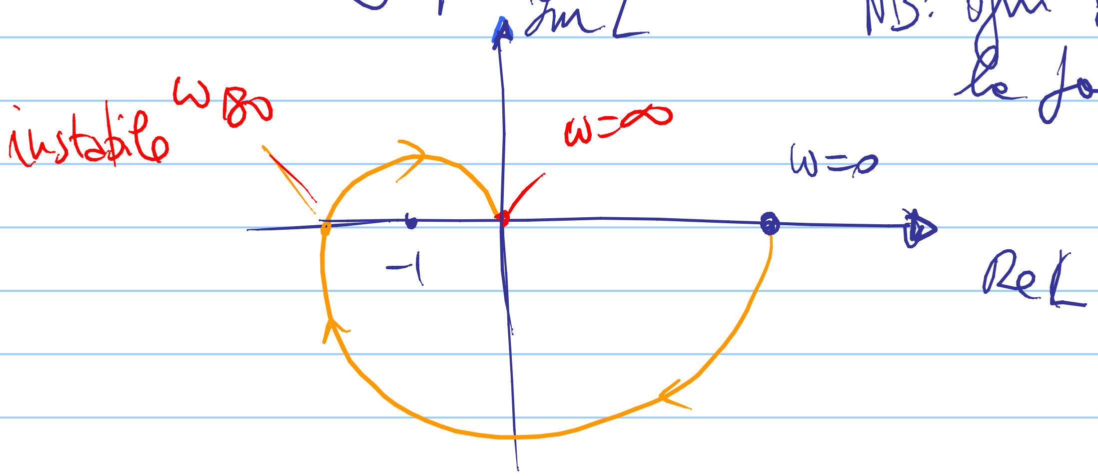
Se $L > -1$ A sarà ancora finito → stabile

Se $L = -1$ $A \to \infty$ per $\omega = \omega_{180}$ → oscilla alla frequenza ω_{180}

Se $L < -1$ → oscilla in modo determinato dalle non linearità.

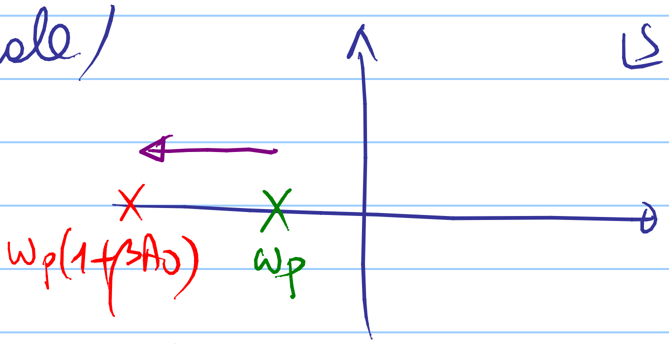
Plot di Nyquist

NB: ogni polo ruota la fase di $-90°$



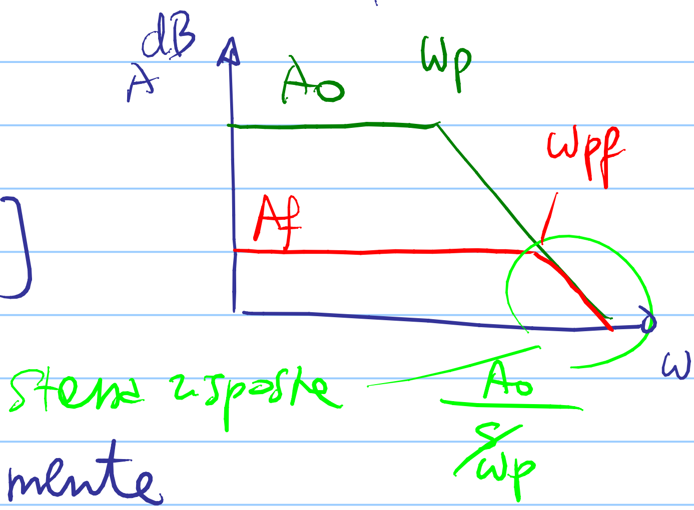
6 Effetto sui poli (β reale)

- Singolo polo



$$A(s) = \frac{A_0}{1 + s/\omega_p}$$

$$A_f(s) = \frac{A_0/(1+\beta A_0)}{1 + s/[\omega_p(1+\beta A_0)]}$$



Stabile incondizionatamente

- Doppio polo reale

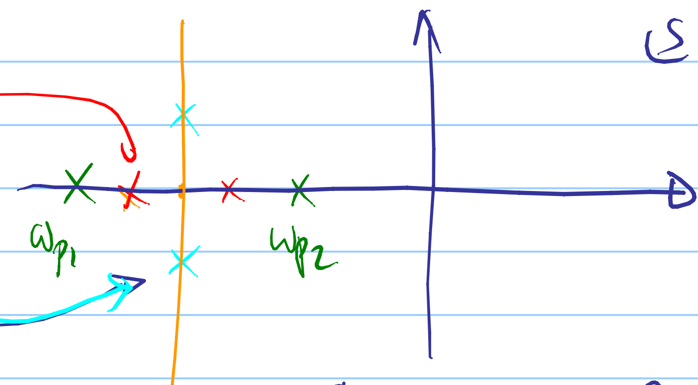
$$A(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+s/\omega_{p2}\right)} = \frac{A_0\,\omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2})}$$

$$A_f(s) = \frac{A_0\,\omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2}) + \beta A_0\omega_{p1}\omega_{p2}} = \frac{A_0\,\omega_{p1}\omega_{p2}}{s^2 + (\omega_{p1}+\omega_{p2})\,s + (1+\beta A_0)\omega_{p1}\omega_{p2}}$$

