Power Analysis

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- Power analysis is a method used to determine the sample size for an measurably accurate experiment:
  - Confidence Intervals
  - Hypothesis Testing
- I'll be covering power analysis for confidence intervals and hypothesis testing separately as these experiments have a different nature.

Aim is different for Confidence Intervals and Hypothesis Testing

- For Confidence intervals the aim of power analysis is to determine sample size n to achieve a specific width for the confidence interval

Aim is different for Confidence Intervals and Hypothesis Testing

- For Hypothesis testing the aim of power analysis is to determine sample size n to reject H0 with a certain probability if it is false
  - This probability is called Power
  - Power=P(Reject H0 | H0 is false)
  - Usually Power is set to 80 %, 90 %

Power Analysis for Confidence Intervals

Theory

- Power analysis is used to determine the sample size needed to achieve a certain level of precision in the confidence interval (determined by the width of the interval)
- Input
  - Precision
  - Standard Deviation
  - Confidence 95 %, 99 %

Theory for 1 sample, Continuous outcome

The margin of error in the one sample confidence interval for μ can be written as follows:

$$E = Z \frac{\sigma}{\sqrt{n}}.$$

Now we need to determine what the right E is for our experiment?

- For weights of cows E=10
- For weigths of cats E=0.5

Our goal is to determine the sample size, n, that ensures that the margin of error, "**E**," does not exceed a specified value. We can take the formula above and, with some algebra, solve for **n**:

$$n = \left(\frac{Z\sigma}{E}\right)^2$$

Example

- https://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_power/bs704_power_print.html
- Input
  - 95% confidence interval estimate, Z=1.96
  - Allowed E, margin of error 5 units.
  - Standard deviation is 20
- Output

$$n = \left(\frac{Z\sigma}{E}\right)^2 = \left(\frac{1.96(20)}{5}\right)^2 = \mathbf{61.5}$$

