In this notebook I try to asses whether the algorithm is *numerically* ill-posed, as the eigenvalues of $e^{-\tau L}$ get really small so there is no difference between any Laplacian when τ goes to ∞ as that is just the matrix $J_n = \{\frac{1}{n}\}_{ij}$, that is why we implement the regularization terms, but then the regularized solution is not the real solution, *is it near the real solution*? I also would not expect the solution to dramatically change for little changes over the regularization parameters.

In [ ]:
```
import learnHeat as lh
import networkx as nx
import numpy as np
from matplotlib import pyplot as plt

np.set_printoptions(precision=5)
np.set_printoptions(suppress=True)

np.random.seed(10)
```

Let $X = \texttt{create\_signal}(L_0, H_0, \tau_0)$ be created from three ground inputs

if I feed the learning algorithm with two of the three ground inputs, I would expect to recover the same solution near the real solution for all three choices of excluded ground inputs

$$\text{res} = \texttt{learnHeat}(X, L_{rand}, H_0, \tau_0)$$
$$= \texttt{learnHeat}(X, L_0, H_{rand}, \tau_0)$$
$$= \texttt{learnHeat}(X, L_0, H_0, \tau_{rand})$$

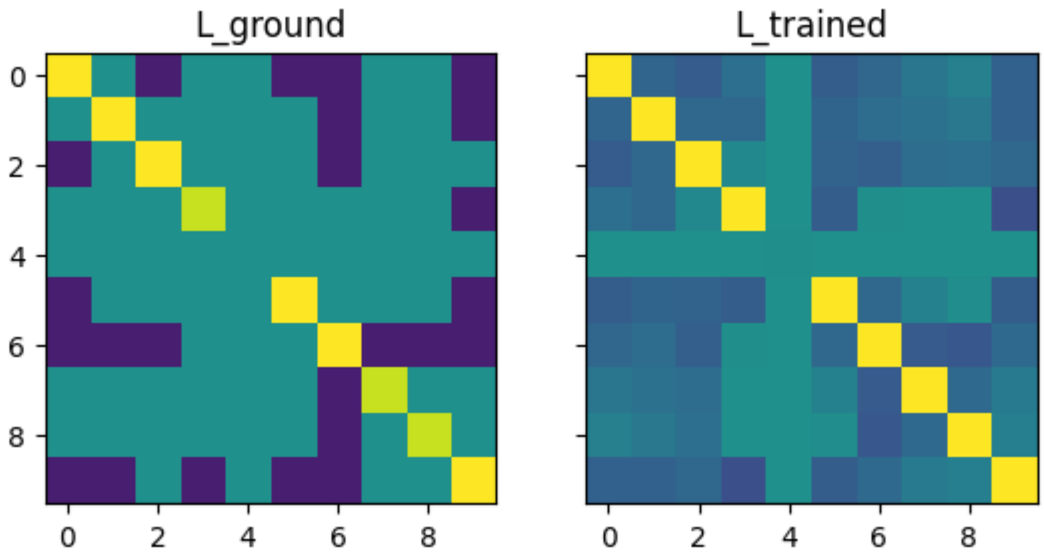
In [ ]:
```
N = 10 # number of vertices
M = 60 # number of signals (I put a lot of signals just in case)
tau_ground = [2.5,4] # as in paper
X, L_ground, H_ground, tau_ground = lh.create_signal(N=N,M=M,p=0.3,tau_gr
```

Now I will not feed the real Laplacian

In [ ]:
```
res = lh.learn_heat(X,H0=H_ground,tau0=tau_ground,beta=0.05)
```

```
Learning progress:   0%|          | 0/100 [00:00<?, ?it/s]
```

In [ ]:
```
vmin = -0.5
vmax = 0.5
f, (ax1, ax2) = plt.subplots(1, 2, sharey=True)
ax1.imshow(L_ground/np.trace(L_ground)*N, vmin = vmin, vmax = vmax)
ax1.set_title('L_ground')
ax2.imshow(res["L"], vmin = vmin, vmax = vmax)
ax2.set_title('L_trained')
plt.show()
```



I think we can't actually assess the quality of the learned Laplacian without the precision and the recall. We can draw the precision-recall curve to be extremelly fair. In the paper, the threshold for which an edge is deprecated (the filter to the entries of the Laplacian) is 0.0001. Of course if they feed 20000 signals like in the paper, the algorithm will work, but that is just plain useless in real applications.

