CS 131 – Problem Set 8

Problems must be submitted by Monday November 10, 2025 at 11:59pm, on Gradescope.

How to submit: This problem set contains a written portion and a lean portion. For the lean portion, download and complete `ps8.lean` as instructed in the problems. Upload the written document to HW8 Written Portion and the lean file to HW8 Programming Portion on gradescope.
Note: In case you haven't realized this on the previous autograded homework assignments, the grade you receive is whatever the autograder gives you. There are no manual adjustments. So wait for the autograder to run and make sure it gives you the grade you think you should be getting. For the lean part, make sure you upload a .lean file. Any other file format will not be graded correctly from the autograder.

Problem 1. (20 points) Prove using math induction that for every $n \in \mathbb{N}$, the inequality $2^n \geq n$ holds. by completing `theorem problem1` in `ps8.lean`.

Problem 2. (30 points, 15 each part) Consider the sequence $(a_n)_{n\in\mathbb{N}}$ defined by

$$a_0 = 3, \qquad a_{n+1} = a_n + 9 \quad \text{for all } n \in \mathbb{N}.$$

a) Prove, by mathematical induction, that for all $n \in \mathbb{N}$,

$$a_n = 3 + 9n.$$

b) Formalize the definition of the sequence **and** the proof in Lean by completing `def problem2seq` and `theorem problem2` in `ps8.lean`

Problem 3. (15 points) Prove that for all integers a and b and for all positive integers n, $a - b$ divides $a^n - b^n$.

Problem 4. (15 points, 5 for each part) For each of the sequences below, give a recurrence relation (with an initial term) and state whether the sequence is arithmetic, geometric, or none of these.

a) $\left\{\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right\}$.

b) $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right\}$.

c) $\{1, 2, 4, 7, 11, \ldots\}$.

Problem 5. (20 points) Given the following summation and it's closed form, Prove that the following equality is true by induction.

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = 1 - \frac{1}{n+1}$$