



**Ejercicio 1.** ★ Calcular las siguientes expresiones, donde  $a, b$  son variables reales,  $i$  una variable entera y  $A$  es una secuencia de reales.

- a)  $\text{def}(a + 1)$ .
- b)  $\text{def}(a/b)$ .
- c)  $\text{def}(\sqrt{a/b})$ .
- d)  $\text{def}(A[i] + 1)$ .
- e)  $\text{def}(A[i + 2])$ .
- f)  $\text{def}(0 \leq i \leq |A|)$ .
- g)  $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0)$ .

## Respuestas

Supongo que  $\text{def}(x) \equiv \text{True}$ , para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

- a)  $\text{def}(a + 1) \equiv \text{def}(a) \wedge \text{def}(1) \equiv \text{True} \wedge \text{True} \equiv \text{True}$
- b)  $\text{def}(a/b) \equiv \text{def}(a) \wedge \text{def}(b) \wedge b \neq 0 \equiv b \neq 0$ .
- c)  $\text{def}(\sqrt{a/b}) \equiv b \neq 0 \wedge (a/b) \geq 0$ .
- d)  $\text{def}(A[i] + 1) \equiv 0 \leq i < |A|$
- e)  $\text{def}(A[i + 2]) \equiv 0 \leq i + 2 < |A|$
- f)  $\text{def}(0 \leq i \leq |A|) \equiv \text{True}$
- g)  $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0) \equiv i < |A|$

**Ejercicio 2.** Calcular las siguientes precondiciones más débiles, donde  $a, b$  son variables reales,  $i$  una variable entera y  $A$  es una secuencia de reales.

- a)  $wp(\mathbf{a} := \mathbf{a} + 1, a \geq 0)$ .
- b)  $wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \geq 0)$ .
- c)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \geq 0)$ .
- d)  $wp(\mathbf{a} := \mathbf{b} * \mathbf{b}, a \geq 0)$ .
- e)  $wp(\mathbf{b} := \mathbf{b} + 1, a \geq 0)$ .

## Respuestas

a)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{a}+1, a \geq 0) &\equiv \text{def}(a+1) \wedge_L (a \geq 0)_{a+1}^a \\ &\equiv \text{True} \wedge_L a+1 \geq 0 \\ &\equiv a \geq -1 \end{aligned}$$

b)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \geq 0) &\equiv \text{def}(a/b) \wedge_L \wedge_L (a \geq 0)_{a/b}^a \\ &\equiv \text{def}(a) \wedge_L \text{def}(b) \wedge_L b \neq 0 \wedge_L (a \geq 0)_{a/b}^a \\ &\equiv \text{True} \wedge_L \text{True} \wedge_L b \neq 0 \wedge_L a/b \geq 0 \\ &\equiv b \neq 0 \wedge_L a \geq 0 \end{aligned}$$

c)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \geq 0) &\equiv \text{def}(A[i]) \wedge_L (a \geq 0)_{A[i]}^a \\ &\equiv (\text{def}(A) \wedge_L \text{def}(i)) \wedge_L 0 \leq i < |A| \wedge_L A[i] \geq 0 \\ &\equiv 0 \leq i < |A| \wedge_L A[i] \geq 0 \end{aligned}$$

d)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{b}*\mathbf{b}, a \geq 0) &\equiv \text{def}(b*b) \wedge_L (a \geq 0)_{b*b}^a \\ &\equiv \text{True} \wedge_L b*b \geq 0 \\ &\equiv b*b \geq 0 \end{aligned}$$

e)

$$\begin{aligned} wp(\mathbf{b} := \mathbf{b}+1, a \geq 0) &\equiv \text{def}(b+1) \wedge_L (a \geq 0)_a^a \\ &\equiv \text{True} \wedge_L a \geq 0 \\ &\equiv a \geq 0 \end{aligned}$$

**Ejercicio 3.** ★ Calcular las siguientes precondiciones más débiles, donde  $a, b$  son variables reales,  $i$  una variable entera y  $A$  es una secuencia de reales.

a)  $wp(\mathbf{a} := \mathbf{a}+1; \mathbf{b} := \mathbf{a}/2, b \geq 0).$

b)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a}*\mathbf{a}, b \neq 2).$

c)  $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{a} := \mathbf{b}*\mathbf{b}, a \geq 0).$

d)  $wp(\mathbf{a} := \mathbf{a}-\mathbf{b}; \mathbf{b} := \mathbf{a}+\mathbf{b}, a \geq 0 \wedge b \geq 0).$

