Actividad 1. Unidad 1: Funciones analíticas

Problem 1 Prove algebraically that for complex numbers,

$$|z_1| - |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|.$$

Problem 2 Show that:

a)
$$\cos n\theta = \cos^n \theta - \binom{n}{2} \cos^{n-2}\theta \sin^2\theta + \binom{n}{4} \cos^{n-4}\theta \sin^4\theta - \dots$$

b)
$$\sin n\theta = \binom{n}{1} \cos^{n-1}\theta \sin\theta - \binom{n}{3} \cos^{n-3}\theta \sin^3\theta + \dots$$

Note. The quantities $\binom{n}{m}$ are binomial coefficients: $\binom{n}{m} = \frac{n!}{m!(n-m)!}$.

Problem 3 Prove that:

a)
$$\sum_{n=0}^{N-1} \cos nx = \frac{\sin(Nx/2)}{\sin x/2} \cos(N-1)\frac{x}{2},$$

b)
$$\sum_{n=0}^{N-1} \sin nx = \frac{\sin(Nx/2)}{\sin x/2} \sin(N-1)\frac{x}{2}.$$

Note. Parts (a) and (b) may be combined to form a geometric series.

Problem 4 Assume that the trigonometric functions ($\sin z$ and $\cos z$) and the hyperbolic functions ($\sinh z$ and $\cosh z$) are defined for complex argument by the appropriate power series. Show that:

a) $i \sin z = \sinh iz$

b) $\sin iz = i \sinh z$

c) $\cos z = \cosh iz$

d) $\cos iz = \cosh z$

Tomado de: Arfken & Weber, Mathematical methods for physicists (6th ed., 2005). Elsevier. Chapter 6: Functions of a Complex Variable I.
Fecha de Entrega: 13 de Septiembre de 2024. Buil in LaTeX