

Lez. 3 Circuito RC, Trasformata di Fourier, Bode-Plot

Argomenti: Circuito RC passabasso e passa alto

Analisi nel dominio del tempo

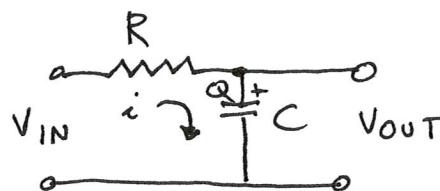
Trasformata di Fourier

Analisi nel dominio delle frequenze

Bode-plot

1. Circuito RC

C. Passa-basso



$\frac{1}{T}$ → Circuito aperto in DC
corto circuito ad alta frequenza

$$\left\{ \begin{array}{l} V_{OUT} = \frac{Q}{C} = \frac{1}{C} \left(Q_0 + \int_0^t i dt \right) \\ i = (V_{IN} - V_{OUT})/R \end{array} \right.$$

$$\frac{dV_{OUT}}{dt} = \frac{i}{C} = \frac{V_{IN} - V_{OUT}}{RC}$$

$\tau = RC$, tempo tipico del circuito

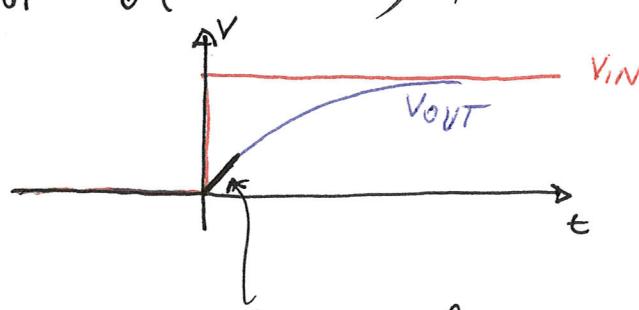
$$\frac{dV_{OUT}}{dt} + \frac{V_{OUT}}{\tau} = \frac{V_{IN}}{\tau}$$

$$1k\Omega \cdot 1nF = 1\mu s$$

- Se $V_{IN} = V_0 \theta(t)$ (gradino di tensione)

e $Q_0 = 0$ (condensatore scarso)

$$V_{OUT} = V_0 \left(1 - e^{-t/\tau} \right) \theta(t) \rightarrow t = -\tau \ln \left(1 - \frac{V_{OUT}}{V_0} \right)$$



per $t << \tau \rightarrow$ lineare
= integratore

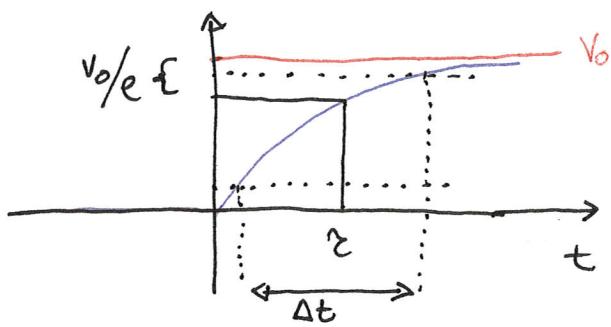
Se $t << \tau$

$$V_{OUT} \approx V_0 \cdot \frac{t}{\tau} = \frac{1}{\tau} \int V_{IN} dt$$

(integratore)

check: $\frac{dV_{OUT}}{dt} = V_0 \left[\frac{e^{-t/\tau}}{\tau} \cdot \theta(t) + (1 - e^{-t/\tau}) \delta(t) \right]$

• Tempo di salita



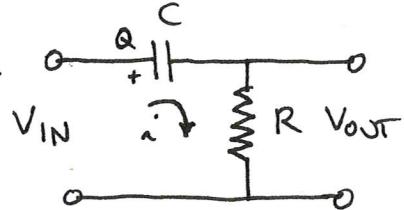
per $t = \tau$

$$V_{\text{OUT}} = V_0 \left(1 - \frac{1}{e}\right)$$

$$\Delta t (10\% \rightarrow 90\%) = t_{90\%} - t_{10\%} = -\tau \ln(0.1) + \tau \ln(0.9) \\ = \tau \ln 9 = 2.2 \tau$$

