2a. Yes, we will show $X \in P(A) \cap P(B) \leftrightarrow X \in P(A \cap B)$.

Suppose $X \in P(A) \cap P(B)$. Then $X \subseteq A$ and $X \subseteq B$. Let $x \in X$. Then $x \in A$ and $x \in B$, so $x \in A \cap B$. Therefore $X \subseteq A \cap B$, so $X \in P(A \cap B)$.

Suppose $X \in P(A \cap B)$. Then $X \subseteq A \cap B$. Let $x \in X$. Then $x \in A \cap B$, so $x \in A$ and $x \in B$. Therefore $X \subseteq A$ and $X \subseteq B$, so $X \in P(A)$ and $X \in P(B)$, thus $X \in P(A) \cap P(B)$.

2c. No

Counterexample: Let $A = \{1\}$ and $B = \{2\}$. Then $P(A) \cup P(B) = \{\varnothing, \{1\}, \{2\}\}$ but $P(A \cup B) = \{\varnothing, \{1\}, \{2\}, \{1,2\}\}$. Since $\{1,2\} \in P(A \cup B)$ but $\{1,2\} \notin P(A) \cup P(B)$, they are not equal.

3a. Not transitive

Counterexample: Let $P_1 = (0, 0)$, $P_2 = (1.6, 1.6)$, $P_3 = (0.5, 0.5)$. $(P_1, P_2) \in C$ since $|0-1.6| = 1.6 > 1$ and $|0-1.6| = 1.6 > 1$. $(P_2, P_3) \in C$ since $|1.6-0.5| = 1.1 > 1$ and $|1.6-0.5| = 1.1 > 1$. $(P_1, P_3) \notin C$ since $|0-0.5| = 0.5 \leq 1$
Therefore C is not transitive.

3b. Transitive

Proof: Suppose $((x,y),(z,t)) \in D$ and $((z,t),(u,v)) \in D$. Then $x > z$, $y > t$, $z > u$, and $t > v$. By transitivity of $>$, we have $x > u$ and $y > v$. Therefore $((x,y),(u,v)) \in D$.

3c. Not transitive

Counterexample: Let $P_1 = (1, 0)$, $P_2 = (0, 1)$, $P_3 = (2, 0)$. $(P_1, P_2) \in E$ since $1 > 0$. $(P_2, P_3) \in E$ since $1 > 0$. $(P_1, P_3) \notin E$ since $1 \leq 2$ and $0 \leq 0$. Therefore E is not transitive.

4a. Reflexivity and Symmetry: By what we proved in the lab.

Transitivity: Suppose $a \equiv_5 b$ and $b \equiv_5 c$. Then $a - b = 5k_1$ and $b - c = 5k_2$ for some integers k_1, k_2. Adding: $a - c = 5(k_1 + k_2)$. Since $k_1 + k_2$ is an integer, $5 \mid (a - c)$, so $a \equiv_5 c$.

4b. Suppose $b \in S_3$. Then $3 - b = 5k$ for some integer k. So $8 - b = 5k + 5 = 5(k + 1)$. Therefore $b \in S_8$.
Suppose $b \in S_8$. Then $8 - b = 5m$ for some integer m. So $3 - b = 5m - 5 = 5(m - 1)$. Therefore $b \in S_3$.

4c. $|S| = 5$

The equivalence classes are determined by remainder when dividing by 5. There are 5 possible remainders (0, 1, 2, 3, 4), so there are 5 distinct equivalence classes.

4d. Property 1: For any $x \in X$, by reflexivity xRx, so $x \in S_x$. Therefore $\cup S = X$.

Property 2: Suppose $S_x \cap S_y \neq \varnothing$. Then there exists $z \in S_x \cap S_y$, so xRz and yRz. By symmetry, zRy. By transitivity, xRy. Now let $w \in S_x$. Then xRw. Since xRy and xRw, by transitivity yRw, so $w \in S_y$. Similarly, $S_y \subseteq S_x$. Therefore $S_x = S_y$.

5. For every $y \in D_{51}$^(>100), set $x = (y/51) - 2$. Since y is divisible by 51 and $y > 100$, we have $y \geq 102$, so $y/51 \geq 2$. Thus $x = (y/51) - 2 \geq 0$ and $x \in \mathbb{Z}$, so $x \in \mathbb{N}$. We verify $f(x) = 51((y/51) - 2 + 2) = y$.

For uniqueness, if $f(x_1) = f(x_2)$, then $51(x_1 + 2) = 51(x_2 + 2)$. Dividing by 51: $x_1 + 2 = x_2 + 2$, so $x_1 = x_2$.

6a: $xyz + xz$

DNF only. Already in DNF.
Truth table:

x	y	z	xyz	xz	xyz + xz
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	0	0	0
1	1	1	1	1	1

CNF = $(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(\bar{x} + \bar{y} + z)$

6b: $yxzw$
Both CNF and DNF. Single product term.

6c: $y(x + z)(y + z)(x + y + z)$
CNF only. Already in CNF.
Truth table:

x	y	z	Result
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

DNF = $\bar{x}yz + xy\bar{z} + xyz$

6d: x + y + z

CNF only. Single clause.
Truth table:

x	y	z	x + y + z
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

DNF = $\bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$