Instituto Tecnológico de Querétaro

Formulario elemental de integración

Formas elementales

1. $\displaystyle\int dx = x + C$
2. $\displaystyle\int [f+g]\,dx = \int f\,dx + \int g\,dx$
3. $\displaystyle\int [\lambda f]\,dx = \lambda \int f\,dx$
4. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
5. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

Integrales de funciones trigonométricas

6. $\displaystyle\int \operatorname{sen}(x)\,dx = -\cos(x) + C$
7. $\displaystyle\int \cos(x)\,dx = \operatorname{sen}(x) + C$
8. $\displaystyle\int \tan(x)\,dx = \ln\left|\sec(x)\right| + C$
9. $\displaystyle\int \cot(x)\,dx = \ln\left|\operatorname{sen}(x)\right| + C$
10. $\displaystyle\int \sec(x)\,dx = \ln\left|\sec(x)+\tan(x)\right| + C = \ln\left(\tan\left(\frac{2x+\pi}{4}\right)\right) + C$
11. $\displaystyle\int \csc(x)\,dx = \ln\left|\csc(x)-\cot(x)\right| + C = \ln\left(\tan\left(\frac{x}{2}\right)\right) + C$

Integrales de funciones trigonométricas inversas

12. $\displaystyle\int \operatorname{sen}^{-1}(x)\,dx = x\operatorname{sen}^{-1}(x) + \sqrt{1-x^2} + C$
13. $\displaystyle\int \cos^{-1}(x)\,dx = x\cos^{-1}(x) - \sqrt{1-x^2} + C$
14. $\displaystyle\int \tan^{-1}(x)\,dx = x\tan^{-1}(x) - \frac{\ln(1+x^2)}{2} + C$
15. $\displaystyle\int \cot^{-1}(x)\,dx = x\cot^{-1}(x) + \frac{\ln(1+x^2)}{2} + C$
16. $\displaystyle\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \ln\left|x+\sqrt{x^2-1}\right| + C$
17. $\displaystyle\int \csc^{-1}(x)\,dx = x\csc^{-1}(x) + \ln\left|x+\sqrt{x^2-1}\right| + C$

Integrales de algunos productos comunes de funciones trigonométricas

18. $\displaystyle\int \operatorname{sen}^2(x)\,dx = \frac{2x-\operatorname{sen}(2x)}{4} + C = \frac{x-\operatorname{sen}(x)\cos(x)}{2} + C$
19. $\displaystyle\int \cos^2(x)\,dx = \frac{2x+\operatorname{sen}(2x)}{4} + C = \frac{x+\operatorname{sen}(x)\cos(x)}{2} + C$
20. $\displaystyle\int \tan^2(x)\,dx = \tan(x) - x + C$
21. $\displaystyle\int \cot^2(x)\,dx = -\cot(x) - x + C$
22. $\displaystyle\int \sec^2(x)\,dx = \tan(x) + C$
23. $\displaystyle\int \csc^2(x)\,dx = -\cot(x) + C$
24. $\displaystyle\int \sec(x)\tan(x)\,dx = \sec(x) + C$
25. $\displaystyle\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

Integrales de funciones logarítmicas

26. $\displaystyle\int \log_a(x)\,dx = \frac{x\ln(x) - x}{\ln(a)} + C$
27. $\displaystyle\int \ln(x)\,dx = x\ln(x) - x + C$

Integrales de funciones exponenciales

28. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln(a)} + C$
29. $\displaystyle\int e^x\,dx = e^x + C$

Integrales de formas racionales que incluyen sumas y diferencias de cuadrados

30. $\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right)+C$

31. $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{a^2+x^2}\right)+C$
$\displaystyle= \operatorname{senh}^{-1}\left(\frac{x}{a}\right)+C$

32. $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\left(\frac{x}{a}\right)+C$; para $a>0$

33. $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)+C$
$\displaystyle= -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C$; para $x^2>a^2$

34. $\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

35. $\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)+C$
$\displaystyle= \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C$; para $x^2<a^2$

36. $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|+C$

37. $\displaystyle\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)+C$

38. $\displaystyle\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)+C$

Integrales de funciones hiperbólicas

39. $\displaystyle\int \operatorname{senh}(x)\,dx = \cosh(x)+C$

40. $\displaystyle\int \cosh(x)\,dx = \operatorname{senh}(x)+C$

41. $\displaystyle\int \tanh(x)\,dx = \ln\big(\cosh(x)\big)+C$

42. $\displaystyle\int \coth(x)\,dx = \ln\big(\operatorname{senh}(x)\big)+C$

43. $\displaystyle\int \operatorname{sech}(x)\,dx = \operatorname{sen}^{-1}\big(\tanh(x)\big)+C$
$\displaystyle= 2\tan^{-1}(e^x)+C$

44. $\displaystyle\int \operatorname{csch}(x)\,dx = \ln\Big(\tanh\left(\frac{x}{2}\right)\Big)+C$
$\displaystyle= -\coth^{-1}(e^x)+C$

Integrales de funciones hiperbólicas inversas

45. $\displaystyle\int \operatorname{senh}^{-1}(x)\,dx = x\operatorname{senh}^{-1}(x)-\sqrt{x^2+1}+C$

46. $\displaystyle\int \cosh^{-1}(x)\,dx = x\cosh^{-1}(x)-\sqrt{x^2-1}+C$

47. $\displaystyle\int \tanh^{-1}(x)\,dx = x\tanh^{-1}(x)+\frac{\ln(1-x^2)}{2}+C$

48. $\displaystyle\int \coth^{-1}(x)\,dx = x\coth^{-1}(x)+\frac{\ln(1-x^2)}{2}+C$

49. $\displaystyle\int \operatorname{sech}^{-1}(x)\,dx = x\operatorname{sech}^{-1}(x)+2\operatorname{sen}^{-1}\sqrt{\frac{x+1}{2}}+C$

50. $\displaystyle\int \operatorname{csch}^{-1}(x)\,dx = x\operatorname{csch}^{-1}(x)+|x|\operatorname{senh}^{-1}(x)+C$

Integrales de algunos productos comunes de funciones hiperbólicas

51. $\displaystyle\int \operatorname{senh}^2(x)\,dx = \frac{\operatorname{senh}(2x)-2x}{4}+C$
$\displaystyle= \frac{\operatorname{senh}(x)\cosh(x)-x}{2}+C$

52. $\displaystyle\int \cosh^2(x)\,dx = \frac{\operatorname{senh}(2x)+2x}{4}+C$
$\displaystyle= \frac{\operatorname{senh}(x)\cosh(x)+x}{2}+C$

53. $\displaystyle\int \tanh^2(x)\,dx = x-\tanh(x)+C$

54. $\displaystyle\int \coth^2(x)\,dx = x-\coth(x)+C$

55. $\displaystyle\int \operatorname{sech}^2(x)\,dx = \tanh(x)+C$

56. $\displaystyle\int \operatorname{csch}^2(x)\,dx = -\coth(x)+C$

57. $\displaystyle\int \operatorname{sech}(x)\tanh(x)\,dx = -\operatorname{sech}(x)+C$

58. $\displaystyle\int \operatorname{csch}(x)\coth(x)\,dx = -\operatorname{csch}(x)+C$