**Fractal Analysis of the Optimal Hydraulic Gradient Surface in Water Distribution Networks**

**Andres Jaramillo**, Researcher, Water Distribution and Sewerage Systems Research Center (CIACUA), Universidad de los Andes, Carrera 1 Este No. 19A-40, Bogotá 111711018, Colombia.

Email: af.jaramillo@uniandes.edu.co

**Juan Saldarriaga**, Professor, Dept. of Civil and Environmental Engineering, Water Distribution and Sewerage Systems Research Center (CIACUA), Universidad de los Andes, Carrera 1 Este No. 19A-40, Bogotá 111711018, Colombia.

Email: jsaldarr@uniandes.edu.co

# Reproducibility Guide

This document provides a list of steps and directions to reproduce figures and tables in the paper. In terms of software, the authors employed the Python programming language (v. 3.7.9), Spyder and EPANET hydraulic modelling software (v. 2.0) to calculate results. All tools are available online for free with the corresponding installation instructions. The user can find useful links to access these resources in the references section of this document. It is also important that the user learns how to run Python scripts before attempting to reproduce figures and tables. Although Spyder was employed by the authors to run Python scripts, the user is free to run the scripts in any way.

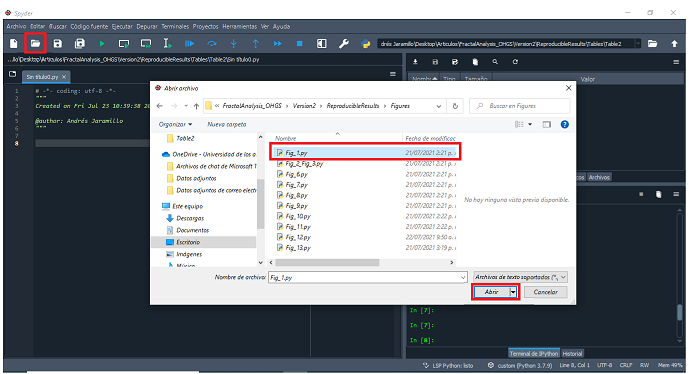
## Figures

To reproduce figures, the user must enter the “Figures” folder in the repository. The user will find ten available Python scripts. The name of each script specifies the figure that will be generated when the user runs the script.





Figure 1 will be generated to exemplify the use of Spyder to run the scripts. First, the user should click on the folder symbol in the upper left corner of the screen. Then, the user must locate and open the “Fig\_1.py” file.



Afterwards, the script can be run by clicking on the run symbol as specified in the following screenshot:

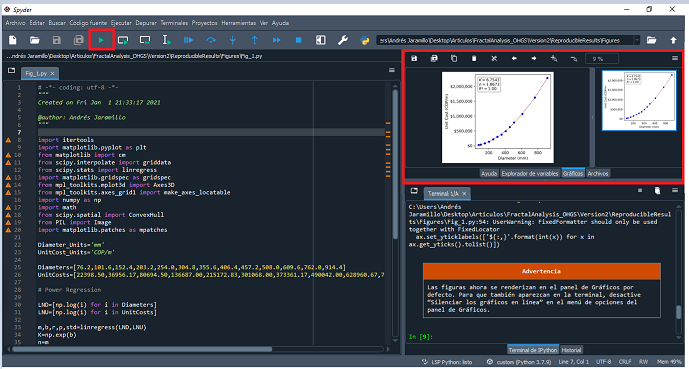
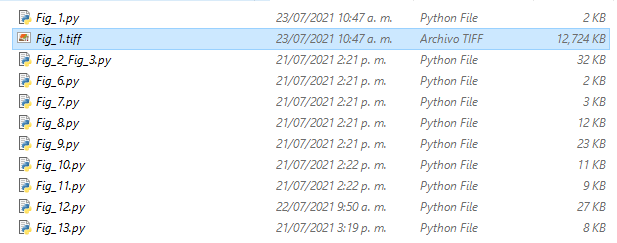


Figure 1 will then be generated in the same folder that contains the “Fig\_1.py” script. The remaining figures can be generated in the same manner.



