Algoritmos y Estructuras de Datos I
Primer Cuatrimestre 2020

Guía Práctica 4
Precondición más débil en SmallLang

Departamento de Computación
Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires

Ejercicio 1. ★ Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) def$(a + 1)$.

b) def(a/b).

c) def$(\sqrt{a/b})$.

d) def$(A[i] + 1)$.

e) def$(A[i + 2])$.

f) def$(0 \leq i \leq |A|)$.

g) def$(0 \leq i \leq |A| \wedge_L A[i] \leq 0)$.

Respuestas

Supongo que def$(x) \equiv True$,para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

a) def$(a + 1) \equiv$def$(a) \wedge$def$(1) \equiv True \wedge True \equiv True$

b) def$(a/b) \equiv$ def(a) $\wedge$ def(b) $\wedge$ $b \neq 0 \equiv b \neq 0$.

c) def$(\sqrt{a/b}) \equiv b \neq 0 \wedge (a/b) \geq 0$.

d) def$(A[i] + 1) \equiv 0 \leq i < |A|$

e) def$(A[i + 2]) \equiv 0 \leq i + 2 < |A|$

f) def$(0 \leq i \leq |A|) \equiv True$

g) def$(0 \leq i \leq |A| \wedge_L A[i] \leq 0) \equiv i < |A|$

Ejercicio 2. Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp($**a := a+1**$, a \geq 0)$.

b) $wp($**a := a/b**$, a \geq 0)$.

c) $wp($**a := A[i]**$, a \geq 0)$.

d) $wp($**a := b*b**$, a \geq 0)$.

e) $wp($**b := b+1**$, a \geq 0)$.

Respuestas

a)

$$\begin{aligned} wp(\textbf{a := a+1}, a \geq 0) &\equiv \text{def}(a+1) \wedge_L (a \geq 0)^a_{a+1} \\ &\equiv \mathit{True} \wedge_L a+1 \geq 0 \\ &\equiv a \geq -1 \end{aligned}$$

b)

$$\begin{aligned} wp(\textbf{a := a/b}, a \geq 0) &\equiv \text{def}(a/b) \wedge_L \wedge_L (a \geq 0)^a_{a/b} \\ &\equiv \text{def}(a) \wedge_L \text{def}(b) \wedge_L b \neq 0 \wedge_L (a \geq 0)^a_{a/b} \\ &\equiv \mathit{True} \wedge_L \mathit{True} \wedge_L b \neq 0 \wedge_L a/b \geq 0 \\ &\equiv b \neq 0 \wedge_L a \geq 0 \end{aligned}$$

c)

$$\begin{aligned} wp(\textbf{a := A[i]}, a \geq 0) &\equiv \text{def}(A[i]) \wedge_L (a \geq 0)^a_{A[i]} \\ &\equiv (\text{def}(A) \wedge_L \text{def}(i)) \wedge_L 0 \leq i < |A| \wedge_L A[i] \geq 0 \\ &\equiv 0 \leq i < |A| \wedge_L A[i] \geq 0 \end{aligned}$$

d)

$$\begin{aligned} wp(\textbf{a := b*b}, a \geq 0) &\equiv \text{def}(b*b) \wedge_L (a \geq 0)^a_{b*b} \\ &\equiv \mathit{True} \wedge_L b*b \geq 0 \\ &\equiv b*b \geq 0 \end{aligned}$$

e)

$$\begin{aligned} wp(\textbf{b := b+1}, a \geq 0) &\equiv \text{def}(b+1) \wedge_L (a \geq 0)^a_a \\ &\equiv \mathit{True} \wedge_L a \geq 0 \\ &\equiv a \geq 0 \end{aligned}$$

Ejercicio 3. ★ Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp(\textbf{a := a+1; b := a/2}, b \geq 0)$.

b) $wp(\textbf{a := A[i] + 1; b := a*a}, b \neq 2)$.

c) $wp(\textbf{a := A[i] + 1; a := b*b}, a \geq 0)$.

d) $wp(\textbf{a := a-b; b := a+b}, a \geq 0 \wedge b \geq 0)$.

