Algoritmos y Estructuras de Datos I
Primer Cuatrimestre 2020

Guía Práctica 4
Precondición más débil en SmallLang

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Ejercicio 1. ★ Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) def$(a + 1)$.

b) def(a/b).

c) def$(\sqrt{a/b})$.

d) def$(A[i] + 1)$.

e) def$(A[i + 2])$.

f) def$(0 \leq i \leq |A|)$.

g) def$(0 \leq i \leq |A| \wedge_L A[i] \leq 0)$.

Respuestas

Supongo que def$(x) \equiv True$,para todas las variables por lo expuesto en la teorica, ya que de este modo se simplifica la notación.

a) def$(a + 1) \equiv$def$(a)\wedge$def$(1) \equiv True \wedge True \equiv True$

b) def$(a/b) \equiv$ def(a) $\wedge$ def(b) $\wedge$ $b \neq 0 \equiv b \neq 0$.

c) def$(\sqrt{a/b}) \equiv b \neq 0 \wedge (a/b) \geq 0$.

d) def$(A[i] + 1) \equiv 0 \leq i < |A|$

e) def$(A[i + 2]) \equiv 0 \leq i + 2 < |A|$

f) def$(0 \leq i \leq |A|) \equiv True$

g) def$(0 \leq i \leq |A| \wedge_L A[i] \leq 0) \equiv i < |A|$

Ejercicio 2. Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp($**a := a+1**$, a \geq 0)$.

b) $wp($**a := a/b**$, a \geq 0)$.

c) $wp($**a := A[i]**$, a \geq 0)$.

d) $wp($**a := b*b**$, a \geq 0)$.

e) $wp($**b := b+1**$, a \geq 0)$.

Respuestas

a)

$$\begin{aligned} wp(\mathbf{a := a+1}, a \geq 0) &\equiv \text{def}(a+1) \wedge_L (a \geq 0)^a_{a+1} \\ &\equiv True \wedge_L a + 1 \geq 0 \\ &\equiv a \geq -1 \end{aligned}$$

b)

$$\begin{aligned} wp(\mathbf{a := a/b}, a \geq 0) &\equiv \text{def}(a/b) \wedge_L \wedge_L (a \geq 0)^a_{a/b} \\ &\equiv \text{def}(a) \wedge_L \text{def}(b) \wedge_L b \neq 0 \wedge_L (a \geq 0)^a_{a/b} \\ &\equiv True \wedge_L True \wedge_L b \neq 0 \wedge_L a/b \geq 0 \\ &\equiv b \neq 0 \wedge_L a \geq 0 \end{aligned}$$

c)

$$\begin{aligned} wp(\mathbf{a := A[i]}, a \geq 0) &\equiv \text{def}(A[i]) \wedge_L (a \geq 0)^a_{A[i]} \\ &\equiv (\text{def}(A) \wedge_L \text{def}(i)) \wedge_L 0 \leq i < |A| \wedge_L A[i] \geq 0 \\ &\equiv 0 \leq i < |A| \wedge_L A[i] \geq 0 \end{aligned}$$

d)

$$\begin{aligned} wp(\mathbf{a := b*b}, a \geq 0) &\equiv \text{def}(b*b) \wedge_L (a \geq 0)^a_{b*b} \\ &\equiv True \wedge_L b * b \geq 0 \\ &\equiv b * b \geq 0 \end{aligned}$$

e)

$$\begin{aligned} wp(\mathbf{b := b+1}, a \geq 0) &\equiv \text{def}(b+1) \wedge_L (a \geq 0)^a_a \\ &\equiv True \wedge_L a \geq 0 \\ &\equiv a \geq 0 \end{aligned}$$

Ejercicio 3. ★ Calcular las siguientes precondiciones más débiles, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $wp(\mathbf{a := a+1;\ b := a/2}, b \geq 0)$.

b) $wp(\mathbf{a := A[i] + 1;\ b := a*a}, b \neq 2)$.

c) $wp(\mathbf{a := A[i] + 1;\ a := b*b}, a \geq 0)$.

d) $wp(\mathbf{a := a\text{-}b;\ b := a+b}, a \geq 0 \wedge b \geq 0)$.