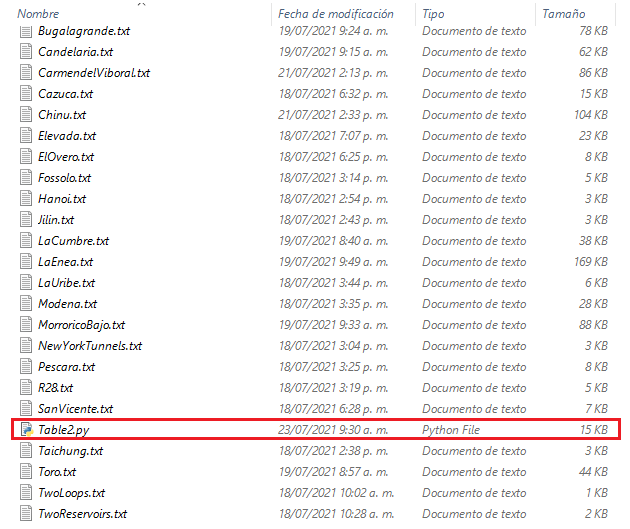
## Table 2

To reproduce Table 2, the user must enter the “Tables” folder and then the “Table2” subfolder in the repository. The user will find multiple folders, text files and a Python script named “Table2.py”.

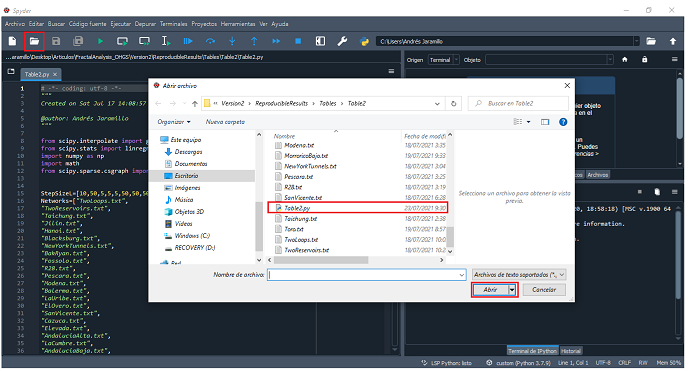




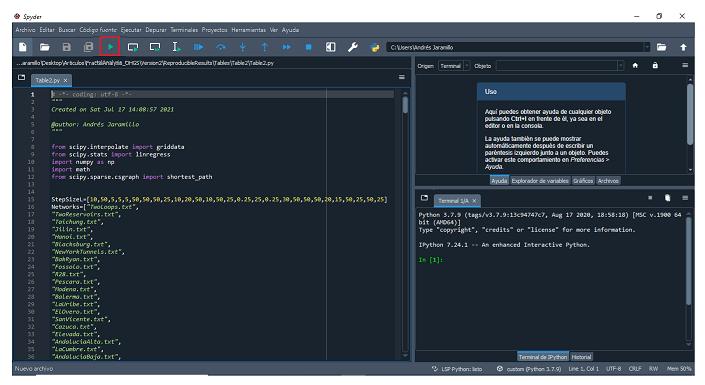
The user can choose between two possible ways to reproduce Table 2. The first one is to run the “Table2.py” script available in the folder. Text files should not be modified or altered in any way. Otherwise, the script may not compile or may estimate different results. In addition, all input text files must be at the same folder as “Table2.py” in order for the script to run.



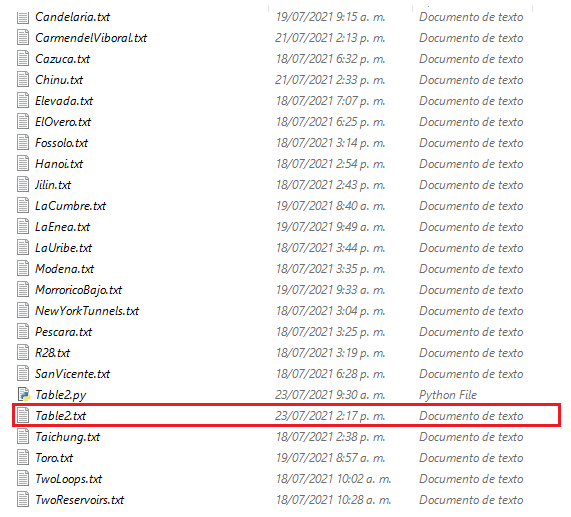
If the user employs Spyder to run the script, the following process should be done. First, the user should click on the folder symbol in the upper left corner of the screen. Then, the user must locate and open the “Table2.py” file.



