Unit III B

86.66

27/03/2021

Talavera Rivera Luis Fernando

- Solve the system of Differential Equations

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1.- $\frac{dx}{dt} = \underbrace{\begin{pmatrix} 0 & 5 & -3 \\ -1 & 6 & -3 \\ -3 & 9 & -4 \end{pmatrix}}_{A} X$

$\det(A-\lambda I)$

$= \begin{vmatrix} -\lambda & 5 & -3 \\ -1 & 6-\lambda & -3 \\ -3 & 9 & -4-\lambda \end{vmatrix} \begin{matrix} -\lambda & 5 & -3 \\ -1 & 6-\lambda & -3 \\ -3 & 9 & -4-\lambda \end{matrix}$

$= (-\lambda)(6-\lambda)(-4-\lambda) + 45 + 27 - (9)(6-\lambda) - (-\lambda)(-27) - (-4-\lambda)(-5)$

$= (-\lambda)(6-\lambda)(-4-\lambda) + 72 - 9(6-\lambda) - 27\lambda + 5(-4-\lambda)$

$= (-\lambda)(6-\lambda)(-4-\lambda) + 72 - 54 + 9\lambda - 27\lambda - 20 - 5\lambda$

$= -\lambda^3 + 2\lambda^2 + 24\lambda + 9\lambda - 27\lambda - 5\lambda + 72 - 74$

$= -\lambda^3 + 2\lambda^2 + \lambda - 2 = -(\lambda - 2)(\lambda+1)(\lambda-1)$

$\lambda_1 = 2 \quad \lambda_2 = 1 \quad \lambda_3 = -1$

$\lambda_1 = 2$

$\left(\begin{array}{ccc|c} -2 & 5 & -3 & 0 \\ -1 & 4 & -3 & 0 \\ -3 & 9 & -6 & 0 \end{array}\right) R_3 \leftrightarrow R_1 \left(\begin{array}{ccc|c} -3 & 9 & -6 & 0 \\ -1 & 4 & -3 & 0 \\ -2 & 5 & -3 & 0 \end{array}\right) \begin{matrix} R_3 \to R_3 - \frac{2}{3}R_1 \\ R_2 \to R_2 - \frac{1}{3}R_1 \end{matrix}$

$\left(\begin{array}{ccc|c} -3 & 9 & -6 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) R_3 \to R_3 + R_2 \left(\begin{array}{ccc|c} -3 & 9 & -6 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$k_3 = 1 \qquad k_2 = k_3 = 1$

$-3k_1 + 9k_2 - 6k_3 = 0 \Rightarrow +k_1 - 3k_2 + 2k_3 = 0$

$k_1 = 3 - 2 = 1$

$X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t}$

$\lambda_2 = 1$

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ -1 & 5 & -3 & 0 \\ -3 & 9 & -5 & 0 \end{array}\right) R_3 \leftrightarrow R_1 \left(\begin{array}{ccc|c} -3 & 9 & -5 & 0 \\ -1 & 5 & -3 & 0 \\ -1 & 5 & -3 & 0 \end{array}\right) R_3 \to R_3 - R_2$$

$$\left(\begin{array}{ccc|c} -3 & 9 & -5 & 0 \\ -1 & 5 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) R_2 \to R_2 - \frac{1}{3}R_1 \left(\begin{array}{ccc|c} -3 & 9 & -5 & 0 \\ 0 & 2 & -4/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$k_3 = 1 \quad 2k_2 - \frac{4}{3}k_3 = 0 \qquad -3k_1 + 9k_2 - 5k_3 = 0$

$k_2 = \frac{2}{3}k_3 = 2/3 \qquad -3k_1 + 6 - 5 = 0$

$k_1 = 1/3$

$$X_2 = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} e^{t}$$

$\lambda_3 = -1$

$$\left(\begin{array}{ccc|c} 1 & 5 & -3 & 0 \\ -1 & 7 & -3 & 0 \\ -3 & 9 & -3 & 0 \end{array}\right) \begin{array}{l} R_2 \Rightarrow R_2 + R_1 \\ R_3 \Rightarrow R_3 + 3R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 5 & -3 & 0 \\ 0 & 12 & -6 & 0 \\ 0 & 24 & -12 & 0 \end{array}\right) R_3 \to R_3 - 2R_2$$

$$\left(\begin{array}{ccc|c} 1 & 5 & -3 & 0 \\ 0 & 12 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad k_3 = 1 \quad \begin{array}{l} 12k_2 - 6k_3 = 0 \\ 2k_2 - k_3 = 0 \\ k_2 = 1/2 \end{array}$$