Respuestas

a)

$$\{wp(\textbf{a := a+1; b := a/2}, b \geq 0)\} \equiv \{(a+1)/2 \geq 0\}$$

a := a+1;

$$\{wp(\textbf{b := a/2}, Q)\} \equiv \{a/2 \geq 0\}$$

b := a/2;

$$\{Q : b \geq 0\}$$

b)

$$\{wp(\textbf{a := A[i] + 1; b := a*a}, b \neq 2)\} \equiv \{0 \leq i < |A| \wedge_L (A[i]+1)*(A[i]+1) \neq 0\}$$

a := A[i] + 1;

$$\{wp(\textbf{b := a*a}, Q)\} \equiv \{a*a \neq 0\}$$

b := a*a;

$$\{Q : b \neq 0\}$$

c)

$$\{wp(\textbf{a := A[i] + 1; a := b*b}, a \geq 0)\} \equiv \{0 \leq i < |A| \wedge b \geq 0\}$$
$$\textbf{a := A[i] + 1;}$$
$$\{wp(\textbf{a := b*b}, Q)\} \equiv \{b \geq 0\}$$
$$\textbf{a := b*b;}$$
$$\{Q : b \geq 0\}$$

d)

$$\{wp(\textbf{a := a-b; b := a+b}, a \geq 0 \wedge b \geq 0)\} \equiv \{a - b \geq 0 \wedge a + b \geq 0\}$$
$$\textbf{a := a-b;}$$
$$\{wp(\textbf{b := a+b}, Q)\} \equiv \{a \geq 0 \wedge a + b \geq 0\}$$
$$\textbf{b := a+b;}$$
$$\{Q : a \geq 0 \wedge b \geq 0\}$$

Ejercicio 4. ★ Sea $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$. Calcular las siguientes precondiciones más débiles, donde i es una variable entera y A es una secuencia de reales.

a) $wp(\textbf{A[i] := 0}, Q)$.

b) $wp(\textbf{A[i+2] := 0}, Q)$.

c) $wp(\textbf{A[i+2] := -1}, Q)$.

d) $wp(\textbf{A[i] := 2 * A[i]}, Q)$.

e) $wp(\textbf{A[i] := A[i-1]}, Q)$.

Respuestas

a)

$$\{wp(\textbf{A[i] := 0}\ , Q)\} \equiv \{0 \leq i < |A| \wedge Q\}$$
$$\textbf{A[i] := 0}$$
$$\{Q\}$$

b)

$$\{wp(\textbf{A[i+2] := 0}\ , Q)\} \equiv \{0 \leq i + 2 < |A| \wedge Q\}$$
$$\textbf{A[i+2] := 0}$$
$$\{Q\}$$

c)

$$\{wp(\textbf{A[i+2] := -1}\ , Q)\} \equiv \{0 \leq i + 2 < |A| \wedge Q \wedge \textbf{Esto no pasa la post nunca}\}$$
$$\textbf{A[i+2] := -1}$$
$$\{Q\}$$

d)

$$\{wp(\textbf{A[i] := 2 * A[i]}\ , Q)\} \equiv \{0 \leq i < |A| \wedge A[i] \geq 0 \wedge Q\}$$
$$\textbf{A[i] := 2 * A[i]}$$
$$\{Q\}$$

e)

$$\{wp(\textbf{A[i] := A[i-1]}\ , Q)\} \equiv \{0 \leq i - 1 < |A| \wedge Q\}$$
$$\textbf{A[i] := A[i-1]}$$
$$\{Q\}$$

Ejercicio 5. Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) $S \equiv i := i + 1$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

b) $S \equiv A[0] := 4$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

c) $S \equiv A[2] := 4$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

d) $S \equiv A[i] := A[i + 1] - 1$
$Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j - 1])$

e) $S \equiv A[i] := A[i + 1] - 1$
$Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j - 1])$

Respuestas

a)

$$\{wp(S, Q)\} \equiv \{Q\}$$
$$S \equiv i := i + 1$$
$$\{Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

Y a mi que me contas, no hace nada esto.

b)

$$\{wp(S, Q)\} \equiv \{0 \leq 4 < |A| \wedge_L Q\}$$
$$S \equiv A[0] := 4$$
$$\{Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

c)

$$\{wp(S, Q)\} \equiv \{0 \leq 2 < |A| \wedge_L Q\}$$
$$S \equiv A[2] := 4$$
$$\{Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

d)

$$\{wp(S, Q)\} \equiv \{0 \leq i \wedge i + 1 < |A| \wedge_L Q\}$$
$$S \equiv A[i] := A[i + 1] - 1$$
$$\{Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j - 1])\}$$

e)

$$\{wp(S, Q)\} \equiv \{0 \leq i \wedge i + 1 < |A| \wedge_L Q\}$$
$$S \equiv A[i] := A[i + 1] - 1$$
$$\{Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j - 1])\}$$

