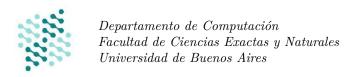
# Algoritmos y Estructuras de Datos I

Primer Cuatrimestre 2020

## Guía Práctica 4 Precondición más débil en SmallLang



Ejercicio 1.  $\bigstar$  Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- a) def(a+1).
- b) def(a/b).
- c)  $def(\sqrt{a/b})$ .
- d) def(A[i] + 1).
- e) def(A[i+2]).
- f)  $def(0 \le i \le |A|)$ .
- g)  $\operatorname{def}(0 \le i \le |A| \land_L A[i] \le 0)$ .

## Respuestas

Supongo que  $def(x) \equiv True$ , para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

- a)  $def(a+1) \equiv def(a) \wedge def(1) \equiv True \wedge True \equiv True$
- b)  $def(a/b) \equiv def(a) \wedge def(b) \wedge b \neq 0 \equiv b \neq 0$ .
- c)  $\operatorname{def}(\sqrt{a/b}) \equiv b \neq 0 \land (a/b) \geq 0$ .
- d)  $def(A[i] + 1) \equiv 0 \le i < |A|$
- e)  $def(A[i+2]) \equiv 0 \le i+2 < |A|$
- f)  $def(0 \le i \le |A|) \equiv True$
- g)  $\operatorname{def}(0 \le i \le |A| \land_L A[i] \le 0) \equiv i < |A|$

**Ejercicio 2.** Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- a)  $wp(\mathbf{a} := \mathbf{a} + \mathbf{1}, a \ge 0)$ .
- b)  $wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \ge 0)$ .
- c)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \ge 0)$ .
- d)  $wp(\mathbf{a} := \mathbf{b*b}, a > 0)$ .
- e)  $wp(\mathbf{b} := \mathbf{b} + \mathbf{1}, a \ge 0)$ .

#### Respuestas

a)

$$wp(\mathbf{a} := \mathbf{a+1}, a \ge 0) \equiv def(a+1) \wedge_L (a \ge 0)_{a+1}^a$$
$$\equiv True \wedge_L a + 1 \ge 0$$
$$\equiv a \ge -1$$

b)

$$wp(\mathbf{a} := \mathbf{a/b}, a \ge 0) \equiv \operatorname{def}(a/b) \wedge_L \wedge_L (a \ge 0)_{a/b}^a$$

$$\equiv \operatorname{def}(a) \wedge_L \operatorname{def}(b) \wedge_L b \ne 0 \wedge_L (a \ge 0)_{a/b}^a$$

$$\equiv True \wedge_L True \wedge_L b \ne 0 \wedge_L a/b \ge 0$$

$$\equiv b \ne 0 \wedge_L a \ge 0$$

c)

$$wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \ge 0) \equiv \operatorname{def}(A[i]) \wedge_L (a \ge 0)_{A[i]}^a$$
$$\equiv (\operatorname{def}(A) \wedge_L \operatorname{def}(i)) \wedge_L 0 \le i < |A| \wedge_L A[i] \ge 0$$
$$\equiv 0 \le i < |A| \wedge_L A[i] \ge 0$$

d)

$$wp(\mathbf{a} := \mathbf{b*b}, a \ge 0) \equiv \operatorname{def}(b*b) \wedge_L (a \ge 0)_{b*b}^a$$
$$\equiv True \wedge_L b*b \ge 0$$
$$\equiv b*b \ge 0$$

e)

$$wp(\mathbf{b} := \mathbf{b+1}, a \ge 0) \equiv \operatorname{def}(b+1) \wedge_L (a \ge 0)_a^a$$
  
 $\equiv True \wedge_L a \ge 0$   
 $\equiv a \ge 0$ 

**Ejercicio 3.**  $\bigstar$  Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- a)  $wp(\mathbf{a} := \mathbf{a} + \mathbf{1}; \mathbf{b} := \mathbf{a}/\mathbf{2}, b \ge 0).$
- b)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a}^*\mathbf{a}, b \neq 2).$
- c)  $wp(\mathbf{a} := \mathbf{A[i]} + \mathbf{1}; \mathbf{a} := \mathbf{b*b}, a \ge 0).$
- d)  $wp(\mathbf{a} := \mathbf{a} \mathbf{b}; \mathbf{b} := \mathbf{a} + \mathbf{b}, a > 0 \land b > 0).$

