1a. 500 CS rooms, C(800, 500)

1b. C(2000,4), C(1996,4), …

1c. Assume that at least one group of students from the same room attended orientation.
By the Pigeonhole Principle, when mapping a domain of 1,556 elements to a target of 500 elements, there must exist at least one element in the target to which at least $\lceil 1,556/500 \rceil$ elements of the domain map.
$\lceil 1556/500 \rceil = \lceil 3.112 \rceil = 4$
Therefore, there exists a CS room to which at least 4 students map to and attend orientation.

1d. 60, since the first student in each 500 rooms would be placed then there would be 60 more that need to be placed for the remaining students into 60 of those rooms.

2. Starting out distributing 131 students among 6 cars where each car can have 0 or more students. Using the stars-and-bars method, 131 stones would be representing students, and we need to place 5 dividers to separate them into 6 groups. Unlike Problem 1 where dividers could only go between stones, here dividers can be placed anywhere to allow for empty cars. This gives us 5 extra positions with 136 total positions and we need to choose which 5 of these 136 positions will be dividers. Therefore, the answer is C(136, 5).

3.
10.5.6 Answer: C(37, 2)
We have 40 computers total, with 5 failures occurring. Three specific computers have the file, and all three must fail to wipe out all copies. Since these 3 specific computers must be among the 5 failures, we choose the remaining 2 failures from the other 37 computers. Therefore, the answer is C(37, 2).

10.6.4 Answer: 20! / (4!)^5
We distribute 20 distinct comic books to 5 kids, with each receiving exactly 4 books. Arrange all 20 books in a line in 20! ways. Give the first 4 to kid 1, the next 4 to kid 2, and so on. Within each group of 4 books, order does not matter. Since there are 4! ways to arrange each group and 5 groups total, we divide by (4!)^5. Therefore, the answer is 20! / (4!)^5.

10.7.3 Answer: $3 \times 11 \times 10 \times 9 \times 8$
We fill five positions: center, power forward, small forward, shooting guard, and point guard. Only 3 of 12 players can play center, giving 3 choices. After selecting the center, 11 players remain. We have 11 choices for power forward, 10 for small forward, 9 for shooting guard, and 8 for point guard. Therefore, the answer is $3 \times 11 \times 10 \times 9 \times 8$.

10.8.2 Answer: $C(52, 5) - C(13, 5) \times 4^5$

Using complement counting, the total number of 5-card hands is $C(52, 5)$. Hands with all different ranks require choosing 5 ranks from 13: $C(13, 5)$ ways. For each rank, we pick one of 4 suits: 4^5 ways. Therefore, hands with at least two matching ranks equals $C(52, 5) - C(13, 5) \times 4^5$.

10.9.5 Proof:

In a round-robin tournament with n players, each player plays n - 1 games. Win totals range from 0 to n - 1, giving n possible values. Since no person loses all games, no player has 0 wins. All players have between 1 and n - 1 wins, giving only n - 1 possible win totals. By the Pigeonhole Principle, with n players and n - 1 possible win totals, at least two players must have the same number of wins.

11.3.5 Answer: $2 \times 2 \times 5! - 3 \times 2 \times 4!$

Using inclusion-exclusion, let A be arrangements with mother next to daughter 1, and B be arrangements with mother next to daughter 2. Treating mother and daughter 1 as one unit gives $2 \times 5!$ arrangements for A. Similarly, B gives $2 \times 5!$ arrangements. For $A \cap B$, mother is next to both daughters, creating a block of three with 3×2 internal arrangements and 4! ways to arrange with the other 3 people. By inclusion-exclusion, the answer is $2 \times 5! + 2 \times 5! - 3 \times 2 \times 4! = 2 \times 2 \times 5! - 3 \times 2 \times 4!$.

4a. Answer: $1179 \times 2556 \times 3 \times 8$ bits

The iPhone 16 has a resolution of 1179×2556 pixels. Each pixel has 3 color components: red, green, and blue. Each component is stored using 8 bits. Therefore, the total number of bits is $1179 \times 2556 \times 3 \times 8 = 72{,}400{,}224$ bits.

4b. Answer: $2^{(1179 \times 2556 \times 3 \times 8)}$

Since the image uses $1179 \times 2556 \times 3 \times 8$ bits total, and each bit can be either 0 or 1, the number of different images that can be represented is 2 raised to the power of the total number of bits. Therefore, the answer is $2^{(72{,}400{,}224)}$.

4c. Answer: $2^0 + 2^1 + 2^2 + ... + 2^{(n-1)} = 2^n - 1$

For bitstrings of length less than n, we count strings of length 0, 1, 2, up to n - 1. A string of length k can be formed in 2^k ways. Summing over all possible lengths from 0 to n - 1 gives $2^0 + 2^1 + 2^2 + ... + 2^{(n-1)}$. Using the geometric series formula, this equals $2^n - 1$.

4d. Proof:

Assume we have a compression function that takes n-bit inputs and always produces outputs of length less than n. The number of possible n-bit inputs is 2^n. From part 4c, the number of possible outputs with length less than n is $2^n - 1$. By the Pigeonhole Principle, when mapping 2^n inputs to $2^n - 1$ outputs, at least two different inputs must map to the same output. Therefore, the compression function cannot be injective, and thus cannot be lossless.

4e. Proof:
Let $n = 1179 \times 2556 \times 3 \times 8 = 72{,}400{,}224$. If the compression saves at least 300 bits, then outputs have length at most $n - 300$. The total number of possible compressed outputs is $2^0 + 2^1 + ... + 2^{(n-300)} = 2^{(n-299)} - 1$. The total number of n-bit inputs is 2^n. By the Pigeonhole Principle, there exists at least one compressed output that corresponds to at least ceiling of 2^n divided by $2^{(n-299)} - 1$ different inputs. Since $2^{(n-299)} - 1 < 2^{(n-299)}$, we have ceiling of 2^n divided by $2^{(n-299)} - 1 >$ ceiling of 2^n divided by $2^{(n-299)} = 2^{299}$. Since $2^{299} > 2^{265}$, there exists a compressed image that can be decompressed in at least 2^{265} ways, which exceeds the number of particles in the visible universe.