



**Ejercicio 1.** ★ Calcular las siguientes expresiones, donde  $a, b$  son variables reales,  $i$  una variable entera y  $A$  es una secuencia de reales.

- a)  $\text{def}(a + 1)$ .
- b)  $\text{def}(a/b)$ .
- c)  $\text{def}(\sqrt{a/b})$ .
- d)  $\text{def}(A[i] + 1)$ .
- e)  $\text{def}(A[i + 2])$ .
- f)  $\text{def}(0 \leq i \leq |A|)$ .
- g)  $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0)$ .

### Respuestas

Supongo que  $\text{def}(x) \equiv \text{True}$ , para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

- a)  $\text{def}(a + 1) \equiv \text{def}(a) \wedge \text{def}(1) \equiv \text{True} \wedge \text{True} \equiv \text{True}$
- b)  $\text{def}(a/b) \equiv \text{def}(a) \wedge \text{def}(b) \wedge b \neq 0 \equiv b \neq 0$ .
- c)  $\text{def}(\sqrt{a/b}) \equiv b \neq 0 \wedge (a/b) \geq 0$ .
- d)  $\text{def}(A[i] + 1) \equiv 0 \leq i < |A|$
- e)  $\text{def}(A[i + 2]) \equiv 0 \leq i + 2 < |A|$
- f)  $\text{def}(0 \leq i \leq |A|) \equiv \text{True}$
- g)  $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0) \equiv i < |A|$

**Ejercicio 2.** Calcular las siguientes precondiciones más débiles, donde  $a, b$  son variables reales,  $i$  una variable entera y  $A$  es una secuencia de reales.

- a)  $wp(\mathbf{a} := \mathbf{a} + 1, a \geq 0)$ .
- b)  $wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \geq 0)$ .
- c)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \geq 0)$ .
- d)  $wp(\mathbf{a} := \mathbf{b} * \mathbf{b}, a \geq 0)$ .
- e)  $wp(\mathbf{b} := \mathbf{b} + 1, a \geq 0)$ .

## Respuestas

a)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{a}+1, a \geq 0) &\equiv \text{def}(a+1) \wedge_L (a \geq 0)_{a+1}^a \\ &\equiv \text{True} \wedge_L a+1 \geq 0 \\ &\equiv a \geq -1 \end{aligned}$$

b)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \geq 0) &\equiv \text{def}(a/b) \wedge_L \wedge_L (a \geq 0)_{a/b}^a \\ &\equiv \text{def}(a) \wedge_L \text{def}(b) \wedge_L b \neq 0 \wedge_L (a \geq 0)_{a/b}^a \\ &\equiv \text{True} \wedge_L \text{True} \wedge_L b \neq 0 \wedge_L a/b \geq 0 \\ &\equiv b \neq 0 \wedge_L a \geq 0 \end{aligned}$$

c)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \geq 0) &\equiv \text{def}(A[i]) \wedge_L (a \geq 0)_{A[i]}^a \\ &\equiv (\text{def}(A) \wedge_L \text{def}(i)) \wedge_L 0 \leq i < |A| \wedge_L A[i] \geq 0 \\ &\equiv 0 \leq i < |A| \wedge_L A[i] \geq 0 \end{aligned}$$

d)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{b}*\mathbf{b}, a \geq 0) &\equiv \text{def}(b*b) \wedge_L (a \geq 0)_{b*b}^a \\ &\equiv \text{True} \wedge_L b*b \geq 0 \\ &\equiv b*b \geq 0 \end{aligned}$$

e)

$$\begin{aligned} wp(\mathbf{b} := \mathbf{b}+1, a \geq 0) &\equiv \text{def}(b+1) \wedge_L (a \geq 0)_a^a \\ &\equiv \text{True} \wedge_L a \geq 0 \\ &\equiv a \geq 0 \end{aligned}$$

**Ejercicio 3.** ★ Calcular las siguientes precondiciones más débiles, donde  $a, b$  son variables reales,  $i$  una variable entera y  $A$  es una secuencia de reales.

a)  $wp(\mathbf{a} := \mathbf{a}+1; \mathbf{b} := \mathbf{a}/2, b \geq 0).$

b)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a}*\mathbf{a}, b \neq 2).$

c)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{a} := \mathbf{b}*\mathbf{b}, a \geq 0).$

d)  $wp(\mathbf{a} := \mathbf{a}-\mathbf{b}; \mathbf{b} := \mathbf{a}+\mathbf{b}, a \geq 0 \wedge b \geq 0).$