Respuestas

a)

$$\{wp(\mathbf{a := a+1;\ b := a/2}, b \geq 0)\} \equiv \{(a+1)/2 \geq 0\}$$

a := a+1;

$$\{wp(\mathbf{b := a/2}, Q)\} \equiv \{a/2 \geq 0\}$$

b := a/2;

$$\{Q : b \geq 0\}$$

b)

$$\{wp(\mathbf{a := A[i] + 1;\ b := a*a}, b \neq 2)\} \equiv \{0 \leq i < |A| \wedge_L (A[i]+1) * (A[i]+1) \neq 0\}$$

a := A[i] + 1;

$$\{wp(\mathbf{b := a*a}, Q)\} \equiv \{a * a \neq 0\}$$

b := a*a;

$$\{Q : b \neq 0\}$$

c)

$$\{wp(\textbf{a := A[i] + 1; a := b*b}, a \geq 0)\} \equiv \{0 \leq i < |A| \wedge b \geq 0\}$$
$$\textbf{a := A[i] + 1;}$$
$$\{wp(\textbf{a := b*b}, Q)\} \equiv \{b \geq 0\}$$
$$\textbf{a := b*b;}$$
$$\{Q : b \geq 0\}$$

d)

$$\{wp(\textbf{a := a-b; b := a+b}, a \geq 0 \wedge b \geq 0)\} \equiv \{a - b \geq 0 \wedge a + b \geq 0\}$$
$$\textbf{a := a-b;}$$
$$\{wp(\textbf{b := a+b}, Q)\} \equiv \{a \geq 0 \wedge a + b \geq 0\}$$
$$\textbf{b := a+b;}$$
$$\{Q : a \geq 0 \wedge b \geq 0\}$$

Ejercicio 4. ★ Sea $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$. Calcular las siguientes precondiciones más débiles, donde i es una variable entera y A es una secuencia de reales.

a) $wp(\textbf{A[i] := 0}, Q)$.

b) $wp(\textbf{A[i+2] := 0}, Q)$.

c) $wp(\textbf{A[i+2] := -1}, Q)$.

d) $wp(\textbf{A[i] := 2 * A[i]}, Q)$.

e) $wp(\textbf{A[i] := A[i-1]}, Q)$.

Respuestas

a)

$$\{wp(\textbf{A[i] := 0}\ , Q)\} \equiv \{0 \leq i < |A| \wedge Q\}$$
$$\textbf{A[i] := 0}$$
$$\{Q\}$$

b)

$$\{wp(\textbf{A[i+2] := 0}\ , Q)\} \equiv \{0 \leq i + 2 < |A| \wedge Q\}$$
$$\textbf{A[i+2] := 0}$$
$$\{Q\}$$

c)

$$\{wp(\textbf{A[i+2] := -1}\ , Q)\} \equiv \{0 \leq i + 2 < |A| \wedge Q \wedge \textbf{Esto no pasa la post nunca}\}$$
$$\textbf{A[i+2] := -1}$$
$$\{Q\}$$

d)

$$\{wp(\textbf{A[i] := 2 * A[i]}\ , Q)\} \equiv \{0 \leq i < |A| \wedge A[i] \geq 0 \wedge Q\}$$
$$\textbf{A[i] := 2 * A[i]}$$
$$\{Q\}$$

e)

$$\{wp(\textbf{A[i] := A[i-1]}\ , Q)\} \equiv \{0 \leq i - 1 < |A| \wedge Q\}$$
$$\textbf{A[i] := A[i-1]}$$
$$\{Q\}$$

Ejercicio 5. Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) $S \equiv i := i + 1$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

b) $S \equiv A[0] := 4$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

c) $S \equiv A[2] := 4$
$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)$

d) $S \equiv A[i] := A[i + 1] - 1$
$Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j - 1])$

e) $S \equiv A[i] := A[i + 1] - 1$
$Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j - 1])$