#### Respuestas

a)

$$\{wp(\mathbf{a} := \mathbf{a} + \mathbf{1}; \ \mathbf{b} := \mathbf{a}/\mathbf{2}, b \ge 0)\} \equiv \{(a+1)/2 \ge 0\}$$

$$\mathbf{a} := \mathbf{a} + \mathbf{1};$$

$$\{wp(\mathbf{b} := \mathbf{a}/\mathbf{2}, Q)\} \equiv \{a/2 \ge 0\}$$

$$\mathbf{b} := \mathbf{a}/\mathbf{2};$$

$$\{Q : b \ge 0\}$$

b)

$$\{wp(\mathbf{a} := \mathbf{A[i]} + \mathbf{1}; \mathbf{b} := \mathbf{a^*a}, b \neq 2)\} \equiv \{0 \le i < |A| \land_L (A[i] + 1) * (A[i] + 1) \neq 0\}$$

$$\mathbf{a} := \mathbf{A[i]} + \mathbf{1};$$

$$\{wp(\mathbf{b} := \mathbf{a^*a}, Q)\} \equiv \{a * a \neq 0\}$$

$$\mathbf{b} := \mathbf{a^*a};$$

$$\{Q : b \neq 0\}$$

```
c)
                                               \{wp(\mathbf{a} := \mathbf{A[i]} + 1; \mathbf{a} := \mathbf{b*b}, a \ge 0)\} \equiv \{0 \le i < |A| \land b \ge 0\}
                                                                                 a := A[i] + 1;
                                                                           \{wp(\mathbf{a} := \mathbf{b*b}, Q)\} \equiv \{b \ge 0\}
                                                                                        a := b*b;
                                                                                       {Q:b\geq 0}
   d)
                                            \{wp(\mathbf{a} := \mathbf{a} - \mathbf{b}; \mathbf{b} := \mathbf{a} + \mathbf{b}, a \ge 0 \land b \ge 0)\} \equiv \{a - b \ge 0 \land a + b \ge 0\}
                                                                                         a := a-b;
                                                                           \{wp(\mathbf{b} := \mathbf{a} + \mathbf{b}, Q)\} \equiv \{a \ge 0 \land a + b \ge 0\}
                                                                                       b := a+b;
                                                                             {Q: a > 0 \land b > 0}
Ejercicio 4. \bigstar Sea Q \equiv (\forall j : \mathbb{Z})(0 \le j < |A| \to_L A[j] \ge 0). Calcular las siguientes precondiciones más débiles, donde i es una
variable entera y A es una secuencia de reales.
   a) wp(A[i] := 0, Q).
   b) wp(A[i+2] := 0, Q).
   c) wp(A[i+2] := -1, Q).
   d) wp(A[i] := 2 * A[i], Q).
   e) wp(\mathbf{A[i]} := \mathbf{A[i-1]}, Q).
Respuestas
   a)
                                                               \{wp(\mathbf{A[i]} := \mathbf{0}, Q)\} \equiv \{0 \le i < |A| \land Q\}
                                                                              A[i] := 0
                                                                                      \{Q\}
   b)
                                                          \{wp(A[i+2] := 0, Q)\} \equiv \{0 \le i + 2 < |A| \land Q\}
                                                                         A[i+2] := 0
                                                                                   \{Q\}
   c)
                                  \{wp(\mathbf{A[i+2]} := -1, Q)\} \equiv \{0 \le i + 2 < |A| \land Q \land \mathbf{Esto} \text{ no pasa la post nunca}\}
                                                 A[i+2] := -1
                                                               \{Q\}
   d)
                                                   \{wp(\mathbf{A[i]} := \mathbf{2} * \mathbf{A[i]}, Q)\} \equiv \{0 \le i < |A| \land A[i] \ge 0 \land Q\}
                                                                 A[i] := 2 * A[i]
                                                                                    \{Q\}
   e)
                                                         \{wp(\mathbf{A[i]} := \mathbf{A[i-1]}, Q)\} \equiv \{0 \le i - 1 < |A| \land Q\}
                                                                       A[i] := A[i-1]
```

 $\{Q\}$ 

Ejercicio 5. Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) 
$$S \equiv i := i + 1$$
  