## Respuestas

a)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{a}+1; \mathbf{b} := \mathbf{a}/2, b \geq 0)\} &\equiv \{(a+1)/2 \geq 0\} \\ &\quad a := a+1; \\ \{wp(\mathbf{b} := \mathbf{a}/2, Q)\} &\equiv \{a/2 \geq 0\} \\ &\quad \mathbf{b} := \mathbf{a}/2; \\ &\quad \{Q : b \geq 0\} \end{aligned}$$

b)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a}*\mathbf{a}, b \neq 2)\} &\equiv \{0 \leq i < |A| \wedge_L (A[i] + 1) * (A[i] + 1) \neq 0\} \\ &\quad \mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \\ \{wp(\mathbf{b} := \mathbf{a}*\mathbf{a}, Q)\} &\equiv \{a * a \neq 0\} \\ &\quad \mathbf{b} := \mathbf{a}*\mathbf{a}; \\ &\quad \{Q : b \neq 0\} \end{aligned}$$

c)

$$\begin{aligned}
\{wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{a} := \mathbf{b} * \mathbf{b}, a \geq 0)\} &\equiv \{0 \leq i < |A| \wedge b \geq 0\} \\
&\quad \mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \\
\{wp(\mathbf{a} := \mathbf{b} * \mathbf{b}, Q)\} &\equiv \{b \geq 0\} \\
&\quad \mathbf{a} := \mathbf{b} * \mathbf{b}; \\
&\quad \{Q : b \geq 0\}
\end{aligned}$$

d)

$$\begin{aligned}
\{wp(\mathbf{a} := \mathbf{a} - \mathbf{b}; \mathbf{b} := \mathbf{a} + \mathbf{b}, a \geq 0 \wedge b \geq 0)\} &\equiv \{a - b \geq 0 \wedge a + b \geq 0\} \\
&\quad \mathbf{a} := \mathbf{a} - \mathbf{b}; \\
\{wp(\mathbf{b} := \mathbf{a} + \mathbf{b}, Q)\} &\equiv \{a \geq 0 \wedge a + b \geq 0\} \\
&\quad \mathbf{b} := \mathbf{a} + \mathbf{b}; \\
&\quad \{Q : a \geq 0 \wedge b \geq 0\}
\end{aligned}$$

**Ejercicio 4.** ★ Sea  $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$ . Calcular las siguientes precondiciones más débiles, donde  $i$  es una variable entera y  $A$  es una secuencia de reales.

- a)  $wp(\mathbf{A}[\mathbf{i}] := 0, Q)$ .
- b)  $wp(\mathbf{A}[\mathbf{i}+2] := 0, Q)$ .
- c)  $wp(\mathbf{A}[\mathbf{i}+2] := -1, Q)$ .
- d)  $wp(\mathbf{A}[\mathbf{i}] := 2 * \mathbf{A}[\mathbf{i}], Q)$ .
- e)  $wp(\mathbf{A}[\mathbf{i}] := \mathbf{A}[\mathbf{i}-1], Q)$ .

## Respuestas

a)

$$\begin{aligned}
wp(\mathbf{A}[\mathbf{i}] := 0, Q) &\equiv wp(\mathbf{A} := \text{setAt}(\mathbf{A}, \mathbf{i}, 0), Q) \\
&\equiv \text{def}(\mathbf{A} := \text{setAt}(\mathbf{A}, \mathbf{i}, 0)) \wedge_L Q_{\text{setAt}(\mathbf{A}, \mathbf{i}, 0)}^{\mathbf{A}} \\
&\equiv \left( (\text{def}(\mathbf{A}) \wedge \text{def}(\mathbf{i})) \wedge_L 0 \leq i < |A| \right) \wedge_L Q_{\text{setAt}(\mathbf{A}, \mathbf{i}, 0)}^{\mathbf{A}} \\
&\equiv 0 \leq i < |\text{setAt}(\mathbf{A}, \mathbf{i}, 0)| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(\mathbf{A}, \mathbf{i}, 0)| \rightarrow_L \left( \text{setAt}(\mathbf{A}, \mathbf{i}, 0) \geq 0 \right) \right) \\
&\equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \rightarrow_L \left( (i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 0 \geq 0) \right) \right)
\end{aligned}$$

b)

