



Ejercicio 1. ★ Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- a) $\text{def}(a + 1)$.
- b) $\text{def}(a/b)$.
- c) $\text{def}(\sqrt{a/b})$.
- d) $\text{def}(A[i] + 1)$.
- e) $\text{def}(A[i + 2])$.
- f) $\text{def}(0 \leq i \leq |A|)$.
- g) $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0)$.

Respuestas

Supongo que $\text{def}(x) \equiv \text{True}$, para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

- a) $\text{def}(a + 1) \equiv \text{def}(a) \wedge \text{def}(1) \equiv \text{True} \wedge \text{True} \equiv \text{True}$
- b) $\text{def}(a/b) \equiv \text{def}(a) \wedge \text{def}(b) \wedge b \neq 0 \equiv b \neq 0$.
- c) $\text{def}(\sqrt{a/b}) \equiv b \neq 0 \wedge (a/b) \geq 0$.
- d) $\text{def}(A[i] + 1) \equiv 0 \leq i < |A|$
- e) $\text{def}(A[i + 2]) \equiv 0 \leq i + 2 < |A|$
- f) $\text{def}(0 \leq i \leq |A|) \equiv \text{True}$
- g) $\text{def}(0 \leq i \leq |A| \wedge_L A[i] \leq 0) \equiv i < |A|$

Ejercicio 2. Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- a) $wp(\mathbf{a} := \mathbf{a} + 1, a \geq 0)$.
- b) $wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \geq 0)$.
- c) $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \geq 0)$.
- d) $wp(\mathbf{a} := \mathbf{b} * \mathbf{b}, a \geq 0)$.
- e) $wp(\mathbf{b} := \mathbf{b} + 1, a \geq 0)$.

Respuestas

a)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{a}+1, a \geq 0) &\equiv \text{def}(a+1) \wedge_L (a \geq 0)_{a+1}^a \\ &\equiv \text{True} \wedge_L a+1 \geq 0 \\ &\equiv a \geq -1 \end{aligned}$$

b)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{a}/\mathbf{b}, a \geq 0) &\equiv \text{def}(a/b) \wedge_L \wedge_L (a \geq 0)_{a/b}^a \\ &\equiv \text{def}(a) \wedge_L \text{def}(b) \wedge_L b \neq 0 \wedge_L (a \geq 0)_{a/b}^a \\ &\equiv \text{True} \wedge_L \text{True} \wedge_L b \neq 0 \wedge_L a/b \geq 0 \\ &\equiv b \neq 0 \wedge_L a \geq 0 \end{aligned}$$

c)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{A}[\mathbf{i}], a \geq 0) &\equiv \text{def}(A[i]) \wedge_L (a \geq 0)_{A[i]}^a \\ &\equiv (\text{def}(A) \wedge_L \text{def}(i)) \wedge_L 0 \leq i < |A| \wedge_L A[i] \geq 0 \\ &\equiv 0 \leq i < |A| \wedge_L A[i] \geq 0 \end{aligned}$$

d)

$$\begin{aligned} wp(\mathbf{a} := \mathbf{b}*\mathbf{b}, a \geq 0) &\equiv \text{def}(b*b) \wedge_L (a \geq 0)_{b*b}^a \\ &\equiv \text{True} \wedge_L b*b \geq 0 \\ &\equiv b*b \geq 0 \end{aligned}$$

e)

$$\begin{aligned} wp(\mathbf{b} := \mathbf{b}+1, a \geq 0) &\equiv \text{def}(b+1) \wedge_L (a \geq 0)_a^a \\ &\equiv \text{True} \wedge_L a \geq 0 \\ &\equiv a \geq 0 \end{aligned}$$

Ejercicio 3. ★ Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp(\mathbf{a} := \mathbf{a}+1; \mathbf{b} := \mathbf{a}/2, b \geq 0).$

b) $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a}*\mathbf{a}, b \neq 2).$

c) $wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{a} := \mathbf{b}*\mathbf{b}, a \geq 0).$

d) $wp(\mathbf{a} := \mathbf{a}-\mathbf{b}; \mathbf{b} := \mathbf{a}+\mathbf{b}, a \geq 0 \wedge b \geq 0).$