## Respuestas

a)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{a}+1; \mathbf{b} := \mathbf{a}/2, b \geq 0)\} &\equiv wp(a := a+1, wp(b := a/2, b \geq 0)) \\ &\equiv wp(a := a+1, \text{def}(a/2) \wedge_L (b \geq 0)_{a/2}^b) \\ &\equiv wp(a := a+1, \text{True} \wedge_L (a/2 \geq 0)) \\ &\equiv \text{def}(a+1) \wedge_L (a/2 \geq 0)_{a+1}^a \\ &\equiv (a+1)/2 \geq 0 \end{aligned}$$

b)

$$\begin{aligned}
\{wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a} * \mathbf{a}, b \neq 2)\} &\equiv wp(a := A[i] + 1, wp(b := a * a, b \neq 2)) \\
&\equiv wp(a := A[i] + 1, def(a * a) \wedge_L (b \neq 2)_{a * a}^b) \\
&\equiv wp(a := A[i] + 1, True \wedge_L (a * a \neq 2)) \\
&\equiv def(A[i] + 1) \wedge_L (a * a \neq 2)_{A[i] + 1}^a \\
&\equiv 0 \leq i < |A| \wedge_L (A[i] + 1)^2 \neq 2
\end{aligned}$$

c)

$$\begin{aligned}
\{wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{a} := \mathbf{b} * \mathbf{b}, a \geq 0)\} &\equiv wp(a := A[i] + 1, wp(a := b * b, a \geq 0)) \\
&\equiv wp(a := A[i] + 1, def(b * b) \wedge_L (a \geq 0)_{a * b}^b) \\
&\equiv wp(a := A[i] + 1, True \wedge_L (b * b \geq 0)) \\
&\equiv def(A[i] + 1) \wedge_L (b * b \geq 0)_{A[i] + 1}^a \\
&\equiv 0 \leq i < |A| \wedge_L b * b \geq 0
\end{aligned}$$

d)

$$\begin{aligned}
\{wp(\mathbf{a} := \mathbf{a} - \mathbf{b}; \mathbf{b} := \mathbf{a} + \mathbf{b}, a \geq 0 \wedge b \geq 0)\} &\equiv wp(a := a - b, wp(b := a + b, a \geq 0 \wedge b \geq 0)) \\
&\equiv wp(a := a - b, def(a + b) \wedge_L (a \geq 0 \wedge b \geq 0)_{a + b}^b) \\
&\equiv wp(a := a - b, True \wedge_L (a \geq 0 \wedge a + b \geq 0)) \\
&\equiv def(a - b) \wedge_L (a \geq 0 \wedge a + b \geq 0)_{a - b}^a \\
&\equiv a - b \geq 0 \wedge a - b + b \geq 0 \\
&\equiv a \geq b \wedge a \geq 0
\end{aligned}$$

**Ejercicio 4.** ★ Sea  $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$ . Calcular las siguientes precondiciones más débiles, donde  $i$  es una variable entera y  $A$  es una secuencia de reales.

- a)  $wp(\mathbf{A}[\mathbf{i}] := 0, Q)$ .
- b)  $wp(\mathbf{A}[\mathbf{i} + 2] := 0, Q)$ .
- c)  $wp(\mathbf{A}[\mathbf{i} + 2] := -1, Q)$ .
- d)  $wp(\mathbf{A}[\mathbf{i}] := 2 * \mathbf{A}[\mathbf{i}], Q)$ .
- e)  $wp(\mathbf{A}[\mathbf{i}] := \mathbf{A}[\mathbf{i} - 1], Q)$ .

## Respuestas

a)

$$\begin{aligned}
wp(\mathbf{A}[\mathbf{i}] := 0, Q) &\equiv wp(A := \text{setAt}(A, i, 0), Q) \\
&\equiv \text{def}(A := \text{setAt}(A, i, 0)) \wedge_L Q_{\text{setAt}(A, i, 0)}^A \\
&\equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i < |A| \right) \wedge_L Q_{\text{setAt}(A, i, 0)}^A \\
&\equiv 0 \leq i < |\text{setAt}(A, i, 0)| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i, 0)| \rightarrow_L (\text{setAt}(A, i, 0) \geq 0) \right) \\
&\equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \rightarrow_L ((i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 0 \geq 0)) \right)
\end{aligned}$$

b)

