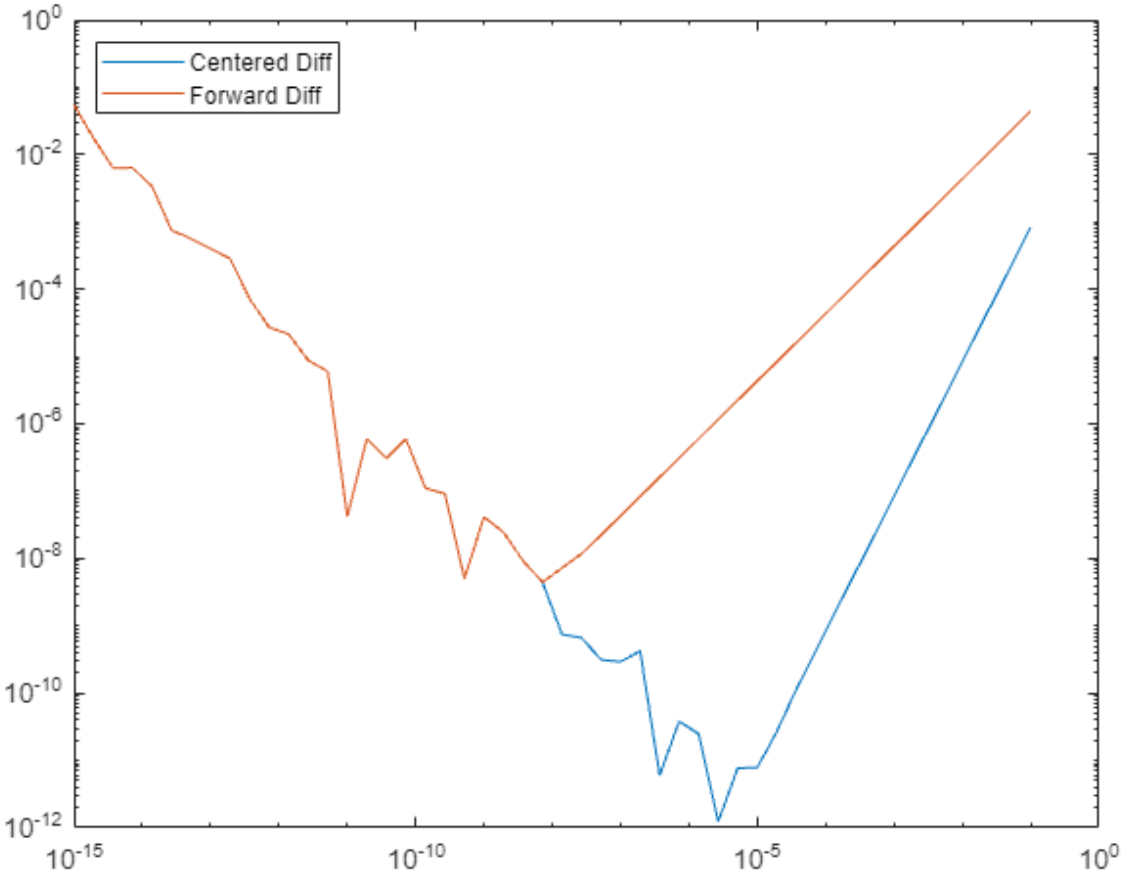
ISC 4220
Algorithms 1
Numerical Differentiation

Part 1:

Construct a log-log plot of absolute error and h:



Optimum value of h for the Centered difference formula (x value at minimum of blueline) : 2.6827*10^(-6)

Optimum value of h for the Forward difference formula (x value at minimum of orange line) : 7.19686*10^(-9)

Compare the absolute errors of the two numerical formulae:

As seen in the graph, the absolute error of the centered differentiation formula, which overlaps perfectly with the forward differentiation error until a certain point, reaches a smaller error value than the forward differentiation one.
From this we can conclude that, if we had to choose from the two, it would be optimal to use the centered differentiation formula in this specific case.

