Algoritmos y Estructuras de Datos I
Primer Cuatrimestre 2020

Guía Práctica 4
Precondición más débil en SmallLang

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Ejercicio 1. ★ Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $\text{def}(a + 1)$.

b) $\text{def}(a/b)$.

c) $\text{def}(\sqrt{a/b})$.

d) $\text{def}(A[i] + 1)$.

e) $\text{def}(A[i + 2])$.

f) $\text{def}(0 \leq i \leq |A|)$.

g) $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0)$.

Respuestas

Supongo que $\text{def}(x) \equiv True$,para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

a) $\text{def}(a + 1) \equiv \text{def}(a) \wedge \text{def}(1) \equiv True \wedge True \equiv True$

b) $\text{def}(a/b) \equiv \text{def}(a) \wedge \text{def}(b) \wedge b \neq 0 \equiv b \neq 0$.

c) $\text{def}(\sqrt{a/b}) \equiv b \neq 0 \wedge (a/b) \geq 0$.

d) $\text{def}(A[i] + 1) \equiv 0 \leq i < |A|$

e) $\text{def}(A[i + 2]) \equiv 0 \leq i + 2 < |A|$

f) $\text{def}(0 \leq i \leq |A|) \equiv True$

g) $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0) \equiv i < |A|$

Ejercicio 2. Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp(\textbf{a := a+1}, a \geq 0)$.

b) $wp(\textbf{a := a/b}, a \geq 0)$.

c) $wp(\textbf{a := A[i]}, a \geq 0)$.

d) $wp(\textbf{a := b*b}, a \geq 0)$.

e) $wp(\textbf{b := b+1}, a \geq 0)$.

Respuestas

a)

$$\begin{aligned} wp(\textbf{a := a+1}, a \geq 0) &\equiv \text{def}(a+1) \wedge_L (a \geq 0)^a_{a+1} \\ &\equiv True \wedge_L a+1 \geq 0 \\ &\equiv a \geq -1 \end{aligned}$$

b)

$$\begin{aligned} wp(\textbf{a := a/b}, a \geq 0) &\equiv \text{def}(a/b) \wedge_L \wedge_L (a \geq 0)^a_{a/b} \\ &\equiv \text{def}(a) \wedge_L \text{def}(b) \wedge_L b \neq 0 \wedge_L (a \geq 0)^a_{a/b} \\ &\equiv True \wedge_L True \wedge_L b \neq 0 \wedge_L a/b \geq 0 \\ &\equiv b \neq 0 \wedge_L a \geq 0 \end{aligned}$$

c)

$$\begin{aligned} wp(\textbf{a := A[i]}, a \geq 0) &\equiv \text{def}(A[i]) \wedge_L (a \geq 0)^a_{A[i]} \\ &\equiv (\text{def}(A) \wedge_L \text{def}(i)) \wedge_L 0 \leq i < |A| \wedge_L A[i] \geq 0 \\ &\equiv 0 \leq i < |A| \wedge_L A[i] \geq 0 \end{aligned}$$

d)

$$\begin{aligned} wp(\textbf{a := b*b}, a \geq 0) &\equiv \text{def}(b*b) \wedge_L (a \geq 0)^a_{b*b} \\ &\equiv True \wedge_L b*b \geq 0 \\ &\equiv b*b \geq 0 \end{aligned}$$

e)

$$\begin{aligned} wp(\textbf{b := b+1}, a \geq 0) &\equiv \text{def}(b+1) \wedge_L (a \geq 0)^a_a \\ &\equiv True \wedge_L a \geq 0 \\ &\equiv a \geq 0 \end{aligned}$$

Ejercicio 3. ★ Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp(\textbf{a := a+1; b := a/2}, b \geq 0)$.

b) $wp(\textbf{a := A[i] + 1; b := a*a}, b \neq 2)$.

c) $wp(\textbf{a := A[i] + 1; a := b*b}, a \geq 0)$.

d) $wp(\textbf{a := a-b; b := a+b}, a \geq 0 \wedge b \geq 0)$.