→ Le presenze di una capacità "annodante" il segnale.
verso masse

b. Pausa-alto

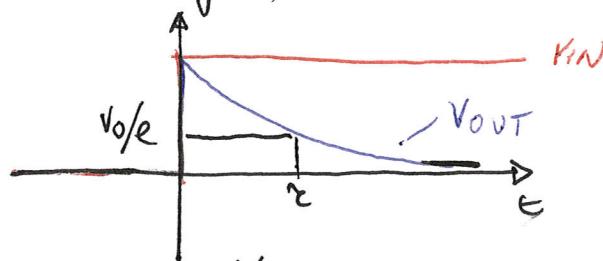


$$\begin{cases} (V_{\text{IN}} - V_{\text{OUT}}) = \frac{Q}{C} = \frac{1}{C} (Q_0 + \int_0^t i dt) \\ i = V_{\text{OUT}} / R \end{cases} \Rightarrow \frac{dV_{\text{IN}}}{dt} - \frac{dV_{\text{OUT}}}{dt} = \frac{i}{C} = \frac{V_{\text{OUT}}}{RC}$$

$$\frac{dV_{\text{OUT}}}{dt} + \frac{V_{\text{OUT}}}{\tau} = \frac{dV_{\text{IN}}}{dt}$$

$$\tau = RC$$

• Se $V_{\text{IN}} = V_0 \theta(t)$ e $Q_0 = 0$



$$\Delta t (90\% \rightarrow 10\%) = 2.2 \tau$$

$$\text{per } t \gg \tau \quad V_0 \approx 0 \Rightarrow \frac{dV_{\text{IN}}}{dt}$$

$$V_{\text{OUT}} = V_0 e^{-t/\tau} \cdot \theta(t)$$

$$\text{Check: } \frac{dV_{\text{OUT}}}{dt} = V_0 \left[-\frac{e^{-t/\tau}}{\tau} \delta(t) + e^{-t/\tau} \delta'(t) \right] = V_0 \left[-\frac{e^{-t/\tau}}{\tau} \theta(t) + \delta(t) \right]$$

2. Trasformate di Fourier

- L'analisi nel dominio del tempo è molto complica^{te}
 - Analisi nel dominio delle frequenze semplifica la soluzione delle equazioni e permette una migliore comprensione del circuito (o sistema fisico)
 - Somma p^{ro}positi^{on}e: risposta complessiva = somma delle risposte a segnali sinusoidali di frequenze definite

- definizione $\hat{f}(t)(\omega) = \textcircled{1} \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$

usando $\int_{-\infty}^{+\infty} e^{j\omega t} d\omega = 2\pi f(t)$

si ottiene l'antitrasformata

j per non confondere con la corrente

- convenzione in elettronica

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{f}(\omega) e^{j\omega t} d\omega$$

→ Ricordare che ω = pulsazione = $2\pi \cdot$ frequenze
 rad/s \downarrow Hz \downarrow

- Propriété : Linéaire $\widetilde{f_1 + f_2}(\omega) = \widetilde{f}_1(\omega) + \widetilde{f}_2(\omega)$
 - Trans. temp. $\widetilde{f(t-t_0)}(\omega) = \widetilde{f}(t)(\omega) \cdot e^{-j\omega t_0}$

$$\text{Trans. temp. } \widetilde{f(t-t_0)}(\omega) = \widetilde{f(t)}(\omega) \cdot e^{-j\omega t_0}$$

$$\text{Scaling} \quad \widetilde{f(\alpha t)}(\omega) = \left(\frac{1}{|\alpha|}\right) \widetilde{f}\left(\frac{\omega}{\alpha}\right)$$

$$\text{Transl. freq.: } \tilde{f}(t) e^{j\omega_0 t} (\omega_F) \quad \tilde{f}(t)/(\omega - \omega_0)$$

$$\text{Trasl. freq. } \tilde{f}(t) e^{j\omega_0 t} (\omega F) \quad \tilde{f}(t)(\omega - \omega_0)$$

Derivative $\tilde{f}'(t)(\omega) = j\omega \tilde{f}(\omega)$; Integral $\int \tilde{f}(t)dt(\omega) = \frac{1}{j\omega} \tilde{f}(t)(\omega)$

