Parcial 2 SYS

Punto 1:

· El sistema masa, resorte, y amortiguador se puede modelar a partir de la conservación de fuerzas

$$F_S(t) + F_F(t) + F_I(t) = F_E(t)$$

donde $F_S(t) = ky(t)$, $F_F(t) = c\frac{dy(t)}{dt}$, y $F_I(t) = m\frac{d^2y(t)}{dt^2}$

Por consiguiente

$$m\frac{d^2y(t)}{dt^2} + c\frac{dy(t)}{dt} + ky(t) = F_E(t) = x(t)$$

Aplicamos la transformada de Laplace

$\mathcal{L}\left\{\frac{d^n x(t)}{dt}\right\} = s^n X(s)$ tenemos que:

$$ms^2 Y(s) + Cs\,Y(s) + KY(s) = X(s)$$

y:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{ms^2 + cs + K}$$

Ahora, para el circuito electrico

$$V_i(s) = Ls\, I_1(s) + (I_1(s) - I_2(s))\, {}^1/_{Cs}$$

$$(I_2(s) - I_1(s))\frac{1}{Cs} + I_2(s)\, R = 0$$

$$V_0(s) = R\, I_2(s)$$

Se despeja $I_1(s)$ respecto a $I_2(s)$:

$$\frac{1}{Cs} I_2(s) - \frac{1}{Cs} I_1(s) + I_2(s)\, R = 0$$

$$\boxed{I_1(s) = I_2(s)(1 + CRs)}$$

Se reemplaza respecto a la 1ra ecuacion:

$$V_i(s) = Ls\, I_2(s)(1 + CRs) + (I_2(s)(1 + CRs) - I_2(s))\, {}^1/_{Cs}$$

$$V_i(s) = Ls\, I_2(s) + CRLs^2\, I_2(s) + I_2(s)\, {}^1/_{Cs} +$$
$$I_2(s)\, R - I_2(s)\, {}^1/_{Cs}$$

$$V_i(s) = I_2(s)(CRLs^2 + Ls + R)$$

$$\frac{I_2(s)}{V_i(s)} = \frac{1}{CRLs^2 + Ls + R}$$

$$\frac{R\, I_2(s)}{V_i(s)} = \frac{V_0(s)}{V_i(s)} = \frac{R}{CRLs^2 + Ls + R}$$

$$\boxed{H_s\ \frac{V_0(s)}{V_i(s)} = \frac{1}{CLs^2 + {}^L/_R\, s + 1}}$$