Respuestas

a)

$$\begin{aligned} \{wp(\textbf{a := a+1; b := a/2}, b \geq 0)\} &\equiv wp(a := a+1, wp(b := a/2, b \geq 0)) \\ &\equiv wp(a := a+1, def(a/2) \wedge_L (b \geq 0)^b_{a/2}) \\ &\equiv wp(a := a+1, True \wedge_L (a/2 \geq 0)) \\ &\equiv def(a+1) \wedge_L (a/2 \geq 0)^a_{a+1} \\ &\equiv (a+1)/2 \geq 0 \end{aligned}$$

b)

$$\begin{aligned}\{wp(\textbf{a := A[i] + 1; b := a*a}, b \neq 2)\} &\equiv wp(a := A[i]+1, wp(b := a*a, b \neq 2))\\ &\equiv wp(a := A[i]+1, def(a*a) \wedge_L (b \neq 2)^b_{a*a})\\ &\equiv wp(a := A[i]+1, True \wedge_L (a*a \neq 2)\\ &\equiv def(A[i]+1) \wedge_L (a*a \neq 2)^a_{A[i]+1}\\ &\equiv 0 \leq i < |A| \wedge_L (A[i]+1)^2 \neq 2\end{aligned}$$

c)

$$\begin{aligned}\{wp(\textbf{a := A[i] + 1; a := b*b}, a \geq 0)\} &\equiv wp(a := A[i]+1, wp(a := b*b, a \geq 0))\\ &\equiv wp(a := A[i]+1, def(b*b) \wedge_L (a \geq 0)^b_{a*a})\\ &\equiv wp(a := A[i]+1, True \wedge_L (b*b \geq 0)\\ &\equiv def(A[i]+1) \wedge_L (b*b \geq 0)^a_{A[i]+1}\\ &\equiv 0 \leq i < |A| \wedge_L b*b \geq 0\end{aligned}$$

d)

$$\begin{aligned}\{wp(\textbf{a := a-b; b := a+b}, a \geq 0 \wedge b \geq 0)\} &\equiv wp(a := a-b, wp(b := a+b, a \geq 0 \wedge b \geq 0))\\ &\equiv wp(a := a-b, def(a+b) \wedge_L (a \geq 0 \wedge b \geq 0)^b_{a+b})\\ &\equiv wp(a := a-b, True \wedge_L (a \geq 0 \wedge a+b \geq 0)\\ &\equiv def(a-b) \wedge_L (a \geq 0 \wedge a+b \geq 0)^a_{a-b}\\ &\equiv a-b \geq 0 \wedge a-b+b \geq 0\\ &\equiv a \geq b \wedge a \geq 0\end{aligned}$$

Ejercicio 4. ★ Sea $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$. Calcular las siguientes precondiciones más débiles, donde i es una variable entera y A es una secuencia de reales.

a) $wp(\textbf{A[i] := 0}, Q)$.

b) $wp(\textbf{A[i+2] := 0}, Q)$.

c) $wp(\textbf{A[i+2] := -1}, Q)$.

d) $wp(\textbf{A[i] := 2 * A[i]}, Q)$.

e) $wp(\textbf{A[i] := A[i-1]}, Q)$.

Respuestas

a)

$$\begin{aligned}&wp(\text{A[i]} := 0\ , Q)\\ &\quad\equiv wp(\text{A} := \text{setAt(A,i,0)}\ , Q)\\ &\quad\equiv \text{def(A := setAt(A,i,0))}\ \wedge_L Q^A_{setAt(A,i,0)}\\ &\quad\equiv \Big((\text{def(A)} \wedge \text{def(i)}) \wedge_L 0 \leq i < |A|\Big) \wedge_L Q^A_{setAt(A,i,0)}\\ &\quad\equiv 0 \leq i < |setAt(A,i,0)| \wedge_L (\forall j : \mathbb{Z})\Big(0 \leq j < |setAt(A,i,0)|\Big) \rightarrow_L \Big(setAt(A,i,0) \geq 0\Big)\\ &\quad\equiv 0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 0 \geq 0)\Big)\end{aligned}$$

b)

$$
\begin{aligned}
&wp(\text{A[i+2] := 0 }, Q)\\
&\quad \equiv wp(\text{A := setAt(A,i+2,0) }, Q)\\
&\quad \equiv \text{def(A := setAt(A,i+2,0))} \ \wedge_L Q^A_{setAt(A,i+2,0)}\\
&\quad \equiv \Big((\text{def(A)} \wedge \text{def(i)}) \wedge_L 0 \leq i+2 < |A|\Big) \wedge_L Q^A_{setAt(A,i+2,0)}\\
&\quad \equiv 0 \leq i+2 < |setAt(A,i+2,0)| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |setAt(A,i+2,0)|\Big) \rightarrow_L \Big(setAt(A,i+2,0) \geq 0\Big)\\
&\quad \equiv 0 \leq i+2 < |A| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i+2 \neq j \wedge A[j] \geq 0) \vee (i+2 = j \wedge 0 \geq 0)\Big)
\end{aligned}
$$