Respuestas

a)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{a}+1; \mathbf{b} := \mathbf{a}/2, b \geq 0)\} &\equiv \{(a+1)/2 \geq 0\} \\ &\quad \mathbf{a} := \mathbf{a}+1; \\ &\quad \{wp(\mathbf{b} := \mathbf{a}/2, Q)\} \equiv \{a/2 \geq 0\} \\ &\quad \mathbf{b} := \mathbf{a}/2; \\ &\quad \{Q : b \geq 0\} \end{aligned}$$

b)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{b} := \mathbf{a}*\mathbf{a}, b \neq 2)\} &\equiv \{0 \leq i < |A| \wedge_L (A[i] + 1) * (A[i] + 1) \neq 0\} \\ &\quad \mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \\ &\quad \{wp(\mathbf{b} := \mathbf{a}*\mathbf{a}, Q)\} \equiv \{a * a \neq 0\} \\ &\quad \mathbf{b} := \mathbf{a}*\mathbf{a}; \\ &\quad \{Q : b \neq 0\} \end{aligned}$$

c)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{A}[\mathbf{i}] + 1; \mathbf{a} := \mathbf{b} * \mathbf{b}, a \geq 0)\} &\equiv \{0 \leq i < |A| \wedge b \geq 0\} \\ \mathbf{a} &:= \mathbf{A}[\mathbf{i}] + 1; \\ \{wp(\mathbf{a} := \mathbf{b} * \mathbf{b}, Q)\} &\equiv \{b \geq 0\} \\ \mathbf{a} &:= \mathbf{b} * \mathbf{b}; \\ \{Q : b \geq 0\} \end{aligned}$$

d)

$$\begin{aligned} \{wp(\mathbf{a} := \mathbf{a} - \mathbf{b}; \mathbf{b} := \mathbf{a} + \mathbf{b}, a \geq 0 \wedge b \geq 0)\} &\equiv \{a - b \geq 0 \wedge a + b \geq 0\} \\ \mathbf{a} &:= \mathbf{a} - \mathbf{b}; \\ \{wp(\mathbf{b} := \mathbf{a} + \mathbf{b}, Q)\} &\equiv \{a \geq 0 \wedge a + b \geq 0\} \\ \mathbf{b} &:= \mathbf{a} + \mathbf{b}; \\ \{Q : a \geq 0 \wedge b \geq 0\} \end{aligned}$$

Ejercicio 4. ★ Sea $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$. Calcular las siguientes precondiciones más débiles, donde i es una variable entera y A es una secuencia de reales.

- a) $wp(\mathbf{A}[\mathbf{i}] := \mathbf{0}, Q)$.
- b) $wp(\mathbf{A}[\mathbf{i} + \mathbf{2}] := \mathbf{0}, Q)$.
- c) $wp(\mathbf{A}[\mathbf{i} + \mathbf{2}] := -\mathbf{1}, Q)$.
- d) $wp(\mathbf{A}[\mathbf{i}] := \mathbf{2} * \mathbf{A}[\mathbf{i}], Q)$.
- e) $wp(\mathbf{A}[\mathbf{i}] := \mathbf{A}[\mathbf{i} - \mathbf{1}], Q)$.

Respuestas

a)

$$\begin{aligned} \{wp(\mathbf{A}[\mathbf{i}] := \mathbf{0}, Q)\} &\equiv \{0 \leq i < |A| \wedge Q\} \\ \mathbf{A}[\mathbf{i}] &:= \mathbf{0} \\ \{Q\} \end{aligned}$$

b)

$$\begin{aligned} \{wp(\mathbf{A}[\mathbf{i} + \mathbf{2}] := \mathbf{0}, Q)\} &\equiv \{0 \leq i + 2 < |A| \wedge Q\} \\ \mathbf{A}[\mathbf{i} + \mathbf{2}] &:= \mathbf{0} \\ \{Q\} \end{aligned}$$

c)

$$\begin{aligned} \{wp(\mathbf{A}[\mathbf{i} + \mathbf{2}] := -\mathbf{1}, Q)\} &\equiv \{0 \leq i + 2 < |A| \wedge Q \wedge \text{Esto no pasa la post nunca}\} \\ \mathbf{A}[\mathbf{i} + \mathbf{2}] &:= -\mathbf{1} \\ \{Q\} \end{aligned}$$

d)

$$\begin{aligned} \{wp(\mathbf{A}[\mathbf{i}] := \mathbf{2} * \mathbf{A}[\mathbf{i}], Q)\} &\equiv \{0 \leq i < |A| \wedge A[i] \geq 0 \wedge Q\} \\ \mathbf{A}[\mathbf{i}] &:= \mathbf{2} * \mathbf{A}[\mathbf{i}] \\ \{Q\} \end{aligned}$$

e)

$$\begin{aligned} \{wp(\mathbf{A}[\mathbf{i}] := \mathbf{A}[\mathbf{i} - \mathbf{1}], Q)\} &\equiv \{0 \leq i - 1 < |A| \wedge Q\} \\ \mathbf{A}[\mathbf{i}] &:= \mathbf{A}[\mathbf{i} - \mathbf{1}] \\ \{Q\} \end{aligned}$$

Ejercicio 5. Calcular $wp(S, Q)$, para los siguientes pares de programas S y postcondiciones Q .

