CS 131 – Lab Worksheet 7
October 29, 2025

Problem 1. Consider sets S and R the relation root $= \{(s, r) \in S \times R : r^2 = s\}$

1. Find S_1 and R_1 for which root is not a function because for some $s \in S_1$ there is no appropriate $r \in R_1$.
2. Find S_2 and R_2 for which root is not a function because for some $s \in S_2$ there are too many $r \in R_2$.
3. Find S_3, R_3 for which root is a function

Solution. There are many possible answers. Here are a few.

1. $S_1 = R_1 = \mathbb{Z}$, because not every integer has an integer square root. Or $S_1 = R_1 = \mathbb{R}$, because not every real number has a real square root (negative numbers do not).
2. Same as above work, because some numbers have two square roots (e.g., $(9, 3)$ and $(9, -3)$ are both in our relation).
3. $S_3 = \mathbb{Z}^+$ and $R_3 = \mathbb{R}^+$ works, because every positive integer has a positive real square root. $S_3 = \mathbb{R}^+$ and $R_3 = \mathbb{R}^-$ also work, because every positive real number has a negative real square root.

Problem 2. Consider the function $f(x) = |x|$. For each of the items below, find a domain D and target R such that

1. $f : D_1 \to R_1$ is neither 1-to-1 nor onto
2. $f : D_2 \to R_2$ is onto but not 1-to-1
3. $f : D_3 \to R_3$ is 1-to-1 but not onto
4. $f : D_4 \to R_4$ 1-to-1 and onto

Solution. There are many possible answers. Here is one example.

1. $D_1 = R_1 = \mathbb{Z}$
2. $D_2 = \mathbb{Z}$, $R_2 = \mathbb{N}$
3. $D_3 = \mathbb{N}$, $R_3 = \mathbb{Z}$
4. $D_4 = R_4 = \mathbb{N}$

Problem 3. Let $\equiv_5$ be the relation on the set of integers $\mathbb{Z}$ defined as $\{(a, b) : 5 \mid (a - b)\}$. (Note that we defined it a little differently in class on October 21; in class, we said $a \equiv_5 b$ if $a \bmod 5 = b \bmod 5$. Those two definitions result in the same relation, but for this problem we want you to work with $5 \mid (a - b)$.)

a) Prove that $\equiv_5$ is reflexive.

Solution. We will show $\forall a \in \mathbb{Z} : a \equiv_5 a$.

Let $a \in \mathbb{Z}$. $a - a = 0 = 5(0)$. Since 0 is an integer, $5|(a-a)$ by definition of divisibility. Therefore, $a \equiv_5 a$.

b) Prove that $\equiv_5$ is symmetric.

Solution. We will show $\forall a, b \in \mathbb{Z} : a \equiv_5 b \to b \equiv_5 a$.

So suppose $a \equiv_5 b$. Then $5 \mid (a-b)$, so $a - b = 5k$ for some integer k. By multiplying both sides by -1, we get $b - a = 5(-k)$. Since $-k$ is an integer, $5|(b-a)$ by definition of divisibility. Therefore, $b \equiv_5 a$.

Problem 4.

a) Let D_5 be the set of all natural numbers that are divisible by 5. Find a bijection $f \colon \mathbb{N} \to D_5$, where $\mathbb{N} = \{0, 1, 2, 3, ...\}$ is the set of natural numbers. Prove that is a bijection.

Solution. $f(x) = 5x$. It is a bijection, because for every $y \in D_5$, there is exactly one $x \in \mathbb{N}$ such that $f(x) = y$. We will prove this fact as follows.

To show that $x \in \mathbb{N}$, we set $x = y/5$. We know it will be a natural number because y is divisible by 5, so $x \in \mathbb{Z}$ by definition of divisibility. Moreover $y \geq 0$, so $y/5 \geq 0$, so $x \geq 0$, so $x \in \mathbb{N}$.

To show that $x \in \mathbb{N}$ is unique, suppose $f(x_1) = f(x_2) = y$. Then $5x_1 = 5x_2$. Dividing both sides by 5, we get $x_1 = x_2$.

b) Let $D_{51}^{>100}$ be the set of all integers that are greater than 100 and are divisible by 51. Find a bijection $f \colon \mathbb{N} \to D_{51}^{>100}$. You will prove that it is a bijection on the problem set.

Solution. $f(x) = 51(x+2)$. (Note $(x+2)$ since 0 needs to map to 102.)