3. Analisi nel dominio delle frequenze

- Si immagine il circuito sollecitato da un segnale di frequenza definita $V = V_0 e^{j\omega t}$ complesso
- Si usano le impedenze complesse

$$\begin{aligned} Z &= R \quad \text{---} \frac{R}{R} \quad Y = \frac{1}{R} \\ Z &= \frac{1}{j\omega C} \quad \text{---} \frac{j\omega C}{C} \quad Y = j\omega C \\ Z &= j\omega L \quad \text{---} \frac{j\omega L}{L} \quad Y = \frac{1}{j\omega L} \end{aligned} \quad \left. \begin{array}{l} Y = \frac{1}{R} \\ Y = j\omega C \\ Y = \frac{1}{j\omega L} \end{array} \right\} \text{Ammettenze complesse}$$

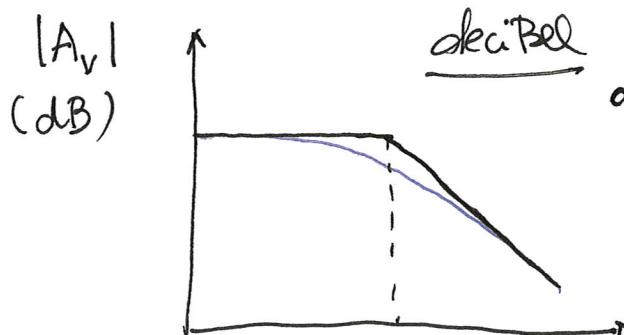
- Note tecniche: si usano segnali a frequenze (non pulsazione) definite

$$\tilde{V}(f) = V_0 \delta(f - f_0)$$

$$v(t) = V_0 \delta(f - f_0) e^{j 2\pi f t} = V_0 e^{j 2\pi f t} = V_0 e^{j\omega t}$$

V_0 contiene ampiezza e fase.
Segnale fisico = $\operatorname{Re}(V)$.

- Plot di Bode



$$A_V = \frac{V_{\text{out}}}{V_{\text{in}}} \quad A_V = \frac{V_{\text{out}}}{V_{\text{in}}}$$

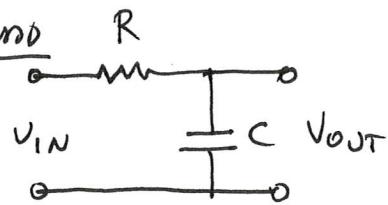
$$\text{dB} = 20 \log_{10} \frac{V_{\text{out}}}{V_{\text{in}}}$$

riferito alle potenze divente 10

$$\text{dB}_W = 10 \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}} \quad P \sim V^2 \rightarrow \text{stesse cose}$$

4. RC nel dominio delle frequenze

A. Passa basso



Puntatore complesso

$$V_{OUT} = \frac{Z_C}{R + Z_C} \cdot V_{IN}$$

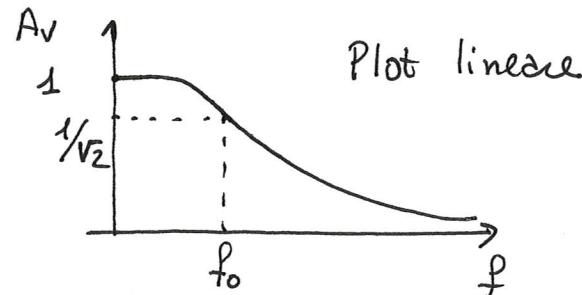
$$\text{Guadagno } A_V = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega^2}$$

$$\text{Se scriviamo } \omega = 1/\omega_0 = 1/2\pi f_0 \quad f_0 = \frac{1}{2\pi RC}$$