c)

$$
\begin{aligned}
&wp(\text{A[i+2] := -1 }, Q)\\
&\quad \overset{Ax,1}{\equiv} wp(\text{A := setAt(A,i+2,-1) }, Q)\\
&\quad \equiv \text{def(A := setAt(A,i+2,0))} \ \wedge_L Q^A_{setAt(A,i+2,-1)}\\
&\quad \equiv \Big((\text{def(A)} \wedge \text{def(i)}) \wedge_L 0 \leq i+2 < |A|\Big) \wedge_L Q^A_{setAt(A,i+2,0)}\\
&\quad \equiv 0 \leq i+2 < |setAt(A,i+2,-1)| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |setAt(A,i+2,-1)|\Big) \rightarrow_L \Big(setAt(A,i+2,-1) \geq 0\Big)\\
&\quad \equiv 0 \leq i+2 < |A| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i+2 \neq j \wedge A[j] \geq 0) \vee (i+2 = j \wedge -1 \geq 0)\Big)\\
&\quad \equiv False
\end{aligned}
$$

d)

$$
\begin{aligned}
&wp(\text{A[i] := 2 * A[i] }, Q)\\
&\quad \equiv wp(\text{A := setAt(A,i,2 * A[i]) }, Q)\\
&\quad \equiv \text{def(A := setAt(A,i,2 * A[i]))} \ \wedge_L Q^A_{setAt(A,i,2*A[i])}\\
&\quad \equiv \Big((\text{def(A)} \wedge \text{def(i)}) \wedge_L 0 \leq i < |A|\Big) \wedge_L Q^A_{setAt(A,i,2*A[i])}\\
&\quad \equiv 0 \leq i < |setAt(A,i,2*A[i])| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |setAt(A,i,2*A[i])|\Big) \rightarrow_L \Big(setAt(A,i,2*A[i]) \geq 0\Big)\\
&\quad \equiv 0 \leq i < |A| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i \neq j \wedge A[j] \geq 0) \vee (i = j \wedge 2*A[i] \geq 0)\Big)
\end{aligned}
$$

e)

$$
\begin{aligned}
&wp(\text{A[i] := A[i-1] }, Q)\\
&\quad \equiv wp(\text{A := setAt(A,i,A[i-1]) }, Q)\\
&\quad \equiv \text{def(A := setAt(A,i,A[i-1]))} \ \wedge_L Q^A_{setAt(A,i,A[i-1])}\\
&\quad \equiv \Big((\text{def(A)} \wedge \text{def(i)}) \wedge_L 0 \leq i-1 < |A|\Big) \wedge_L Q^A_{setAt(A,i,A[i-1])}\\
&\quad \equiv 0 \leq i-1 < |setAt(A,i,A[i-1])| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |setAt(A,i,A[i-1])|\Big) \rightarrow_L \Big(setAt(A,i,0) \geq 0\Big)\\
&\quad \equiv 0 \leq i-1 < |A| \wedge_L (\forall j:\mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i-1 \neq j \wedge A[j] \geq 0) \vee (i-1 = j \wedge 0 \geq 0)\Big)
\end{aligned}
$$

Ejercicio 5. Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) $S \equiv i := i+1$
   $Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

b) $S \equiv A[0] := 4$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

c) $S \equiv A[2] := 4$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

d) $S \equiv A[i] := A[i+1] - 1$
$Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j-1])$

e) $S \equiv A[i] := A[i+1] - 1$
$Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j-1])$

