Homework 3 **Root Finding Methods** Due October 8, 2021

1: Root finding methods (10 pts)

Explain the difference between a bracketing method and an open root finding method. List 2 examples of each method.

1) Bracketing methods are based on two initial guesses that"bracket" the root one on either side of the root.

   Ex: Incremental search, Bisection

2) Open methods can involve one or more initial guesses, but they don't bracket the root.

   Ex: Newton-Raphson, Secant Method

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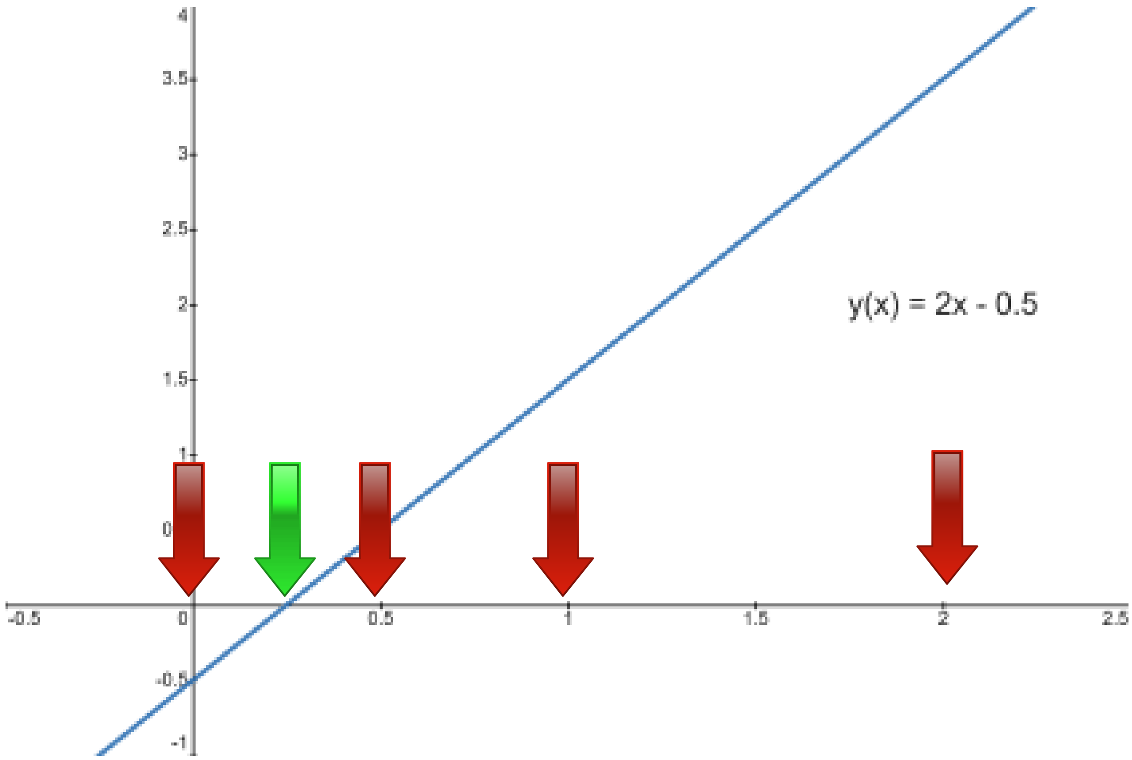
2: Bisection Method (15 pts)

The Bisection method is a variation of the incremental search method in which the interval is always divided in half. If a function changes sign over the interval, the function at the midpoint is evaluated. The root is then determined to lie in the interval where the sign change occurs. That subinterval becomes the new interval for the next iteration. The process is repeated until the root is known to a required precision.

Use this method to find the root of $y(x) = 2x - 0.5$. Iterate 5 times with initial bounds $a = 0$, $b = 2$. Draw your intervals on the figure below. Report x_r and the percent relative error for each interval where applicable. Remember, once a solution has been found, there is no need to continue iterating.



iteration 1:

root: 1

Relative error: 1

iteration 2:

root: 0.5

Relative error:1

iteration 3:

root: 0.25

Relative error: 1

iteration 4: not needed

iteration 5: not needed

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3: Bisection Method (15 pts)

Modify the/submit your own bisection method code developed in lecture to determine the drag coefficient, c_d, needed so that a 95*kg* bungee jumper has a velocity of 46*m/s* after 9*s* of free fall. Note: The acceleration of gravity is 9.81m/s^2. Start with initial guesses of x_l = 0.2 and x_u = 0.5 and iterate until the approximate relative error falls below 1%. Report the value of c_d and the number of iterations it took to find the root. Discuss your results. Submit your code.

The root is 0.39453125, the number of iterations was: 8

4: Simple fixed-point iteration (15 pts)/(25 pts)

Open methods employ a formula to predict the root. Use simple fixed-point iteration to locate the root of $f(x) = \sin(\sqrt{x}) - x$. Start with an initial guess of $x_0 = 0.5$ and iterate 6 times. Fill in the table below. You are free to calculate the values of x by hand or write a program to do it for you. 10 bonus points will be awarded for the submission of a working program. Whatever you choose, show all your work for full credit.

i	xi	xi+1
0	0.500000000	0.823591179
1	0.823591179	0.769696432
2	0.769696432	0.768649277
3	0.768649277	0.768648857 ← root found
4	0.768648857	not needed
5	not needed	not needed

The root of this function is ≈ 0.7686488567609. What do you notice about your results?
My results find the root of the equation before the 6 iterations are done.

