CS 131 – Problem Set 7

Problems must be submitted by Monday November 3, 2025 at 11:59pm, on Gradescope.

Problem 1. (20 points) Prove that $A-(B\cap A) = A-B$ by completing `ps7p1.lean` and submitting the file to the autograder (assignment HW7 Programming Portion on gradescope). **Note 1:** Lean uses `X\Y` instead of $X-Y$ to denote elements of X that are not in Y. **Note 2:** In case you haven't realized this on the previous autograded homework assignments, the grade you receive is whatever the autograder gives you. There are no manual adjustments. So wait for the autograder to run and make sure it gives you the grade you think you should be getting.

Problem 2. (20 points total, at 10 each) Recall that the powerset $P(A)$ of some set A is the **set of** all possible **subsets** of A.

Note: In the problems below, you may need to prove that two sets are equal. In order to do so, you need to prove that any element belongs to one set if and only if it belongs to the other set.

a) Is it true that for any sets A and B, $P(A) \cap P(B) = P(A \cap B)$? If so, prove it. If not, provide a counterexample.

b) Is it true that for any sets A and B, that $P(A) \cup P(B) = P(A \cup B)$? If so, prove it. If not, provide a counterexample.

Problem 3. (15 points total, at 5 each) Consider the following relations on the set $\mathbb{R}^2$. For each of them, state whether it is transitive and prove your answer either way. (Recall that $\mathbb{R}$ is a set of real numbers. $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ is the set of all points (x, y) where x and y are real numbers.)

a) $C = \{((x, y), (z, t)) \colon (|x - z| > 1) \wedge (|y - t| > 1)\}$

b) $D = \{((x, y), (z, t)) \colon (x > z) \wedge (y > t)\}$

c) $E = \{((x, y), (z, t)) \colon (x > z) \vee (y > t)\}$

Problem 4. (22 points total) We are continuing from Problem 3 in this week's lab. Let $\equiv_5$ be the relation on the set of integers $\mathbb{Z}$ defined as $\{(a, b) : 5 \mid (a - b)\}$. (Note that we defined it a little differently in class on October 21; in class, we said $a \equiv_5 b$ if $a \bmod 5 = b \bmod 5$. Those two definitions result in the same relation, but for this problem we want you to work with $5 \mid (a - b)$.)

a) (5 points) Prove that $\equiv_5$ is an equivalence relation on $\mathbb{Z}$. Note: you already proved reflexivity and symmetry in Lab on October 29; there is no need to repeat those parts — just state "by what we proved in lab."

b) (5 points) For every integer a, we can consider the set $S_a = \{b \in \mathbb{Z} : a \equiv_5 b\}$ (i.e., all elements related to a under relation $\equiv_5$). Prove that $S_3 = S_8$. Recall that to prove two sets are the same, you can prove that an element is in one if and only if it is in the other. You can use the previous part in your proof.

c) (4 points) Now, consider a set of all parts $S = \{S_a : a \in \mathbb{Z}\}$. Note that using this set notation, it is not obvious that parts repeat, because we have a part for every integer; however, many parts will be the same (like $S_3 = S_8$ proven above), so in fact this set is finite (recall that a set contains an element no more than once, so repeated elements do not increase the set size).

What is $|S|$?

d) (8 points) For any equivalence relation R on X (not just $\equiv_5$ on $\mathbb{Z}$), we can consider a part S_x for every element x of X (defined as $S_x = \{b \in X : xRb\}$), and the set of $S = \{S_x : x \in X\}$. Prove that this set of parts is a partition, i.e., satisfies the following properties:

- the union of parts equals X
- if two parts S_x and S_y intersect, they are the same set (i.e., $\forall x, y \in X : S_x \cap S_y \neq \emptyset \to S_x = S_y$)

Problem 5. (7 points) Consider the bijective function you created in Lab 7 Problem 4b. Prove that it is bijective.

Problem 6. (16 points, at 4 each) For each of the formulas below, state whether it is in CNF, DNF, both, or neither. If it's not in CNF, make a truth table for the formula, and write down a CNF expression for it. If it's not in DNF, make a truth table and write down a DNF expression for it.

a) $x\overline{y}z + x\overline{z}$

b) $\overline{y}x\overline{z}w$

c) $y(\overline{x} + z)(\overline{y} + \overline{z})(x + y + \overline{z})$

d) $x + \overline{y} + z$