Elasticities in AIDS Models

Richard Green and Julian M. Alston

In the literature, a variety of approaches have been used to calculate demand elasticities in almost ideal demand system (AIDS) models of demand. It is common to estimate the linear approximate almost ideal demand system (LA/AIDS) instead of the AIDS. When the LA/AIDS is estimated, all of the previously reported approaches to compute elasticities are theoretically incorrect. This paper presents correct formulas for LA/AIDS elasticities and illustrates the potential errors from using incorrect computing formulas.

Key words: AIDS and LA/AIDS models, computing formulas, elasticities.

The almost ideal demand system (AIDS) of Deaton and Muellbauer (1980a,b) has become popular in recent years (Anderson and Blundell; Blanciforti and Green 1983a,b; Blanciforti, Green, and King; Chalfant; Eales and Unnevehr; Fujii, Khaled, and Mak; Fulponi, Heien and Willett; Murray; Parsons; Ray). A variety of approaches to computing elasticities has been used, and some of the approaches may lead to significant errors. This paper clarifies the differences between alternative approaches to estimating demand elasticities in AIDS models.

The AIDS Model

The AIDS model is usually specified as

$$(1)\qquad w_i = \alpha_i + \sum_j \gamma_{ij}\ln P_j + \beta_i \ln\left(\frac{X}{P}\right),$$

where X is total expenditure on the group of goods being analyzed, P is the price index for the group, P_j is the price of the jth good within the group, w_i is the share of total expenditure allocated to the ith good (i.e. $w_i = P_iQ_i/X$), and the price index (P) is defined as

$$(2)\qquad \ln P = \alpha_0 + \sum_j \alpha_j \ln P_j + \frac{1}{2}\sum_j \sum_i \gamma_{ij}\ln P_i \ln P_j.$$

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The Linear Approximate AIDS (LA/AIDS) Model

Using the price index from equation (2) often raises empirical difficulties, especially when aggregate annual time-series data are used, and it is common to use Stone's (geometric) price index (P^*) instead of P:

$$(3)\qquad \ln P^* = \sum_k w_k \ln P_k.$$

The model that uses Stone's index is called the "linear approximate AIDS" (LA/AIDS) following Blanciforti and Green (1983a). If prices are highly collinear, P may be approximately proportional to P^*, i.e., $P \cong \zeta P^*$. In the extreme case when P is exactly (linearly) proportional to P^*, the LA/AIDS model can be used to estimate the parameters of the AIDS model because, then, the LA/AIDS can be written (in terms of the AIDS model parameters) as

$$(4)\qquad w_i = (\alpha_i - \beta_i \ln\zeta) + \sum_j \gamma_{ij}\ln P_j + \beta_i \ln\left(\frac{X}{P^*}\right).$$

More generally, however, the relationshp between the parameters of the AIDS and the corresponding parameters of the LA/AIDS is not known.[1] In addition, it is not known whether the LA/AIDS has satisfactory theoretical properties. These issues notwithstanding, the LA/AIDS is very popular.

[1] The AIDS and the LA/AIDS are actually nonnested systems. However, as the Stone's index becomes a better and better proxy for the price index in (2), the estimates from the LA/AIDS would approach the estimates for the AIDS except for the intercept term.

Price Elasticities

A general definition of the uncompensated elasticities of demand from the AIDS and LA/AIDS (η_{ij}) is

$$(5)\quad \eta_{ij} = \frac{d\ln Q_i}{d\ln P_j} = -\delta_{ij} + \frac{d\ln w_i}{d\ln P_j} = -\delta_{ij} + \left\{\gamma_{ij} - \beta_i \frac{d\ln P}{d\ln P_j}\right\} \Big/ w_i,$$

where these elasticities refer to allocations within the group holding constant total group expenditures (X) and all other prices (P_k, $k \neq j$), δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$; $\delta_{ij} = 0$ for $i \neq j$) and for the LA/AIDS we use P^* from (3) instead of P from (2).

