ISC 4220
Continuous Algorithms
Interpolation

1. The gamma function for $x > 0$ is given by the integral:

•Use divided differences to determine a polynomial of degree 4 that interpolates through the 5 points.

table of resulting Divided differences: (Code submitted in canvas)

1	0	0.5	0.333	0.25
1	1	1.5	1.333	0
2	4	5.5	0	0
6	16	0	0	0
21	0	0	0	0

For plotting I will turn the first column of the matrix into a row vector and use that:

1	1	2	6	21

4th degree polynomial:
0.25x^4 - 2.1667x^3 + 7.25x^2 - 10.333x + 6

•Use the in-built cubic spline (for example, interp1 with splines) to interpolate the same data.

Resulting Divided differences

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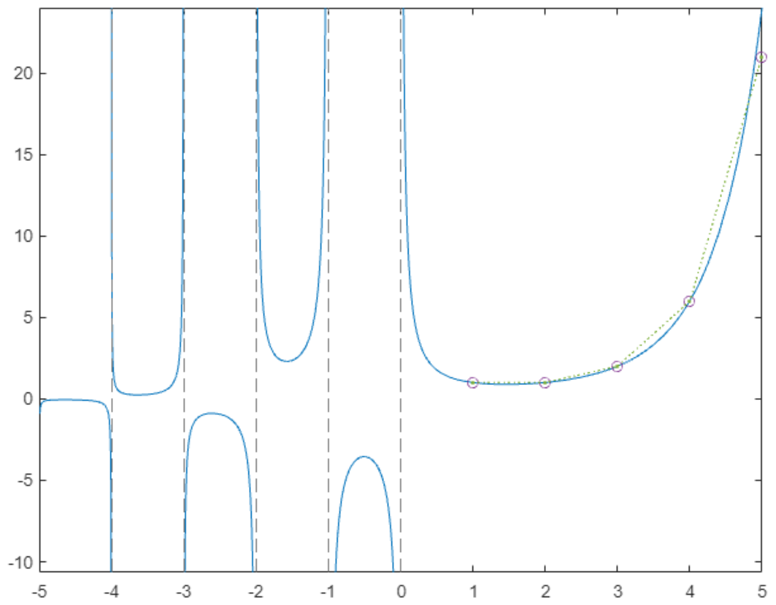
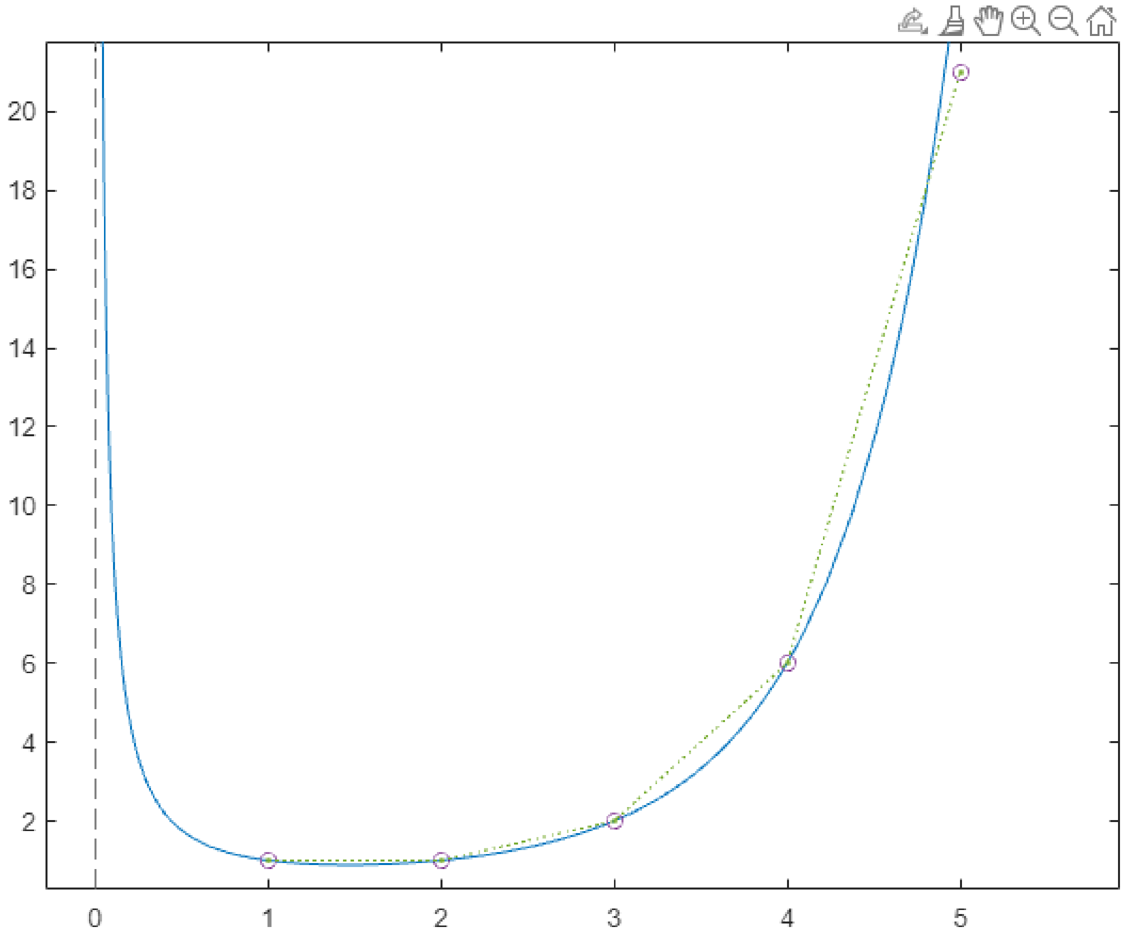
1     1     2     6    21
```

•Plot and compare the interpolated polynomials, and the intrinsic Matlab function gamma.



A closer look:



My polynomial overlaps with matlab's polynomial in the graph since we obtained the same values for the divided differences.
They both seem to follow the shape of the gamma function, if you were to draw it with straight lines instead of a smooth curve.

2. We recently studied successive quadratic interpolation for 1D optimization. We claimed that the maximum of a quadratic polynomial p2(x) passing through the points (x0, f0), (x1, f1), and (x2, f2), is:

•Find the quadratic interpolating polynomial, p2(x), which passes through the three points (1,3), (2,5), (3,3).

Polynomial found using matlab's polyfit function:
p2(x) = -2x^{2}+8x-3

•Find the maximum of p2(x) by solving for p′2(x) = 0.

p'2(x) = -4x+8

x when p'2(x) = 0:
-4x+8 = 0 → x = 2

•How does this compare with the formula for xmax above?

Using the formula we get:

xmax = (3*-5 + 5*8 + 3*-3) / (6*-1 + 10*2 + 6*-1) = 16/8 = 2
xmax = 2

Conclusion:
we get the same maximum either way.