Measuring Planck's constant with LEDs

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CHEM321: Physical Chemistry II

Planck's constant (h) is of fundamental importance in quantum mechanics. Indeed, virtually every equation in this class involves Planck's constant in some fashion. Therefore, knowing the value of h in SI units is of fundamental importance to the practice of quantum mechanics.

In this experiment your goal is to measure Planck's constant using a collection of light emitting diodes (LEDs). You will have two weeks in which to build and perform this experiment. Your report is due at the beginning of lab on 2007-01-30.

1 The energy of a photon

Each of the various LED's available to you emits photons of different frequencies. The color we perceive when observing a given LED is related to the frequency of the maximum intensity in this spectrum. Thus, a red LED might show a peak at around 650nm, while a blue LED might have its peak around 470nm. The energy of a particular photon is given by

$$E = h\nu \tag{1}$$

Therefore, if we know the frequency of the photons (ν) coming from an LED, we can find their energy using Eq. 1

2 How LEDs work

So, how does an LED produce photons? And how is it different LEDs produce different photons?

LEDs are semiconductor devices. Nearly all such devices are constructed from arrangements of doped semiconductor materials. It is the geometry and electronic structure of these doped regions that gives a particular device its operational characteristics. In the case of an LED, p-type and n-type semiconductors are arranged as shown in Figure 1.

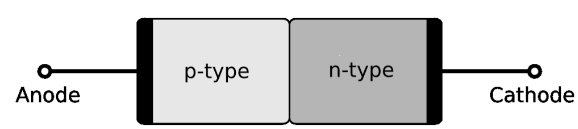


Figure 1: A diode - Modified from `http://en.wikipedia.org/wiki/Image:PN_Junction_Open_Circuited.svg`

N-type semiconductor is doped with atoms with more valence electrons than the undoped material. P-type semiconductor is doped with atoms with fewer valence electrons than the undoped material. For example, if silicon is doped with aluminum you get a p-type semiconductor. If silicon is doped with phosphorous you get n-type. N-type semiconductors are "electron rich", while p-type semiconductors are "hole rich". Holes are simply empty orbitals. It is the movement of electrons (and holes) that is controlled by a semiconductor device.

There are two ways you can hook up a diode: forward biased or reverse biased. In the reverse bias configuration the p-type side is connected to negative voltage and the n-type side to positive.

When reverse biased the electrons in the n-type region are attracted to the positive potential away from the junction while the holes are attracted by the negative potential away from the junction (see bottom of Figure 2).

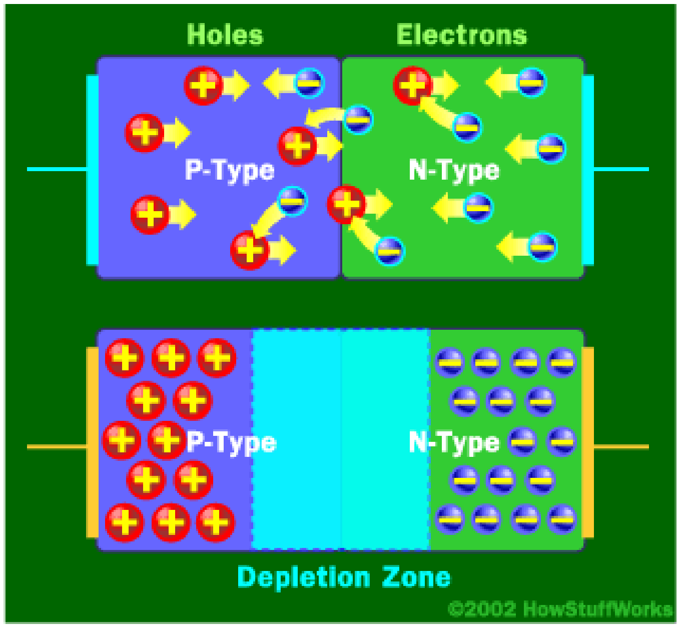


Figure 2: Forward bias (top) and reverse bias (bottom) `http://static.howstuffworks.com/gif/led-depletion.gif`

In the forward biased configuration the p-type side is connected to positive voltage and the n-type side to negative. When forward biased the electrons in the n-type region are repelled from by the negative potential towards the junction while the holes are repelled by the positive potential towards the junction. At the junction the holes and electrons meet and current flows (see top of Figure 2). In certain cases the energy of this recombination of electrons and holes is released as light energy. Then you have a light emitting diode.

3 Band gap and photon energy

So, how much energy is released/does it take to recombine these electrons and holes? Well, one way to measure it is by Eq. 1; measure the frequency/wavelength of the photons and multiply by h to get E. But the purpose of this experiment is to *measure* h; so Eq. 1 in a sense has *two* unknowns: E and h.