The script can be run by clicking on the run symbol as specified in the following screenshot:

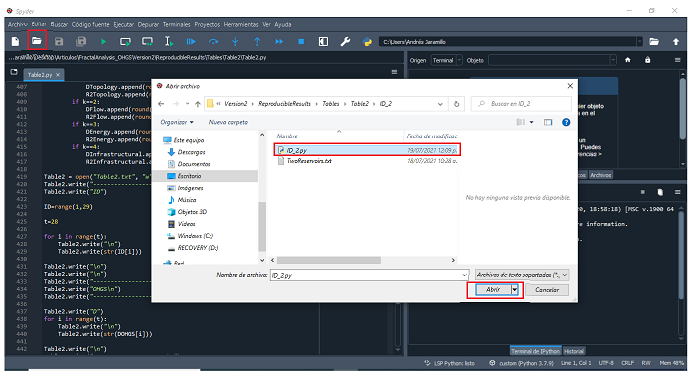


After running the script, a text file is generated. Each column of the table is separated by a dashed line.

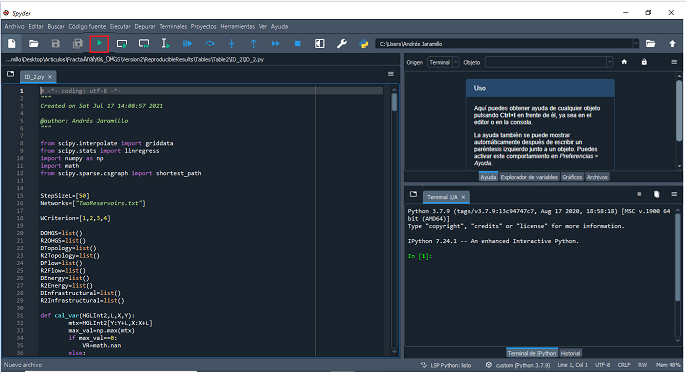


The user should be aware that “Table2.py” script takes a long time to be processed. Therefore, if the user only desires to reproduce the information for certain case studies, it is more appropriate to obtain the results in an alternative way. The “Table2” subfolder contains twenty-eight subfolders, named after the nomenclature in which networks are differentiated in the paper. Each “ID” subfolder contains the necessary files to obtain a specific row of Table 2.

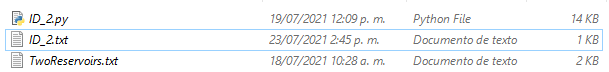
For example, if the user desires to obtain the results for Two Reservoirs network, i.e., the second row of Table 2, the “ID\_2” subfolder should be opened. The user will find a Python script named after the ID of the network and a text file with the input information. The text file should not be modified or altered in any way. If the user employs Spyder to run the script, the following process should be done. First, the user should click on the folder symbol in the upper left corner of the screen. Then, the user must locate and open the “ID\_2.py” file.



The script can be run by clicking on the run symbol as specified in the following screenshot:



After running the script, a text file is generated (“ID\_2.txt”). If the user desires to obtain additional rows of Table 2 it is necessary to run the Python scripts inside the corresponding “ID” subfolders in the same manner.



## Table 3

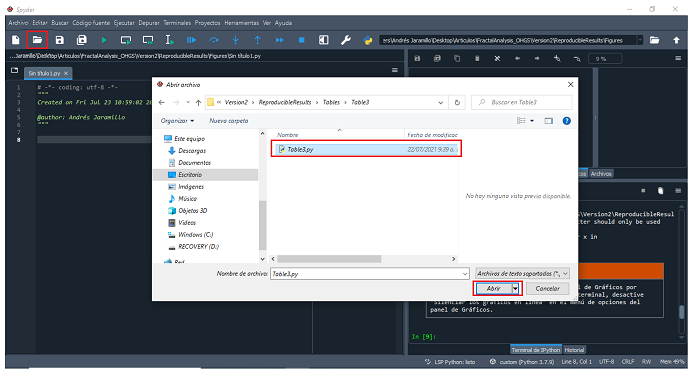
To reproduce Table 3, the user must enter the “Tables” folder and then the “Table3” subfolder in the repository. The user will find one available Python script.