Respuestas

a)

$$\{wp(S, Q)\} \equiv \{Q\}$$
$$S \equiv i := i + 1$$
$$\{Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

Y a mi que me contas, no hace nada esto.

b)

$$\{wp(S, Q)\} \equiv \{0 \leq 4 < |A| \wedge_L Q\}$$
$$S \equiv A[0] := 4$$
$$\{Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

c)

$$\{wp(S, Q)\} \equiv \{0 \leq 2 < |A| \wedge_L Q\}$$
$$S \equiv A[2] := 4$$
$$\{Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \neq 0)\}$$

d)

$$\{wp(S, Q)\} \equiv \{0 \leq i \wedge i + 1 < |A| \wedge_L Q\}$$
$$S \equiv A[i] := A[i + 1] - 1$$
$$\{Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \geq A[j - 1])\}$$

e)

$$\{wp(S, Q)\} \equiv \{0 \leq i \wedge i + 1 < |A| \wedge_L Q\}$$
$$S \equiv A[i] := A[i + 1] - 1$$
$$\{Q \equiv (\forall j : Z)(0 < j < |A| \rightarrow_L A[j] \leq A[j - 1])\}$$

Ejercicio 6. . Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (inout a: $\mathbb{Z}$)
Pre $\{a = a_0 \wedge a \geq 0\}$
Post $\{a = a_0 + 2\}$

b) **proc problema2** (in a: $\mathbb{Z}$, out b: $\mathbb{Z}$)
  Pre $\{a \neq 0\}$
  Post $\{b = a + 3\}$

c) **proc problema3** (in a: $\mathbb{Z}$, in b: $\mathbb{Z}$, out c: $\mathbb{Z}$)
  Pre $\{true\}$
  Post $\{c = a + b\}$

d) **proc problema4** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
  Pre $\{0 \leq i < |a|\}$
  Post $\{result = 2 * a[i]\}$

e) **proc problema5** (in a: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, out result: $\mathbb{Z}$)
  Pre $\{0 \leq i \wedge i + 1 < |a|\}$
  Post $\{result = a[i] + a[i + 1]\}$

Respuestas

Para probar la correctitud de la tripla **{Pre} S {Post}** alcanza probar que
Pre $\rightarrow wp(\mathbf{S}, Post)$

a)

$$\{wp(S, Post)\} \equiv \{a_0 + 2 = a_0 + 2\}$$
$$\textbf{S: } a := a_0 + 2$$
$$\{\textbf{Post: } a = a_0 + 2\}$$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{a_0 + 2 = a_0 + 2\}$$
$$\{a = a_0 \wedge a \geq 0\} \rightarrow \{True\}$$
$$True$$

b) 1. Calculamos $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + 3 = a + 3\}$$
$$\equiv True$$
$$\textbf{S: } b := a + 3$$
$$\{\textbf{Post: } b = a + 3\}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{a \neq 0\} \rightarrow \{True\}$$
$$True$$

c) 1. Calculamos $\{wp(S, Post)\}$

$$\{wp(S, Post)\} \equiv \{a + b = a + b\}$$
$$\equiv True$$
$$\textbf{S: } c := a + b$$
$$\{\textbf{Post: } c = a + b\}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{True\} \rightarrow \{True\}$$
$$True$$

d) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned}\{wp(S, Post)\} &\equiv \{\text{def}(2 * a[i]) \wedge_L 2 * a[i] = 2 * a[i]\}\\ &\equiv \text{def}(a[i]) \wedge_L 0 \leq i < |a| \wedge True\\ &\equiv True \wedge_L 0 \leq i < |a|\\ &\equiv 0 \leq i < |a|\\ \textbf{S: } & result := 2 * a[i]\\ \{\textbf{Post: } & result = 2 * a[i]\}\end{aligned}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{0 \leq i < |a|\} \rightarrow \{0 \leq i < |a|\}$$
$$True$$