$$\begin{aligned}
& wp(A[i+2] := 0, Q) \\
& \equiv wp(A := \text{setAt}(A, i+2, 0), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i+2, 0)) \wedge_L Q_{\text{setAt}(A, i+2, 0)}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i+2 < |A| \right) \wedge_L Q_{\text{setAt}(A, i+2, 0)}^A \\
& \equiv 0 \leq i+2 < |\text{setAt}(A, i+2, 0)| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i+2, 0)| \right) \rightarrow_L \left( \text{setAt}(A, i+2, 0) \geq 0 \right) \\
& \equiv 0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i+2 \neq j \wedge A[j] \geq 0) \vee (i+2 = j \wedge 0 \geq 0) \right)
\end{aligned}$$

c)

$$\begin{aligned}
& wp(A[i+2] := -1, Q) \\
& \stackrel{Ax,1}{\equiv} wp(A := \text{setAt}(A, i+2, -1), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i+2, -1)) \wedge_L Q_{\text{setAt}(A, i+2, -1)}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i+2 < |A| \right) \wedge_L Q_{\text{setAt}(A, i+2, -1)}^A \\
& \equiv 0 \leq i+2 < |\text{setAt}(A, i+2, -1)| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i+2, -1)| \right) \rightarrow_L \left( \text{setAt}(A, i+2, -1) \geq 0 \right) \\
& \equiv 0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i+2 \neq j \wedge A[j] \geq 0) \vee (i+2 = j \wedge -1 \geq 0) \right) \\
& \equiv \text{False}
\end{aligned}$$

d)

$$\begin{aligned}
& wp(A[i] := 2 * A[i], Q) \\
& \equiv wp(A := \text{setAt}(A, i, 2 * A[i]), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i, 2 * A[i])) \wedge_L Q_{\text{setAt}(A, i, 2 * A[i])}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i < |A| \right) \wedge_L Q_{\text{setAt}(A, i, 2 * A[i])}^A \\
& \equiv 0 \leq i < |\text{setAt}(A, i, 2 * A[i])| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i, 2 * A[i])| \right) \rightarrow_L \left( \text{setAt}(A, i, 2 * A[i]) \geq 0 \right) \\
& \equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 2 * A[i] \geq 0) \right)
\end{aligned}$$

e)

$$\begin{aligned}
& wp(A[i] := A[i-1], Q) \\
& \equiv wp(A := \text{setAt}(A, i, A[i-1]), Q) \\
& \equiv \text{def}(A := \text{setAt}(A, i, A[i-1])) \wedge_L Q_{\text{setAt}(A, i, A[i-1])}^A \\
& \equiv \left( (\text{def}(A) \wedge \text{def}(i)) \wedge_L 0 \leq i-1 < |A| \right) \wedge_L Q_{\text{setAt}(A, i, A[i-1])}^A \\
& \equiv 0 \leq i-1 < |\text{setAt}(A, i, A[i-1])| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |\text{setAt}(A, i, A[i-1])| \right) \rightarrow_L \left( \text{setAt}(A, i, 0) \geq 0 \right) \\
& \equiv 0 \leq i-1 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i-1 \neq j \wedge A[j] \geq 0) \vee (i-1 = j \wedge 0 \geq 0) \right)
\end{aligned}$$

**Ejercicio 5.** Calcular  $wp(S, Q)$ , para los siguientes pares de programas  $S$  y postcondiciones  $Q$ .

$$\begin{aligned}
\text{a) } S & \equiv i := i + 1 \\
Q & \equiv (\forall j : \mathbb{Z}) (0 \leq j < |A| \rightarrow_L A[j] \neq 0)
\end{aligned}$$

- b)  $S \equiv A[0] := 4$   
 $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- c)  $S \equiv A[2] := 4$   
 $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- d)  $S \equiv A[i] := A[i+1] - 1$   
 $Q \equiv (\forall j : \mathbb{Z})(0 < j < |A| \rightarrow_L A[j] \geq A[j-1])$
- e)  $S \equiv A[i] := A[i+1] - 1$   
 $Q \equiv (\forall j : \mathbb{Z})(0 < j < |A| \rightarrow_L A[j] \leq A[j-1])$