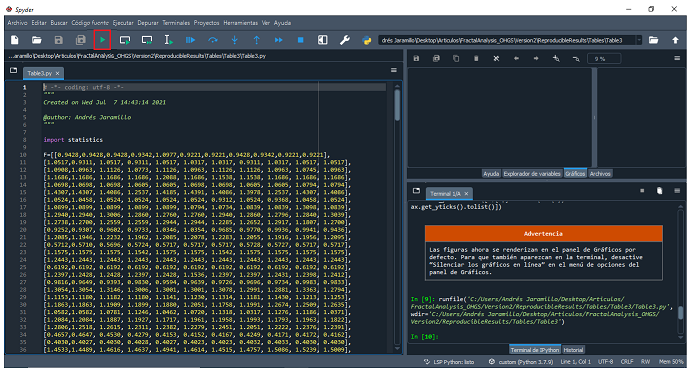




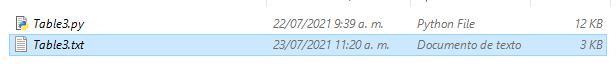
If the user employs Spyder to run the script, the following process should be done. First, the user should click on the folder symbol in the upper left corner of the screen. Then, the user must locate and open the “Table3.py” file.



The script can be run by clicking on the run symbol as specified in the following screenshot:



After running the script, a text file is generated. Each column of the table is separated by a dashed line.



## Supplemental Materials

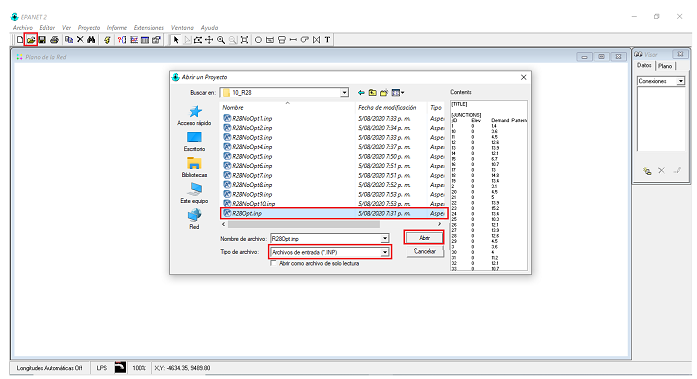
Supplemental Materials were also included in the repository.

