CS131 Fall 2025 Final Exam Practice Problems

Problem 1. Suppose $s \in \mathbb{Z}$ is a perfect square not divisible by 3. Prove that $s \bmod 3 = 1$.

Problem 2. Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Suppose g is injective. Let $h : X \to Z$ be the function defined as $h(x) = g(f(x))$.

a) Give an example of f, g, X, Y, Z such that h is *not* injective. No justification is needed; simply specify these five objects.

b) Prove that if f is injective, then h is injective

Problem 3. Let the domain of discourse be BU undergraduates. Let

- C, D, R, S, W be the sets of Clafflin, Danielsen, Rich Hall, Sleeper Hall, and Warren Towers residents, respectively
- $F(a, b)$ denote that a and b are friends (the relation is F is symmetric and anti-reflexive)

a) Use set-builder notation to describe the set of all students who live in Warren Towers and have no friends in Danielsen.

Translate the following into quantified statements.

b) Everyone in Rich Hall has a friend in Clafflin

c) x has friends only in Warren Towers

d) There is only one person in Danielsen who has a friend in Sleeper and a friend in Clafflin

e) The only students who have a friend in Rich Hall are the ones who also have a friend in Clafflin Hall.

Problem 4. You are attending a holiday party of 75 people with a strict dress code. Because of supply chain disruptions created by recently imposed tariffs on imports, the choices of holiday outfits are very limited: everyone at the party must choose

- one out of 6 possible ugly holiday sweaters,
- and one out of 4 possible unflattering pants.

Prove that at least 4 attendees at the party will have the same outfit.

Problem 5. Consider the sequence $(a_n)_{\{n \in N\}}$ defined as follows:

$$\{2, 3, 12, 21, 30, 39, \ldots\}$$

a) (6 points) Fill in the following definition of a function `my_fun`: $\mathbb{N} \to \mathbb{N}$ such that

$$\texttt{my\_fun}\ n = a_n$$

for all $n \in \mathbb{N}$.

```
def myfun : Nat -> Nat
```

Consider the following proof (we use `<=` for the less or equal symbol $\leq$):

```
1.  theorem myfuncbound (n : Nat ) :
2.  myfun n <= 3 + 9 * n := by
3.  cases n with
4.    | zero =>
5.      simp [myfun]
6.    | succ m =>
7.      induction m with
8.        | zero =>
9.          simp [myfun]
10.       | succ k IH =>
11.         unfold myfun
12.         simp
13.         apply le_trans
14.         apply add_le_add_right
15.         apply IH
16          ring
17          apply Nat.le_refl
```

b) (6 points) What is the proof state at the end of line 8?

c) (8 points) What is the proof state at the end of line 10?

Problem 6. (16 points total, 4 points each) In Western musical notation, there are 12 notes. Chords are either a set of three notes or a set of four notes. For example the sets of notes {C, E, G}, {E, G, C} and {C, E, G, C} all represent the same chord. {C, E, G, B} is a different chord.

Each chord has a type. If a chord has three notes, it is either major or minor. Adding a fourth note changes the type. The chord *remains* either major or minor while *also becoming* one of

- 7th
- dominant 7th
- diminished
- half-diminished

Examples:

- {C, E, G} is a major chord
- {C, E, G, B} is a major seventh chord
- {D, F, A} is a minor chord
- {D, F, A, C} is a minor half-diminished chord

The above is close to being true. If you are familiar with music theory, please know that it irks us as well.

Remember: You may use exponentials, factorials, $C(n, r)$, and $P(n, r)$ in your answers. There is no need to calculate things out. For example, it is perfectly okay to leave your answer in the form $2^5 \cdot 7! \cdot P(10, 3)$. No explanation needed.

a) How many chords are there? (e.g. {C, E, G})

b) How many chord *types* are there? (e.g. minor half-diminished)

c) A cadence is a *sequence* of four chord types. Each chord type can be different, or one of the chord types can be used twice while the rest are only used once. No chord type can be used three or four times.

Examples

- major, minor diminished, minor, major dominant 7th
- major, major dominant 7th, minor half-diminished, major

How many cadences are there?

d) How many cadences are there that do not have a major diminished chord and a major half-diminished chord as their first and second chord type?

Problem 7. Prove by induction that the following statement is true:

$$\forall n \geq 3,\ 3^n > 3 \cdot n + 3$$