$$A_V = \frac{1}{1 + j\omega/\omega_0} = \frac{1}{1 + j f/f_0}$$

$$|A_V| = \frac{1}{\sqrt{1 + f^2/f_0^2}}$$

f_0 = frequenza di taglio



Meglio plot di Bode con i $dB = 20 \log_{10} \frac{V_{OUT}}{V_{IN}}$

AV	dB
1	0 dB
10	20 dB
0.1	-20 dB
$1/\sqrt{2}$	-3 dB
$1/2$	-6 dB

Se $f \ll f_0$ $|A_V| \sim 1 = 0 \text{ dB}$

Se $f \gg f_0$ $|A_V| \sim \frac{f_0}{f}$

$A_V(dB) > 0 \rightarrow$ guadagno

$A_V(dB) < 0 \rightarrow$ attenuazione

Per RC $A_V(dB) < 0$ sempre

Pendenze $1/f$

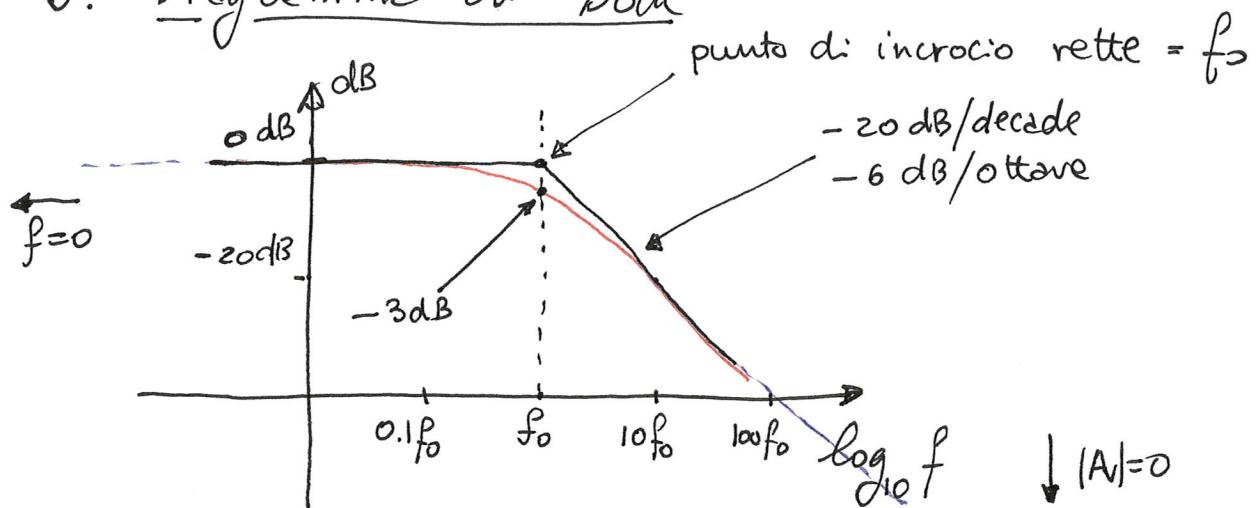
- 20 dB/decade ($f \rightarrow 10f$)

- 6 dB/octave ($f \rightarrow 2f$)

Note: errore sulle misure in dB:

$$\text{se } z = k \log_b x \quad \sigma_z = \frac{K}{\ln b} \frac{\sigma_x}{x} \quad \frac{20 \text{ dB}}{\ln 10} = 8.7 \text{ dB}$$

b. Diagramma di Bode



Punti notevoli: • per $f = f_0$ $|A_{v1}| = \frac{1}{\sqrt{2}} = -3\text{dB}$

- se considero le due rette:

$$f \ll f_0 \quad |A_{v1}| = 1 = 0\text{dB}$$

$$f \gg f_0 \quad |A_{v1}| = \frac{f_0}{f} \rightarrow |A_{v1}|\text{(dB)} = 20(\log_{10} f_0 - \log_{10} f)$$

Queste si incontrano per $f = f_0$.