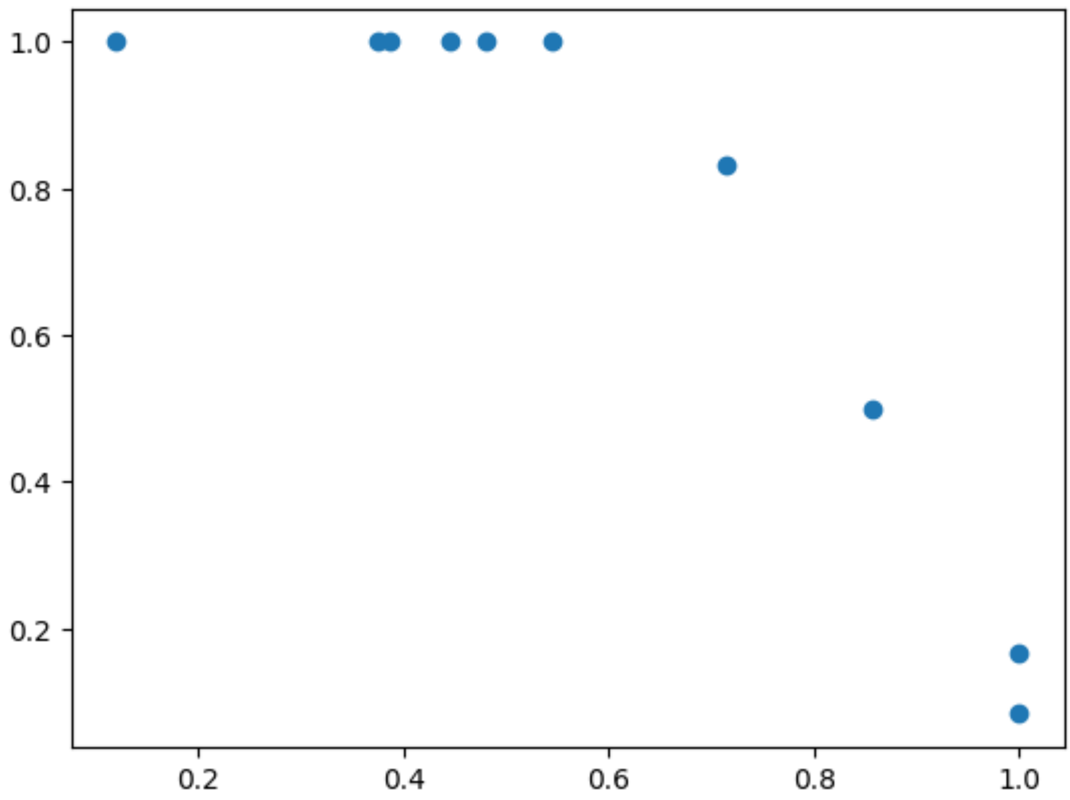
In [ ]:
```
# checking the least squares loss for the learned laplacian
np.linalg.norm(X-lh.D(res["L"],tau_ground)@H_ground,'fro')**2
# is not bad!!
```

Out[ ]:
```
0.05803559073094958
```

In [ ]:
```
precision, recall = lh.heat_scores(L1=res["L"],L2=L_ground)
```

In [ ]:
```
plt.scatter(precision,recall)
```

Out[ ]:
```
<matplotlib.collections.PathCollection at 0x7fc79808b5b0>
```

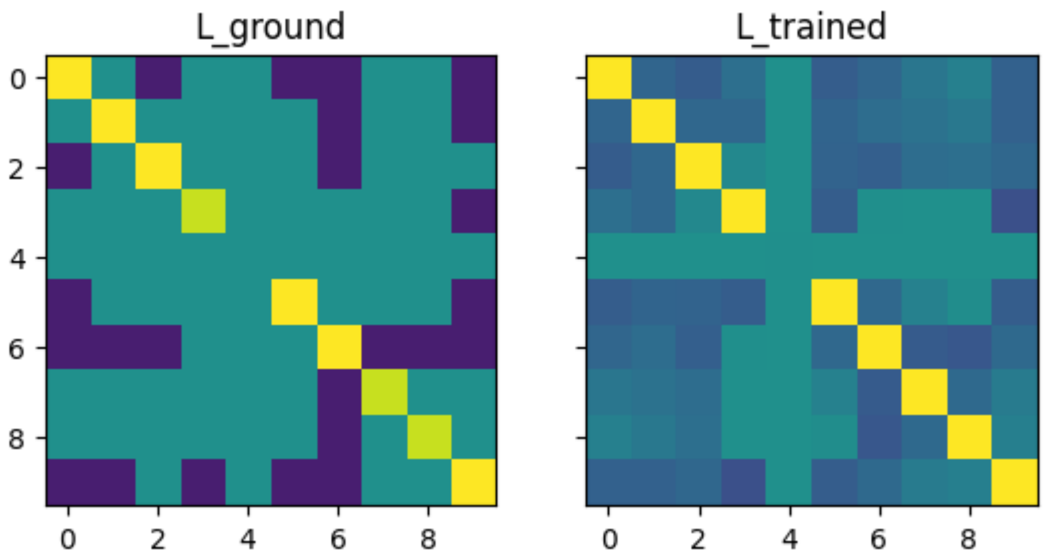


So there is a threshold for which the precision recall is circa 0.8,0.8, but the threshold is not known without the L_ground, so the algorithm usefulness is questionable. Let me feed the algorithm with the ground values and let me set the regularization terms to 0.