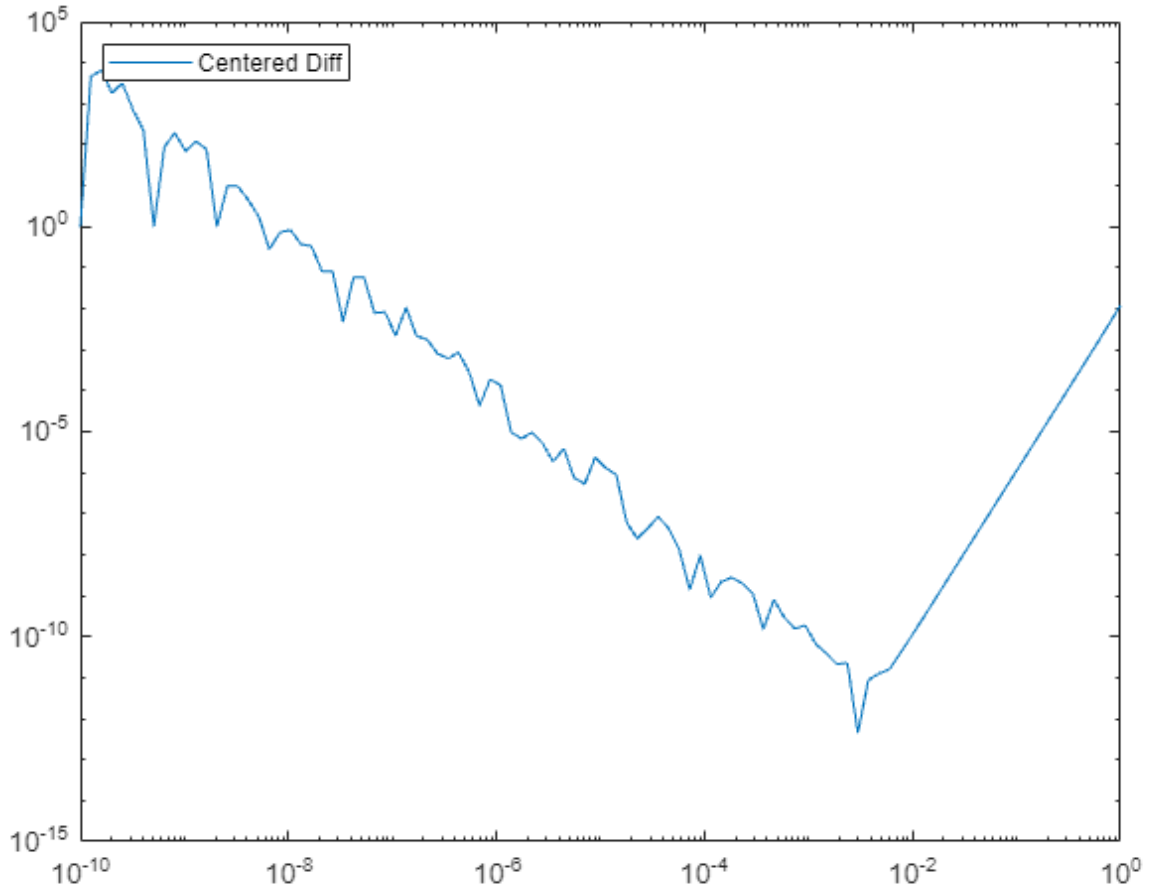
Part 2:

Write an approximate expression for the total error:

$$d/dh\ [\ (\text{leading truncation error term}) + (\text{roundoff error})\ ] = 0$$

$$d/dh\ [\ (\ (h^4)/90\ )*u^6(x) + (\ (C*emach*u(x))/2h\ )\ ] = 0$$

Consider 100 logarithmically equispaced values for h between 10^−10 and 10^0. Define the error as the difference between the true second derivative of u(x) and that numerically computed using eqn. 1 at x = 0.
Find the value of h and the at which this error is minimum, and the minimum value of the error. (15 pts)



Value of h and the at which this error is minimum(x value at minimum of blueline):
0.00298365

Minimum value of the error:
4.4742e^-13