• Fase

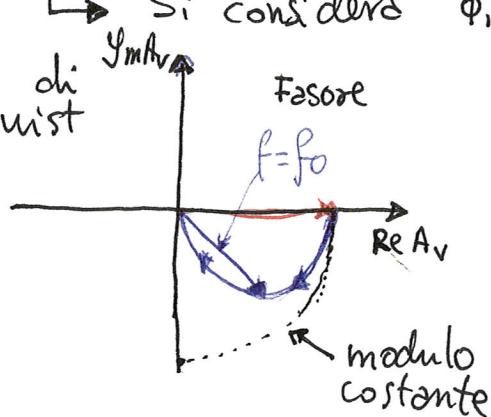
Il segnale esce sfasato dal filtro RC

$$V_{\text{OUT}} = A_v V_{\text{IN}} \rightarrow |V_{\text{OUT}}| e^{j\phi_{\text{OUT}}} e^{j\omega t} = |A_v| e^{j\phi} |V_{\text{IN}}| e^{j\phi_{\text{IN}}} e^{j\omega t}$$

$$\phi_{\text{OUT}} = \phi_{\text{IN}} + \phi$$

Si considera $\phi_{\text{IN}} = 0$

Plot di Nyquist
 $y_{\text{m}A_v}$



$$A_v = \frac{1}{1+j\frac{\omega}{f_0}}$$

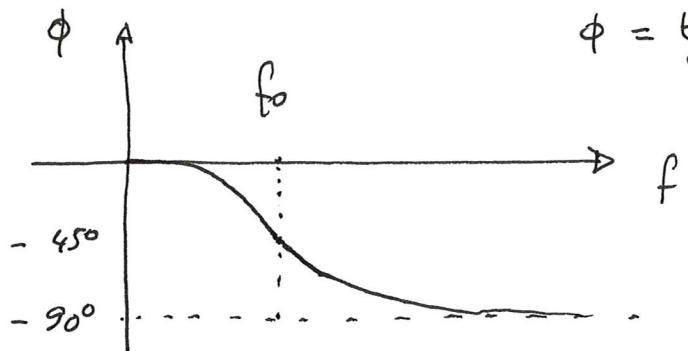
$$\frac{y_{\text{m}Av}}{\text{Re } A_v} = \tan \phi = -\frac{f}{f_0}$$

$$f=0 \quad \tan \phi = 0$$

$$f=0 \quad \tan \phi = -1 \quad \phi = -45^\circ$$

$$f \rightarrow \infty \quad \tan \phi \rightarrow -\infty \quad \phi = -90^\circ$$

Ancora su fase

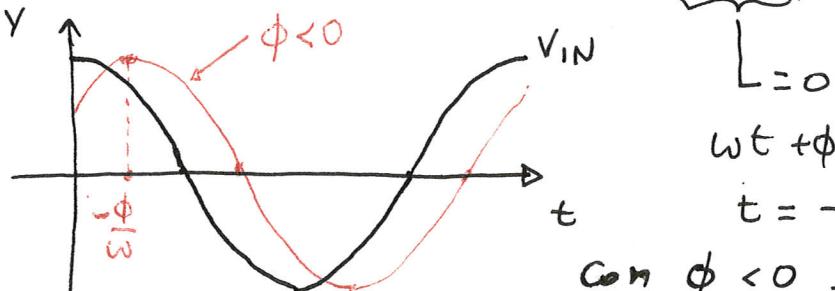


$$\phi = \operatorname{tg}^{-1}\left(-\frac{f}{f_0}\right)$$

H segnale anticipa o ritarda?

$$V_{IN} = |V_{IN}| e^{j\omega t} \rightarrow |V_{IN}| \cos \omega t$$

$$V_{OUT} = |V_{OUT}| e^{j(\omega t + \phi)} \rightarrow |V_{OUT}| \cos(\omega t + \phi)$$