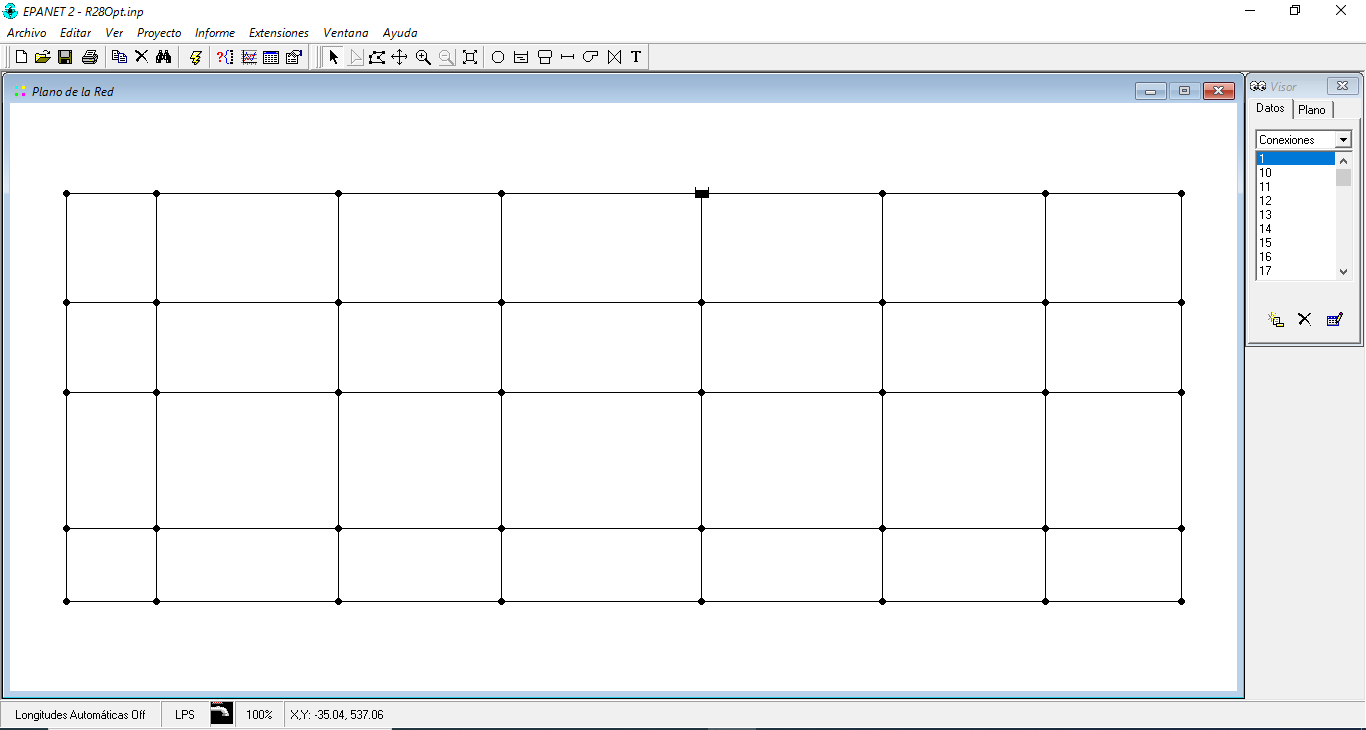


EPANET “.inp” data files of the corresponding optimal and non-optimal designs can be found in the folder “EPANET\_Designs”.

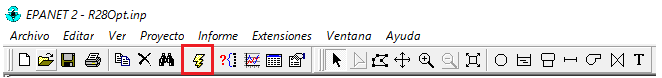


These files can be processed by importing them in EPANET. For example, if the user desires to perform the hydraulic simulation of R28 optimal design, the following process should be followed. First, the user should click on the folder symbol in the upper left corner of the screen. Then, the user must locate and open the “R28Opt.inp” file.

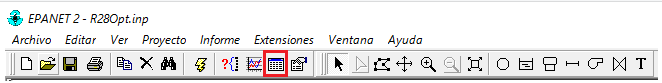


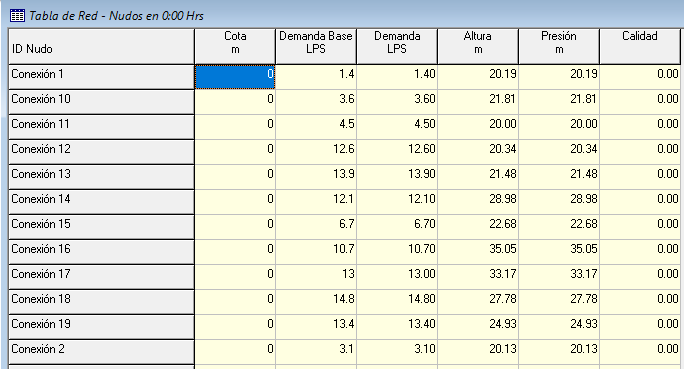


The hydraulic simulation can be performed by clicking on the lightning symbol in the upper toolbar.



