1 Statistical Inference Quiz 1

1.1 Question 1

- Consider influenza epidemics for two parent heterosexual families.
- Suppose that the probability is 17% that at least one of the parents has contracted the disease.
- The probability that the father has contracted influenza is 12% while the probability that both the mother and father have contracted the disease is 6%.
- What is the probability that the mother has contracted influenza?

Express your answer as a percentage to the nearest percentage point. **Answer for Question 1**

1.2 Question 2

A random variable, X is uniform, a box from 0 to 1 of height 1. (So that it's density is $f(x) = 1$ for $0 \leq x \leq 1$.) What is it's 75th percentile?

Express your answer to two decimal places.

1.2.1 Using `R`

```
> X<-runif(20)
> sort(X)
 [1] 0.04072102 0.06952471 0.07260518 0.10968119 0.11164281
 [6] 0.24502397 0.26483832 0.33702175 0.41157263 0.44588699
[11] 0.46461517 0.52120201 0.63389649 0.64879975 0.74126249
[16] 0.76873648 0.85552313 0.92665064 0.97780178 0.97977485
> X<-runif(20)
> sort(X)
 [1] 0.009058821 0.025291159 0.107304686 0.110596883 0.225791117
 [6] 0.247623788 0.316807160 0.340841283 0.496293882 0.562722453
[11] 0.627004402 0.639200010 0.668796364 0.676932374 0.765855561
[16] 0.767786264 0.780400117 0.795518954 0.903478133 0.922662708
```

Answer for Question 2

1.3 Question 3

You are playing a game with a friend where you flip a coin and if it comes up heads you give her X dollars and if it comes up tails she gives you Y dollars. The probability that the coin is heads in p (some number between 0 and 1.) What has to be true about X and Y to make so that both of your expected total earnings is 0. (The game would then be called fair.)

(i) p1p=XY

(ii) p1p=YX

(iii) X=Y

(iv) p=XY

1.4 Question 4

You are playing a game with a friend where you flip a coin and if it comes up heads you give her 1 dollar and if it comes up tails she gives you one dollar. What would be the variance of your earnings?

Express your answer to two decimal places. **Answer for Question 4**

```
#Generate a sequence of 1s or -1s
Winnings <- 2*floor(2*runif(100))-1

#> Winnings
#  [1] -1  1 -1 -1  1  1 -1  1  1 -1 -1 -1 -1 -1 -1  1  1  1  1 -1  1
# [22]  1  1 -1 -1 -1 -1 -1 -1 -1  1 -1 -1  1 -1 -1 -1  1 -1  1 -1  1
# [43] -1 -1  1  1  1  1  1  1  1  1  1 -1 -1 -1  1 -1 -1 -1  1 -1 -1
# [64]  1  1  1  1  1 -1  1  1 -1 -1 -1  1 -1 -1 -1 -1 -1  1 -1 -1  1
# [85] -1 -1 -1  1 -1 -1 -1 -1  1  1 -1 -1 -1  1 -1 -1

var(Winnings)
# [1] 0.9842424
```

1.5 Question 5

Let X1,,Xn1 be random variables independent of Y1,,Yn2, where both groups are iid with associated population means $\mu 1$ and $\mu 2$ and population variances σ_1^2 and σ_2^2, respectively. Let $\bar{X}$ and $\bar{Y}$ be their sample means. What is the variance of $\bar{X} + \bar{Y}$?

- $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
- $\sigma_1^2 \sigma_2^2$
- $\sigma_1^2 + \sigma_2^2$
- $\frac{\sigma_1^2}{n_1} - \frac{\sigma_2^2}{n_2}$

1.6 Question 6

Suppose a random variable, X, is such that E[X]=μ and $Var(X) = \sigma^2$. What is the mean and variance of $Z = \frac{X\mu}{\sigma}$

- Z has mean 0 and variance 1

- Z has mean μ and variance σ^2
- Z has mean 0 and variance σ^2
- Z has mean μ and variance 1

Theory Question : Standard Normal (Z) Distribution.

1.7 Question 7

If a continuous density that never touches the horizontal axis is symmetric about zero, can we say that its associated median is zero?

- We can't conclude that the median is 0.
- Yes, the median must be 0.
- No, the median is definitely not 0.

Question 8

Consider the following PMF generated in `R`

```
x <- 1:4
p <- x/sum(x)
temp <- rbind(x, p)
rownames(temp) <- c("X", "Prob")
temp
##      [,1] [,2] [,3] [,4]
## X     1.0  2.0  3.0  4.0
## Prob  0.1  0.2  0.3  0.4
```

What is the mean?

Express your answer to one decimal place.

Answer for Question 8

Expected Value for Discrete Random Variables

- The expected value (i.e. the mean) of a discrete random variable X is symbolized by E(X).

- If X is a discrete random variable with possible values $\{x_1, x_2, x_3, \ldots, x_n\}$, and $p(x_i)$ denotes $P(X = x_i)$, then the expected value of X is defined by:
$$E(X) = \sum x_i \times p(x_i)$$
 where the elements are summed over all values of the random variable X.

```
> temp[1,]
[1] 1 2 3 4
> temp[2,]
[1] 0.1 0.2 0.3 0.4
> temp[1,] * temp[2,]
[1] 0.1 0.4 0.9 1.6
> sum(temp[1,] * temp[2,])
[1] 3
>
```