

$$E_k = \frac{1}{2} m v^2 \quad \text{tg } \vartheta_B = \frac{w_2}{w_1} = w_{21} \quad pV = nRT \quad \vec{\psi} = \iint \vec{D} d\vec{S} = AD \quad H_\lambda = \frac{\Delta M_e}{\Delta \lambda}$$

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V\psi = E\psi \quad M_e = \sigma T^4 \quad \Phi_e = \frac{L}{4\pi r^2} \int \frac{\Delta \varphi}{2\pi} = \frac{\Delta x}{\lambda_1} = \frac{x_2 - x_1}{\lambda} S_2 \quad V = c/\lambda \quad \Phi = NBS$$

$$U_{ef} = \frac{U_m}{E} \quad E = \hbar \omega \quad \Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad X_L = \frac{U_m}{I_m} = \omega L = 2\pi f L \quad F = \frac{m_1 m_2}{r^2} \quad \mathcal{H}$$

$$\vec{B} = \mu \frac{NI\sqrt{2}}{2\pi r m_e} \quad v = \frac{wh}{2\pi r m_e} \quad \varphi_E = \frac{E_e}{\varphi_0} = k \frac{\varphi}{r^2} \quad \varphi = |\varphi_A - \varphi_B| \quad T = \frac{4 n_1 n_2}{(n_2 + n_1)^2} \quad g = \frac{m_1 m_2}{r^2} \quad \mathcal{H}$$

$$k = \rho^2 / 2m \quad m_o = \frac{M_m}{N_A} = \frac{M_r \cdot 10^{-3}}{N_A} \quad \lambda = \frac{h}{\sqrt{2eUm_e}} \quad R = \rho \frac{L}{S}$$

$$f_o = \frac{1}{2\pi} \frac{1}{\frac{L}{e}} \quad \psi(x) = \sqrt{2/L} \sin \frac{n\pi x}{L}$$

$$\oint \vec{B} d\vec{\ell} = \mu \iint_S \vec{J} d\vec{S} \quad \vec{S} =$$

$$C(s) \quad v_k = \sqrt{\frac{3kT}{m_o}} = \sqrt{\frac{3kTN_A}{M_m}} = \sqrt{\frac{3R_m T}{M_r \cdot 10^{-3}}}$$

$$\lambda = \frac{\ln 2}{T} \quad F_h = Sh\rho g$$

$$\left(\frac{E_t}{E_o} \right)_{\parallel} = \frac{2 \cos \vartheta_1 \cos \vartheta_2}{\cos(\vartheta_1 - \vartheta_2) \sin(\vartheta_1 + \vartheta_2)}$$

$$E_y = E_o \sin(kx - \omega t) \quad R = R_o \sqrt[3]{A} \quad c(s) \rightarrow s$$

$$S = \frac{1}{A} \frac{d\omega}{dt} \quad \omega = U_m \sin \omega(t - \tau) = U_m \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