 $Q \equiv (\forall j : Z)(0 \le j < |A| \to_L A[j] \ne 0)$   
b)  $S \equiv A[0] := 4$   
 $Q \equiv (\forall j : Z)(0 \le j < |A| \to_L A[j] \ne 0)$   
c)  $S \equiv A[2] := 4$   
 $Q \equiv (\forall j : Z)(0 \le j < |A| \to_L A[j] \ne 0)$   
d)  $S \equiv A[i] := A[i + 1] - 1$   
 $Q \equiv (\forall j : Z)(0 < j < |A| \to_L A[j] \ge A[j - 1])$   
e)  $S \equiv A[i] := A[i + 1] - 1$   
 $Q \equiv (\forall j : Z)(0 < j < |A| \to_L A[j] \le A[j - 1])$ 

### Respuestas

a)

$$\{wp(S,Q)\} \equiv \{Q\}$$
 
$$S \equiv i := i+1$$
 
$$\{Q \equiv (\forall j:Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

Y a mi que me contas, no hace nada esto.

b)

$$\{wp(S,Q)\} \equiv \{0 \le 4 < |A| \land_L Q\}$$
 
$$S \equiv A[0] := 4$$
 
$$\{Q \equiv (\forall j : Z)(0 \le j < |A| \to_L A[j] \ne 0)\}$$

c)

$$\{wp(S,Q)\} \equiv \{0 \le 2 < |A| \land_L Q\}$$
$$S \equiv A[2] := 4$$
$$\{Q \equiv (\forall j : Z)(0 \le j < |A| \to_L A[j] \ne 0)\}$$

d)

$$\{wp(S,Q)\} \equiv \{0 \le i \land i + 1 < |A| \land_L Q\}$$
 
$$S \equiv A[i] := A[i+1] - 1$$
 
$$\{Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \ge A[j-1])\}$$

e)

$$\{wp(S,Q)\} \equiv \{0 \le i \land i + 1 < |A| \land_L Q\}$$
 
$$S \equiv A[i] := A[i+1] - 1$$
 
$$\{Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \le A[j-1])\}$$

**Ejercicio 6.** . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (inout a: 
$$\mathbb{Z}$$
) **Pre**  $\{a = a_0 \land a \ge 0\}$  **Post**  $\{a = a_0 + 2\}$ 

b)  $\mathbf{proc}\ \mathbf{problema2}\ (\mathrm{in}\ \mathrm{a:}\ \mathbb{Z},\ \mathrm{out}\ \mathrm{b:}\ \mathbb{Z})$ 

$$\mathbf{Pre} \left\{ a \neq 0 \right\}$$

**Post**  $\{b = a + 3\}$ 

c) **proc problema3** (in a:  $\mathbb{Z}$ , in b:  $\mathbb{Z}$ , out c:  $\mathbb{Z}$ )

$$\mathbf{Pre} \ \{true\}$$

 $\mathbf{Post} \{c = a + b\}$ 

d) **proc problema4** (in a:  $seq\langle \mathbb{Z} \rangle$ , in i:  $\mathbb{Z}$ , out result:  $\mathbb{Z}$ )

**Pre** 
$$\{0 \le i < |a|\}$$

Post  $\{result = 2 * a[i]\}$ 

e) **proc problema5** (in a:  $seq\langle \mathbb{Z} \rangle$ , in i:  $\mathbb{Z}$ , out result:  $\mathbb{Z}$ )

**Pre** 
$$\{0 \le i \land i + 1 < |a|\}$$

**Post**  $\{result = a[i] + a[i+1]\}$ 

## Respuestas

Para probar la correctitud de la tripla  $\{Pre\}\ S\ \{Post\}$  alcanza probar que  $Pre \to wp(S, Post)$ 

a)

$$\{wp(S, Post)\} \equiv \{a_0 + 2 = a_0 + 2\}$$
  
**S:**  $a := a_0 + 2$ 

{**Post:** 
$$a = a_0 + 2$$
}

$$Pre \rightarrow \{wp(S, Post)\}$$

$${a = a_0 \land a \ge 0} \to {a_0 + 2 = a_0 + 2}$$

$$\{a = a_0 \land a \ge 0\} \to \{True\}$$

True

b) 1. Calculamos  $\{wp(S, Post)\}$ 

$$\{wp(S, Post)\} \equiv \{a+3 = a+3\}$$

$$\equiv True$$

**S:** 
$$b := a + 3$$

{**Post:** b = a + 3}

2. Chequeamos  $Pre \to \{wp(S, Post)\}\$ 

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\{a \neq 0\} \rightarrow \{True\}$$