When we have two unknowns we need two equations. Therefore we need another relationship here. That relationship is the band gap. The property that gives semiconductors their name is their band gap. The band gap is essentially the energy difference between the highest occupied orbital, AKA the valence band, (where the electrons live) and the lowest unoccupied orbital, AKA the conduction band (the holes). In a conductor the band gap is very small/zero; electrons move through the conduction band freely. In an insulator the band gap is very large; it takes a lot of energy (i.e., voltage difference) to put electrons up in the conduction band. A semiconductor has a small band gap.

With an LED we can measure the band gap. It is related to the minimum voltage required to produce light, i.e., get the electrons and holes together. Different color LEDs have different band gaps and there for have different minimum voltages before they "turn on". If one measures this threshold voltage, the band gap is then given by:

$$E = e \cdot V_{\text{thresh}} \tag{2}$$

where e is the charge on an electron and V_{thresh} is the threshold voltage.

We can now wave our hands a bit more than we have already and equate Eq. 1 and Eq. 2. By measuring ν (practically speaking λ_{max}, which is related simply to ν by $\lambda\nu = c$) and V_{thresh}, and plugging in the appropriate value for e, our only unknown is h. Therefore, we can use a set of LEDs to measure Planck's constant.

4 Experimental

The first thing you need to do is build a circuit that will light up an LED without destroying it. How can you destroy an LED you ask? There are several ways: by applying a large reverse bias (i.e., plugging it in wrong), by applying a large forward bias (i.e., running it the right way, but too much.), by overheating it while soldering, etc.

The first two methods of killing an LED are both related to putting to much voltage to the leads. Additionally, you are going to have to control the voltage across your LEDs to make the band gap measurement. Therefore you need to build a circuit that lets you control the voltage from zero to some maximum value. The circuit in Figure 3 will let you do exactly that.

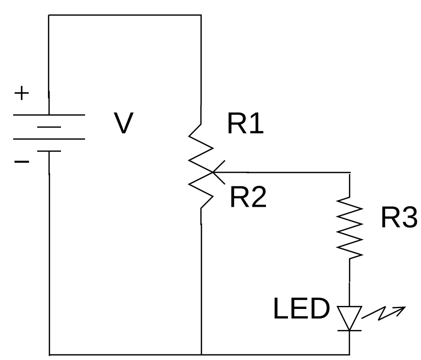


Figure 3: LED circuit with voltage divider. The variable resistor should be about 10 kΩ and the resistor in series with the LED about 1 kΩ. You can use a 9-volt battery as the voltage source.

Determine both λ_{max} (Section 4.1) and V_{thresh} (Section 4.2) for at least one LED of each color: red, orange, yellow, green, and blue. Not all of the LEDs are identical (i.e., there are a few kinds of red and blue LEDs I know we have), and it will probably help to have as many unique data points as possible. ***Also, obtain a white LED and determine λ_{max} and V_{thresh} for it as well.***

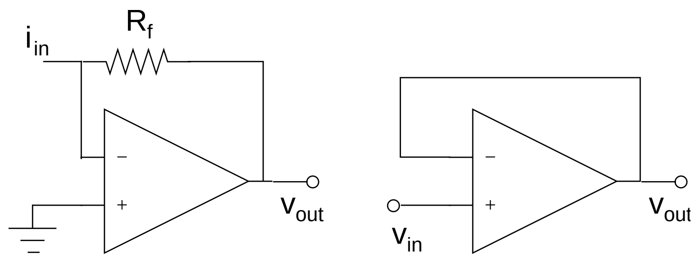
4.1 Determining λ_{max}

It should be possible to construct a setup in which light from an LED is captured by the collector lens on the end of the fiber optic cable included with the Ocean Optics spectrometer. Then it is simply a matter of adjusting the potentiometer (variable resistor) to apply enough voltage to light the LED and then take a spectrum.

One problem you may encounter is that the light is too intense (i.e., λ_{max} is off the scale). This may be alleviated by either tilting the LED away from the collection lens, the use of polarization filters, or some combination thereof.

4.2 Determining V_{thresh}

The easiest way to do this is to attach leads to a multimeter to the legs of the LED and cranking up the applied voltage until you see the LED light up. The voltage just before the LED turns on is V_{thresh}. This is easy, but crude. Collect your data this way first, and then try to improve your V_{thresh} measurements. Some suggestions are using a current-to-voltage converter (Figure 4(a)) to detect the current flow through the LED (essentially no current should flow until V_{thresh}), or using a voltage follower (Figure 4(b)) to remove the error caused by the loading of the LED.



(a) current-to-voltage converter $v_{\text{out}} = i_{\text{in}} \cdot R_f$

(b) voltage follower $v_{\text{out}} = v_{\text{in}}$

Figure 4: Some potentially useful op-amp circuits

5 Analysis

At this point you should have at least one set of at least 5 λ_{max} and V_{thresh}. Now you need to calculate h, the error in your measurement, and compare to the literature value of h. Based on this analysis, you may, if there is time, go back and measure some points again or in a different manner to improve your results. Below is an example analysis for you to follow.