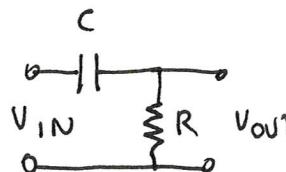


$$\omega t + \phi = 0 \quad \text{per}$$

$$t = -\frac{\phi}{\omega}$$

con $\phi < 0$ è l'onda
ritarda

C Passo alto



$$V_{OUT} = \frac{R}{R + Z_C} V_{IN}$$

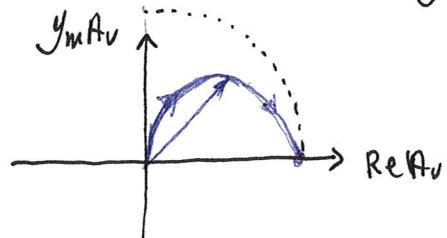
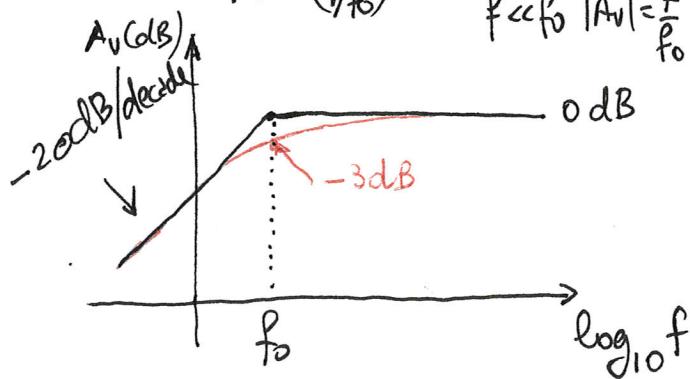
$$f_0 = \frac{1}{2\pi RC}$$

$$f \gg f_0 \quad |A_v| \approx 1$$

$$f \ll f_0 \quad |A_v| = \frac{f}{f_0}$$

$$A_v = \frac{j\omega RC}{1 + j\omega RC} = \frac{j f/f_0}{1 + j f/f_0}$$

$$\operatorname{tg} \phi = \frac{Y_m A_v}{R_e A_v} = \frac{f_0/f}{1 + f/f_0}$$



d. Integratore e derivatore

- I circuiti RC possono essere usati come derivatori e integratori.

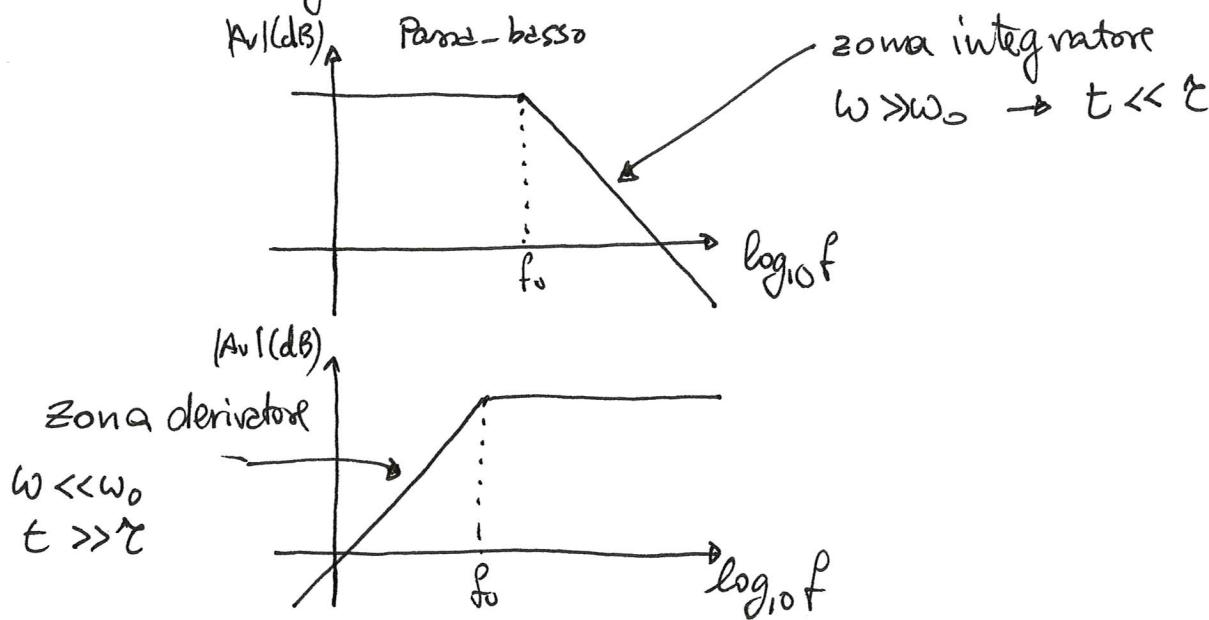
- In trasformata di Fourier

$$\frac{d}{dt} \rightarrow j\omega \quad \int dt \rightarrow \frac{1}{j\omega}$$

- Pass - basso $A_v = \frac{1}{1+j\omega/\omega_0} \xrightarrow{\omega \gg \omega_0} \frac{\omega_0}{j\omega} \rightarrow \text{int.}$