$$\begin{aligned}
wp(\mathbf{A}[\mathbf{i}+2] := 0, Q) &\equiv wp(\mathbf{A} := \text{setAt}(\mathbf{A}, \mathbf{i}+2, 0), Q) \\
&\equiv \text{def}(\mathbf{A} := \text{setAt}(\mathbf{A}, \mathbf{i}+2, 0)) \wedge_L Q_{\text{setAt}(\mathbf{A}, \mathbf{i}+2, 0)}^{\mathbf{A}} \\
&\equiv \left( (\text{def}(\mathbf{A}) \wedge \text{def}(\mathbf{i})) \wedge_L 0 \leq i+2 < |A| \right) \wedge_L Q_{\text{setAt}(\mathbf{A}, \mathbf{i}+2, 0)}^{\mathbf{A}} \\
&\equiv 0 \leq i+2 < |\text{setAt}(\mathbf{A}, \mathbf{i}+2, 0)| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(\mathbf{A}, \mathbf{i}+2, 0)| \rightarrow_L \left( \text{setAt}(\mathbf{A}, \mathbf{i}+2, 0) \geq 0 \right) \right) \\
&\equiv 0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \rightarrow_L \left( (i+2 \neq j \wedge A[j] \geq 0) \vee (i+2 = j \wedge 0 \geq 0) \right) \right)
\end{aligned}$$

c)

$$\begin{aligned}
& wp(A[i+2] := -1, Q) \\
& \stackrel{Ax,1}{\equiv} wp(A := \text{setAt}(A, i+2, -1), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i+2, 0)) \wedge_L Q_{\text{setAt}(A, i+2, -1)}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i+2 < |A| \right) \wedge_L Q_{\text{setAt}(A, i+2, 0)}^A \\
& \equiv 0 \leq i+2 < |\text{setAt}(A, i+2, -1)| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i+2, -1)| \right) \rightarrow_L \left( \text{setAt}(A, i+2, -1) \geq 0 \right) \\
& \equiv 0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i+2 \neq j \wedge A[j] \geq 0) \vee (i+2 = j \wedge -1 \geq 0) \right) \\
& \equiv \text{False}
\end{aligned}$$

d)

$$\begin{aligned}
& wp(A[i] := 2 * A[i], Q) \\
& \equiv wp(A := \text{setAt}(A, i, 2 * A[i]), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i, 2 * A[i])) \wedge_L Q_{\text{setAt}(A, i, 2 * A[i])}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i < |A| \right) \wedge_L Q_{\text{setAt}(A, i, 2 * A[i])}^A \\
& \equiv 0 \leq i < |\text{setAt}(A, i, 2 * A[i])| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i, 2 * A[i])| \right) \rightarrow_L \left( \text{setAt}(A, i, 2 * A[i]) \geq 0 \right) \\
& \equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 2 * A[i] \geq 0) \right)
\end{aligned}$$

e)

$$\begin{aligned}
& wp(A[i] := A[i-1], Q) \\
& \equiv wp(A := \text{setAt}(A, i, A[i-1]), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i, A[i-1])) \wedge_L Q_{\text{setAt}(A, i, A[i-1])}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i-1 < |A| \right) \wedge_L Q_{\text{setAt}(A, i, A[i-1])}^A \\
& \equiv 0 \leq i-1 < |\text{setAt}(A, i, A[i-1])| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i, A[i-1])| \right) \rightarrow_L \left( \text{setAt}(A, i, 0) \geq 0 \right) \\
& \equiv 0 \leq i-1 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i-1 \neq j \wedge A[j] \geq 0) \vee (i-1 = j \wedge 0 \geq 0) \right)
\end{aligned}$$

**Ejercicio 5.** Calcular  $wp(S, Q)$ , para los siguientes pares de programas  $S$  y postcondiciones  $Q$ .

- a)  $S \equiv i := i + 1$   
 $Q \equiv (\forall j : \mathbb{Z}) (0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- b)  $S \equiv A[0] := 4$   
 $Q \equiv (\forall j : \mathbb{Z}) (0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- c)  $S \equiv A[2] := 4$   
 $Q \equiv (\forall j : \mathbb{Z}) (0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- d)  $S \equiv A[i] := A[i+1] - 1$   
 $Q \equiv (\forall j : \mathbb{Z}) (0 < j < |A| \rightarrow_L A[j] \geq A[j-1])$
- e)  $S \equiv A[i] := A[i+1] - 1$   
 $Q \equiv (\forall j : \mathbb{Z}) (0 < j < |A| \rightarrow_L A[j] \leq A[j-1])$

