

# 1 Statistical Inference Quiz 1

## 1.1 Question 1

- Consider influenza epidemics for two parent heterosexual families.
- Suppose that the probability is 17% that at least one of the parents has contracted the disease.
- The probability that the father has contracted influenza is 12% while the probability that both the mother and father have contracted the disease is 6%.
- What is the probability that the mother has contracted influenza?

Express your answer as a percentage to the nearest percentage point. **Answer for Question 1**

## 1.2 Question 2

A random variable,  $X$  is uniform, a box from 0 to 1 of height 1. (So that it's density is  $f(x) = 1$  for  $0 \leq x \leq 1$ .) What is it's 75th percentile?

Express your answer to two decimal places.

### 1.2.1 Using R

```
> X<-runif(20)
> sort(X)
[1] 0.04072102 0.06952471 0.07260518 0.10968119 0.11164281
[6] 0.24502397 0.26483832 0.33702175 0.41157263 0.44588699
[11] 0.46461517 0.52120201 0.63389649 0.64879975 0.74126249
[16] 0.76873648 0.85552313 0.92665064 0.97780178 0.97977485
> X<-runif(20)
> sort(X)
[1] 0.009058821 0.025291159 0.107304686 0.110596883 0.225791117
[6] 0.247623788 0.316807160 0.340841283 0.496293882 0.562722453
[11] 0.627004402 0.639200010 0.668796364 0.676932374 0.765855561
[16] 0.767786264 0.780400117 0.795518954 0.903478133 0.922662708
```

**Answer for Question 2**

## 1.3 Question 3

You are playing a game with a friend where you flip a coin and if it comes up heads you give her  $X$  dollars and if it comes up tails she gives you  $Y$  dollars. The probability that the coin is heads is  $p$  (some number between 0 and 1.) What has to be true about  $X$  and  $Y$  to make so that both of your expected total earnings is 0. (The game would then be called fair.)

- (i)  $p_{1p}=XY$
- (ii)  $p_{1p}=YX$
- (iii)  $X=Y$
- (iv)  $p=XY$

#### 1.4 Question 4

You are playing a game with a friend where you flip a coin and if it comes up heads you give her 1 dollar and if it comes up tails she gives you one dollar. What would be the variance of your earnings?

Express your answer to two decimal places. **Answer for Question 4**

```
#Generate a sequence of 1s or -1s
Winnings <- 2*floor(2*runif(100))-1

#> Winnings
# [1] -1  1 -1 -1  1  1 -1  1  1 -1 -1 -1 -1 -1  1  1  1  1 -1  1
# [22]  1  1 -1 -1 -1 -1 -1 -1 -1  1 -1 -1  1 -1 -1 -1  1 -1  1 -1  1
# [43] -1 -1  1  1  1  1  1  1  1  1  1 -1 -1 -1  1 -1 -1 -1  1 -1 -1
# [64]  1  1  1  1  1 -1  1  1 -1 -1 -1  1 -1 -1 -1 -1 -1  1 -1 -1  1
# [85] -1 -1 -1  1 -1 -1 -1 -1  1  1 -1 -1 -1  1 -1 -1
```

```
var(Winnings)
# [1] 0.9842424
```

#### 1.5 Question 5

Let  $X_1, \dots, X_{n1}$  be random variables independent of  $Y_1, \dots, Y_{n2}$ , where both groups are iid with associated population means  $\mu_1$  and  $\mu_2$  and population variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively. Let  $\bar{X}$  and  $\bar{Y}$  be their sample means. What is the variance of  $\bar{X} + \bar{Y}$ ?

- $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
- $\sigma_1^2 \sigma_2^2$
- $\sigma_1^2 + \sigma_2^2$
- $\frac{\sigma_1^2}{n_1} - \frac{\sigma_2^2}{n_2}$

#### 1.6 Question 6

Suppose a random variable,  $X$ , is such that  $E[X]=\mu$  and  $Var(X) = \sigma^2$ . What is the mean and variance of  $Z = \frac{X-\mu}{\sigma}$ ?

- $Z$  has mean 0 and variance 1

- Z has mean  $\mu$  and variance  $\sigma^2$
- Z has mean 0 and variance  $\sigma^2$
- Z has mean  $\mu$  and variance 1

Theory Question : Standard Normal (Z) Distribution.

### 1.7 Question 7

If a continuous density that never touches the horizontal axis is symmetric about zero, can we say that its associated median is zero?

- We can't conclude that the median is 0.
- Yes, the median must be 0.
- No, the median is definitely not 0.

### 1.8 Question 8

Consider the following PMF generated in R

```
x <- 1:4
p <- x/sum(x)
temp <- rbind(x, p)
rownames(temp) <- c("X", "Prob")
temp
##      [,1] [,2] [,3] [,4]
## X      1.0  2.0  3.0  4.0
## Prob  0.1  0.2  0.3  0.4
```

What is the mean? Express your answer to one decimal place.

**Answer for Question 8**

```
> temp[1,]
[1] 1 2 3 4
> temp[2,]
[1] 0.1 0.2 0.3 0.4
> temp[1,] * temp[2,]
[1] 0.1 0.4 0.9 1.6
> sum(temp[1,] * temp[2,])
[1] 3
>
```