- Pass - alto $A_v = \frac{j\omega/\omega_0}{1+j\omega/\omega_0} \xrightarrow{\omega \ll \omega_0} j\frac{\omega}{\omega_0} \rightarrow \text{deriv.}$

- Sul diagramma di Bode

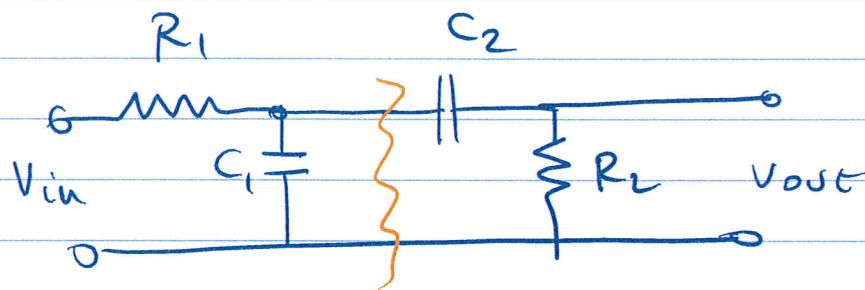


- N.B.: I filtri passivi funzionano come integratore e derivatore nelle zone a basso gradagno.

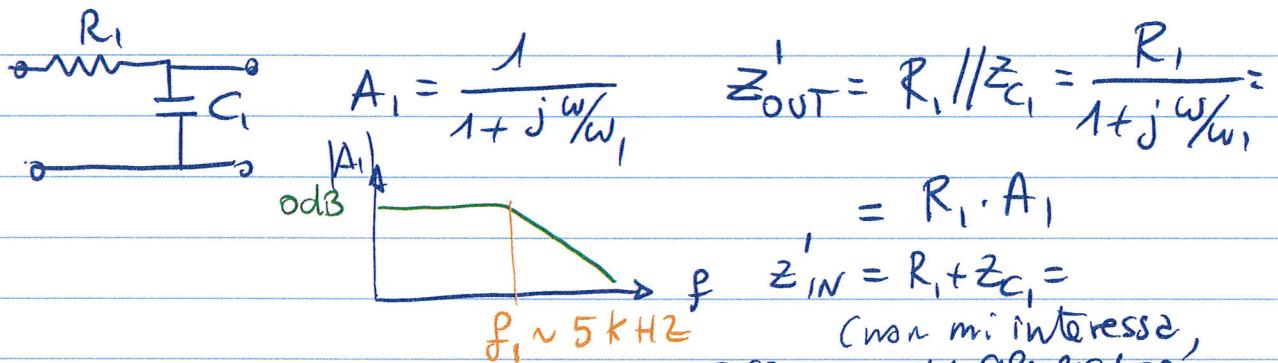
Filtro passabande con RC

Titolo nota

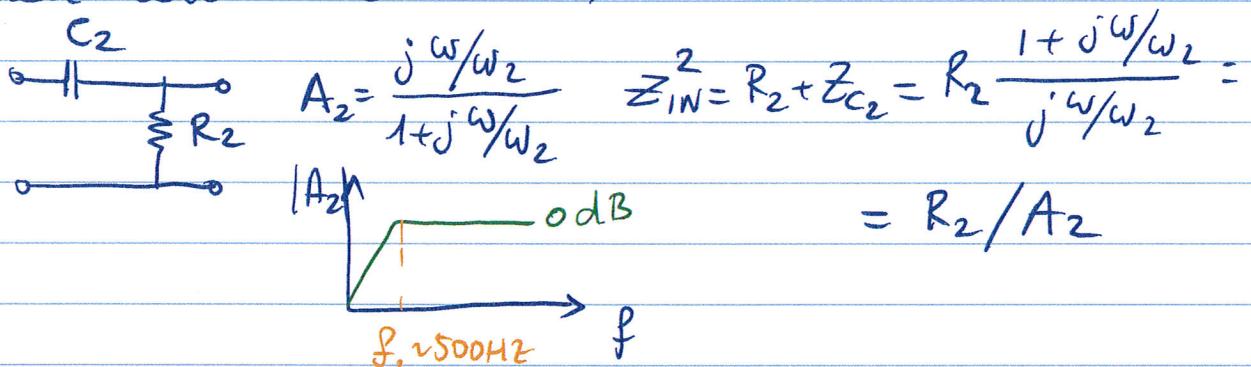
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Passa basso $\omega_1 = (R_1 C_1)^{-1}$