True

c) 1. Calculamos  $\{wp(S, Post)\}$ 

$$\{wp(S, Post)\} \equiv \{a+b=a+b\}$$
  
 $\equiv True$ 

**S:** 
$$c := a + b$$

{**Post:** 
$$c = a + b$$
}

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$ 

$$Pre \rightarrow \{wp(S, Post)\}\$$
  $\{True\} \rightarrow \{True\}\$   $True$ 

d) 1. Calculamos  $\{wp(S, Post)\}$ 

$$\{wp(S, Post)\} \equiv \{ def(2 * a[i]) \land_L 2 * a[i] = 2 * a[i] \}$$

$$\equiv def(a[i]) \land_L 0 \le i < |a| \land True$$

$$\equiv True \land_L 0 \le i < |a|$$

$$\equiv 0 \le i < |a|$$

$$\mathbf{S:} \ result := 2 * a[i]$$

$$\{ \mathbf{Post:} \ result = 2 * a[i] \}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$ 

$$Pre \rightarrow \{wp(S, Post)\}$$
 
$$\{0 \leq i < |a|\} \rightarrow \{0 \leq i < |a|\}$$
 
$$True$$

e) 1. Calculamos  $\{wp(S, Post)\}$ 

```
 \{wp(S, Post)\} \equiv \{ def(a[i] + a[i+1]) \land_L a[i] + a[i+1] = a[i] + a[i+1] \} 
 \equiv def(a[i]) \land_L def(a[i+1]) \land_L 0 \le i \land i+1 < |a| \land a[i] + a[i+1] = a[i] + a[i+1] 
 \equiv True \land_L True \land_L True \land_L 0 \le i \land i+1 < |a| \land True 
 \equiv 0 \le i \land i+1 < |a| 
 \mathbf{S:} \ result := a[i] + a[i+1] 
 \{ \mathbf{Post:} \ result = a[i] + a[i+1] \}
```

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}\$ 

$$Pre \rightarrow \{wp(S, Post)\}$$
 
$$\{0 \leq i \wedge i + 1 < |a|\} \rightarrow \{0 \leq i \wedge i + 1 < |a|\}$$
 
$$True$$

Ejercicio 7. ★ Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) 
$$S \equiv$$
if  $(a < 0)$ 
 $b := a$ 
else
 $b := -a$ 
endif
$$Q \equiv (b = -|a|)$$
b)  $S \equiv$ 
if  $(a < 0)$ 
 $b := a$ 
else

$$Q \equiv (b = |a|)$$

endif

b := -a

c) 
$$S \equiv$$
if  $(i > 0)$ 
 $s[i] := 0$ 
else
 $s[0] := 0$ 
endif

 $Q \equiv (\forall j : Z)(0 \le j < |s| \to_L s[j] \ge 0)$ 

d) 
$$S \equiv$$
if  $(i > 1)$ 
 $s[i] := s[i - 1]$ 
else
 $s[i] := 0$ 

$$s[i] := 0$$
 endif

$$Q \equiv (\forall j : Z)(1 \le j < |s| \to_L s[j] = s[j-1])$$

e) 
$$S \equiv$$
if  $(s[i] < 0)$ 
 $s[i] := -s[i]$ 
else
 $skip$ 
endif

$$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$$

$$\begin{array}{l} \mathbf{f}) \;\; S \equiv \\ & \mathbf{if} \;\; (s[i] > 0) \\ & s[i] := -s[i] \\ & \mathbf{else} \\ & skip \\ & \mathbf{endif} \end{array}$$

$$Q \equiv (\forall j:Z) (0 \leq j < |s| \to_L s[j] \geq 0)$$

### Respuestas

Axioma 4. Si S = if B then S1 else S2 endif, entonces

$$wp(\mathbf{S}, Q) \equiv def(B) \wedge_L ((B \wedge wp(\mathbf{S1}, Q)) \vee (\neg B \wedge wp(\mathbf{S1}, Q)))$$

a) S: if 
$$(a < 0)$$
 then  $b := a$  else  $b := -a$  endif 
$$Q \equiv (b = -|a|)$$

$$\begin{split} wp(\mathbf{S},Q) &\equiv \operatorname{def}(a<0) \wedge_L \left( \left( (a<0) \wedge (a=-|a|) \right) \vee \left( \neg (a<0) \wedge (-a=-|a|) \right) \right) \\ &\equiv True \wedge_L \left( \left( (a<0) \wedge (a=a) \right) \vee \left( (a\geq0) \wedge (-a=-a) \right) \right) \\ &\equiv \left( \left( (a<0) \wedge True \right) \vee \left( (a\geq0) \wedge true \right) \right) \\ &\equiv \left( (a<0) \vee (a\geq0) \right) \\ &\equiv True \end{split}$$