## Respuestas

a)

$$\begin{aligned} wp(S, Q) &\equiv wp(i := i + 1, Q_{i+1}^i) \\ &\equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \neq 0) \end{aligned}$$

b)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[0] := 4, Q_{setAt(A, 0, 4)}^A) \\ &\equiv 0 \leq 0 < |setAt(A, 0, 4)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, 0, 4)| \rightarrow_L setAt(A, 0, 4)[j] \neq 0) \\ &\equiv 0 \leq 0 < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A|) \rightarrow_L ((0 \neq j \wedge A[j] \neq 0) \vee (0 = j \wedge 4 \neq 0)) \end{aligned}$$

c)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[2] := 4, Q_{setAt(A, 2, 4)}^A) \\ &\equiv 0 \leq 2 < |setAt(A, 2, 4)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, 2, 4)| \rightarrow_L setAt(A, 2, 4)[j] \neq 0) \\ &\equiv 0 \leq 2 < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A|) \rightarrow_L ((2 \neq j \wedge A[j] \neq 0) \vee (2 = j \wedge 4 \neq 0)) \end{aligned}$$

d)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[i] := A[i + 1] - 1, Q_{setAt(A, i, A[i+1]-1)}^A) \\ &\equiv 0 \leq i + 1 < |setAt(A, i, A[i + 1] - 1)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, i, A[i + 1] - 1)| \\ &\rightarrow_L setAt(A, i, A[i + 1] - 1)[j] \neq 0) \\ &\equiv 0 \leq i + 1 < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A|) \rightarrow_L ((i \neq j \wedge A[j] \geq A[j - 1]) \vee (i = j \wedge A[i + 1] - 1 \geq A[i - 1])) \end{aligned}$$

e)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[i] := A[i + 1] - 1, Q_{setAt(A, i, A[i+1]-1)}^A) \\ &\equiv 0 \leq i + 1 < |setAt(A, i, A[i + 1] - 1)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, i, A[i + 1] - 1)| \\ &\rightarrow_L setAt(A, i, A[i + 1] - 1)[j] \neq 0) \\ &\equiv 0 \leq i + 1 < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A|) \rightarrow_L ((i \neq j \wedge A[j] \leq A[j - 1]) \vee (i = j \wedge A[i + 1] - 1 \leq A[i - 1])) \end{aligned}$$

**Ejercicio 6.** . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

a) **proc problema1** (inout a:  $\mathbb{Z}$ )

**Pre**  $\{a = a_0 \wedge a \geq 0\}$

**Post**  $\{a = a_0 + 2\}$

b) **proc problema2** (in a:  $\mathbb{Z}$ , out b:  $\mathbb{Z}$ )

**Pre**  $\{a \neq 0\}$

**Post**  $\{b = a + 3\}$

c) **proc problema3** (in a:  $\mathbb{Z}$ , in b:  $\mathbb{Z}$ , out c:  $\mathbb{Z}$ )

**Pre**  $\{true\}$

**Post**  $\{c = a + b\}$

d) **proc problema4** (in a:  $seq\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$  , out result:  $\mathbb{Z}$ )

**Pre**  $\{0 \leq i < |a|\}$

**Post**  $\{result = 2 * a[i]\}$

e) **proc problema5** (in a:  $seq\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$  , out result:  $\mathbb{Z}$ )

**Pre**  $\{0 \leq i \wedge i + 1 < |a|\}$

**Post**  $\{result = a[i] + a[i + 1]\}$

## Respuestas

Para probar la correctitud de la tripla **{Pre} S {Post}** alcanza probar que

$$Pre \rightarrow wp(\mathbf{S}, Post)$$

a)

$$\{wp(S, Post)\} \equiv \{a_0 + 2 = a_0 + 2\}$$

$$\mathbf{S}: a := a_0 + 2$$

$$\{\mathbf{Post}: a = a_0 + 2\}$$

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{a_0 + 2 = a_0 + 2\}$$

$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{True\}$$

$$True$$

b) 1. Calculamos  $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + 3 = a + 3\}$$

$$\equiv True$$

$$\mathbf{S}: b := a + 3$$

$$\{\mathbf{Post}: b = a + 3\}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\{a \neq 0\} \rightarrow \{True\}$$

$$True$$

c) 1. Calculamos  $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + b = a + b\}$$

$$\equiv True$$

$$\mathbf{S}: c := a + b$$

$$\{\mathbf{Post}: c = a + b\}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\{True\} \rightarrow \{True\}$$