Respuestas

a)

$$\begin{aligned} wp(S,Q) &\equiv wp(i := i+1, Q^i_{i+1}) \\ &\equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0) \end{aligned}$$

b)

$$\begin{aligned} wp(S,Q) &\equiv wp(A[0] := 4, Q^A_{setAt(A,0,4)}) \\ &\equiv 0 \leq 0 < |setAt(A,0,4)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A,0,4)| \rightarrow_L setAt(A,0,4)[j] \neq 0) \\ &\equiv 0 \leq 0 < |A| \wedge_L (\forall j : \mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((0 \neq j \wedge A[j] \neq 0) \vee (0 = j \wedge 4 \neq 0)\Big) \end{aligned}$$

c)

$$\begin{aligned} wp(S,Q) &\equiv wp(A[2] := 4, Q^A_{setAt(A,2,4)}) \\ &\equiv 0 \leq 2 < |setAt(A,2,4)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A,2,4)| \rightarrow_L setAt(A,2,4)[j] \neq 0) \\ &\equiv 0 \leq 2 < |A| \wedge_L (\forall j : \mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((2 \neq j \wedge A[j] \neq 0) \vee (2 = j \wedge 4 \neq 0)\Big) \end{aligned}$$

d)

$$\begin{aligned} wp(S,Q) &\equiv wp(A[i] := A[i+1] - 1, Q^A_{setAt(A,i,A[i+1]-1)}) \\ &\equiv 0 \leq i+1 < |setAt(A,i,A[i+1]-1)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A,i,A[i+1]-1)| \\ &\rightarrow_L setAt(A,i,A[i+1]-1)[j] \neq 0) \\ &\equiv 0 \leq i+1 < |A| \wedge_L (\forall j : \mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i \neq j \wedge A[j] \geq A[j-1]) \vee (i = j \wedge A[i+1] - 1 \geq A[i-1])\Big) \end{aligned}$$

e)

$$\begin{aligned} wp(S,Q) &\equiv wp(A[i] := A[i+1] - 1, Q^A_{setAt(A,i,A[i+1]-1)}) \\ &\equiv 0 \leq i+1 < |setAt(A,i,A[i+1]-1)| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |setAt(A,i,A[i+1]-1)| \\ &\rightarrow_L setAt(A,i,A[i+1]-1)[j] \neq 0) \\ &\equiv 0 \leq i+1 < |A| \wedge_L (\forall j : \mathbb{Z})\Big(0 \leq j < |A|\Big) \rightarrow_L \Big((i \neq j \wedge A[j] \leq A[j-1]) \vee (i = j \wedge A[i+1] - 1 \leq A[i-1])\Big) \end{aligned}$$

Ejercicio 6. . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (inout a: $\mathbb{Z}$)
   Pre $\{a = a_0 \wedge a \geq 0\}$
   Post $\{a = a_0 + 2\}$

b) **proc problema2** (in a: $\mathbb{Z}$, out b: $\mathbb{Z}$)
   Pre $\{a \neq 0\}$
   Post $\{b = a + 3\}$

c) **proc problema3** (in a: $\mathbb{Z}$, in b: $\mathbb{Z}$, out c: $\mathbb{Z}$)
   Pre $\{true\}$
   Post $\{c = a + b\}$

d) **proc problema4** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
   Pre $\{0 \leq i < |a|\}$
   Post $\{result = 2 * a[i]\}$

e) **proc problema5** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
   Pre $\{0 \leq i \wedge i + 1 < |a|\}$
   Post $\{result = a[i] + a[i + 1]\}$

Respuestas

Para probar la correctitud de la tripla **{Pre} S {Post}** alcanza probar que

$$\text{Pre} \rightarrow wp(\mathbf{S}, Post)$$

a)

$$\{wp(S, Post)\} \equiv \{a_0 + 2 = a_0 + 2\}$$
$$\textbf{S: } a := a_0 + 2$$
$$\{\textbf{Post: } a = a_0 + 2\}$$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{a_0 + 2 = a_0 + 2\}$$
$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{True\}$$
$$True$$

b) 1. Calculamos $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + 3 = a + 3\}$$
$$\equiv True$$
$$\textbf{S: } b := a + 3$$
$$\{\textbf{Post: } b = a + 3\}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{a \neq 0\} \rightarrow \{True\}$$
$$True$$

c) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned} \{wp(S, Post)\} &\equiv \{a + b = a + b\} \\ &\equiv True \\ \textbf{S: } & c := a + b \\ \{\textbf{Post: } & c = a + b\} \end{aligned}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$\begin{gathered} Pre \to \{wp(S, Post)\} \\ \{True\} \to \{True\} \\ True \end{gathered}$$

d) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned} \{wp(S, Post)\} &\equiv \{\mathrm{def}(2 * a[i]) \wedge_L 2 * a[i] = 2 * a[i]\} \\ &\equiv \mathrm{def}(a[i]) \wedge_L 0 \leq i < |a| \wedge True \\ &\equiv True \wedge_L 0 \leq i < |a| \\ &\equiv 0 \leq i < |a| \\ \textbf{S: } & result := 2 * a[i] \\ \{\textbf{Post: } & result = 2 * a[i]\} \end{aligned}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$\begin{gathered} Pre \to \{wp(S, Post)\} \\ \{0 \leq i < |a|\} \to \{0 \leq i < |a|\} \\ True \end{gathered}$$

e) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned} \{wp(S, Post)\} &\equiv \{\mathrm{def}(a[i] + a[i+1]) \wedge_L a[i] + a[i+1] = a[i] + a[i+1]\} \\ &\equiv \mathrm{def}(a[i]) \wedge_L \mathrm{def}(a[i+1]) \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge a[i] + a[i+1] = a[i] + a[i+1] \\ &\equiv True \wedge_L True \wedge_L True \wedge_L 0 \leq i \wedge i + 1 < |a| \wedge True \\ &\equiv 0 \leq i \wedge i + 1 < |a| \\ \textbf{S: } & result := a[i] + a[i+1] \\ \{\textbf{Post: } & result = a[i] + a[i+1]\} \end{aligned}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$\begin{gathered} Pre \to \{wp(S, Post)\} \\ \{0 \leq i \wedge i + 1 < |a|\} \to \{0 \leq i \wedge i + 1 < |a|\} \\ True \end{gathered}$$

Ejercicio 7. ★ Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) $S \equiv$

if $(a < 0)$
    $b := a$
else
    $b := -a$
endif

$Q \equiv (b = -|a|)$

b) $S \equiv$
if $(a < 0)$
$b := a$
else
$b := -a$
endif

$Q \equiv (b = |a|)$

c) $S \equiv$
if $(i > 0)$
$s[i] := 0$
else
$s[0] := 0$
endif

$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

d) $S \equiv$
if $(i > 1)$
$s[i] := s[i-1]$
else
$s[i] := 0$
endif

$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j-1])$

e) $S \equiv$
if $(s[i] < 0)$
$s[i] := -s[i]$
else
skip
endif

$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

f) $S \equiv$
if $(s[i] > 0)$
$s[i] := -s[i]$
else
skip
endif

$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

Respuestas

Axioma 4. Si **S = if B then S1 else S2 endif**, entonces

$$wp(\mathbf{S}, Q) \equiv \text{def}(B) \wedge_L ((B \wedge wp(\mathbf{S1}, Q)) \vee (\neg B \wedge wp(\mathbf{S1}, Q)))$$

a) **S: if** $(a < 0)$ **then** $b := a$ **else** $b := -a$ **endif**
$Q \equiv (b = -|a|)$

$$
\begin{aligned}
wp(\mathbf{S},Q) &\equiv \mathrm{def}(a<0) \land_L \Big(\big((a<0)\land(a=-|a|)\big)\lor\big(\neg(a<0)\land(-a=-|a|)\big)\Big)\\
&\equiv True \land_L \Big(\big((a<0)\land(a=a)\big)\lor\big((a\geq 0)\land(-a=-a)\big)\Big)\\
&\equiv \Big(\big((a<0)\land True\big)\lor\big((a\geq 0)\land True\big)\Big)\\
&\equiv (a<0)\lor(a\geq 0)\\
&\equiv True
\end{aligned}
$$

b) **S: if** $(a<0)$ **then** $b:=a$ **else** $b:=-a$ **endif**
$Q\equiv(b=|a|)$

$$
\begin{aligned}
wp(\mathbf{S},Q) &\equiv \mathrm{def}(a<0) \land_L \Big(\big((a<0)\land(a=|a|)\big)\lor\big(\neg(a<0)\land(-a=|a|)\big)\Big)\\
&\equiv True \land_L \Big(\big((a<0)\land(a=-a)\big)\lor\big((a\geq 0)\land(a=-a)\big)\Big)\\
&\equiv \Big(\big((a<0)\land False\big)\lor\big((a\geq 0)\land False\big)\Big)\\
&\equiv False\lor False\\
&\equiv False
\end{aligned}
$$

c) **S: if** $(i>0)$ **then** $s[i]:=0$ **else** $s[0]:=0$ **endif**
$Q\equiv(\forall j:Z)(0\leq j<|s|\rightarrow_L s[j]\geq 0)$

