Taller Metodo De Gauss

sábado, 23 de agosto de 2025 10:07

b) $x - 2y + 5z - u = -6$
$x + y - z + u = 6$
$x + 2y - z - u = 2$
$2x + y + u = 7$

$$\left[\begin{array}{cccc|c} 1 & -2 & 5 & -1 & -6 \\ 1 & 1 & -1 & 1 & 6 \\ 1 & 2 & -1 & -1 & 2 \\ 2 & 1 & 0 & 1 & 7 \end{array}\right]$$

$F_2 \leftarrow F_2 - F_1:\ (1-1,\ 1-(-2),\ -1-5,\ 1-(-1),\ 6-(-6)) = (0, 3, -6, 2, 12)$
$F_3 \leftarrow F_3 - F_1:\ (1-1,\ 2-(-2),\ -1-5,\ -1-(-1),\ 2-(-6)) = (0, 4, -6, 0, 8)$
$F_4 \leftarrow F_4 - 2F_1:\ (2-2,\ 1-2(-2),\ 0-2\cdot 5,\ 1-2(-1),\ 7-2(-6)) = (0, 5, -10, 3, 19)$

$$\left[\begin{array}{cccc|c} 1 & -2 & 5 & -1 & -6 \\ 0 & 3 & -6 & 2 & 12 \\ 0 & 4 & -6 & 0 & 8 \\ 0 & 5 & -10 & 3 & 19 \end{array}\right]$$

$F_2 \leftarrow \frac{1}{3}F_2:\ (0,\ 3,\ -6,\ 2,\ 12)/3 = (0, 1, -2, \frac{2}{3}, 4)$

$$\left[\begin{array}{cccc|c} 1 & -2 & 5 & -1 & -6 \\ 0 & 1 & -2 & \frac{2}{3} & 4 \\ 0 & 4 & -6 & 0 & 8 \\ 0 & 5 & -10 & 3 & 19 \end{array}\right]$$

$F_3 \leftarrow F_3 - 4F_2:\ (0, 4, -6, 0, 8) - 4(0, 1, -2, \frac{2}{3}, 4) = (0, 0, 2, -\frac{8}{3}, -8)$
$F_4 \leftarrow F_4 - 5F_2:\ (0, 5, -10, 3, 19) - 5(0, 1, -2, \frac{2}{3}, 4) = (0, 0, 0, -\frac{1}{3}, -1)$

$$\left[\begin{array}{cccc|c} 1 & -2 & 5 & -1 & -6 \\ 0 & 1 & -2 & \frac{2}{3} & 4 \\ 0 & 0 & 2 & -\frac{8}{3} & -8 \\ 0 & 0 & 0 & -\frac{1}{3} & -1 \end{array}\right]$$

$F_3 \leftarrow \frac{1}{2}F_3:\ (0, 0, 2, -\frac{8}{3}, -8)/2 = (0, 0, 1, -\frac{4}{3}, -4)$
$F_4 \leftarrow -3F_4:\ (0, 0, 0, -\frac{1}{3}, -1)\cdot(-3) = (0, 0, 0, 1, 3)$

$$\left[\begin{array}{cccc|c} 1 & -2 & 5 & -1 & -6 \\ 0 & 1 & -2 & \frac{2}{3} & 4 \\ 0 & 0 & 1 & -\frac{4}{3} & -4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

Solución: (x, y, z, μ) = (1, 2, 0, 3)

Comprobación:

1 - 2(2) + 5(0) - 3 = -6

1 + 2 - 0 + 3 = 6

1 + 2(2) - 0 - 3 = 2

2(1) + 2 + 3 = 7

d) $x + 2y - z + u = -3$
$2x - y + 2z - u = 8$
$-x + 3y - 2z + u = -7$
$3x - 4y + 3z - 2u = 13.$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -3 \\ 2 & -1 & 2 & -1 & 8 \\ -1 & 3 & -2 & 1 & -7 \\ 3 & -4 & 3 & -2 & 13 \end{array}\right]$$

$F_2 = (2, -1, 2, -1, 8) - 2(1, 2, -1, 1, -3)$
$F_3 = (-1, 3, -2, 1, -7) + (1, 2, -1, 1, -3)$
$F_4 = (3, -4, 3, -2, 13) - 3(1, 2, -1, 1, -3)$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -3 \\ 0 & -5 & 4 & -3 & 14 \\ 0 & 5 & -3 & 2 & -10 \\ 0 & -10 & 6 & -5 & 22 \end{array}\right]$$

$F_2 = -\frac{1}{5}(0, -5, 4, -3, 14)$
$F_3 = (0, 5, -3, 2, -10) - 5(0, 1, -\frac{4}{5}, \frac{3}{5}, -\frac{14}{5})$
$F_4 = (0, -10, 6, -5, 22) + 10(0, 1, -\frac{4}{5}, \frac{3}{5}, -\frac{14}{5})$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -3 \\ 0 & 1 & -\frac{4}{5} & \frac{3}{5} & -\frac{14}{5} \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & -2 & 1 & -6 \end{array}\right]$$

$F_4 = (0, 0, -2, 1, -6) + 2(0, 0, 1, -1, 4)$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -3 \\ 0 & 1 & -\frac{4}{5} & \frac{3}{5} & -\frac{14}{5} \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & -1 & 2 \end{array}\right]$$

$F_4 = -1(0, 0, 0, -1, 2)$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -3 \\ 0 & 1 & -\frac{4}{5} & \frac{3}{5} & -\frac{14}{5} \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$u = -2$

$$\begin{aligned} z - u &= 4 \\ z - (-2) &= 4 \\ z + 2 &= 4 \\ z &= 2 \end{aligned}$$

$$\begin{aligned} y - \tfrac{4}{5}z + \tfrac{3}{5}u &= -\tfrac{14}{5} \\ y - \tfrac{4}{5}\cdot 2 + \tfrac{3}{5}\cdot(-2) &= -\tfrac{14}{5} \\ y - \tfrac{8}{5} - \tfrac{6}{5} &= -\tfrac{14}{5} \\ y - \tfrac{14}{5} &= -\tfrac{14}{5} \\ y &= 0 \end{aligned}$$

$$\begin{aligned} x + 2y - z + u &= -3 \\ x + 2\cdot 0 - 2 + (-2) &= -3 \\ x - 4 &= -3 \\ x &= 1 \end{aligned}$$

Solucion: $(x, y, z, u) = (1,\ 0,\ 2,\ -2)$

Comprobacion

$$x + 2y - z + u = -3$$

$$1 + 2(0) - 2 + (-2) = 1 - 2 - 2 = -3$$

$$2x - y + 2z - u = 8$$

$$2(1) - 0 + 2(2) - (-2) = 2 + 4 + 2 = 8$$

$$-x + 3y - 2z + u = -7$$

$$-(1) + 3(0) - 2(2) + (-2) = -1 - 4 - 2 = -7$$

$$3x - 4y + 3z - 2u = 13$$

$$3(1) - 4(0) + 3(2) - 2(-2) = 3 + 6 + 4 = 13$$