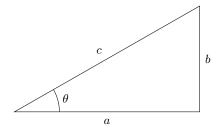
# Instituto Tecnológico de Querétaro



# Trigonometría

# RESUMEN DE DEFINICIONES, FÓRMULAS Y VALORES

#### **DEFINICIONES**



1. 
$$\operatorname{sen}(\theta) := \frac{b}{c}$$

4. 
$$\csc(\theta) := \frac{c}{b}$$

$$2. \cos(\theta) := \frac{a}{c}$$

4. 
$$\csc(\theta) := \frac{c}{b}$$
  
5.  $\sec(\theta) := \frac{c}{a}$   
6.  $\cot(\theta) := \frac{a}{b}$ 

3. 
$$\tan(\theta) := \frac{b}{a}$$

6. 
$$\cot(\theta) := \frac{\theta}{\theta}$$

# PERIODICIDAD DE LAS FUNCIONES

Si n es un número entero  $(n \in \mathbb{Z})$ , entonces:

7. 
$$sen(\alpha + 2n\pi) \equiv sen(\alpha)$$

8. 
$$\cos(\alpha + 2n\pi) \equiv \cos(\alpha)$$

9. 
$$\tan(\alpha + n\pi) \equiv \cos(\alpha)$$

10. 
$$\cot(\alpha + n\pi) \equiv \cot(\alpha)$$

11. 
$$\sec(\alpha + 2n\pi) \equiv \sec(\alpha)$$

12. 
$$\csc(\alpha + 2n\pi) \equiv \csc(\alpha)$$

#### FÓRMULAS PARA LAS COFUNCIONES

13. 
$$\operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) \equiv \cos(\alpha)$$

14. 
$$\cos\left(\frac{\pi}{2} - \alpha\right) \equiv \operatorname{sen}(\alpha)$$

15. 
$$\tan\left(\frac{\pi}{2} - \alpha\right) \equiv \cot(\alpha)$$

16. 
$$\cot\left(\frac{\pi}{2} - \alpha\right) \equiv \tan(\alpha)$$

17. 
$$\sec\left(\frac{\pi}{2} - \alpha\right) \equiv \csc(\alpha)$$

18. 
$$\csc\left(\frac{\pi}{2} - \alpha\right) \equiv \sec(\alpha)$$

#### FÓRMULAS PARA EL ÁNGULO NEGATIVO

19. 
$$sen(-\alpha) \equiv -sen(\alpha)$$

20. 
$$\cos(-\alpha) \equiv \cos(\alpha)$$

21. 
$$tan(-\alpha) \equiv -tan(\alpha)$$

22. 
$$\cot(-\alpha) \equiv -\cot(\alpha)$$

23. 
$$\sec(-\alpha) \equiv \sec(\alpha)$$

24. 
$$\csc(-\alpha) \equiv -\csc(\alpha)$$

# IDENTIDADES PITAGÓRICAS

25. 
$$\operatorname{sen}^2(\alpha) + \cos^2(\alpha) \equiv 1$$

26. 
$$1 + \cot^2(\alpha) \equiv \csc^2(\alpha)$$

27. 
$$\tan^2(\alpha) + 1 \equiv \sec^2(\alpha)$$

#### FÓRMULAS PARA LA SUMA DE ÁNGULOS

28. 
$$sen(\alpha + \beta) \equiv sen(\alpha)cos(\beta) + cos(\alpha)sen(\beta)$$

29. 
$$\cos(\alpha + \beta) \equiv \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

30. 
$$\tan(\alpha + \beta) \equiv \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

#### FÓRMULAS PARA LA DIFERENCIA DE ÁNGULOS

31. 
$$\operatorname{sen}(\alpha - \beta) \equiv \operatorname{sen}(\alpha) \cos(\beta) - \cos(\alpha) \operatorname{sen}(\beta)$$

32. 
$$\cos(\alpha - \beta) \equiv \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

33. 
$$\tan(\alpha - \beta) \equiv \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

#### FÓRMULAS PARA PRODUCTO DE FUNCIONES

34. 
$$\operatorname{sen}(\alpha)\operatorname{sen}(\beta) \equiv \frac{1}{2} \left[ \cos(\alpha - \beta) - \cos(\alpha + \beta) \right]$$

35. 
$$\cos(\alpha)\cos(\beta) \equiv \frac{1}{2} \left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

