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Ejercicio 1. Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

- $\text{def}(\sqrt{a/b})$.
- $\text{def}(A[i+2])$.

Respuesta:

Supongo que $\text{def}(x) \equiv \text{True}$, para todas las variables por lo expuesto en la teorica; ya que de este modo se simplifica la notación.

- $\text{def}(\sqrt{a/b}) \stackrel{Ax,1}{\equiv} b \neq 0 \wedge_L (a/b) \geq 0$.
- $\text{def}(A[i+2]) \stackrel{Ax,1}{\equiv} 0 \leq i+2 < |A|$

Ejercicio 6.e Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil.

- **proc problema5** (in a: $\text{seq}(\mathbb{Z})$, in i: \mathbb{Z} , out result: \mathbb{Z})
Pre $\{0 \leq i \wedge i+1 < |a|\}$
Post $\{\text{result} = a[i] + a[i+1]\}$

Respuesta:

S: $\text{result} := a[i] + a[i+1]$

1. Calculamos $\{wp(S, \text{Post})\}$

$$\begin{aligned}
 \{wp(S, \text{Post})\} &\equiv wp(\text{result} := a[i] + a[i+1], \text{Post}) \\
 &\stackrel{Ax,1}{\equiv} \text{def}(a[i] + a[i+1]) \wedge_L \text{Post}_{a[i]+a[i+1]}^{\text{result}} \\
 &\equiv \left(((\text{def}(a) \wedge \text{def}(i)) \wedge_L 0 \leq i+1 < |a|) \wedge_L \text{Post}_{a[i]+a[i+1]}^{\text{result}} \right) \\
 &\equiv \left(((\text{True} \wedge \text{True}) \wedge_L 0 \leq i+1 < |a|) \wedge_L \text{Post}_{a[i]+a[i+1]}^{\text{result}} \right) \\
 &\equiv \left(0 \leq i+1 < |a| \right) \wedge_L \text{Post}_{a[i]+a[i+1]}^{\text{result}} \\
 &\equiv \left(0 \leq i+1 < |a| \right) \wedge_L \left(a[i] + a[i+1] = a[i] + a[i+1] \right) \\
 &\equiv 0 \leq i+1 < |a| \wedge_L \text{True} \\
 &\equiv 0 \leq i+1 < |a|
 \end{aligned}$$

2. Chequeamos $\text{Pre} \rightarrow \{wp(S, \text{Post})\}$

$$\begin{aligned}
 &\text{Pre} \rightarrow \{wp(S, \text{Post})\} \\
 &\{0 \leq i \wedge i+1 < |a|\} \rightarrow \{0 \leq i \wedge i+1 < |a|\} \\
 &\text{True}
 \end{aligned}$$

Ejercicio 8.d Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondition más débil

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■ proc problema4 (in s: seq⟨ℤ⟩, in i: ℤ, inout a: ℤ) {
    Pre {0 ≤ i < |s| ∧L a = ∑j=0i-1 (if s[j] ≠ 0 then 1 else 0 fi)}
    Post {a = ∑j=0i (if s[j] ≠ 0 then 1 else 0 fi)}
}

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Respuesta:

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S: if (s[i] ≠ 0) then a := a + 1 else skip endif
    Post ≡ a = ∑j=0i (if s[j] ≠ 0 then 1 else 0 fi)

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1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned}
 wp(\mathbf{S}, Post) &\stackrel{Ax,4}{\equiv} \text{def}(s[i] \neq 0) \wedge_L \left(\left((s[i] \neq 0) \wedge (wp(a := a + 1, Post)) \right) \vee \left(\neg(s[i] \neq 0) \wedge (wp(skip, Post)) \right) \right) \\
 &\stackrel{Ax,2}{\equiv} \text{def}(s[i] \neq 0) \wedge_L \left(\left((s[i] \neq 0) \wedge (wp(a := a + 1, Post)) \right) \vee \left((s[i] = 0) \wedge Post \right) \right) \\
 &\equiv \left(((\text{def}(s) \wedge \text{def}(i)) \wedge_L 0 \leq i < |s|) \wedge_L \left(\left((s[i] \neq 0) \wedge (wp(a := a + 1, Post)) \right) \vee \left((s[i] = 0) \wedge Post \right) \right) \right) \\
 &\stackrel{Ax,1}{\equiv} \left(((True \wedge True) \wedge_L 0 \leq i < |s|) \wedge_L \left(\left((s[i] \neq 0) \wedge (\text{def}(a + 1) \wedge_L Post_{a+1}^a) \right) \vee \left((s[i] = 0) \wedge Post \right) \right) \right) \\
 &\equiv 0 \leq i < |s| \wedge_L \left(\left((s[i] \neq 0) \wedge (True \wedge_L Post_{a+1}^a) \right) \vee \left((s[i] = 0) \wedge Post \right) \right) \\
 &\equiv 0 \leq i < |s| \wedge_L \left(\left((s[i] \neq 0) \wedge Post_{a+1}^a \right) \vee \left((s[i] = 0) \wedge Post \right) \right)
 \end{aligned}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$\begin{aligned}
 &Pre \rightarrow \{wp(S, Post)\} \\
 \left\{ 0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \right\} &\rightarrow \left\{ 0 \leq i < |s| \wedge_L \left(\left((s[i] \neq 0) \wedge Post_{a+1}^a \right) \vee \left((s[i] = 0) \wedge Post \right) \right) \right\} \\
 0 \leq i < |s| &\rightarrow 0 \leq i < |s| \quad \blacklozenge \\
 a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) &\rightarrow \left(\left((s[i] \neq 0) \wedge Post_{a+1}^a \right) \vee \left((s[i] = 0) \wedge Post \right) \right)
 \end{aligned}$$

Separo en casos.

$$1.(s[i] = 0) = True$$

$$\begin{aligned}
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow \left(\left((False) \wedge Post_{a+1}^a \right) \vee \left((s[i] = 0) \wedge Post \right) \right) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow \left((False) \vee \left((s[i] = 0) \wedge Post \right) \right) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow (True \wedge Post) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow Post \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow a = \sum_{j=0}^i (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \quad \blacklozenge
\end{aligned}$$

$$2.(s[i] \neq 0) = True$$

$$\begin{aligned}
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow \left(\left((True) \wedge Post_{a+1}^a \right) \vee \left((False) \wedge Post \right) \right) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow \left(\left(Post_{a+1}^a \right) \vee (False) \right) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow Post_{a+1}^a \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow a + 1 = \sum_{j=0}^i (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \\
a &= \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \rightarrow a = \sum_{j=0}^{i-1} (\text{if } s[j] \neq 0 \text{ then } 1 \text{ else } 0 \text{ fi}) \quad \blacklozenge
\end{aligned}$$