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Una Acreditación con Rostro Humano

Hacia una Universidad comprometida con la paz territorial

Metaheuristic

Iterated Local Search

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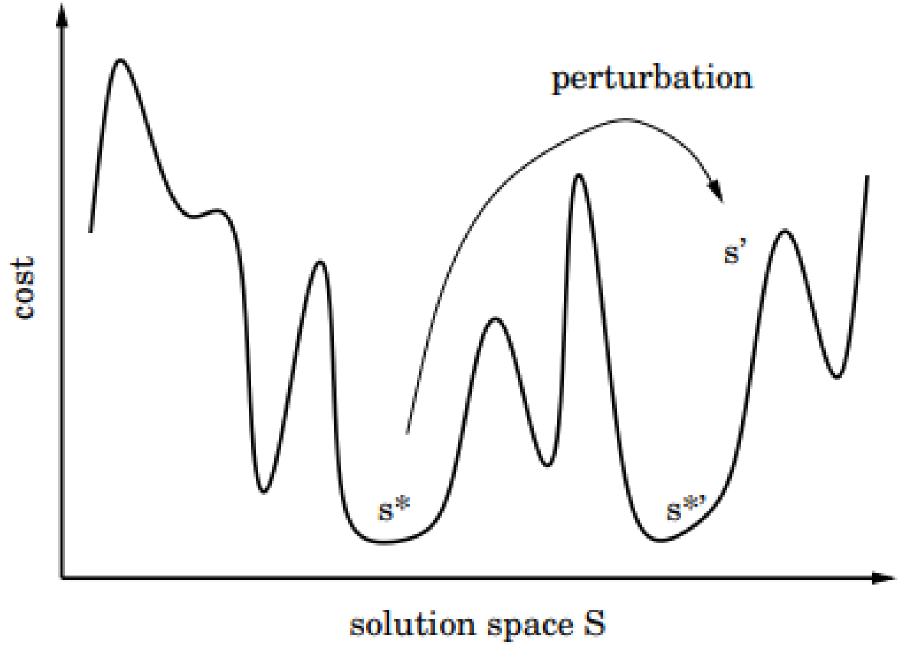
Iterated Local Search or also called iterated descent, large-step Markov chains, iterated Lin-Kernighan or chained local optimization is a metaheuristic algorithm that will allow to evaluate different local searches, depending on an acceptance criterion and a perturbation.



Algorithm local iterated search

```
for i =0→ length (tw) do
   for j=0 → length (n) do
      for z=0 → length (p) do
          for  k =0 → E do
              F(s) ← Choose objective function
              S ← Some initial random candidate solution
              H← S
              Best ←S
              repeat
                     N ← n [ j ]
                     repeat
                            R ← Tweak(Copy(S))
                            if (Quality(R) < Quality(S) then
                                   S ← R
                     until N
                     if (Quality(S) < Quality(Best)) then
                            Best ← S
                     if (Quality (S) < Quality(H)) then
                            H ← S
                     S ← Tweak(Copy(H))
              until Maximum objective function evaluations: 5000
              return MeanBest
          end for
      end for
end for
```