$$5 = 1.96\frac{20}{\sqrt{62}}$$

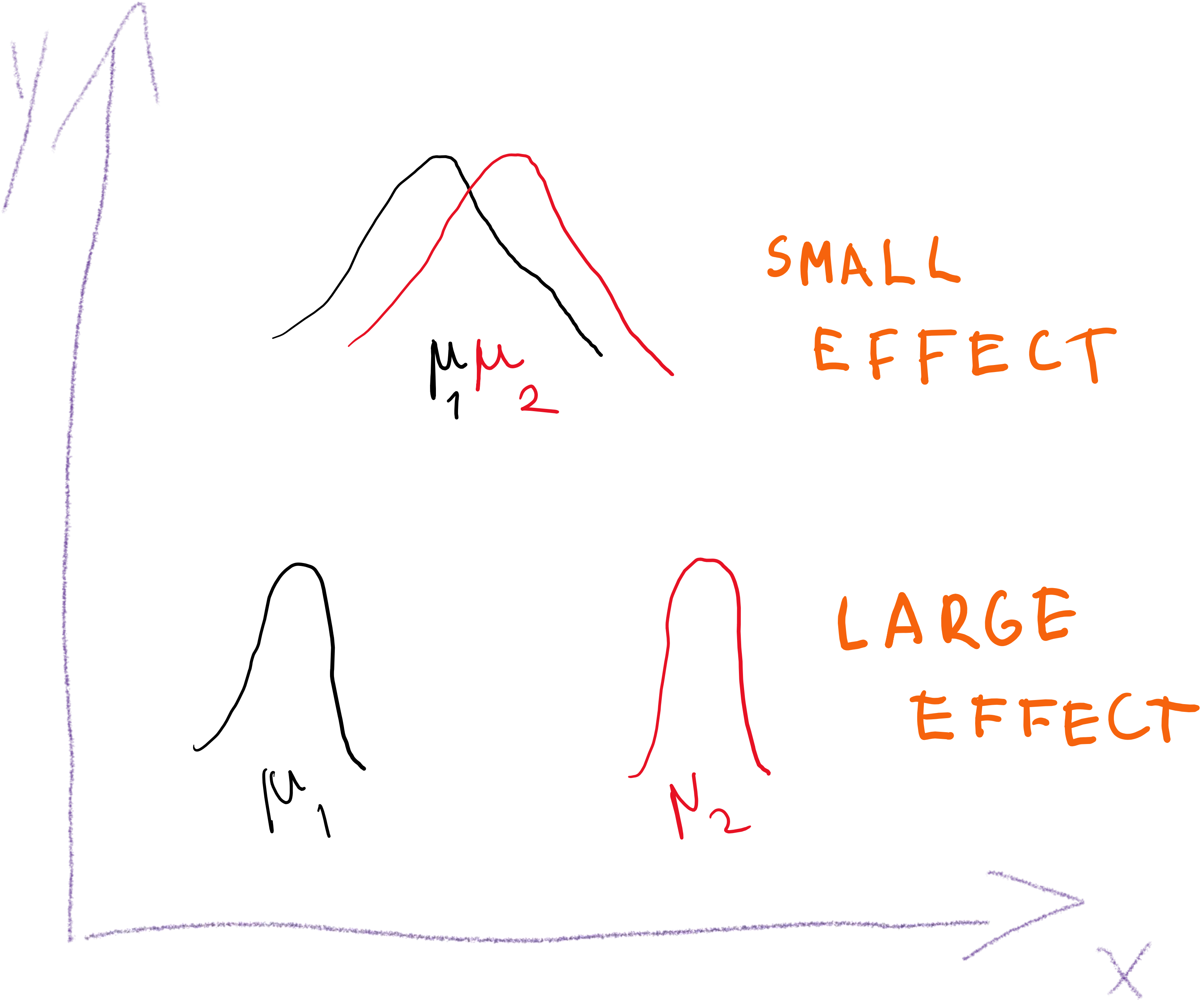
Power Analysis for Hypothesis testing

Intro

- For Hypothesis testing the aim of power analysis is to determine sample size **n** to **reject H0** with a **certain probability (Power) if it is false**
- Input variables
  - Effect Size – The difference we want to be able to classify as significant with probability=Power
  - Standard Deviation of data
  - Test Confidence 95 %, 99 %
    - Type I error P(Reject H0 | H0 is True)
  - Power (80%/90%) – P(Rejecting H0 | H0 is False)
    - Power can be also called the probability of rejecting Type II error = P(Not Rejecting H0 |H0 is False)

Effect Size

- $E = \frac{\mu_1 - \mu_2}{\sigma}$
- More Materials



Power

$$\textit{\textbf{Effect Size}} = ES = \frac{|\mu_1 - \mu_0|}{\sigma}$$

Distribution of $\bar{X}$ Under H_0: μ = 90 and Under H_1: μ = 94

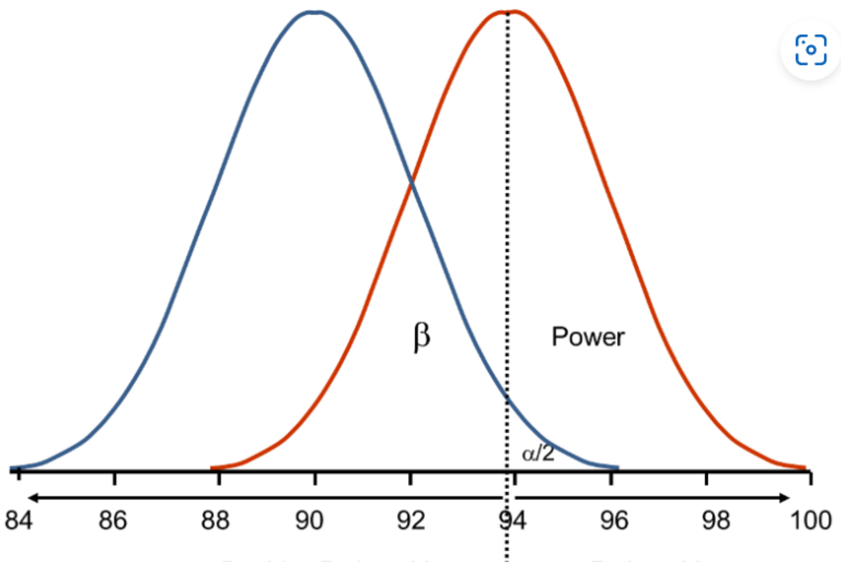
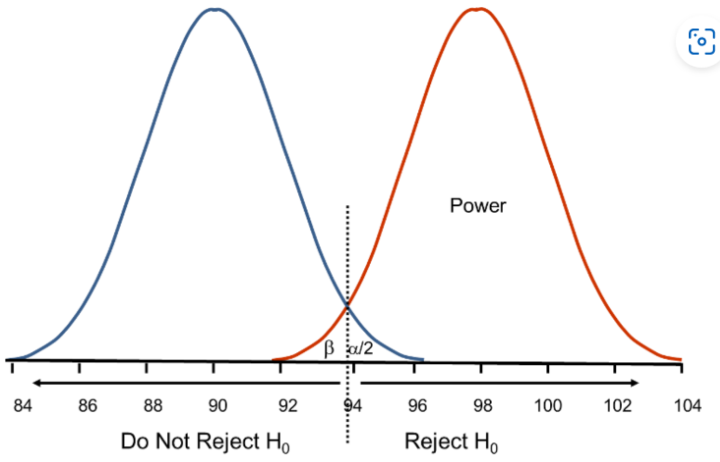


Figure - Distribution of $\bar{X}$ Under H_0: μ = 90 and Under H_1: μ = 98.



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References

- Lisa Sullivan, PhD,Professor of Biosatistics,Boston Univeristy School of Public Health, Power and Sample Size Determination (bu.edu)