CS 131 – Practice Problems for Midterm 1.
Note: the actual midterm will not be quite as long as this set.

Problem 1. Give truth tables for the following propositions (intermediate columns are allowed but not required; only the final column is graded):

a) $\neg p \vee q$

b) $p \wedge \neg q$

c) $(p \wedge q) \rightarrow r$

Problem 2. Prove that the proposition $\Big((P \rightarrow Q) \vee (P \rightarrow R)\Big) \rightarrow \Big(P \rightarrow (Q \vee R)\Big)$ is a tautology.

Problem 3. If you want more practice with rules of propositional logic: ZyBook 1.5.2.

Problem 4. Let the domain of discourse be BU students. Let

- C be the set of CAS students
- Q be the set of Questrom students
- E be the set of ENG students
- $F(x, y)$ be the statement "x is a friend of y"

Translate the following into quantified statements. Also, negate and simplify the resulting quantified statements for parts a–c.

a) Anyone in CAS is not in Questrom.

b) There is someone who is in CAS and in ENG.

c) Any student in ENG but not in Questrom is not in CAS.

d) Every student in CAS has a friend in Questrom

e) Every ENG student has a friend in CAS who has no friends in Questrom

f) Every Questrom student has exactly one friend in ENG

g) Everyone except Leo has at least two distinct friends

Problem 5. Prove that if $5|x$ and $5|y$ then $5|(x + y)$.

Problem 6. For all $m, n \in \mathbb{Z}$, if $m^2 + n^2$ is odd, then either m is odd, or n is odd. **Note:** For the purposes of this problem, a number x is odd if it is not even.

Problem 7. Prove that $\frac{a+b}{2} \geq \min(a, b)$

Problem 8. Prove that if $n \in \mathbb{N}$ is composite, then it has at least one factor greater than 1 and no greater than $\sqrt{n}$.