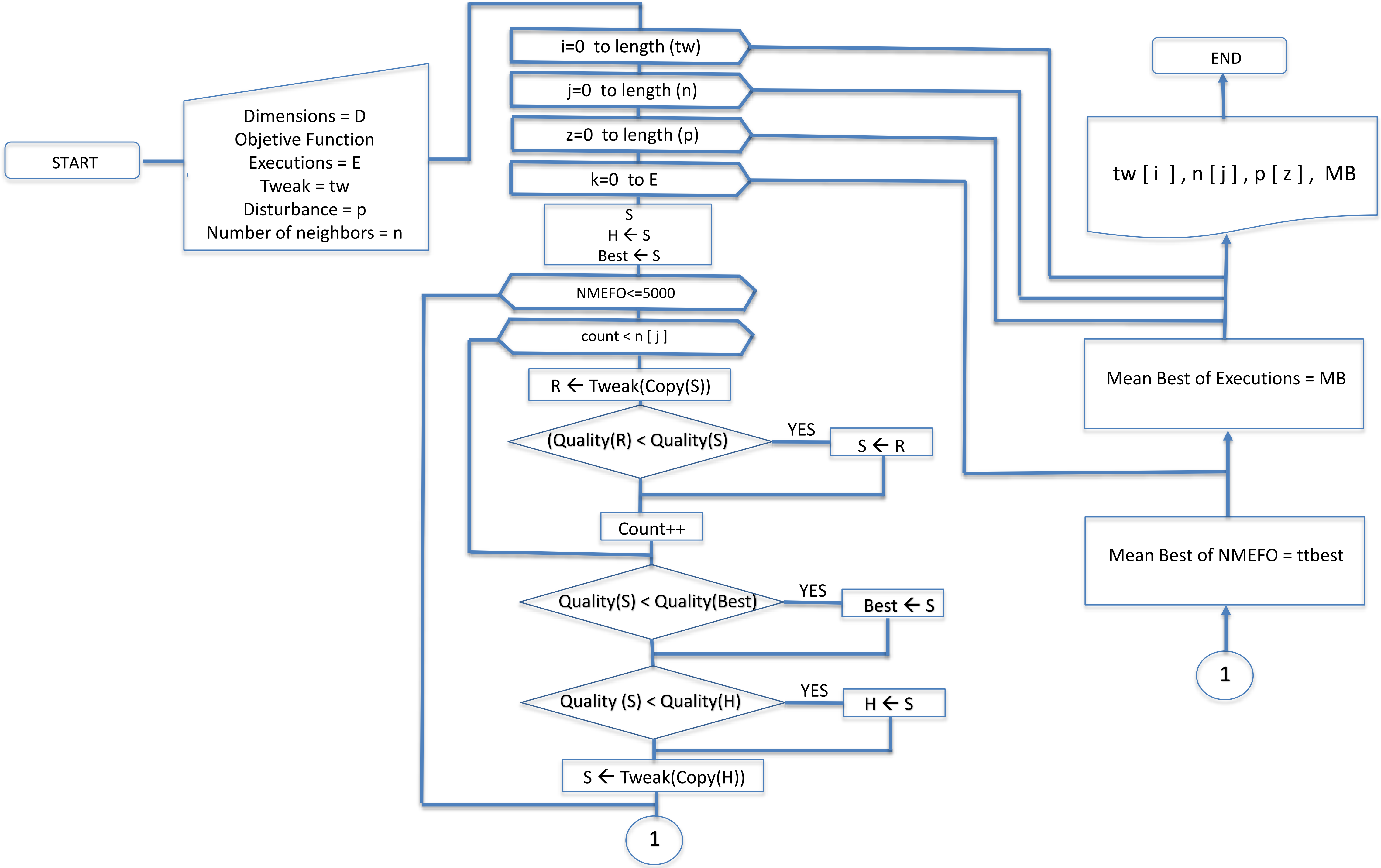
Local search

Value of tweak: tw [i]

Acceptance criteria: “better”

Perturbation

Value of tweak: p [z]


START
Dimensions = D
Objetive Function
Executions = E
Tweak = tw
Disturbance = p
Number of neighbors = n
i=0 to length (tw)
j=0 to length (n)
z=0 to length (p)
k=0 to E
S
H ← S
Best ← S
NMEFO<=5000
count < n [j]
R ← Tweak(Copy(S))
(Quality(R) < Quality(S)
YES
S ← R
Count++
Quality(S) < Quality(Best)
YES
Best ← S
Quality (S) < Quality(H)
YES
H ← S
S ← Tweak(Copy(H))
1
END
tw [i] , n [j] , p [z] , MB
Mean Best of Executions = MB
Mean Best of NMEFO = ttbest
1

Parameters

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Objective Function

Separable unimodal: Sphere [-100, 100]

No-separable unimodal: Schwefel [-100, 100]

Separable multimodal: Rastrigin [-5.12, 5.12]

No-separable multimodal: Griewank [-600, 600]

Parameters

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Tweak : 0.2 0.6 1.0

Dimensions: 20 50 100

Number of neighbors: 10 20

Disturbance : 1.0 2.0 3.0

Evaluation of results
Local Iterated search

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12 experiments with variable dimensionality and the four objective functions.