## Respuestas

a)

$$\begin{aligned} wp(S, Q) &\equiv wp(i := i + 1, Q_{i+1}^i) \\ &\equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \neq 0) \end{aligned}$$

b)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[0] := 4, Q_{setAt(A,0,4)}^A) \\ &\equiv 0 \leq 0 < |setAt(A, 0, 4)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, 0, 4)| \rightarrow_L setAt(A, 0, 4)[j] \neq 0) \\ &\equiv 0 \leq 0 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (0 \neq j \wedge A[j] \neq 0) \vee (0 = j \wedge 4 \neq 0) \right) \end{aligned}$$

c)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[2] := 4, Q_{setAt(A,2,4)}^A) \\ &\equiv 0 \leq 2 < |setAt(A, 2, 4)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, 2, 4)| \rightarrow_L setAt(A, 2, 4)[j] \neq 0) \\ &\equiv 0 \leq 2 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (2 \neq j \wedge A[j] \neq 0) \vee (2 = j \wedge 4 \neq 0) \right) \end{aligned}$$

d)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[i] := A[i+1] - 1, Q_{setAt(A,i,A[i+1]-1)}^A) \\ &\equiv 0 \leq i+1 < |setAt(A, i, A[i+1] - 1)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, i, A[i+1] - 1)| \\ &\rightarrow_L setAt(A, i, A[i+1] - 1)[j] \neq 0) \\ &\equiv 0 \leq i+1 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i \neq j \wedge A[j] \geq A[j-1]) \vee (i = j \wedge A[i+1] - 1 \geq A[i-1]) \right) \end{aligned}$$

e)

$$\begin{aligned} wp(S, Q) &\equiv wp(A[i] := A[i+1] - 1, Q_{setAt(A,i,A[i+1]-1)}^A) \\ &\equiv 0 \leq i+1 < |setAt(A, i, A[i+1] - 1)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A, i, A[i+1] - 1)| \\ &\rightarrow_L setAt(A, i, A[i+1] - 1)[j] \neq 0) \\ &\equiv 0 \leq i+1 < |A| \wedge_L (\forall j : \mathbb{Z}) \left( 0 \leq j < |A| \right) \rightarrow_L \left( (i \neq j \wedge A[j] \leq A[j-1]) \vee (i = j \wedge A[i+1] - 1 \leq A[i-1]) \right) \end{aligned}$$

**Ejercicio 6.** . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

- a) **proc problema1** (inout a:  $\mathbb{Z}$ )  
**Pre**  $\{a = a_0 \wedge a \geq 0\}$   
**Post**  $\{a = a_0 + 2\}$
- b) **proc problema2** (in a:  $\mathbb{Z}$ , out b:  $\mathbb{Z}$ )  
**Pre**  $\{a \neq 0\}$   
**Post**  $\{b = a + 3\}$
- c) **proc problema3** (in a:  $\mathbb{Z}$ , in b:  $\mathbb{Z}$ , out c:  $\mathbb{Z}$ )  
**Pre**  $\{true\}$   
**Post**  $\{c = a + b\}$
- d) **proc problema4** (in a:  $seq\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , out result:  $\mathbb{Z}$ )  
**Pre**  $\{0 \leq i < |a|\}$   
**Post**  $\{result = 2 * a[i]\}$
- e) **proc problema5** (in a:  $seq\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , out result:  $\mathbb{Z}$ )  
**Pre**  $\{0 \leq i \wedge i + 1 < |a|\}$   
**Post**  $\{result = a[i] + a[i + 1]\}$

## Respuestas

Para probar la correctitud de la tripla **{Pre} S {Post}** alcanza probar que

$$Pre \rightarrow wp(\mathbf{S}, Post)$$

a)

$$\{wp(S, Post)\} \equiv \{a_0 + 2 = a_0 + 2\}$$

$$\mathbf{S}: a := a_0 + 2$$

$$\{\mathbf{Post}: a = a_0 + 2\}$$

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{a_0 + 2 = a_0 + 2\}$$

$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{True\}$$

$$True$$

b) 1. Calculamos  $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + 3 = a + 3\}$$

$$\equiv True$$

$$\mathbf{S}: b := a + 3$$

$$\{\mathbf{Post}: b = a + 3\}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\{a \neq 0\} \rightarrow \{True\}$$

$$True$$

c) 1. Calculamos  $\{wp(S, Post)\}$

$$\begin{aligned}\{wp(S, Post)\} &\equiv \{a + b = a + b\} \\ &\equiv True \\ \mathbf{S}: c &:= a + b \\ \{\mathbf{Post}: c &= a + b\}\end{aligned}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$\begin{aligned}Pre &\rightarrow \{wp(S, Post)\} \\ \{True\} &\rightarrow \{True\} \\ True\end{aligned}$$