The differences in the literature can be represented in terms of different expressions for the elasticity of the group price index with respect to the jth price (i.e., $d\ln P/d\ln P_j$ or $d\ln P^*/d\ln P_j$). These differences carry over directly into the computation of compensated elasticities (η^*_{ij}), which are

$$(6)\qquad \eta^*_{ij} = \eta_{ij} + w_j\left(1 + \frac{\beta_i}{w_i}\right).$$

In the AIDS model, the correct expression for the elasticity of the group price with respect to the jth price is

$$(7)\qquad \frac{d\ln P}{d\ln P_j} = \alpha_j + \sum_k \gamma_{kj} \ln P_k.$$

Substituting (7) into (5) yields the correct elasticities for the AIDS. Several authors (e.g., Anderson and Blundell; Blanciforti and Green 1983a,b); Fulponi; Heien and Willett) have used the resulting computing formula for AIDS elasticities with parameters from the LA/AIDS.

To obtain the correct formula for the LA/AIDS, we differentiate Stone's price index with respect to the jth commodity price and get

$$(8)\quad d\ln P^*/d\ln P_j = w_j + \sum_k w_k \ln P_k \frac{d\ln w_k}{d\ln P_j}.$$

From (5) we have $d\ln w_i/d\ln P_j = \delta_{ij} + \eta_{ij}$, so that (8) may be written as

$$(8')\quad d\ln P^*/d\ln P_j = w_j + \sum_k w_k \ln P_k(\eta_{kj} + \delta_{kj}).$$

Notice the similarity between this expression for the LA/AIDS and equation (7) for the AIDS. An important difference is that when equation (8) is substituted into (5) or (6) the elasticity of interest is expressed in terms of itself and all of the other elasticities. In the case of n goods we have n^2 simultaneous equations for uncompensated demand elasticities of the form

$$(9)\quad \eta_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i}\left\{w_j + \sum_k w_k \ln P_k(\eta_{kj} + \delta_{kj})\right\}.$$

Equation (9) can be expressed in matrix form as

$$(10)\qquad E = A - (BC)(E + I),$$

where the typical elements are $a_{ij} = -\delta_{ij} + (\gamma_{ij}/w_i) - \beta_i(w_j/w_i)$ in A (an $n \times n$ matrix); $b_i = (\beta_i/w_i)$ in B (an $n \times 1$ vector); $c_j = w_j \ln P_j$ in C (a $1 \times n$ vector); and η_{ij} in E (an $n \times n$ matrix). Solving for the elasticities [η_{ij}] yields, after some simplifications,[2]

$$(11)\qquad E = [BC + I]^{-1}[A + I] - I.$$

Several authors have used computing formulas for the LA/AIDS that may be seen as special cases of this result. A common approach (e.g., Chalfant; Fujii, Khaled, and Mak) is to use equations for elasticities of the form $\eta_{ij} = -\delta_{ij} + \{\gamma_{ij} - \beta_i w_j\}/w_i$, which obtain in the special case when $d\ln P^*/d\ln P_j = w_j$ (i.e., expenditure shares are constant). As a further special case, Eales and Unnevehr use an elasticity formula corresponding to $\eta_{ij} = -\delta_{ij} + \gamma_{ij}/w_i$. This is compatible with both AIDS and LA/AIDS under the assumption that either preferences are homothetic ($\beta_i = 0\ \forall i$) or the group price is constant, independent of individual goods' prices (i.e., $d\ln P^*/d\ln P_j = 0$).[3] In their application the latter assumption is appropriate.

Table 1 summarizes the different approaches to estimating uncompensated price elasticities from the AIDS and the LA/AIDS. Clearly, all of the formulas are identical when budget shares are invariant with income (i.e., $\beta_i = 0\ \forall i$; pref-

[2] The steps are as follows. First, add an identity matrix (I) to both sides of (10) to get $(E + I) = (A + I) - (BC)(E + I)$. Then add $(BC)(E + I)$ to both sides to get $(BC + I)(E + I) = A + I$. Then premultiply by $(BC + I)^{-1}$ to get $(E + I) = (BC + I)^{-1}(A + I)$.

[3] Parsons uses a formula for compensated elasticities that is the same as Eales and Unnevehr's formula for uncompensated elasticities. We cannot see conditions under which Parson's formula could be correct.