36. 
$$\operatorname{sen}(\alpha) \cos(\beta) \equiv \frac{1}{2} \left[ \operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta) \right]$$

# FÓRMULAS PARA LAS SUMAS DE FUNCIONES

37. 
$$\operatorname{sen}(\alpha) + \operatorname{sen}(\beta) \equiv 2 \operatorname{sen}\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

38. 
$$\cos(\alpha) + \cos(\beta) \equiv 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

#### FÓRMULAS PARA LAS DIFERENCIAS DE FUNCIONES

39. 
$$\operatorname{sen}(\alpha) - \operatorname{sen}(\beta) \equiv 2 \cos\left(\frac{\alpha + \beta}{2}\right) \operatorname{sen}\left(\frac{\alpha - \beta}{2}\right)$$

40. 
$$\cos(\alpha) - \cos(\beta) \equiv -2 \operatorname{sen}\left(\frac{\alpha + \beta}{2}\right) \operatorname{sen}\left(\frac{\alpha - \beta}{2}\right)$$

# FÓRMULAS PARA EL ÁNGULO DOBLE

41. 
$$sen(2\alpha) \equiv 2 sen(\alpha) cos(\alpha)$$

42. 
$$\cos(2\alpha) \equiv \cos^2(\alpha) - \sin^2(\alpha)$$
  
 $\equiv 2\cos^2(\alpha) - 1$   
 $\equiv 1 - 2\sin^2(\alpha)$ 

43. 
$$\tan(2\alpha) \equiv \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$$

#### FÓRMULAS PARA EL ÁNGULO MEDIO

44. 
$$\operatorname{sen}\left(\frac{\alpha}{2}\right) \equiv \pm \sqrt{\frac{1 - \cos(\alpha)}{2}}$$

45. 
$$\cos\left(\frac{\alpha}{2}\right) \equiv \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$$

46. 
$$\tan\left(\frac{\alpha}{2}\right) \equiv \pm \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}}$$

#### RELACIÓNES ENTRE LAS FUNCIONES INVERSAS

47. 
$$sen^{-1}(\alpha) + cos^{-1}(\alpha) \equiv \frac{\pi}{2}$$

48. 
$$\tan^{-1}(\alpha) + \cot^{-1}(\alpha) \equiv \frac{\pi}{2}$$

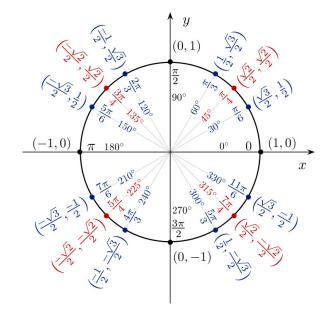
49. 
$$\sec^{-1}(\alpha) + \csc^{-1}(\alpha) \equiv \frac{\pi}{2}$$

50. 
$$\csc^{-1}(\alpha) \equiv \sec^{-1}(\frac{1}{\alpha})$$

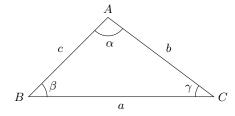
51. 
$$\sec^{-1}(\alpha) \equiv \cos^{-1}(\frac{1}{\alpha})$$

52. 
$$\cot^{-1}(\alpha) \equiv \tan^{-1}\left(\frac{1}{\alpha}\right)$$

# VALORES EXACTOS DE LAS FUNCIONES TRIGONOMÉTRICAS



# LEY DE SENOS



53. 
$$\frac{a}{\operatorname{sen}(\alpha)} \equiv \frac{b}{\operatorname{sen}(\beta)} \equiv \frac{c}{\operatorname{sen}(\gamma)}$$

#### LEY DE COSENOS

54. 
$$a^2 \equiv b^2 + c^2 - 2bc\cos(\alpha)$$

55. 
$$b^2 \equiv a^2 + c^2 - 2ac\cos(\beta)$$

$$56. \ c^2 \equiv a^2 + b^2 - 2ab\cos(\gamma)$$

#### LEY DE TANGENTES

57. 
$$\frac{a+b}{a-b} \equiv \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

58. 
$$\frac{a+c}{a-c} \equiv \frac{\tan\left(\frac{\alpha+\gamma}{2}\right)}{\tan\left(\frac{\alpha-\gamma}{2}\right)}$$

59. 
$$\frac{b+c}{b-c} \equiv \frac{\tan\left(\frac{\beta+\gamma}{2}\right)}{\tan\left(\frac{\beta-\gamma}{2}\right)}$$