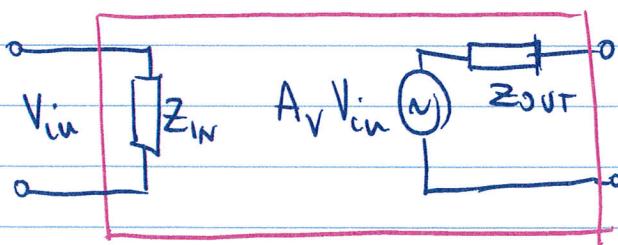


Passa alto $\omega_2 = (R_2 C_2)^{-1}$

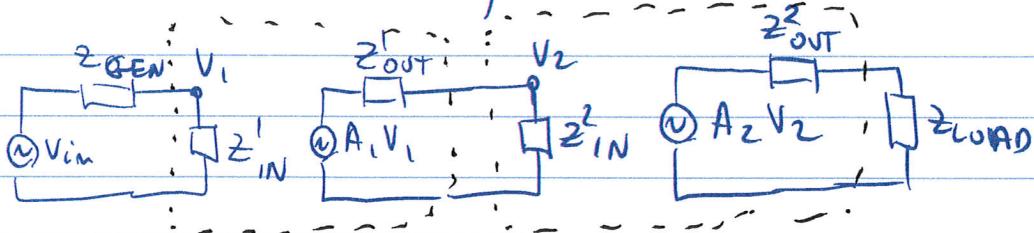


Se li metto in serie $A_{TOT} \neq A_1 \cdot A_2$

Qualunque quadruolo lineare può essere rappresentato come:



Nelle serie si può scrivere:



Considera $Z_{GEN} = 0 \quad Z_{LOAD} = \infty$

Rimane la parte centrale

$$V_{OUT} = A_2 V_2 = A_2 \cdot A_1 V_{IN} \cdot \frac{Z_{IN}^2}{Z_{OUT}^1 + Z_{IN}^2}$$

$$A_{TOT} = A_1 A_2 \cdot \frac{Z_{IN}^2}{Z_{OUT}^1 + Z_{IN}^2} = A_1 A_2 \frac{R_2 / A_2}{R_1 A_1 + R_2 / A_2} = A_1 A_2 \frac{1}{1 + \frac{R_1}{R_2} A_1 A_2}$$

Se $R_1 \ll R_2 \rightarrow A_{TOT} = A_1 A_2$

$$\text{Se } R_1 = R_2 \rightarrow A_{TOT} = \frac{A_1 A_2}{1 + A_1 A_2} \quad A_{TOT}^{\text{MAX}} = \frac{1}{2}$$

$$\text{Se } \omega \ll \omega_1 \quad A_1 \approx 1 \rightarrow A_{TOT} = \frac{A_2}{1 + A_2} = \frac{1}{1 + j \frac{\omega}{\omega_2}} \quad \frac{j \frac{\omega}{\omega_2}}{1 + j \frac{\omega}{\omega_2}}$$

$$= \frac{j \frac{\omega}{\omega_2}}{1 + 2j \frac{\omega}{\omega_2}} = \frac{j \frac{\omega}{\omega_2}}{2} \frac{1}{1 + j \frac{\omega}{\omega_2}} \xrightarrow{\text{guadagno } 1/2}$$

$$\xrightarrow{\text{taglio a } \frac{\omega_2}{2}}$$

$$\text{Se } \omega \gg \omega_2 \quad A_2 \approx 1 \rightarrow A_{TOT} = \frac{A_1}{1 + A_1} =$$

$$= \frac{1}{2 + j \frac{\omega}{\omega_1}} = \frac{1}{2} \frac{1}{1 + j \frac{\omega}{(2\omega_1)}}$$

