1) Sea $x_1(t)=Ae^{-jn\omega_0 t}$ y $x_2(t)=Be^{jm\omega_0 t}$

Con $A, B \in \mathbb{R}^+$, $n, m \in \mathbb{Z}$, $\omega_0 = 2\pi/T$

Tenemos

$$d^2(x_1,x_2)=\frac{1}{T}\int_0^T |x_1(t)-x_2(t)|^2\,dt$$

• Expandimos el modulo:

$$|x_1-x_2|^2=(x_1-x_2)(x_1-x_2)^*=x_1x_1^*+x_2x_2^*-x_1x_2^*-x_1^*x_2$$

• Se sustituye x_1, x_2:

$x_1x_1^*=A^2$ $\quad x_2x_2^*=B^2$ $\quad x_1x_2^*=ABe^{-j(n+m)\omega_0 t}$

$x_1^*x_2=ABe^{j(n+m)\omega_0 t}$

De donde

$$|x_1-x_2|^2=A^2+B^2-AB\left(e^{-j(n+m)\omega_0 t}+e^{j(n+m)\omega_0 t}\right)$$

$$=A^2+B^2-2AB\cos((n+m)\omega_0 t)$$

• Integrando

$$d^2=A^2+B^2-\frac{2AB}{T}\int_0^T \cos((n+m)\omega_0 t)\,dt$$

Si $n+m\neq 0$

$$\int_0^T \cos((n+m)\omega_0 t)\,dt=\left.\frac{\operatorname{sen}((n+m)\omega_0 t)}{(n+m)\omega_0}\right|_0^T$$

$$= \frac{\operatorname{Sen}((n+m)2\pi)}{(n+m)w_0} = 0$$

$d^2 = A^2 + B^2$

Si $n+m=0$ (es decir $m=-n$)

$\operatorname{Cos}((n+m)w_0 t) = \operatorname{Cos} 0 = 1 \Rightarrow \int_0^T 1\, dt = T$

entonces $d^2 = A^2 + B^2 - 2AB$

$= (A-B)^2$

Conclusion

$$d(x_1, x_2) = \begin{cases} |A-B|, & \text{Se } n = -m \\ \sqrt{A^2+B^2}, & \text{si } n \neq -m \end{cases}$$