Exámen 2

Matemática Discreta

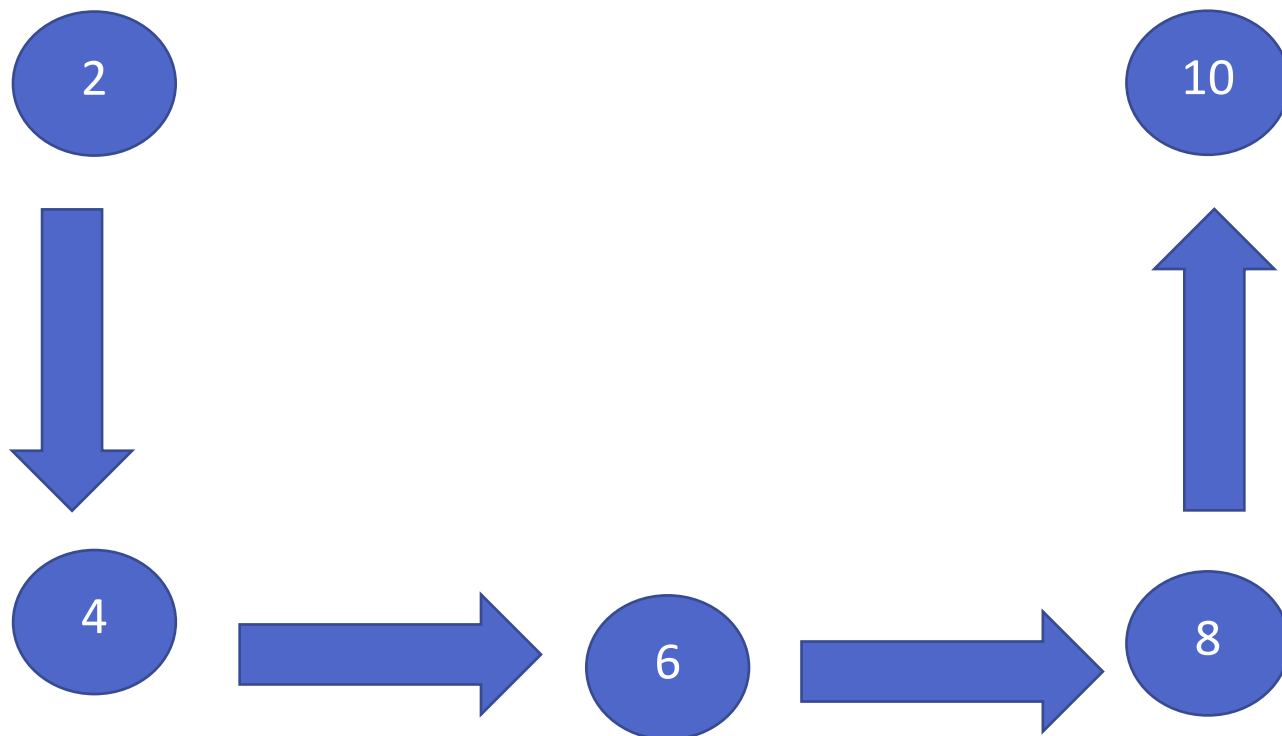
Andrés Montenegro

UTC

Problema 1

1. gráficamente el dígrafo $D = (Z^+, R)$, donde R es la relación definida sobre el conjunto de los números naturales consistente en todos los pares de números de la forma $(x, x + 2)$.

a. Trace una idea del dígrafo correspondiente al menos con 5 pares de ordenados que pertenecen a la relación. 5pts



Problema 2

2. Para $U = \mathbb{Z}^+$, $A = \{2, 3, 4, 5, 6, 7\}$, $B = \{10, 11, 12, 13, 14\}$, escribir los elementos de la relación $R \rightarrow A \times B$, donde

aRb si y solo si **a** divide (exactamente) a **b**.

a. Calcule G_R de la relación R 5pts

$A \times B =$

$\{\{2,10\},\{2,11\},\{2,12\},\{2,13\},\{2,14\},\{3,10\},\{3,11\},\{3,12\},\{3,13\},\{3,14\},\{4,10\},$
 $\{4,11\},\{4,12\},\{4,13\},\{4,14\},\{5,10\},\{5,11\},\{5,12\},\{5,13\},\{5,14\},\{6,10\},\{6,11\},\{$
 $6,12\},\{5,13\},\{6,14\},\{7,10\},\{7,11\},\{7,12\},\{7,13\},\{7,14\}\}$

$B\%A = 0$

$G_R =$

$\{\{2,10\},\{2,12\},\{2,14\},\{3,12\},\{4,12\},\{5,10\},\{6,10\},\{7,14\}\}$

b .De la matriz relación correspondiente de R 3pts

	10	11	12	13	14
2	1	0	1	0	1
3	0	0	1	0	0
4	0	0	1	0	0
5	1	0	0	0	0
6	1	0	0	0	0
7	0	0	0	0	1

Problema 3

3 .A = {huevos, leche, maíz} y B = {vacas, cabras, gallinas}. Escribir la relación R de A a B definida por:

(a, b) e R \leftrightarrow a es producido por b.

Calcule G_R de la relación R. 4puntos.

$$G_R =$$

{{huevos,gallinas},{leche,vacas},{leche,cabras}}

De la matriz relación correspondiente de R.3puntos.