Ejercicio 6. . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (inout a: $\mathbb{Z}$)
Pre $\{a = a_0 \wedge a \geq 0\}$
Post $\{a = a_0 + 2\}$

b) **proc problema2** (in a: $\mathbb{Z}$, out b: $\mathbb{Z}$)
Pre $\{a \neq 0\}$
Post $\{b = a + 3\}$

c) **proc problema3** (in a: $\mathbb{Z}$, in b: $\mathbb{Z}$, out c: $\mathbb{Z}$)
Pre $\{true\}$
Post $\{c = a + b\}$

d) **proc problema4** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
Pre $\{0 \leq i < |a|\}$
Post $\{result = 2 * a[i]\}$

e) **proc problema5** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
Pre $\{0 \leq i \land i + 1 < |a|\}$
Post $\{result = a[i] + a[i + 1]\}$

Respuestas

Para probar la correctitud de la tripla **{Pre} S {Post}** alcanza probar que
Pre $\rightarrow wp(\mathbf{S}, Post)$

a)

$$\{wp(S, Post)\} \equiv \{a_0 + 2 = a_0 + 2\}$$
$$\mathbf{S:}\ a := a_0 + 2$$
$$\{\mathbf{Post:}\ a = a_0 + 2\}$$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{a = a_0 \land a \geq 0\} \rightarrow \{a_0 + 2 = a_0 + 2\}$$
$$\{a = a_0 \land a \geq 0\} \rightarrow \{True\}$$
$$True$$

b) 1. Calculamos $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + 3 = a + 3\}$$
$$\equiv True$$
$$\mathbf{S:}\ b := a + 3$$
$$\{\mathbf{Post:}\ b = a + 3\}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{a \neq 0\} \rightarrow \{True\}$$
$$True$$

c) 1. Calculamos $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + b = a + b\}$$
$$\equiv True$$
$$\mathbf{S:}\ c := a + b$$
$$\{\mathbf{Post:}\ c = a + b\}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$\begin{gathered} Pre \to \{wp(S, Post)\} \\ \{True\} \to \{True\} \\ True \end{gathered}$$

d) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned} \{wp(S, Post)\} &\equiv \{\mathrm{def}(2 * a[i]) \wedge_L 2 * a[i] = 2 * a[i]\} \\ &\equiv \mathrm{def}(a[i]) \wedge_L 0 \leq i < |a| \wedge True \\ &\equiv True \wedge_L 0 \leq i < |a| \\ &\equiv 0 \leq i < |a| \\ \textbf{S: } & result := 2 * a[i] \\ \{\textbf{Post: } & result = 2 * a[i]\} \end{aligned}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$\begin{gathered} Pre \to \{wp(S, Post)\} \\ \{0 \leq i < |a|\} \to \{0 \leq i < |a|\} \\ True \end{gathered}$$

e) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned} \{wp(S, Post)\} &\equiv \{\mathrm{def}(a[i] + a[i+1]) \wedge_L a[i] + a[i+1] = a[i] + a[i+1]\} \\ &\equiv \mathrm{def}(a[i]) \wedge_L \mathrm{def}(a[i+1]) \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge a[i] + a[i+1] = a[i] + a[i+1] \\ &\equiv True \wedge_L True \wedge_L True \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge True \\ &\equiv 0 \leq i \wedge i + 1 < |a| \\ \textbf{S: } & result := a[i] + a[i+1] \\ \{\textbf{Post: } & result = a[i] + a[i+1]\} \end{aligned}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$\begin{gathered} Pre \to \{wp(S, Post)\} \\ \{0 \leq i \wedge i + 1 < |a|\} \to \{0 \leq i \wedge i + 1 < |a|\} \\ True \end{gathered}$$

Ejercicio 7. ★ Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) $S \equiv$
if $(a < 0)$
    $b := a$
else
    $b := -a$
endif

$Q \equiv (b = -|a|)$

b) $S \equiv$
if $(a < 0)$
    $b := a$
else
    $b := -a$
endif

$Q \equiv (b = |a|)$

c) $S \equiv$

if $(i > 0)$

$\quad s[i] := 0$

else

$\quad s[0] := 0$

endif

$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

d) $S \equiv$

if $(i > 1)$

$\quad s[i] := s[i-1]$

else

$\quad s[i] := 0$

endif

$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j-1])$

e) $S \equiv$

if $(s[i] < 0)$

$\quad s[i] := -s[i]$

else

$\quad skip$

endif

$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

f) $S \equiv$

if $(s[i] > 0)$

$\quad s[i] := -s[i]$

else

$\quad skip$

endif

$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

Respuestas

Axioma 4. Si **S** = **if B then S1 else S2 endif**, entonces

$$wp(\mathbf{S}, Q) \equiv \text{def}(B) \wedge_L ((B \wedge wp(\mathbf{S1}, Q)) \vee (\neg B \wedge wp(\mathbf{S1}, Q)))$$