In [ ]:
```
res2 = lh.learn_heat(X,L0=L_ground,H0=H_ground,tau0=tau_ground,alpha=0,be
```

```
Learning progress:   0%|          | 0/100 [00:00<?, ?it/s]
```

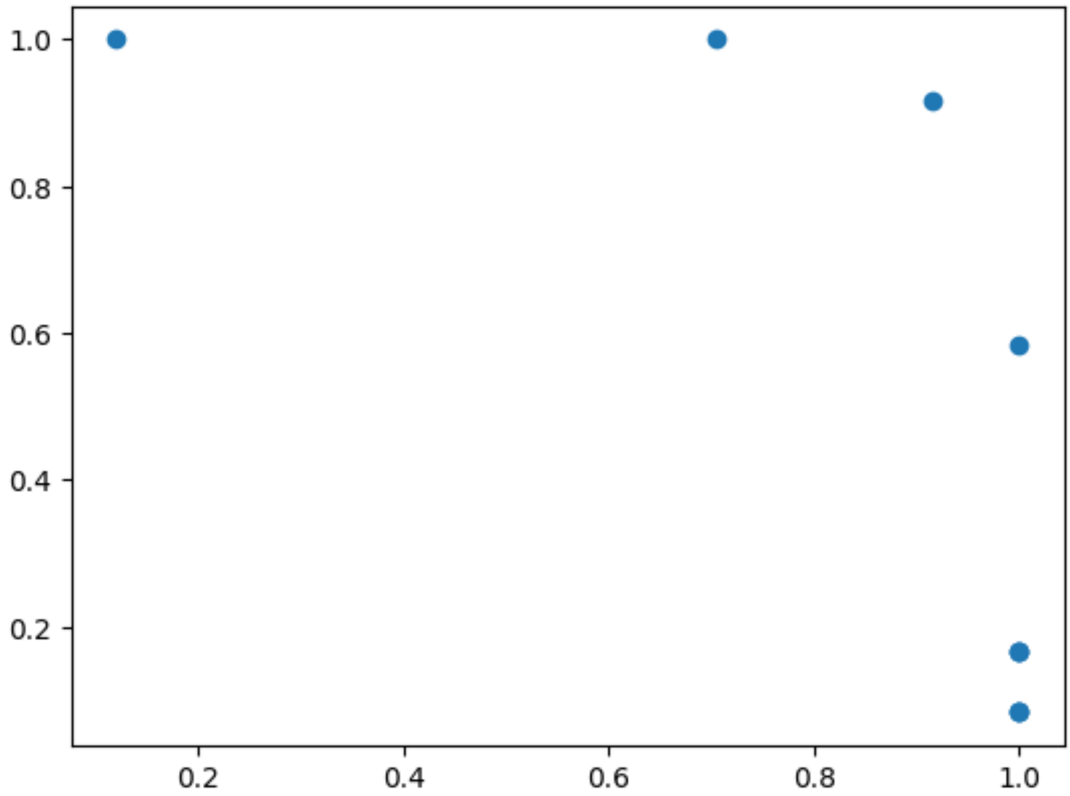
In [ ]:
```
vmin = -0.5
vmax = 0.5
f, (ax1, ax2) = plt.subplots(1, 2, sharey=True)
ax1.imshow(L_ground/np.trace(L_ground)*N, vmin = vmin, vmax = vmax)
ax1.set_title('L_ground')
ax2.imshow(res["L"], vmin = vmin, vmax = vmax)
ax2.set_title('L_trained')
plt.show()
```



Again, can't assess without precision recall.

In [ ]:
```
precision2, recall2 = lh.heat_scores(L1=res2["L"],L2=L_ground)
plt.scatter(precision2, recall2)
```

Out[ ]: <matplotlib.collections.PathCollection at 0x7fc798021400>



Hey it works, but again, how do I even know the threshold, do I need to feed it 20000 signals so I only need a threshold of 10e-4? How do I find 20000 signals in real life? Big data? The thing about the uber voyages?

Last try, feeding all correctly except τ (suppose I have no clue of what the real τ is) that means H_0 is not feeded because I would need to know S also

In [ ]:
```
res3 = lh.learn_heat(X,L0=L_ground,tau0=[1],alpha=0.1,beta=0.05)
```

```
Learning progress:   0%|          | 0/100 [00:00<?, ?it/s]
```

In [ ]:
```
precision3, recall3 = lh.heat_scores(L1=res3["L"],L2=L_ground)
plt.scatter(precision3, recall3)
```

Out[ ]: <matplotlib.collections.PathCollection at 0x7fc80837bf70>