	vacas	cabras	gallinas
huevos	0	0	1
leche	1	1	0
maíz	0	0	0

Problema 4

4.Sea A = {1,2,3,4} y definimos la relación

$aRb \Leftrightarrow b$ es múltiplo de a , $\forall a, b \in A$

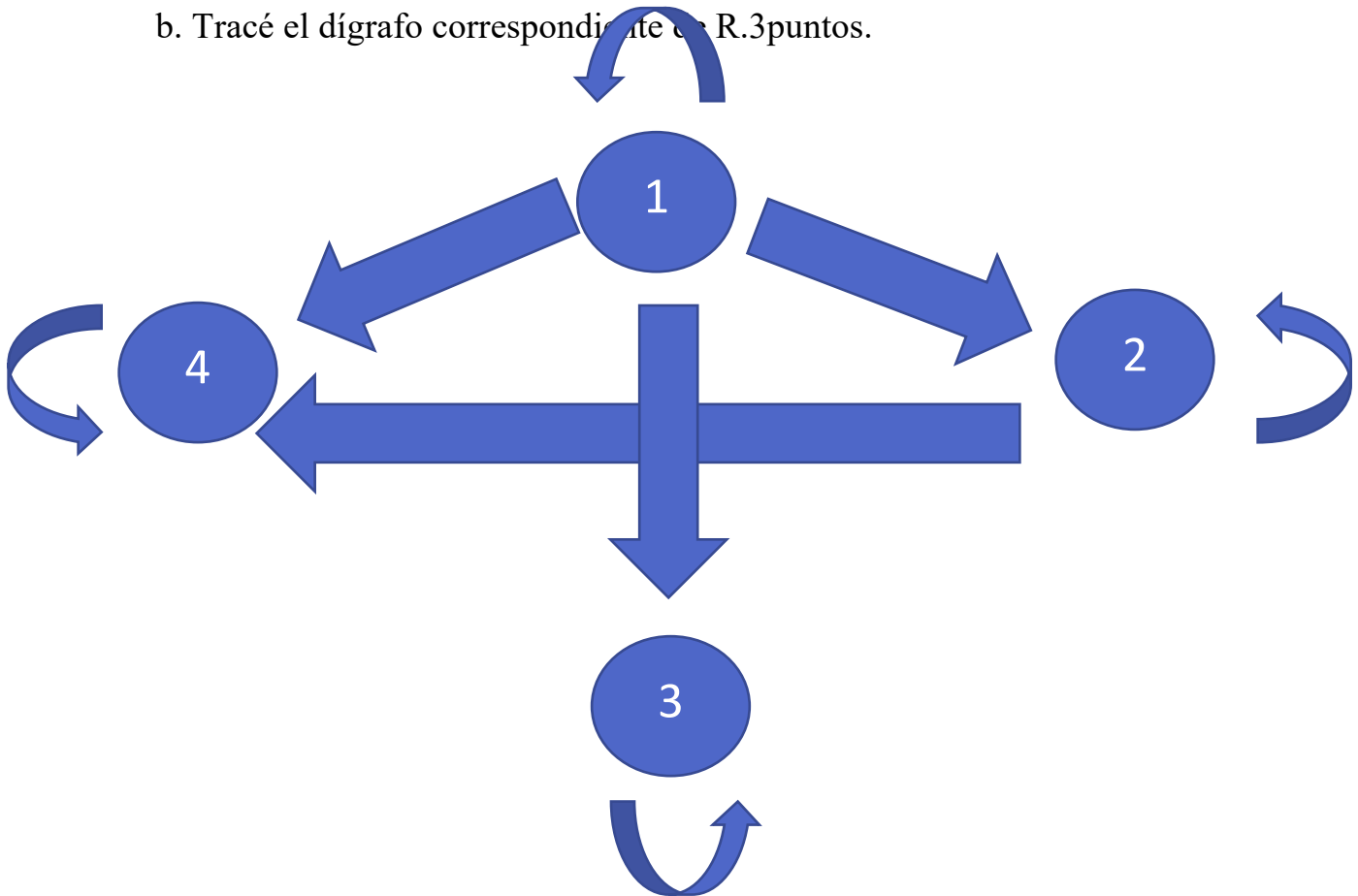
$$G_R =$$

{{1,1},{1,2},{1,3},{1,4},{2,2},{2,4},{3,3},{4,4}}

a. Calcule la matriz de la relación R. 4 puntos.

	1	2	3	4
1	1	1	1	1
2	0	1	0	1
3	0	0	1	0
4	0	0	0	1

b. Tracé el dígrafo correspondiente a R. 3 puntos.



C. Encuentre todas las trayectorias de longitud 2. USANDO PRODUCTO BOOLEANO DE MATRICES.. 4 puntos.

Problema 5

5. Dados los conjuntos:

$$A = \{x \in \mathbb{N} / 0 < x < 15\}$$

$$B = \{y \in \mathbb{Z} / -5 < y < 25\}$$

Se define la relación:

$$R = \{(x,y) \in A \times B / y = 1 + x^2\}$$

$$A = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$$

$$B = \{-4,-3,-2,-$$

$$1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24\}$$

$$A \times B = \{\{1,-4\},\{1,-3\},\{1,-2\},\{1,-$$

$$1\},\{1,0\},\{1,1\},\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{1,6\},\{1,7\},\{1,8\},\{1,9\},\{1,10\},\{1,11\},\{1,12\},$$
$$\{1,13\},\{1,14\},\{1,15\},\{1,16\},\{1,17\},\{1,18\},\{1,19\},\{1,20\},\{1,21\},\{1,22\},\{1,23\},\{1,$$
$$24\},\{2,-4\},\{2,-3\},\{2,-2\},\{2,-$$