d) 1. Calculamos  $\{wp(S, Post)\}$

$$\begin{aligned}\{wp(S, Post)\} &\equiv \{\text{def}(2 * a[i]) \wedge_L 2 * a[i] = 2 * a[i]\} \\ &\equiv \text{def}(a[i]) \wedge_L 0 \leq i < |a| \wedge True \\ &\equiv True \wedge_L 0 \leq i < |a| \\ &\equiv 0 \leq i < |a| \\ \mathbf{S}: result &:= 2 * a[i] \\ \{\mathbf{Post}: result &= 2 * a[i]\}\end{aligned}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$\begin{aligned}Pre &\rightarrow \{wp(S, Post)\} \\ \{0 \leq i < |a|\} &\rightarrow \{0 \leq i < |a|\} \\ True\end{aligned}$$

e) 1. Calculamos  $\{wp(S, Post)\}$

$$\begin{aligned}\{wp(S, Post)\} &\equiv \{\text{def}(a[i] + a[i + 1]) \wedge_L a[i] + a[i + 1] = a[i] + a[i + 1]\} \\ &\equiv \text{def}(a[i]) \wedge_L \text{def}(a[i + 1]) \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge a[i] + a[i + 1] = a[i] + a[i + 1] \\ &\equiv True \wedge_L True \wedge_L True \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge True \\ &\equiv 0 \leq i \wedge i + 1 < |a| \\ \mathbf{S}: result &:= a[i] + a[i + 1] \\ \{\mathbf{Post}: result &= a[i] + a[i + 1]\}\end{aligned}$$

2. Chequeamos  $Pre \rightarrow \{wp(S, Post)\}$

$$\begin{aligned}Pre &\rightarrow \{wp(S, Post)\} \\ \{0 \leq i \wedge i + 1 < |a|\} &\rightarrow \{0 \leq i \wedge i + 1 < |a|\} \\ True\end{aligned}$$

**Ejercicio 7.** ★ Calcular  $wp(S, Q)$ , para los siguientes pares de programas S y postcondiciones Q.

a)  $S \equiv$   
 if ( $a < 0$ )  
    $b := a$   
 else  
    $b := -a$   
 endif

$Q \equiv (b = -|a|)$

b)  $S \equiv$   
**if**  $(a < 0)$   
 $\quad b := a$   
**else**  
 $\quad b := -a$   
**endif**

$$Q \equiv (b = |a|)$$

c)  $S \equiv$   
**if**  $(i > 0)$   
 $\quad s[i] := 0$   
**else**  
 $\quad s[0] := 0$   
**endif**

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

d)  $S \equiv$   
**if**  $(i > 1)$   
 $\quad s[i] := s[i - 1]$   
**else**  
 $\quad s[i] := 0$   
**endif**

$$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j - 1])$$

e)  $S \equiv$   
**if**  $(s[i] < 0)$   
 $\quad s[i] := -s[i]$   
**else**  
 $\quad skip$   
**endif**

$$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$$

f)  $S \equiv$   
**if**  $(s[i] > 0)$   
 $\quad s[i] := -s[i]$   
**else**  
 $\quad skip$   
**endif**

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

## Respuestas

**Axioma 4.** Si  $S = \text{if } B \text{ then } S1 \text{ else } S2 \text{ endif}$ , entonces

$$wp(S, Q) \equiv \text{def}(B) \wedge_L ((B \wedge wp(S1, Q)) \vee (\neg B \wedge wp(S2, Q)))$$

a) **S: if**  $(a < 0)$  **then**  $b := a$  **else**  $b := -a$  **endif**  
 $\quad Q \equiv (b = -|a|)$



$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(a < 0) \wedge_L \left( \left( (a < 0) \wedge (a = -|a|) \right) \vee \left( \neg(a < 0) \wedge (-a = -|a|) \right) \right) \\
&\equiv \text{True} \wedge_L \left( \left( (a < 0) \wedge (a = a) \right) \vee \left( (a \geq 0) \wedge (-a = -a) \right) \right) \\
&\equiv \left( \left( (a < 0) \wedge \text{True} \right) \vee \left( (a \geq 0) \wedge \text{True} \right) \right) \\
&\equiv (a < 0) \vee (a \geq 0) \\
&\equiv \text{True}
\end{aligned}$$