- a) $S \equiv i := i + 1$
 $Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- b) $S \equiv A[0] := 4$
 $Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- c) $S \equiv A[2] := 4$
 $Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$
- d) $S \equiv A[i] := A[i + 1] - 1$
 $Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j - 1])$
- e) $S \equiv A[i] := A[i + 1] - 1$
 $Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j - 1])$

Respuestas

a)

$$\begin{aligned} \{wp(S, Q)\} &\equiv \{Q\} \\ S &\equiv i := i + 1 \\ \{Q &\equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\} \end{aligned}$$

Y a mi que me contas, no hace nada esto.

b)

$$\begin{aligned} \{wp(S, Q)\} &\equiv \{\} \\ S &\equiv A[0] := 4 \\ \{Q &\equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\} \end{aligned}$$

c)

$$\begin{aligned} \{wp(S, Q)\} &\equiv \{\} \\ S &\equiv A[2] := 4 \\ \{Q &\equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\} \end{aligned}$$

d)

$$\begin{aligned} \{wp(S, Q)\} &\equiv \{\} \\ S &\equiv A[i] := A[i + 1] - 1 \\ \{Q &\equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j - 1])\} \end{aligned}$$

e)

$$\begin{aligned} \{wp(S, Q)\} &\equiv \{\} \\ S &\equiv A[i] := A[i + 1] - 1 \\ \{Q &\equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j - 1])\} \end{aligned}$$

Ejercicio 6. . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

- a) **proc problema1** (inout a: Z)
Pre $\{a = a_0 \wedge a \geq 0\}$
Post $\{a = a_0 + 2\}$

- b) **proc problema2** (in a: Z, out b: Z)
 Pre $\{a6 = 0\}$
 Post $\{b = a + 3\}$
- c) **proc problema3** (in a: Z, in b: Z, out c: Z)
 Pre $\{true\}$
 Post $\{c = a + b\}$
- d) **proc problema4** (in a: seqhZi, in i: Z, out result: Z)
 Pre $\{0 \leq i < |a|\}$
 Post $\{result = 2 * a[i]\}$
- e) **proc problema5** (in a: seqhZi, in i: Z, out result: Z)
 Pre $\{0 \leq i \wedge i + 1 < |a|\}$
 Post $\{result = a[i] + a[i + 1]\}$

Respuestas

- a)
b)
c)
d)
e)

Ejercicio 7. ★ Calcular $wp(S, Q)$, para los siguientes pares de programas S y postcondiciones Q.

- a) $S \equiv$
 if $(a < 0)$
 $b := a$
 else
 $b := -a$
 endif

$$Q \equiv (b = -|a|)$$

- b) $S \equiv$
 if $(a < 0)$
 $b := a$
 else
 $b := -a$
 endif

$$Q \equiv (b = |a|)$$

- c) $S \equiv$
 if $(i > 0)$
 $s[i] := 0$
 else
 $s[0] := 0$
 endif

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

d) $S \equiv$ if ($i > 1$)
 $s[i] := s[i - 1]$
 else
 $s[i] := 0$
 endif

$$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j - 1])$$

e) $S \equiv$
 if ($s[i] < 0$)
 $s[i] := -s[i]$
 else
skip
 endif

$$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$$

f) $S \equiv$
 if ($s[i] > 0$)
 $s[i] := -s[i]$
 else
skip
 endif

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

Respuestas

- a)
- b)
- c)
- d)
- e)

Ejercicio 8. ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

a) **proc problema1** (in s: seqhZi, in i: Z, inout a: Z)

$$\mathbf{Pre} \{0 \leq i < |s| \wedge_L a = j = 0s[j]\}$$

$$\mathbf{Post} \{a = j = 0s[j]\}$$

b) **proc problema2** (in s: seqhZi, in i: Z, inout a: Z)

$$\mathbf{Pre} \{0 \leq i < |s| \wedge a = j = 0s[j]\}$$

$$\mathbf{Post} \{a = j = 1s[j]\}$$

c) **proc problema3** (in s: seqhZi, in i: Z, out res: Bool)

$$\mathbf{Pre} \{0 \leq i < |s| \wedge (\forall j : Z)(0 \leq j < i \rightarrow_L s[j] \geq 0)\}$$

$$\mathbf{Post} \{res = true \leftrightarrow (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0)\}$$

d) **proc problema4** (in s: seqhZi, in i: Z, inout a: Z)

Pre $0 \leq i < |s| \wedge_L a = j = 0 (if s[j]6 = 0 then 1 else 0 fi)$

Post $\{a = j = 0 (if s[j]6 = 0 then 1 else 0 fi)\}$

e) **proc problema5** (in s: seqhZi, in i: Z, inout a: Z)

Pre $\{0 < i \leq |s| \wedge_L a = j = 1 (if s[j]6 = 0 then 1 else 0 fi)\}$

Post $\{a = j = 0 (if s[j]6 = 0 then 1 else 0 fi)\}$

Respuestas

a)

b)

c)

d)

e)