Experiment #1

Number of executions → E = 30
Dimensions → D=20
Objective Function = Sphere

TW	N	P	MEAN FO
0.2	10	1.0	5598.377467483199
0.2	10	2.0	5690.514083039108
0.2	10	3.0	5061.077172508967
0.2	20	1.0	2950.8273803573456
0.2	20	2.0	2842.929340226294
0.2	20	3.0	2942.888479859467
0.6	10	1.0	5511.620915959865
0.6	10	2.0	5681.537711067781
0.6	10	3.0	5541.859005608835

TW	N	P	MEAN FO
0.6	20	1.0	2899.3755971036685
0.6	20	2.0	3084.4825468080253
0.6	20	3.0	2914.392050505634
1.0	10	1.0	5583.66148743505
1.0	10	2.0	5285.38253617788
1.0	10	3.0	5285.42789762258
1.0	20	1.0	2887.637149061792
1.0	20	2.0	2998.48817844319
1.0	20	3.0	2528.63711550738

Table 1. Experiment number 1 with 20 dimensions and sphere as objective function.

Experiment #2

Number of executions → E = 30
Dimensions → D=50
Objective Function = Sphere

TW	N	P	MEAN FO
0.2	10	1.0	14967.193795690613
0.2	10	2.0	14691.206376676484
0.2	10	3.0	15814.560294743367
0.2	20	1.0	7746.780197999574
0.2	20	2.0	7902.129904064993
0.2	20	3.0	7753.2193655409565
0.6	10	1.0	15324.448796903716
0.6	10	2.0	15237.237604630247
0.6	10	3.0	14919.64551751639

TW	N	P	MEAN FO
0.6	20	1.0	7780.447010884757
0.6	20	2.0	7625.107575792074
0.6	20	3.0	7387.71943645275
1.0	10	1.0	14836.258447539132
1.0	10	2.0	15210.015167243107
1.0	10	3.0	14316.937056155246
1.0	20	1.0	7658.228533145157
1.0	20	2.0	7361.933157591401
1.0	20	3.0	7427.13501300498

Table 2. Experiment number 1 with 50 dimensions and sphere as objective function.

Experiment #3

Number of executions → E = 30
Dimensions → D=100
Objective Function = Sphere

TW	N	P	MEAN FO
0.2	10	1.0	29800.84839392656
0.2	10	2.0	29196.728175082422
0.2	10	3.0	30119.306277540767
0.2	20	1.0	15647.949907638329
0.2	20	2.0	15779.069076991042
0.2	20	3.0	15274.559400876902
0.6	10	1.0	29909.48313221186
0.6	10	2.0	30458.227024413176
0.6	10	3.0	30078.224769797238

TW	N	P	MEAN FO
0.6	20	1.0	15931.27284940564
0.6	20	2.0	15357.386217698147
0.6	20	3.0	15318.722411849329
1.0	10	1.0	29244.714205245313
1.0	10	2.0	29401.852740056504
1.0	10	3.0	29567.862882942656
1.0	20	1.0	15601.25271903842
1.0	20	2.0	15505.227268715847
1.0	20	3.0	15621.57692892849

Table 3. Experiment number 1 with 100 dimensions and sphere as objective function.

Experiment #4

Number of executions → E = 30
Dimensions → D=20
Objective Function = Schwefel

TW	N	P	MEAN FO
0.2	10	1.0	55565.99938408953
0.2	10	2.0	39162.19473212718
0.2	10	3.0	45799.99960291476
0.2	20	1.0	28212.0667890106
0.2	20	2.0	20017.378278344328
0.2	20	3.0	20298.291116245196
0.6	10	1.0	31511.10649850265
0.6	10	2.0	35566.32212588335
0.6	10	3.0	62536.940766613116

TW	N	P	MEAN FO
0.6	20	1.0	22824.980432747165
0.6	20	2.0	38011.46398490285
0.6	20	3.0	25206.245018941605
1.0	10	1.0	36105.34425080723
1.0	10	2.0	31706.367650032815
1.0	10	3.0	31922.34745250216
1.0	20	1.0	23300.064882302948
1.0	20	2.0	21910.467749617255
1.0	20	3.0	15961.973745419262

Table 4. Experiment number 1 with 20 dimensions and schwefel as objective function.