5: Newton-Raphson Method (15 pts)/(20 pts)

Use the Newton-Raphson method to locate the root of $f(x) = \sin(\sqrt{x}) - x$. Start with an initial guess of $x_0 = 0.5$ and iterate 6 times. Fill in the table below. Note: $f'(x) = \frac{1}{2\sqrt{x}}\cos(\sqrt{x}) - 1$.

You are free to calculate the values of x by hand or write a program to do it for you. 5 bonus points will be awarded for the submission of a working program. Whatever you choose, show all your work for full credit.

i	xi	xi+1
0	0.500000000	0.823591179
1	0.823591179	0.769696432
2	0.769696432	0.768649277
3	0.768649277	0.768648857 ← root found
4	0.768648857	not needed
5	not needed	not needed

The root of this function is ≈ 0.7686488567609. What do you notice about your results?
My results find the root of the equation before the 6 iterations are done. Also, I got the same results for question 4.

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6: Newton-Raphson Method (15 pts)

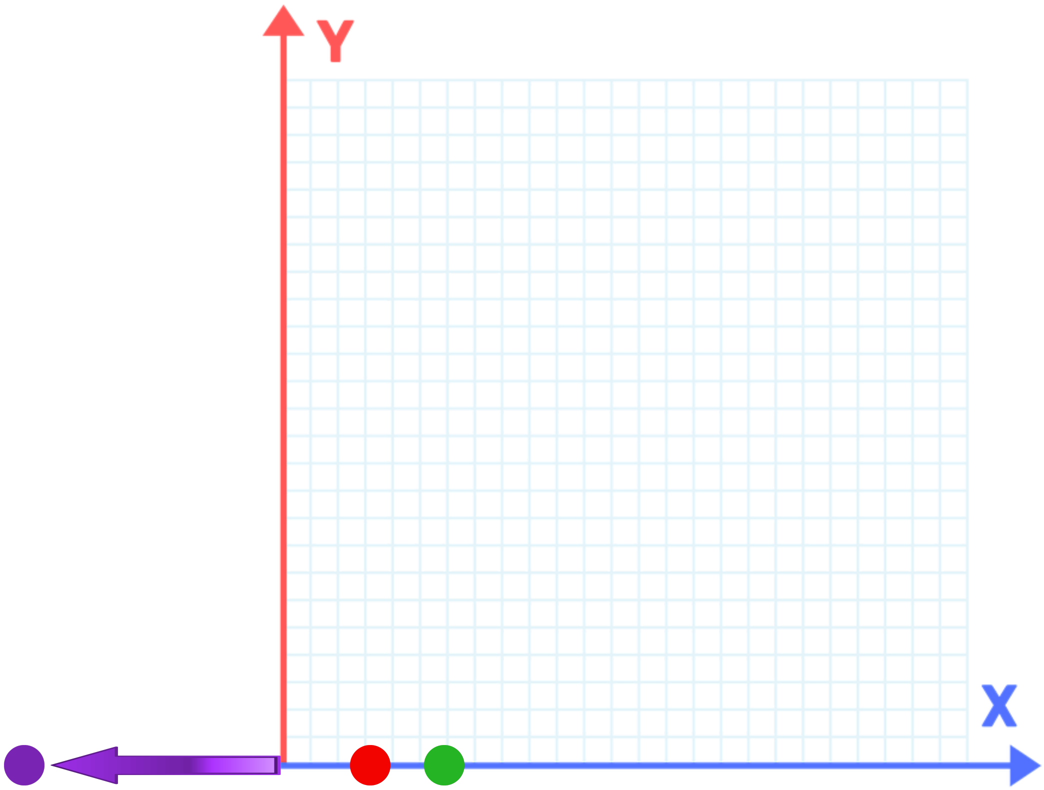
Modify the/submit your own Newton-Raphson method code developed in lecture and apply it to the function $f(x) = \tanh(x^2 - 9)$ to evaluate its known real root at $x = 3$. Use an initial guess of $x_0 = 3.2$ and take a minimum of three iterations. Submit your code.

- $x_1 = 2.73681558$
- $x_2 = 3.67019683$
- $x_3 = -256.413291$

Did the method converge to its real root?

No, the Newton Raphson method led to a divergent root, since at x4 equals infinity.

Sketch the plot with the results for each iteration labeled.



7: Newton-Raphson Method (15 pts)

Modify the/submit your own Newton-Raphson method code developed in lecture to determine the drag coefficient, c_d, needed so that a 95*kg* bungee jumper has a velocity of 46*m/s* after 9*s* of free fall. Note: The acceleration of gravity is 9.81m/s^2. Start with initial guesses of x_l = 0.2 and x_u = 0.5 and iterate until the approximate relative error falls below 1%. Report the value of c_d and the number of iterations it took to find the root. Compare your result to Question 3. Submit your code.

The root is 0.394229721, the number of iterations was: 4.

With this method we managed to get (almost) the same result for the root as in question 3 in fewer iterations.