Scribe

3 Punto) $x_1(t), x_2(t) \in \mathbb{R}\ \mathbb{C}$

$$d(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

Sean $x_1(t)$ y $x_2(t)$

$x_1(t) = A\cos(w_0 t)$, $w_0 = 2\pi/T$, $T, A \in \mathbb{R}^+$

$$x_2(t) = \begin{cases} 1 & \text{si } 0 \leq t < T/4 \\ -1 & \text{si } T/4 \leq t < 3T/4 \\ 1 & \text{si } 3T/4 \leq t < T \end{cases}$$

Señal 1:

$x_1(t) = A\cos(w_0 t)$, $w_0 = 2\pi/T$

Señal 2:

$$x_2(t) \begin{cases} 1 & \text{Si } 0 \leq t \quad T/4 \\ -1 & \text{Si } T/4 \leq t < 3T/4 \\ 1 & \text{Si } 3T/4 \leq t < T \end{cases}$$

Division por tramos: Porque la señal 2 no es continua.

- Tramo 1: $0 \leq t < T/4$ $x_2(t) = 1$

$$|x_1(t) - x_2(t)|^2 = |A\cos(w_0 t) - 1|^2 = (A\cos(w_0 t) - 1)^2$$

- Tramo 2: $T/4 \leq t < 3T/4$ $x_2(t) = -1$

$$|x_1(t) - x_2(t)|^2 = |A\cos(w_0 t) + 1|^2 = (A\cos(w_0 t) + 1)^2$$

·Tramo 3: $3T/4 \leq t < T \qquad x_2(t) = 1$

$$|x_1(t) - x_2(t)|^2 = |A\cos(w_0 t) - 1|^2 = (A\cos(w_0 t) + 1)^2$$

$$\int_0^T |x_1(t) - x_2(t)|^2 dt = \int_0^{T/4} (A\cos(w_0 t) - 1)^2 dt + \int_{T/4}^{3T/4} (A\cos(w_0 t) + 1)^2 dt$$

$$+ \int_{3T/4}^{T} (A\cos(w_0 t) - 1)^2 dt$$

Expandir los cuadrados:

1 y 3 Tramo (son iguales):

$$(A\cos(w_0 t) - 1)^2 = A^2\cos^2(w_0 t) - 2A\cos(w_0 t) + 1$$

2 Tramo

$$(A\cos(w_0 t) + 1)^2 = A^2\cos^2(w_0 t) + 2A\cos(w_0 t) + 1$$

Identidad trigonometrica

Se sabe que: $\cos^2(w_0 t) = \frac{1 + \cos(2w_0 t)}{2}$

Entonces: $A^2\cos^2(w_0 t) = \frac{A^2}{2} + \frac{A^2}{2}\cos(2w_0 t)$

Se Integra cada tramo:

1 tramo) $\int_0^{T/4} (A^2\cos^2(w_0 t) - 2A\cos(w_0 t) + 1)\, dt$

$$= \int_0^{T/4} \left(\frac{A^2}{2} + \frac{A^2}{2}\cos(2w_0 t) - 2A\cos(w_0 t) + 1\right) dt$$

Separando

$$= A^2/2 \int_0^{T/4} dt + A^2/2 \int_0^{T/4} \cos(2w_0 t)\, dt - 2A\int_0^{T/4} \cos(w_0 t)\, dt + \int_0^{T/4} dt$$

$$A/2 \cdot T/4 + A/2 \int_0^{T/4} Cos(2w_0t)\,dt - 2A \int_0^{T/4} Cos(w_0t)\,dt + T/4$$

Ahora usamos $\int Cos(Kt)\,dt = sen(Kt)/K$

Entonces:

- $\int_0^{T/4} Cos(2w_0t)\,dt = \left[\dfrac{Sen(2w_0t)}{2w_0}\right]_0^{T/4} = \dfrac{Sen(\pi) - Sen(0)}{2w_0}$

$$= \frac{0-0}{2w_0} = 0$$

- $\int_0^{T/4} Cos(w_0t)\,dt = \left[\dfrac{Sen(w_0t)}{w_0}\right]_0^{T/4} = \dfrac{Sen(\pi/2)}{w_0} - \dfrac{Sen(0)}{w_0}$

$$= \frac{1-0}{w_0} = 1/w$$

El primera tramo da:

$$A^2T/8 + 0 - 2A/w_0 + T/4 = \boxed{A^2T/8 + T/4 - 2A/w_0}$$

2 tramo)

$$\int_{T/4}^{3T/4} (A^2Cos^2(w_0t) + 2ACos(w_0t) + 1)\,dt$$

recordemos:

$$Cos^2(w_0t) = \frac{1+cos(2w_0t)}{2} \Rightarrow A^2Cos^2(w_0t)$$

$$= A^2/2 + A^2/2\ Cos(2w_0t)$$

$$\int_{T/4}^{3T/4}\left(\frac{A^2}{2}+\frac{A^2}{2}\cos(2w_0t)+2A\cos(w_0t)+1\right)dt$$

Separar la integral:

$$=\underbrace{\int_{T/4}^{3T/4}\frac{A^2}{2}\,dt}_{1}+\overbrace{\int_{T/4}^{3T/4}\frac{A^2}{2}\cos(2w_0t)\,dt}^{2}+\underbrace{\int_{T/4}^{3T/4}2A\cos(w_0t)\,dt}_{3}+\underbrace{\int_{T/4}^{3T/4}1\,dt}_{4}$$

Evaluar Cada Integral:

1) $$\int_{T/4}^{3T/4}\frac{A^2}{2}\,dt=\frac{A^2}{2}\cdot\left(\frac{3T}{4}\cdot\frac{T}{4}\right)=\frac{A^2}{2}\cdot\frac{T}{2}=\frac{A^2T}{4}$$

2) $$\int_{T/4}^{3T/4}\frac{A^2}{2}\cos(2w_0t)\,dt=\left[\frac{\operatorname{Sen}(2w_0t)}{2w_0}\right]_{T/4}^{3T/4}$$

recordemos: $w_0=2\pi/T \Rightarrow 2w_0=4\pi/T$

$2w_0\cdot T/4=\pi$ y $2w_0\cdot 3T/4=3\pi$

$$\int_{T/4}^{3T/4}\cos(2w_0t)\,dt=\frac{\operatorname{Sen}(3\pi-\operatorname{Sen}(\pi)}{2w_0}=\frac{0-0}{2w_0}=0$$

multiplicado por $A^2/2\cdot 0=0$

3) $$\int_{T/4}^{3T/4}\cos(w_0t)\,dt=\left[\frac{\operatorname{Sen}(w_0t)}{w_0}\right]_{T/4}^{3T/4}$$

$\omega_0 \cdot T/4 = \pi/2$, $\omega_0 \cdot 3T/4 = 3\pi/2$

$\Rightarrow \text{Sen}(3\pi/2) - \text{Sen}(\pi/2) = \frac{-1-1}{\omega_0} = -2/\omega_0$

$= 2A(-2/\omega_0) = -4A/\omega_0$

4) $\int_{T/4}^{3T/4} 1\,dt = 3T/4 - T/4 = T/2$

Resultado: $\boxed{A^2T/4 - 4A/\omega_0 + T/2}$

$E = A^2 - \frac{8A + T\omega_0(A^2+2)}{2\omega_0}$