$k_1 + 5k_2 - 3k_3 = 0 \Rightarrow k_1 + 5/2 - 3 = 0 \Rightarrow k_1 = 1/2$

$$X_3 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix} e^{-t}$$

$$X = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} e^{t} + c_3 \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix} e^{-t}$$

2. $\frac{dx}{dt} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 1 & 1 \\ -4 & 0 & -1 \end{pmatrix} X$

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$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 0 & 1 \\ 2 & 1-\lambda & 1 \\ -4 & 0 & -1-\lambda \end{vmatrix} \begin{matrix} 3-\lambda & 0 & 1 \\ 2 & 1-\lambda & 1 \\ -4 & 0 & -1-\lambda \end{matrix}$

$\det(A - \lambda I) = (3-\lambda)(1-\lambda)(-1-\lambda) + 4(1-\lambda)$

$= (1-\lambda)[(3-\lambda)(1-\lambda) + 4]$

$= (1-\lambda)[\lambda^2 - 2\lambda + 1]$

$\lambda_1 = \lambda_2 = \lambda_3 = 1$

$\left(\begin{matrix} 2 & 0 & 1 \\ 2 & 0 & 1 \\ -4 & 0 & -2 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right)$ $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{matrix}$ $\left(\begin{matrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right)$

$2k_1 + k_3 = 0$

$2k_1 = -k_3$

$X_1 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} e^t$

If $k_3 = -2 \Rightarrow k_1 = 1$

$X_2 = \begin{pmatrix} -1/2 \\ -2 \\ 1 \end{pmatrix} e^t$

$k_2 = 1$

If $k_3 = 1 \Rightarrow k_1 = -1/2$

$k_2 = -2$

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ -4 & 0 & -2 & -2 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$X_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} t e^t$$

$$X_4 = \begin{pmatrix} -1/2 \\ -2 \\ 1 \end{pmatrix} t e^t$$

$2k_1 + k_3 = 0$

$2k_1 = -k_3$

If $k_3 = -2 \Rightarrow k_1 = 1$

$k_2 = 1$

If $k_3 = 1 \Rightarrow k_1 = -1/2$

$k_2 = -2$

$$X = c_1 \begin{pmatrix} 1 \\ 1 \\ -1/2 \end{pmatrix} e^t + c_2 \begin{pmatrix} -1/2 \\ -2 \\ 1 \end{pmatrix} e^t \qquad c_1 \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} e^t$$

$$+ c_3 \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} t e^t + c_4 \begin{pmatrix} -1/2 \\ -2 \\ 1 \end{pmatrix} t e^t$$

3.- $\frac{dx}{dt} = \begin{pmatrix} 5 & -3 \\ 15 & -7 \end{pmatrix} X$

$$\det(A-\lambda I) = \begin{pmatrix} 5-\lambda & -3 \\ 15 & -7-\lambda \end{pmatrix} = (5-\lambda)(-7-\lambda) + 45$$
$$= \lambda^2 + 2\lambda - 35 + 45$$
$$\lambda^2 + 2\lambda + 10$$

$$\lambda = \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm \sqrt{-36}}{2} = -1 \pm 3i$$

$\lambda_1 = -1 + 3i$

$$\left(\begin{array}{cc|c} 6-3i & -3 & 0 \\ 15 & -6-3i & 0 \end{array}\right) \quad R_2 \Rightarrow -\frac{1}{3}(6+3i)R_1 + R_2$$

$-\frac{1}{3}(45) = -15$

$-\frac{1}{3}(6+3i)(-3) = 6+3i$

$$(6-3i)(6+3i) = (36+9) + (0)i = 45$$

$$\left(\begin{array}{cc|c} 6-3i & -3 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad (6-3i)k_1 - 3k_2 = 0$$
$$k_2 = \frac{(6-3i)}{3}k_1$$

if $k_1 = 3 \Rightarrow k_2 = 6-3i$

$B_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

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$$\boxed{\begin{aligned} X = {} & C_1 e^{-t}\left[\begin{pmatrix} 3 \\ 6 \end{pmatrix}\cos(3t) - \begin{pmatrix} 0 \\ -3 \end{pmatrix}\sin(3t)\right] \\ & + C_2 e^{-t}\left[\begin{pmatrix} 0 \\ -3 \end{pmatrix}\cos(3t) + \begin{pmatrix} 3 \\ 6 \end{pmatrix}\sin(3t)\right] \end{aligned}}$$