a) **S: if** $(a < 0)$ **then** $b := a$ **else** $b := -a$ **endif**

$Q \equiv (b = -|a|)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(a < 0) \wedge_L \left(\Big((a < 0) \wedge (a = -|a|) \Big) \vee \Big(\neg(a < 0) \wedge (-a = -|a|) \Big) \right) \\
&\equiv \mathit{True} \wedge_L \left(\Big((a < 0) \wedge (a = a) \Big) \vee \Big((a \geq 0) \wedge (-a = -a) \Big) \right) \\
&\equiv \left(\Big((a < 0) \wedge \mathit{True} \Big) \vee \Big((a \geq 0) \wedge \mathit{True} \Big) \right) \\
&\equiv (a < 0) \vee (a \geq 0) \\
&\equiv \mathit{True}
\end{aligned}
$$

b) **S: if** $(a < 0)$ **then** $b := a$ **else** $b := -a$ **endif**
$Q \equiv (b = |a|)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(a < 0) \wedge_L \left(\Big((a < 0) \wedge (a = |a|) \Big) \vee \Big(\neg(a < 0) \wedge (-a = |a|) \Big) \right) \\
&\equiv \mathit{True} \wedge_L \left(\Big((a < 0) \wedge (a = -a) \Big) \vee \Big((a \geq 0) \wedge (a = -a) \Big) \right) \\
&\equiv \left(\Big((a < 0) \wedge \mathit{False} \Big) \vee \Big((a \geq 0) \wedge \mathit{False} \Big) \right) \\
&\equiv \mathit{False} \vee \mathit{False} \\
&\equiv \mathit{False}
\end{aligned}
$$

c) **S: if** $(i > 0)$ **then** $s[i] := 0$ **else** $s[0] := 0$ **endif**
$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(i > 0) \wedge_L \left(\Big((i > 0) \wedge S1 \Big) \vee \Big(\neg(i > 0) \wedge S2) \Big) \right) \\
&\equiv \mathit{True} \wedge_L \left(\Big(\Big) \vee \Big(\Big) \right) \\
&\equiv ()
\end{aligned}
$$

d) **S: if** $(i > 1)$ **then** $s[i] := s[i-1]$ **else** $s[i] := 0$ **endif**
$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j-1])$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(i > 1) \wedge_L \left(\Big((i > 1) \wedge S1 \Big) \vee \Big(\neg(i > 1) \wedge S2) \Big) \right) \\
&\equiv \mathit{True} \wedge_L \left(\Big(\Big) \vee \Big(\Big) \right) \\
&\equiv ()
\end{aligned}
$$

e) **S: if** $(s[i] < 0)$ **then** $s[i] := -s[i]$ **else** *skip* **endif**
$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \mathrm{def}(s[i] < 0) \wedge_L \left(\Big((s[i] < 0) \wedge S1 \Big) \vee \Big(\neg (s[i] < 0) \wedge S2 \Big) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left(\Big(\Big) \vee \Big(\Big) \right) \\
&\equiv ()
\end{aligned}
$$

f) **S: if** $(s[i] > 0)$ **then** $s[i] := -s[i]$ **else** *skip* **endif**
$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \mathrm{def}(s[i] > 0) \wedge_L \left(\Big((s[i] > 0) \wedge S1 \Big) \vee \Big(\neg (s[i] > 0) \wedge S2 \Big) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left(\Big(\Big) \vee \Big(\Big) \right) \\
&\equiv ()
\end{aligned}
$$

Ejercicio 8. ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (in s: $seq\langle \mathbb{Z} \rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {
Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j])\}$
Post $\{a = \sum_{j=0}^{i} s[j]\}$
}

b) **proc problema2** (in s: $seq\langle \mathbb{Z} \rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {
Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i} s[j])\}$
Post $\{a = \sum_{j=1}^{i} s[j]\}$
}

c) **proc problema3** (in s: $seq\langle \mathbb{Z} \rangle$, in i: $\mathbb{Z}$, out res: **Bool**)
Pre $\{0 \leq i < |s| \wedge (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L s[j] \geq 0)\}$
Post $\{res = true \leftrightarrow (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L s[j] \geq 0)\}$

d) **proc problema4** (in s: $seq\langle \mathbb{Z} \rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {
Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$
Post $\{a = \sum_{j=0}^{i} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$
}

e) **proc problema5** (in s: $seq\langle \mathbb{Z} \rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {
Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=1}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$
Post $\{a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$
}

Respuestas

a)

b)

c)

d)

e)