$$True$$

d) 1. Calculamos  $\{wp(S, Post)\}$

$$\begin{aligned}
\{wp(S, Post)\} &\equiv \{\text{def}(2 * a[i]) \wedge_L 2 * a[i] = 2 * a[i]\} \\
&\equiv \text{def}(a[i]) \wedge_L 0 \leq i < |a| \wedge True \\
&\equiv True \wedge_L 0 \leq i < |a| \\
&\equiv 0 \leq i < |a| \\
\mathbf{S}: result &:= 2 * a[i] \\
\{\mathbf{Post}: result &= 2 * a[i]\}
\end{aligned}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$\begin{aligned}
&Pre \rightarrow \{wp(S, Post)\} \\
\{0 \leq i < |a|\} &\rightarrow \{0 \leq i < |a|\} \\
&True
\end{aligned}$$

e) 1. Calculamos  $\{wp(S, Post)\}$

$$\begin{aligned}
\{wp(S, Post)\} &\equiv \{\text{def}(a[i] + a[i + 1]) \wedge_L a[i] + a[i + 1] = a[i] + a[i + 1]\} \\
&\equiv \text{def}(a[i]) \wedge_L \text{def}(a[i + 1]) \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge a[i] + a[i + 1] = a[i] + a[i + 1] \\
&\equiv True \wedge_L True \wedge_L True \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge True \\
&\equiv 0 \leq i \wedge i + 1 < |a| \\
\mathbf{S}: result &:= a[i] + a[i + 1] \\
\{\mathbf{Post}: result &= a[i] + a[i + 1]\}
\end{aligned}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$\begin{aligned}
&Pre \rightarrow \{wp(S, Post)\} \\
\{0 \leq i \wedge i + 1 < |a|\} &\rightarrow \{0 \leq i \wedge i + 1 < |a|\} \\
&True
\end{aligned}$$

**Ejercicio 7.** ★ Calcular  $wp(S, Q)$ , para los siguientes pares de programas S y postcondiciones Q.

a)  $S \equiv$   
**if** ( $a < 0$ )  
     $b := a$   
**else**  
     $b := -a$   
**endif**

$$Q \equiv (b = -|a|)$$

b)  $S \equiv$   
**if** ( $a < 0$ )  
     $b := a$   
**else**  
     $b := -a$   
**endif**

$$Q \equiv (b = |a|)$$

c)  $S \equiv$   
**if** ( $i > 0$ )

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    s[i] := 0
else
    s[0] := 0
endif

```

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

d)  $S \equiv$

```

if (i > 1)
    s[i] := s[i - 1]
else
    s[i] := 0
endif

```

$$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j - 1])$$

e)  $S \equiv$

```

if (s[i] < 0)
    s[i] := -s[i]
else
    skip
endif

```

$$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$$

f)  $S \equiv$