b) S: if (a < 0) then b := a else b := -a endif  $Q \equiv (b = |a|)$ 

$$wp(\mathbf{S}, Q) \equiv \operatorname{def}(a < 0) \wedge_{L} (((a < 0) \wedge (a = |a|)) \vee (\neg (a < 0) \wedge (-a = |a|)))$$

$$\equiv True \wedge_{L} (((a < 0) \wedge (a = -a)) \vee ((a \ge 0) \wedge (a = -a)))$$

$$\equiv (((a < 0) \wedge False) \vee ((a \ge 0) \wedge False))$$

$$\equiv (False \vee False)$$

$$\equiv False$$

c) S: if (i > 0) then s[i] := 0 else s[0] := 0 endif  $Q \equiv (\forall j : Z)(0 \le j < |s| \to_L s[j] \ge 0)$ 

$$wp(\mathbf{S}, Q) \equiv \operatorname{def}(i > 0) \wedge_{L} (((i > 0) \wedge S1) \vee (\neg(i > 0) \wedge S2)))$$
  
$$\equiv True \wedge_{L} () \vee ()$$
  
$$\equiv ()$$

d) S: if (i > 1) then s[i] := s[i-1] else s[i] := 0 endif  $Q \equiv (\forall j: Z)(1 \le j < |s| \rightarrow_L s[j] = s[j-1])$ 

$$wp(\mathbf{S}, Q) \equiv \operatorname{def}(i > 1) \wedge_{L} (((i > 1) \wedge S1) \vee (\neg(i > 1) \wedge S2)))$$

$$\equiv True \wedge_{L} () \vee ()$$

$$\equiv ()$$

e) S: if (s[i] < 0) then s[i] := -s[i] else skip endif  $Q \equiv 0 \le i < |s| \land_L s[i] \ge 0$ 

$$wp(\mathbf{S}, Q) \equiv \operatorname{def}(s[i] < 0) \wedge_L (((s[i] < 0) \wedge S1) \vee (\neg(s[i] < 0) \wedge S2)))$$
  
$$\equiv 0 \le i < |s| \wedge_L () \vee ()$$
  
$$\equiv ()$$

f) S: if (s[i] > 0) then s[i] := -s[i] else skip endif  $Q \equiv (\forall j: Z)(0 \le j < |s| \to_L s[j] \ge 0)$ 

$$wp(\mathbf{S}, Q) \equiv \operatorname{def}(s[i] > 0) \wedge_L (((s[i] > 0) \wedge S1) \vee (\neg(s[i] > 0) \wedge S2)))$$
  
$$\equiv 0 \leq i < |s| \wedge_L () \vee ()$$
  
$$\equiv ()$$

Ejercicio 8. ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (in s:  $seq(\mathbb{Z})$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ )

**Pre** 
$$\{0 \le i < |s| \land_L a = j = 0s[j]\}$$

**Post** 
$$\{a = j = 0s[j]\}$$

b) **proc problema2** (in s:  $seq\langle \mathbb{Z} \rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ )

**Pre** 
$$\{0 \le i < |s| \land a = j = 0s[j]\}$$

**Post** 
$$\{a = j = 1s[j]\}$$

c) **proc problema3** (in s:  $seq\langle \mathbb{Z} \rangle$ , in i:  $\mathbb{Z}$ , out res: **Bool**)

Pre 
$$\{0 \le i < |s| \land (\forall j : Z)(0 \le j < i \rightarrow_L s[j] \ge 0)\}$$
  
Post  $\{res = true \leftrightarrow (\forall j : Z)(0 \le j \le i \rightarrow_L s[j] \ge 0)\}$ 

d) **proc problema4** (in s:  $seq\langle \mathbb{Z} \rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ )

**Pre** 
$$0 \le i < |s| \land_L a = j = 0 (ifs[j]6 = 0 then1else0 fi)$$

Post 
$$\{a = j = 0(ifs[j]6 = 0then1else0fi)\}$$

e) **proc problema5** (in s:  $seq\langle \mathbb{Z} \rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ )

**Pre** 
$$\{0 < i \le |s| \land_L a = j = 1(ifs[j]6 = 0then1else0fi)\}$$

Post 
$$\{a = j = 0(ifs[j]6 = 0then1else0fi)\}$$

## Respuestas

- a)
- b)
- c)
- d)
- e)