

Andres Candido

ISC 4220
Continuous Algorithms
Numerical Integration

1. Consider the integral

For $n=1$:

$$I = 0$$

Rel error = 1

Abs error = $4/9$

For $n=2$:

$$I = -0.2451$$

Rel error = 0.4485

Abs error = 0.199

For $n=4$:

$$I = -0.3581$$

Rel error = 0.1943

Abs error = 0.086

For $n=8$:

$$I = -0.4081$$

Rel error = 0.08178

Abs error = 0.036

2. Evaluate the integral:

Using Simpson's 1/3 rule: 1.3230

abs error: 0.19

Using Gauss Quadrature: 1.4937

abs error: 0.365

3. From 2015 Exam

(i) Use 4-point Gauss quadrature formula to compute the integrals:

$$\text{From } -1 \text{ to } 1: I = 1.298$$

From 0 to 2: $I = 0.9763$

(ii) Suppose we wish to evaluate the integral: Using one of the methods that we studied in class. How would you deal with the “infinity” in the upper limit of integration? Be as specific as possible.

Instead of using infinity I would use a very large number, 10000000 for example. This is because the given integral will converge to a point if we use infinity as the upper limit, and by using a large number we can get an approximation of that point/value.

(iii) Consider two integrals. where D is the region described by $x_1^2 + 2x_2^2 \leq 1$. Would you recommend using Monte Carlo to evaluate both integrals? Explain your choice in detail.

I would recommend using Monte Carlo only on the first integral since it has closed boundaries (from 0 to 1 on both dimensions). However, since the second integral has boundaries that go from less or equal to 1, meaning they go from -infinity to 1, I wouldn't recommend Monte Carlo.

Recall the throwing darts example, we throw darts on a grid of a set size that encompasses an region of interest. We can then estimate the proportion of area of that region, in reference to the grid's area, by comparing the number of darts that landed in the region vs the number of the ones that didn't.

We can only find the area of the region of interest if the grid has a set size, if it was infinitely large, then the region of interest would become infinitely small in comparison. A good estimate of the area could not be made.