```

if (s[i] > 0)
    s[i] := -s[i]
else
    skip
endif

```

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

## Respuestas

**Axioma 4.** Si  $S = \text{if } B \text{ then } S1 \text{ else } S2 \text{ endif}$ , entonces

$$wp(S, Q) \equiv \text{def}(B) \wedge_L ((B \wedge wp(S1, Q)) \vee (\neg B \wedge wp(S2, Q)))$$

a)  $S: \text{if } (a < 0) \text{ then } b := a \text{ else } b := -a \text{ endif}$   
 $Q \equiv (b = -|a|)$

$$\begin{aligned}
 wp(S, Q) &\equiv \text{def}(a < 0) \wedge_L \left( \left( (a < 0) \wedge (a = -|a|) \right) \vee \left( \neg(a < 0) \wedge (-a = -|a|) \right) \right) \\
 &\equiv \text{True} \wedge_L \left( \left( (a < 0) \wedge (a = a) \right) \vee \left( (a \geq 0) \wedge (-a = -a) \right) \right) \\
 &\equiv \left( \left( (a < 0) \wedge \text{True} \right) \vee \left( (a \geq 0) \wedge \text{True} \right) \right) \\
 &\equiv (a < 0) \vee (a \geq 0) \\
 &\equiv \text{True}
 \end{aligned}$$



b) **S: if**  $(a < 0)$  **then**  $b := a$  **else**  $b := -a$  **endif**

$$Q \equiv (b = |a|)$$

$$\begin{aligned} wp(\mathbf{S}, Q) &\equiv \text{def}(a < 0) \wedge_L \left( \left( (a < 0) \wedge (a = |a|) \right) \vee \left( \neg(a < 0) \wedge (-a = |a|) \right) \right) \\ &\equiv \text{True} \wedge_L \left( \left( (a < 0) \wedge (a = -a) \right) \vee \left( (a \geq 0) \wedge (a = -a) \right) \right) \\ &\equiv \left( \left( (a < 0) \wedge \text{False} \right) \vee \left( (a \geq 0) \wedge \text{False} \right) \right) \\ &\equiv \text{False} \vee \text{False} \\ &\equiv \text{False} \end{aligned}$$

c) **S: if**  $(i > 0)$  **then**  $s[i] := 0$  **else**  $s[0] := 0$  **endif**

$$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

$$\begin{aligned} wp(\mathbf{S}, Q) &\equiv \text{def}(i > 0) \wedge_L \left( \left( (i > 0) \wedge wp(s[i] := 0, Q) \right) \vee \left( \neg(i > 0) \wedge wp(s[0] := 0, Q) \right) \right) \\ &\equiv \text{True} \wedge \left( \left( (i > 0) \wedge wp(\text{setAt}(s, i, 0), Q) \right) \vee \left( i \leq 0 \wedge wp(\text{setAt}(s, 0, 0), Q) \right) \right) \\ &\equiv \left( (i > 0) \wedge wp(\text{setAt}(s, i, 0), Q) \right) \vee \left( i \leq 0 \wedge wp(\text{setAt}(s, 0, 0), Q) \right) \\ &\equiv \left( (i > 0) \wedge wp(\text{setAt}(s, i, 0), Q) \right) \vee \text{True} \\ &\equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 < j < |A|) \rightarrow_L \left( (i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 0 \geq 0) \right) \end{aligned}$$

d) **S: if**  $(i > 1)$  **then**  $s[i] := s[i - 1]$  **else**  $s[i] := 0$  **endif**

$$Q \equiv (\forall j : \mathbb{Z})(1 \leq j < |s| \rightarrow_L s[j] = s[j - 1])$$

$$\begin{aligned} wp(\mathbf{S}, Q) &\equiv \text{def}(i > 1) \wedge_L \left( \left( (i > 1) \wedge S1 \right) \vee \left( \neg(i > 1) \wedge S2 \right) \right) \\ &\equiv \text{True} \wedge_L \left( () \vee () \right) \\ &\equiv () \end{aligned}$$

e) **S: if**  $(s[i] < 0)$  **then**  $s[i] := -s[i]$  **else** *skip* **endif**

$$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$$

$$\begin{aligned} wp(\mathbf{S}, Q) &\equiv \text{def}(s[i] < 0) \wedge_L \left( \left( (s[i] < 0) \wedge S1 \right) \vee \left( \neg(s[i] < 0) \wedge \text{True} \right) \right) \\ &\equiv 0 \leq i < |s| \wedge_L \left( () \vee (s[i] \geq 0) \right) \\ &\equiv () \end{aligned}$$

f) **S: if**  $(s[i] > 0)$  **then**  $s[i] := -s[i]$  **else** *skip* **endif**

$$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(s[i] > 0) \wedge_L \left( \left( (s[i] > 0) \wedge S1 \right) \vee \left( \neg(s[i] > 0) \wedge \text{True} \right) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left( () \vee (s[i] \leq 0) \right) \\
&\equiv ()
\end{aligned}$$

**Ejercicio 8.** ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

- a) **proc problema1** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j]\}$   
**Post**  $\{a = \sum_{j=0}^i s[j]\}$   
}
- b) **proc problema2** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^i s[j]\}$   
**Post**  $\{a = \sum_{j=1}^i s[j]\}$   
}
- c) **proc problema3** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , out res: **Bool**)  
**Pre**  $\{0 \leq i < |s| \wedge (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L s[j] \geq 0)\}$   
**Post**  $\{res = \text{true} \leftrightarrow (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L s[j] \geq 0)\}$
- d) **proc problema4** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
**Post**  $\{a = \sum_{j=0}^i (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
}
- e) **proc problema5** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=1}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
**Post**  $\{a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
}

## Respuestas

- a)
- b)
- c)
- d)
- e)

FIN.

## Violeta De Otoño

♩ = 66

The musical score is written for a single melodic line on a treble clef staff in 3/4 time. It consists of three systems of music. The first system (measures 1-8) begins with a treble clef and a tempo marking of ♩ = 66. The melody is composed of eighth and sixteenth notes, with some measures containing triplets. The second system (measures 9-17) continues the melody, featuring a repeat sign at the beginning of measure 9. The third system (measures 18-20) concludes the piece with a final cadence. The score includes various musical notations such as stems, beams, and rests.

8

9

18