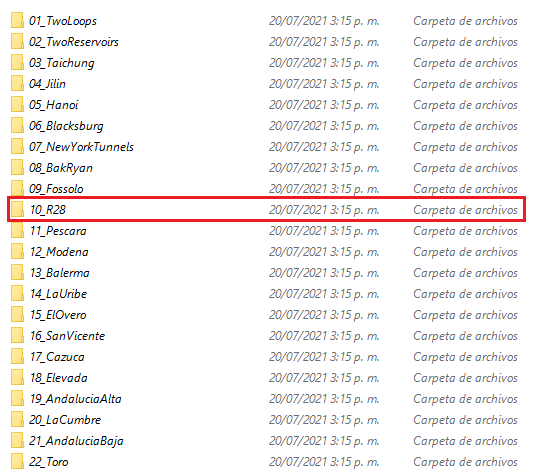
To obtain a tabular listing of results, the user should click the Table button and specify the desired parameters to print out.





In addition, the hydraulic simulation of each design was stored in a separate Excel spreadsheet and can be found in the “Excel\_Designs” subfolder. The following screenshots exemplify how the R28 optimal design Excel spreadsheet can be located:





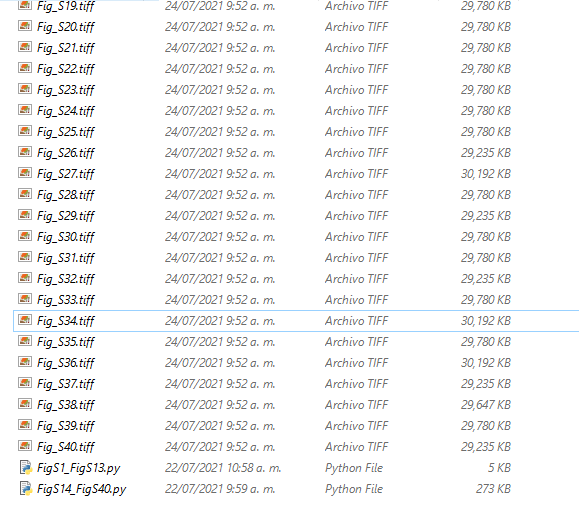


In terms of the supplemental figures, the user must enter the “SupplementalFigures” subfolder. The user will find two Python scripts inside. “FigS1\_FigS13.py” reproduces from Fig. S1 to Fig. S13, while “FigS14\_FigS40.py” reproduces from Fig. S14 to Fig. S40.



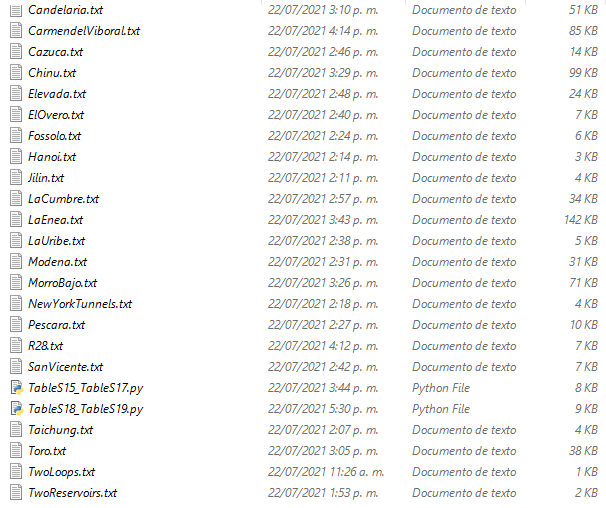


Each script can be run in Spyder in the same manner. Once the scripts are run, supplemental figures are generated.

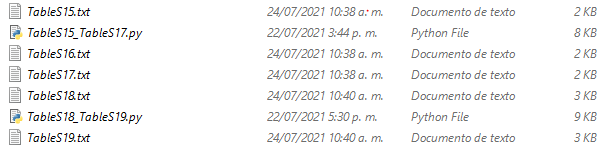


Finally, Table S15 to Table S19 can be reproduced by accessing the “SupplementalTables” subfolder. The user will find two Python scripts and several text files. “TableS15\_FigTable.py” reproduces from Table S15 to Table S17, while “TableS18\_Table19.py” reproduces Table S18 and Table S19. The text files should not be modified or altered in any way. In addition, these input text files must be located in the same folder as the “TableS15\_FigTable.py” and “TableS18\_Table19.py” files. Both scripts can be run in the same way as in previous cases.





Once the scripts are run, tables are printed in text files. Dashed lines are used to separate each column of the tables.



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