Factorización no-negativa

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1. Sean x_j, h_j las j-ésimas cols. de X y H

$$\|x_j - Wh_j\|_2^2 = (x_j - Wh_j)^T(x_j - Wh_j)$$

$$= x_j^T x_j + \underbrace{(Wh_j)^T(Wh_j) - 2x_j^T(Wh_j)}_{q(y)}$$

$$q(y) = (Wh_j)^T(Wh_j) - 2x_j^T(Wh_j)$$

$$= h_j^T(W^TW)h_j - 2(W^Tx_j)^Th_j$$

$$= \tfrac{1}{2}h_j(2W^TW)h_j - 2(W^Tx_j)^Th_j$$

$$\Rightarrow Q = 2W^TW,\ c = -2(W^Tx_j)$$

2. Sean x_i, w_i las i-ésimas filas de X y W
(x_i, w_i vectores columna)

$$\|x_i - H^Tw_i\|_2^2 = x_i^Tx_i + \underbrace{(H^Tw_i)^T(H^Tw_i) - 2x_i^T(H^Tw_i)}_{q(y)}$$

$$q(y) = w_i^T(HH^T)w_i - 2(Hx_i)^Tw_i$$

$$= \tfrac{1}{2}w_i(2HH^T)w_i - 2(Hx_i)^Tw_i$$

$$\Rightarrow Q = 2HH^T,\ c = -2(Hx_i)$$