Table 1. Uncompensated Price Elasticities for the AIDS and the LA/AIDS

		Estimates for Food Group			
Model	Uncompensated Price Elasticity Formula η_{ij}	Meats	Fruits & Vegetables	Cereal & Bakery Products	Misc. Foods
AIDS[a]					
	$-\delta_{ij} + \gamma_{ij}/w_i - \beta_i\alpha_j/w_i - \frac{\beta_i}{w_i}\sum_k \gamma_{kj}\ln P_k$	−0.994	−0.256	−0.799	−0.787
LA/AIDS[b]					
(i)	$-\delta_{ij} + \gamma_{ij}/w_i - \beta_i\alpha_j/w_i - \frac{\beta_i}{w_i}\sum_k \gamma_{kj}\ln P_k$	−0.411	−0.229	−0.736	−0.325
(ii)	$-\delta_{ij} + \gamma_{ij}/w_i$	−0.664	−0.200	−0.888	−1.066
(iii)	$-\delta_{ij} + \gamma_{ij}/w_i - \beta_i w_j/w_i$	−0.988	−0.255	−0.811	−0.764
(iv)	$-\delta_{ij} + \gamma_{ij}/w_i - \beta_i w_j/w_i - \frac{\beta_i}{w_i}\left[\sum_k w_k \ln P_k(\eta_{kj} + \delta_{kj})\right]$	−0.996	−0.255	−0.810	−0.761

[a] Estimates from Blanciforti, Green, and King.
[b] All of these models refer to parameters estimated by the LA/AIDS. Model (i) uses the AIDS formula (e.g., Anderson and Blundell); model (ii) corresponds to Eales and Unnevehr with $d\ln P^*/d\ln P_j = 0$; model (iii) assumes $d\ln P^*/d\ln P_j = w_j$ (e.g., Chalfant); model (iv) is the correct formula for LA/AIDS.

erences are homothetic). When preferences are not homothetic the formulas differ, perhaps in important ways.

Empirical Work

Next, we compare the empirical results from the AIDS model and the four alternative approaches to estimate elasticities from LA/AIDS parameter estimates. Full information maximum likelihood estimates of the AIDS applied to U.S. food consumption data are reported in Blanciforti, Green, and King (p. 36) and those results were used to calculate the AIDS elasticities. Using the same data we estimated the LA/AIDS.

Table 1 reports the alternative measures of uncompensated elasticities (at the means of the sample data) implied by these estimates using the alternative formulas.[4] As can be seen from table 1, the elasticity estimates for any commodity are similar across the AIDS and the LA/AIDS models (iii) and (iv); but the estimates are quite different using the LA/AIDS models (i) and (ii).

In this example, the correct LA/AIDS model (iv) provided similar elasticities to the AIDS model, and the error from using model (iii) (which treats shares as exogenous) was small. However, large errors result from using model (i) (i.e., the AIDS formula with LA/AIDS parameters) or model (ii) (i.e., assuming the price index is exogenous).

We checked the sensitivity of these results for autocorrelation. In the LA/AIDS model, the estimated autocorrelation coefficient was 0.9017 with an asymptotic t-value of 22.0. When the estimates are corrected for autocorrelation the estimated β_i values for the various food groups are −0.002, 0.002, −0.095, and 0.094. Thus, correcting for autocorrelation in this particular application essentially reduces the real income effect to zero and the differences in the various elasticity expressions vanish.

Conclusions and Future Extensions

When the LA/AIDS is estimated, we recommend using the theoretically correct formula which has been developed here. Researchers should employ the AIDS price elasticity formula only when they have estimated the AIDS. In our empirical work these two approaches led to essentially identical elasticity estimates. However, that may be an artifact of the particular data set we analyzed. Monte Carlo studies would be needed in order to establish the conditions under which the LA/AIDS provides a close approximation to the AIDS. The authors are currently engaged in further work exploring some of these issues.

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[4] Only the own-price uncompensated elasticities are reported. The cross-price elasticities and other details of the estimation are available from the authors.

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