b) **S:** if  $(a < 0)$  then  $b := a$  else  $b := -a$  endif  
 $Q \equiv (b = |a|)$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(a < 0) \wedge_L \left( \left( (a < 0) \wedge (a = |a|) \right) \vee \left( \neg(a < 0) \wedge (-a = |a|) \right) \right) \\
&\equiv \text{True} \wedge_L \left( \left( (a < 0) \wedge (a = -a) \right) \vee \left( (a \geq 0) \wedge (a = -a) \right) \right) \\
&\equiv \left( \left( (a < 0) \wedge \text{False} \right) \vee \left( (a \geq 0) \wedge \text{False} \right) \right) \\
&\equiv \text{False} \vee \text{False} \\
&\equiv \text{False}
\end{aligned}$$

c) **S:** if  $(i > 0)$  then  $s[i] := 0$  else  $s[0] := 0$  endif  
 $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(i > 0) \wedge_L \left( \left( (i > 0) \wedge wp(s[i] := 0, Q) \right) \vee \left( \neg(i > 0) \wedge wp(s[0] := 0, Q) \right) \right) \\
&\equiv \text{True} \wedge \left( \left( (i > 0) \wedge wp(\text{setAt}(s, i, 0), Q) \right) \vee \left( i \leq 0 \wedge wp(\text{setAt}(s, 0, 0), Q) \right) \right) \\
&\equiv \left( (i > 0) \wedge wp(\text{setAt}(s, i, 0), Q) \right) \vee \left( i \leq 0 \wedge wp(\text{setAt}(s, 0, 0), Q) \right) \\
&\equiv \left( (i > 0) \wedge wp(\text{setAt}(s, i, 0), Q) \right) \vee \text{True} \\
&\equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 < j < |A|) \rightarrow_L \left( (i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 0 \geq 0) \right)
\end{aligned}$$

d) **S:** if  $(i > 1)$  then  $s[i] := s[i-1]$  else  $s[i] := 0$  endif  
 $Q \equiv (\forall j : \mathbb{Z})(1 \leq j < |s| \rightarrow_L s[j] = s[j-1])$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(i > 1) \wedge_L \left( \left( (i > 1) \wedge S1 \right) \vee \left( \neg(i > 1) \wedge S2 \right) \right) \\
&\equiv \text{True} \wedge_L \left( \left( \right) \vee \left( \right) \right) \\
&\equiv ()
\end{aligned}$$

e) **S:** if  $(s[i] < 0)$  then  $s[i] := -s[i]$  else skip endif  
 $Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(s[i] < 0) \wedge_L \left( \left( (s[i] < 0) \wedge S1 \right) \vee \left( \neg(s[i] < 0) \wedge \text{True} \right) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left( () \vee (s[i] \geq 0) \right) \\
&\equiv ()
\end{aligned}$$

f) **S:** **if**  $(s[i] > 0)$  **then**  $s[i] := -s[i]$  **else** *skip* **endif**  
 $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(s[i] > 0) \wedge_L \left( \left( (s[i] > 0) \wedge S1 \right) \vee \left( \neg(s[i] > 0) \wedge \text{True} \right) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left( () \vee (s[i] \leq 0) \right) \\
&\equiv ()
\end{aligned}$$

**Ejercicio 8.** ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

- a) **proc problema1** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j]\}$   
**Post**  $\{a = \sum_{j=0}^i s[j]\}$   
}
- b) **proc problema2** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^i s[j]\}$   
**Post**  $\{a = \sum_{j=1}^i s[j]\}$   
}
- c) **proc problema3** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , out res: **Bool**)  
**Pre**  $\{0 \leq i < |s| \wedge (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L s[j] \geq 0)\}$   
**Post**  $\{res = \text{true} \leftrightarrow (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L s[j] \geq 0)\}$
- d) **proc problema4** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
**Post**  $\{a = \sum_{j=0}^i (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
}
- e) **proc problema5** (in s:  $\text{seq}\langle\mathbb{Z}\rangle$ , in i:  $\mathbb{Z}$ , inout a:  $\mathbb{Z}$ ) {  
**Pre**  $\{0 \leq i < |s| \wedge_L a = \sum_{j=1}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
**Post**  $\{a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$   
}

## Respuestas

- a)
- b)
- c)
- d)
- e)

FIN.

## Violeta De Otoño

♩ = 66

The musical score is written for a single melodic line on a treble clef staff in 3/4 time. It consists of three systems of music. The first system (measures 1-8) begins with a treble clef and a tempo marking of ♩ = 66. The melody is composed of eighth and quarter notes, with some measures containing triplets. The second system (measures 9-17) continues the melody, featuring a repeat sign at the beginning of measure 9. The third system (measures 18-20) concludes the piece with a final cadence. The score includes various musical notations such as stems, beams, and rests.

8

9

18