$$
\begin{aligned}
wp(\mathbf{S},Q) &\equiv \mathrm{def}(i>0) \land_L \Big(\big((i>0)\land wp(s[i]:=0,Q))\big)\lor\big(\neg(i>0)\land wp(s[0]:=0,Q)\big)\Big)\\
&\equiv True \land \Big(\big((i>0)\land wp(setAt(s,i,0),Q))\big)\lor\big(i\leq 0\land wp(setAt(s,0,0),Q)\big)\Big)\\
&\equiv \big((i>0)\land wp(setAt(s,i,0),Q))\big)\lor\big(i\leq 0\land wp(setAt(s,0,0),Q)\big)\\
&\equiv \big((i>0)\land wp(setAt(s,i,0),Q))\big)\lor True\\
&\equiv 0\leq i<|A|\land_L(\forall j:\mathbb{Z})\big(0<j<|A|\big)\rightarrow_L\big((i\neq j\land A[j]\geq 0)\lor(i=j\land 0\geq 0)\big)
\end{aligned}
$$

d) **S: if** $(i>1)$ **then** $s[i]:=s[i-1]$ **else** $s[i]:=0$ **endif**
$Q\equiv(\forall j:Z)(1\leq j<|s|\rightarrow_L s[j]=s[j-1])$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(i > 1) \wedge_L \left(\Big((i > 1) \wedge wp(s[i] := s[i-1], Q) \Big) \vee \Big(\neg(i > 1) \wedge wp(s[i] := 0, Q)) \Big) \right) \\
&\equiv True \wedge_L \left(\Big((i > 1) \wedge wp(s[i] := s[i-1], Q) \Big) \vee \Big((i \leq 1) \wedge wp(s[i] := 0, Q)) \Big) \right) \\
&\equiv \Big((i > 1) \wedge wp(s[i] := s[i-1], Q) \Big) \vee \Big((i \leq 1) \wedge wp(s[i] := 0, Q)) \Big) \\
&\equiv \Big((i > 1) \wedge (def(setAt(s, i, s[i-1]) \wedge_L Q^s_{setAt(s,i,s[i-1])}) \Big) \\
&\vee \Big((i \leq 1) \wedge (def(setAt(s, i, 0) \wedge_L Q^s_{setAt(s,i,0)}) \Big) \\
&//galerazomagico \\
&\equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = 0)
\end{aligned}
$$

e) **S: if** $(s[i] < 0)$ **then** $s[i] := -s[i]$ **else** *skip* **endif**
$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(s[i] < 0) \wedge_L \left(\Big((s[i] < 0) \wedge S1 \Big) \vee \Big(\neg(s[i] < 0) \wedge True \Big) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left(\Big(\Big) \vee \Big(s[i] \geq 0 \Big) \right) \\
&\equiv ()
\end{aligned}
$$

f) **S: if** $(s[i] > 0)$ **then** $s[i] := -s[i]$ **else** *skip* **endif**
$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv \text{def}(s[i] > 0) \wedge_L \left(\Big((s[i] > 0) \wedge S1 \Big) \vee \Big(\neg(s[i] > 0) \wedge True \Big) \right) \\
&\equiv 0 \leq i < |s| \wedge_L \left(\Big(\Big) \vee \Big(s[i] \leq 0 \Big) \right) \\
&\equiv ()
\end{aligned}
$$

Ejercicio 8. ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {
   Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j])\}$
   Post $\{a = \sum_{j=0}^{i} s[j]\}$
}

b) **proc problema2** (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {
   Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i} s[j])\}$
   Post $\{a = \sum_{j=1}^{i} s[j]\}$
}

c) **proc problema3** (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out res: **Bool**)
   Pre $\{0 \leq i < |s| \wedge (\forall j : \mathbb{Z})(0 \leq j < i \rightarrow_L s[j] \geq 0)\}$
   Post $\{res = true \leftrightarrow (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L s[j] \geq 0)\}$

d) **proc problema4** (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {

  Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$

  Post $\{a = \sum_{j=0}^{i}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$

}

e) **proc problema5** (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {

  Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=1}^{i-1}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$

  Post $\{a = \sum_{j=0}^{i-1}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$

}

Respuestas

a)

b)

c)

d)

e)

FIN.

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