Table 1: Data off the box

color	λ_{max} (nm)	V_{thresh} (V)
red	660	1.7
yellow	587	2.1
green	565	2.1
blue	430	5

Table 1 shows data for four LEDs taken off the boxes. By equating Eq. 1 and Eq. 2, along with the fact that $\lambda\nu = c$:

$$e \cdot V_{\text{thresh}} = h\nu_{\text{max}} \tag{3}$$

$$V_{\text{thresh}} = \frac{h}{e}\nu_{\text{max}} \tag{4}$$

$$V_{\text{thresh}} = \frac{hc}{e}\frac{1}{\lambda_{\text{max}}} \tag{5}$$

Eq. 5 implies that graphing V_{thresh} versus $\frac{1}{\lambda_{\max}}$ should yield a line with slope $\frac{hc}{e}$ (Figure 5). For this data, the slope is $4.3_0 \times 10^{-6} \pm 6._1 \times 10^{-7}\,\frac{\text{J}\cdot\text{m}}{\text{C}}$. Therefore, $h = 2.2_9 \times 10^{-33} \pm 3._3 \times 10^{-34}\,\text{J}\cdot\text{s}$. This is about 3.5 times the accepted value of $6.626 \times 10^{-34}\,\text{J}\cdot\text{s}$, which isn't so bad for not doing any actual lab work. However, t_{calc} for this data is around 7 whereas $t_{\text{table}} = 4.303$ for two degrees of freedom at 95% confidence. This means that statistically speaking the value for h obtained from the data in Table 1 is not the same as the accepted value. In fact, there is only a 5% chance they are the same! I am sure you will get better results from your data...

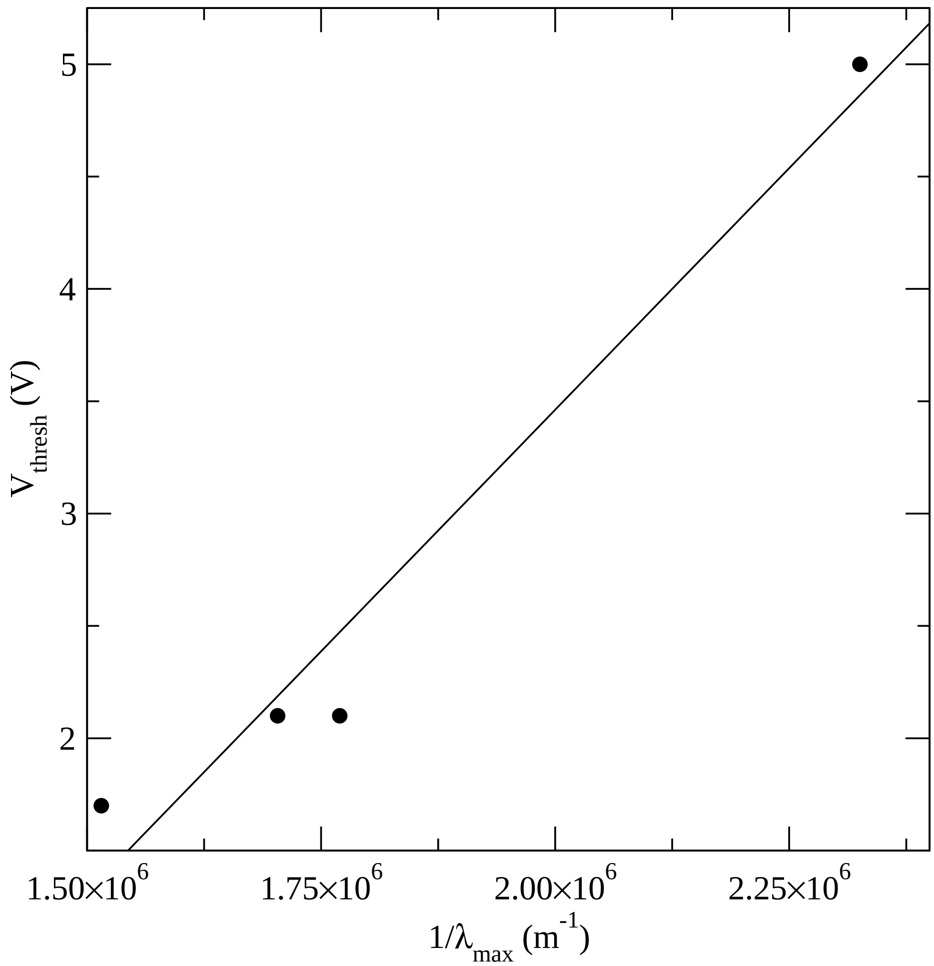


Figure 5: Data for four LEDs

6 Write-up

For your report (due at the beginning of lab on 2007-01-30), include an abstract, short introduction, detailed description of your experimental set-up(s), your data in publication quality tables and/or graphs, discussion, and conclusions.