poli $s = -\frac{1}{2}(\omega_{p1}+\omega_{p2}) \pm \frac{1}{2}\sqrt{(\omega_{p1}+\omega_{p2})^2 - 4(1+\beta A_0)\omega_{p1}\omega_{p2}}$

poco feedback: reali

molto feedback immaginari



Stabile $\to$ max $\varphi = -90-90 = -180$ @ $\omega=\infty$

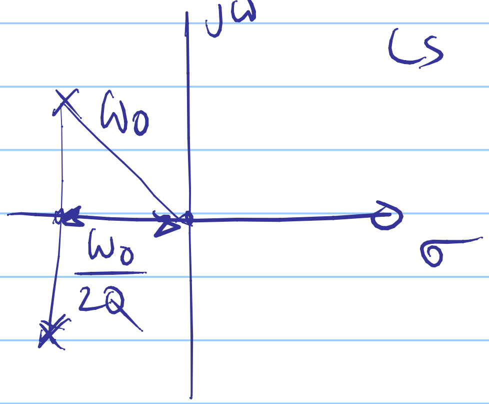
Scrivo l'eq come $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0$

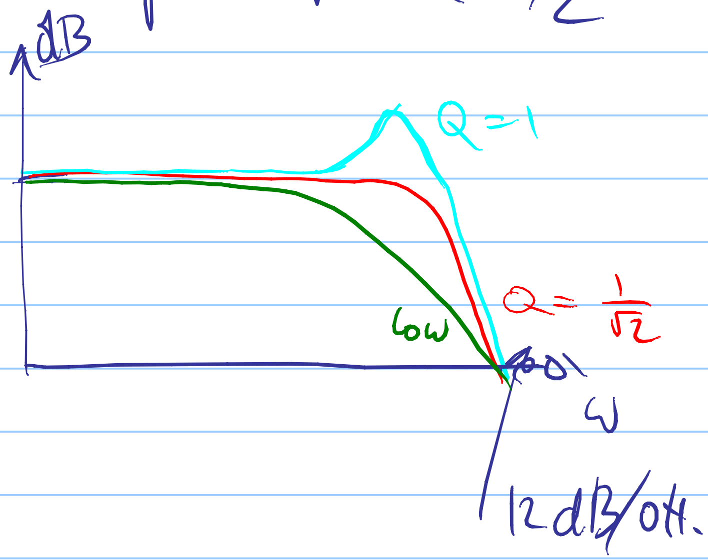
$$\omega_0^2 = (1+\beta A_0)\,\omega_{p1}\omega_{p2}$$

$$Q = \frac{\omega_0}{\omega_{p1}+\omega_{p2}} = \frac{\sqrt{(1+\beta A_0)\,\omega_{p1}\omega_{p2}}}{\omega_{p1}+\omega_{p2}}$$

$$s = -\frac{1}{2}\frac{\omega_0}{Q} \pm \frac{1}{2}\sqrt{\frac{\omega_0^2}{Q^2} - 4\omega_0^2} = -\frac{1}{2}\frac{\omega_0}{Q}\left[1 \pm \sqrt{1-4Q^2}\right]$$

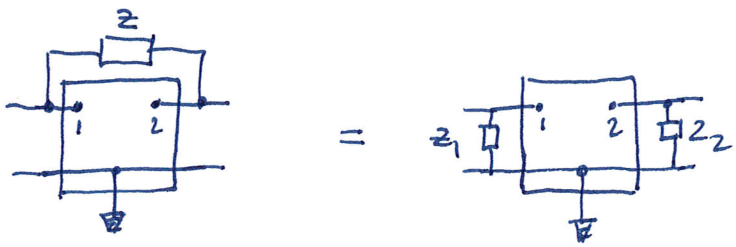
Reali $Q < \frac{1}{2}$ complessi per $Q > \frac{1}{2}$





Teorema di Miller

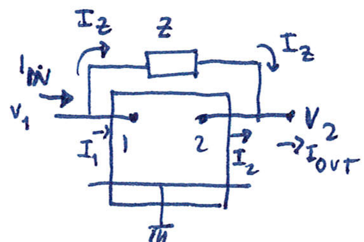
In un sistema lineare si può riportare l'impedenza della rete di feedback in ingresso e in uscita :

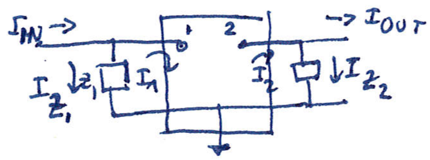


$K = \frac{V_2}{V_1} =$ guadagno con $Z_1 = Z/(1-K)$

$Z_2 = -K\,Z/(1-K)$

DIM.





$I_{Z_2} = -I_Z$

$$I_{IN} = \underset{\substack{\uparrow \\ \text{NON CAMBIA}}}{I_1} + I_Z \rightarrow I_Z = \frac{V_1 - V_2}{Z} = V_1 \frac{(1 - V_2/V_1)}{Z} = V_1 \frac{(1-K)}{Z} = \frac{V_1}{Z_1} = I_{Z_1}$$

$$I_{OUT} = \underset{\substack{\uparrow \\ \text{NON CAMBIA}}}{I_2} + I_Z \rightarrow -I_Z = -\frac{V_1 - V_2}{Z} = -V_2 \cdot \frac{V_1}{V_2} \frac{(1 - V_2/V_1)}{Z} = V_2\left[-\frac{1}{K}\frac{(1-K)}{Z}\right]$$

$$= \frac{V_2}{Z_2} = I_{Z_2}$$