e) 1. Calculamos $\{wp(S, Post)\}$

$$\begin{aligned}\{wp(S, Post)\} &\equiv \{\text{def}(a[i] + a[i+1]) \wedge_L a[i] + a[i+1] = a[i] + a[i+1]\}\\ &\equiv \text{def}(a[i]) \wedge_L \text{def}(a[i+1]) \wedge_L 0 \leq i \wedge i+1 < |a| \wedge a[i] + a[i+1] = a[i] + a[i+1]\\ &\equiv True \wedge_L True \wedge_L True \wedge_L 0 \leq i \wedge i+1 < |a| \wedge True\\ &\equiv 0 \leq i \wedge i+1 < |a|\\ \textbf{S: } & result := a[i] + a[i+1]\\ \{\textbf{Post: } & result = a[i] + a[i+1]\}\end{aligned}$$

2. Chequeamos $Pre \rightarrow \{wp(S, Post)\}$

$$Pre \rightarrow \{wp(S, Post)\}$$
$$\{0 \leq i \wedge i+1 < |a|\} \rightarrow \{0 \leq i \wedge i+1 < |a|\}$$
$$True$$

Ejercicio 7. ★ Calcular wp(S, Q), para los siguientes pares de programas S y postcondiciones Q.

a) $S \equiv$
if $(a < 0)$
$b := a$
else
$b := -a$
endif

$Q \equiv (b = -|a|)$

b) $S \equiv$
if $(a < 0)$
$b := a$
else
$b := -a$
endif

$Q \equiv (b = |a|)$

c) $S \equiv$
if $(i > 0)$
$s[i] := 0$
else
$s[0] := 0$
endif

$Q \equiv (\forall j : Z)(0 \leq j < |s| \to_L s[j] \geq 0)$

d) $S \equiv$ if $(i > 1)$
$s[i] := s[i-1]$
else
$s[i] := 0$
endif

$Q \equiv (\forall j : Z)(1 \leq j < |s| \to_L s[j] = s[j-1])$

e) $S \equiv$
if $(s[i] < 0)$
$s[i] := -s[i]$
else
skip
endif

$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

f) $S \equiv$
if $(s[i] > 0)$
$s[i] := -s[i]$
else
skip
endif

$Q \equiv (\forall j : Z)(0 \leq j < |s| \to_L s[j] \geq 0)$

Respuestas

a)

b)

c)

d)

e)

Ejercicio 8. ★ Escribir programas para los siguientes problemas y demostrar formalmente su corrección usando la precondición más débil.

a) **proc problema1** (in s: seqhZi, in i: Z, inout a: Z)

Pre $\{0 \leq i < |s| \wedge_L a = j = 0s[j]\}$

Post $\{a = j = 0s[j]\}$

b) **proc problema2** (in s: seqhZi, in i: Z, inout a: Z)

Pre $\{0 \leq i < |s| \land a = j = 0s[j]\}$

Post $\{a = j = 1s[j]\}$

c) **proc problema3** (in s: seqhZi, in i: Z, out res: Bool)
Pre $\{0 \leq i < |s| \land (\forall j : Z)(0 \leq j < i \rightarrow_L s[j] \geq 0)\}$
Post $\{res = true \leftrightarrow (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0)\}$

d) **proc problema4** (in s: seqhZi, in i: Z, inout a: Z)

Pre $0 \leq i < |s| \land_L a = j = 0(ifs[j]6 = 0then1else0fi)$

Post $\{a = j = 0(ifs[j]6 = 0then1else0fi)\}$

e) **proc problema5** (in s: seqhZi, in i: Z, inout a: Z)

Pre $\{0 < i \leq |s| \land_L a = j = 1(ifs[j]6 = 0then1else0fi)\}$

Post $\{a = j = 0(ifs[j]6 = 0then1else0fi)\}$

Respuestas

a)

b)

c)

d)

e)