$$1\},\{2,0\},\{2,1\},\{2,2\},\{2,3\},\{2,4\},\{2,5\},\{2,6\},\{2,7\},\{2,8\},\{2,9\},\{2,10\},\{2,11\},\{2,12\},$$
$$\{2,13\},\{2,14\},\{2,15\},\{2,16\},\{2,17\},\{2,18\},\{2,19\},\{2,20\},\{2,21\},\{2,22\},\{2,23\},\{2,$$
$$24\},\{3,-4\},\{3,-3\},\{3,-2\},\{3,-$$

$$1\},\{3,0\},\{3,1\},\{3,2\},\{3,3\},\{3,4\},\{3,5\},\{3,6\},\{3,7\},\{3,8\},\{3,9\},\{3,10\},\{3,11\},\{3,12\},$$
$$\{3,13\},\{3,14\},\{3,15\},\{3,16\},\{3,17\},\{3,18\},\{3,19\},\{3,20\},\{3,21\},\{3,22\},\{3,23\},\{3,$$
$$24\},\{4,-4\},\{4,-3\},\{4,-2\},\{4,-$$

$$1\},\{4,0\},\{4,1\},\{4,2\},\{4,3\},\{4,4\},\{4,5\},\{4,6\},\{4,7\},\{4,8\},\{4,9\},\{4,10\},\{4,11\},\{4,12\},$$
$$\{4,13\},\{4,14\},\{4,15\},\{4,16\},\{4,17\},\{4,18\},\{4,19\},\{4,20\},\{4,21\},\{4,22\},\{4,23\},\{4,$$
$$24\},\{5,-4\},\{5,-3\},\{5,-2\},\{5,-$$

$$1\},\{5,0\},\{5,1\},\{5,2\},\{5,3\},\{5,4\},\{5,5\},\{5,6\},\{5,7\},\{5,8\},\{5,9\},\{5,10\},\{5,11\},\{5,12\},$$
$$\{5,13\},\{5,14\},\{5,15\},\{5,16\},\{5,17\},\{5,18\},\{5,19\},\{5,20\},\{5,21\},\{5,22\},\{5,23\},\{5,$$
$$24\},\{6,-4\},\{6,-3\},\{6,-2\},\{6,-$$

1},{6,0},{6,1},{6,2},{6,3},{6,4},{6,5},{6,6},{6,7},{6,8},{6,9},{6,10},{6,11},{6,12},
 {6,13},{6,14},{6,15},{6,16},{6,17},{6,18},{6,19},{6,20},{6,21},{6,22},{6,23},{6,
 24},{7,-4},{7,-3},{7,-2},{7,-
 1},{7,0},{7,1},{7,2},{7,3},{7,4},{7,5},{7,6},{7,7},{7,8},{7,9},{7,10},{7,11},{7,12},
 {7,13},{7,14},{7,15},{7,16},{7,17},{7,18},{7,19},{7,20},{7,21},{7,22},{7,23},{7,
 24},{8,-4},{8,-3},{8,-2},{8,-
 1},{8,0},{8,1},{8,2},{8,3},{8,4},{8,5},{8,6},{8,7},{8,8},{8,9},{8,10},{8,11},{8,12},
 {8,13},{8,14},{8,15},{8,16},{8,17},{8,18},{8,19},{8,20},{8,21},{8,22},{8,23},{8,
 24},{9,-4},{9,-3},{9,-2},{9,-
 1},{9,0},{9,1},{9,2},{9,3},{9,4},{9,5},{9,6},{9,7},{9,8},{9,9},{9,10},{9,11},{9,12},
 {9,13},{9,14},{9,15},{9,16},{9,17},{9,18},{9,19},{9,20},{9,21},{9,22},{9,23},{9,
 24},{10,-4},{10,-3},{10,-2},{10,-
 1},{10,0},{10,1},{10,2},{10,3},{10,4},{10,5},{10,6},{10,7},{10,8},{10,9},{10,10}
 ,{10,11},{10,12},{10,13},{10,14},{10,15},{10,16},{10,17},{10,18},{10,19},{10,
 20},{10,21},{10,22},{10,23},{10,24},{11,-4},{11,-3},{11,-2},{11,-
 1},{11,0},{11,1},{11,2},{11,3},{11,4},{11,5},{11,6},{11,7},{11,8},{11,9},{11,10}
 ,{11,11},{11,12},{11,13},{11,14},{11,15},{11,16},{11,17},{11,18},{11,19},{11,
 20},{11,21},{11,22},{11,23},{11,24},{12,-4},{12,-3},{12,-2},{12,-
 1},{12,0},{12,1},{12,2},{12,3},{12,4},{12,5},{12,6},{12,7},{12,8},{12,9},{12,10}
 ,{12,11},{12,12},{12,13},{12,14},{12,15},{12,16},{12,17},{12,18},{12,19},{12,
 20},{12,21},{12,22},{12,23},{12,24},{13,-4},{13,-3},{13,-2},{13,-
 1},{13,0},{13,1},{13,2},{13,3},{13,4},{13,5},{13,6},{13,7},{13,8},{13,9},{13,10}
 ,{13,11},{13,12},{13,13},{13,14},{13,15},{13,16},{13,17},{13,18},{13,19},{13,
 20},{13,21},{13,22},{13,23},{13,24},{14,-4},{14,-3},{14,-2},{14,-
 1},{14,0},{14,1},{14,2},{14,3},{14,4},{14,5},{14,6},{14,7},{14,8},{14,9},{14,10}
 ,{14,11},{14,12},{14,13},{14,14},{14,15},{14,16},{14,17},{14,18},{14,19},{14,
 20},{14,21},{14,22},{14,23},{14,24}}

