Algoritmos y Estructuras de Datos I
Primer Cuatrimestre 2020

Guía Práctica 4
Resolución de los Ejercicios Entregables

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Ejercicio 1. Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- $\text{def}(\sqrt{a/b})$.
- $\text{def}(A[i+2])$.

Respuesta:

Suponemos que $\text{def}(x) \equiv True$, para todas las variables por lo expuesto en la teorica; ya que de este modo se simplifica la notación.

- $\text{def}(\sqrt{a/b}) \overset{Ax,1}{\equiv} b \neq 0 \wedge_L (a/b) \geq 0$.
- $\text{def}(A[i+2]) \overset{Ax,1}{\equiv} 0 \leq i+2 < |A|$

Ejercicio 6.e Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

- **proc problema5** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
  Pre $\{0 \leq i \wedge i+1 < |a|\}$
  Post $\{result = a[i] + a[i+1]\}$

Respuesta:

$$\textbf{S: } result := a[i] + a[i+1]$$

1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned}
\{wp(S, Post)\} &\equiv wp(result := a[i] + a[i+1], Post) \\
&\overset{Ax,1}{\equiv} \text{def}(a[i] + a[i+1]) \wedge_L Post^{result}_{a[i]+a[i+1]} \\
&\equiv \Big(((\text{def(a)} \wedge \text{def(i)}) \wedge_L 0 \leq i < |a| \wedge 0 \leq i+1 < |a|\Big) \wedge_L Post^{result}_{a[i]+a[i+1]} \\
&\equiv \Big(((True \wedge True) \wedge_L 0 \leq i \wedge i+1 < |a|\Big) \wedge_L Post^{result}_{a[i]+a[i+1]} \\
&\equiv \Big(0 \leq i \wedge i+1 < |a|\Big) \wedge_L Post^{result}_{a[i]+a[i+1]} \\
&\equiv \Big(0 \leq i \wedge i+1 < |a|\Big) \wedge_L \Big(a[i] + a[i+1] = a[i] + a[i+1]\Big) \\
&\equiv 0 \leq i \wedge i+1 < |a| \wedge_L True \\
&\equiv 0 \leq i \wedge i+1 < |a|
\end{aligned}$$

2. Chequeamos $Pre \to \{wp(S, Post)\}$

$$Pre \to \{wp(S, Post)\}$$
$$\{0 \leq i \wedge i+1 < |a|\} \to \{0 \leq i \wedge i+1 < |a|\}$$
$$True$$

Ejercicio 8.d Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil

- **proc problema4** (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$) {

  Pre $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$

  Post $\{a = \sum_{j=0}^{i}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})\}$

  }

Respuesta:

S: if $(s[i] \neq 0)$ **then** $a := a + 1$ **else** *skip* **endif**

$Post \equiv a = \sum_{j=0}^{i}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})$

1. Calculamos $\{wp(S, Post)\}$

$$wp(\mathbf{S}, Post) \overset{Ax,4}{\equiv} \text{def}(s[i] \neq 0) \wedge_L \Big(\big((s[i] \neq 0) \wedge (wp(a := a+1, Post))\big) \vee \big(\neg(s[i] \neq 0) \wedge (wp(skip, Post))\big)\Big)$$

$$\overset{Ax,2}{\equiv} \text{def}(s[i] \neq 0) \wedge_L \Big(\big((s[i] \neq 0) \wedge (wp(a := a+1, Post))\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

$$\equiv \big(((\text{def(s)} \wedge \text{def(i)}) \wedge_L 0 \leq i < |s|\big) \wedge_L \Big(\big((s[i] \neq 0) \wedge (wp(a := a+1, Post))\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

$$\overset{Ax,1}{\equiv} \big(((True \wedge True) \wedge_L 0 \leq i < |s|\big) \wedge_L \Big(\big((s[i] \neq 0) \wedge (\text{def}(a+1) \wedge_L Post^a_{a+1})\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

$$\equiv 0 \leq i < |s| \wedge_L \Big(\big((s[i] \neq 0) \wedge (True \wedge_L Post^a_{a+1})\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

$$\equiv 0 \leq i < |s| \wedge_L \Big(\big((s[i] \neq 0) \wedge Post^a_{a+1}\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$

$$\left\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi })\right\} \rightarrow \left\{0 \leq i < |s| \wedge_L \Big(\big((s[i] \neq 0) \wedge Post^a_{a+1}\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)\right\}$$

$$0 \leq i < |s| \rightarrow 0 \leq i < |s| \quad \blacklozenge$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow \Big(\big((s[i] \neq 0) \wedge Post^a_{a+1}\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

Separamos en casos.

1. $(s[i] = 0) = True$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow \Big(\big((False) \wedge Post^a_{a+1}\big) \vee \big((s[i] = 0) \wedge Post)\big)\Big)$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow \Big(\big(False\big) \vee \big((s[i] = 0) \wedge Post\big)\Big)$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow \big(True \wedge Post\big)$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow Post$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow a = \sum_{j=0}^{i}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow a = \sum_{j=0}^{i-1}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \quad \blacklozenge$$

2. $(s[i] \neq 0) = True$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow \Big(\big((True) \wedge Post^a_{a+1}\big) \vee \big((False) \wedge Post)\big)\Big)$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow \Big(\big(Post^a_{a+1}\big) \vee \big(False\big)\Big)$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow Post^a_{a+1}$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow a + 1 = \sum_{j=0}^{i}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi})$$

$$a = \sum_{j=0}^{i-1}(\text{ if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi }) \rightarrow a = \sum_{j=0}^{i-1}(\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \quad \blacklozenge$$