a. Calcular el CONJUNTO G_R . 5 puntos.

$G_R =$

{{1,2},{2,5},{3,10},{4,17}}

b. Calcular la matriz relación. . 5 puntos.

Resumido por motivos de espacio (AXB se detalla completo arriba):

	2	5	10	13	17
1	1	0	0	0	1
2	0	1	0	0	0
3	0	0	1	0	0
4	0	0	0	0	1
5	0	0	0	0	0

Problema 6

6. Sean los conjuntos: $A = \{2; 4; 5\}$ y

$B = \{3; 4\}$, y la relación $R: A \rightarrow B$, definida por “.b.. es mayor que a...”.

a) Elabora la matriz relación .4 puntos

	3	4
2	1	1
4	0	0
5	0	0

b)Determina G_R . 5 puntos

$$G_R =$$

$$\{\{2,3\},\{2,4\}\}$$

c) Hallar Dom(R) y Ran (R).4 puntos.

Problema 7

7. Sean los conjuntos:

$$A = \{a/a \text{ es impar positivo} \wedge a < 8\}$$

$$A = \{1,3,5,7\}$$

$$B = \{b/b \text{ es par} \wedge 0 \leq b < 6\}$$

$$B = \{0,2,4\}$$

y una relación binaria: $R: A \rightarrow B$

definida por: $R = \{(a, b) / a < b\}$

$$A \times B = \{\{1,2\},\{1,4\},\{3,4\}\}$$

Indique si son verdaderas (V) o Falsas (F), según corresponda JUSTIFIQUE cada una :

- a. $\text{Dom}(R) =$ 2 puntos
- b. $n[\text{Dom}(R) \cap \text{Ran}(R)] = 1.$ 2 puntos
- c. $\text{Ran}(R) = \{2, 4\}$ 2 puntos
- d. $\text{Dom}(R) \cup \text{Ran}(R) = \{0, 1, 2, 3, 4\}$ 2 puntos

Problema 8 .6PUNTOS.

8. Sean los conjuntos: $A = \{12; 8; 5\}$ y $B = \{2; 3; 4; 5\}$ y la relación " R ": $A \rightarrow B$, definida por "...a es múltiplo de . .b."

$A \times B =$

$\{\{12,2\},\{12,3\},\{12,4\},\{12,5\},\{8,2\},\{8,3\},\{8,4\},\{8,5\},\{5,2\},\{5,3\},\{5,4\},\{5,5\}\}$

Determina G_R 4 pts

$G_R =$

$\{\{12,2\},\{12,3\},\{12,4\},\{8,2\},\{8,4\},\{8,5\}\}$

b) Halla el $\text{Dom}(R)$ y $\text{Ran}(R)$ 2 pts

Problema 9

9. Sean $A = \{2, 3, 5, 1\}$; $B = \{9, 2, 8, 4\}$

y la relación $P = \{ (a ; b) \in A \times B / a^2 = b \}$

Hallar: $n [\text{Dom}(R)] + n [\text{Ran}(R)]$ 4 puntos.

$$G_R =$$

$$\{\{2,4\},\{3,9\}\}$$

Nota: Entiéndase, **n** como la cardinalidad.