Experiment #5

Number of executions → E = 30
Dimensions → D=50
Objective Function = Schwefel

TW	N	P	MEAN FO
0.2	10	1.0	312863.7273383261
0.2	10	2.0	311684.55065931653
0.2	10	3.0	230987.6958059623
0.2	20	1.0	223036.21816329838
0.2	20	2.0	170909.85718064735
0.2	20	3.0	103593.58311036756
0.6	10	1.0	271532.1242208134
0.6	10	2.0	148858.1274256526
0.6	10	3.0	229759.74680195216

TW	N	P	MEAN FO
0.6	20	1.0	139670.30076291782
0.6	20	2.0	154389.5206331803
0.6	20	3.0	114431.34182023093
1.0	10	1.0	298016.63888820854
1.0	10	2.0	275972.2109649719
1.0	10	3.0	224885.40529452515
1.0	20	1.0	91400.18318329443
1.0	20	2.0	148282.31237518086
1.0	20	3.0	160469.80461465195

Table 5. Experiment number 1 with 50 dimensions and schwefel as objective function.

Experiment #6

Number of executions → E = 30
Dimensions → D=100
Objective Function = Schwefel

TW	N	P	MEAN FO
0.2	10	1.0	1200697.9418782894
0.2	10	2.0	1310716.7846743271
0.2	10	3.0	1060017.0757262947
0.2	20	1.0	402427.2636799963
0.2	20	2.0	546604.1836141835
0.2	20	3.0	472690.46206290455
0.6	10	1.0	1139114.6328458611
0.6	10	2.0	1058630.1578555887
0.6	10	3.0	907722.5692915345

TW	N	P	MEAN FO
0.6	20	1.0	728796.278408703
0.6	20	2.0	875008.3399764813
0.6	20	3.0	571571.2399931364
1.0	10	1.0	1161467.602190389
1.0	10	2.0	873495.5360555351
1.0	10	3.0	770133.9886530695
1.0	20	1.0	592219.2577516924
1.0	20	2.0	880308.4087991358
1.0	20	3.0	556408.4013965973

Table 6. Experiment number 1 with 100 dimensions and schwefel as objective function.

Experiment #7

Number of executions → E = 30
Dimensions → D=20
Objective Function = Griewank

TW	N	P	MEAN FO
0.2	10	1.0	54.03163288965206
0.2	10	2.0	52.5761493213057
0.2	10	3.0	54.79821816419963
0.2	20	1.0	27.962919801904306
0.2	20	2.0	29.36844606607154
0.2	20	3.0	29.174064435051694
0.6	10	1.0	53.75382327766706
0.6	10	2.0	57.55858957682607
0.6	10	3.0	52.27281292075346

TW	N	P	MEAN FO
0.6	20	1.0	25.876104274297255
0.6	20	2.0	27.157106947113004
0.6	20	3.0	28.407394249058935
1.0	10	1.0	53.31865502011475
1.0	10	2.0	51.6005534368226
1.0	10	3.0	49.802238776832006
1.0	20	1.0	27.175080668978296
1.0	20	2.0	27.212784895397405
1.0	20	3.0	26.162688959984646

Table 7. Experiment number 1 with 20 dimensions and griewank as objective function.

Experiment #8

Number of executions → E = 30
Dimensions → D = 50
Objective Function = Griewank

TW	N	P	MEAN FO
0.2	10	1.0	138.50463780452898
0.2	10	2.0	131.15085196710365
0.2	10	3.0	132.734794974919
0.2	20	1.0	71.6061327820453
0.2	20	2.0	71.43810430517554
0.2	20	3.0	69.13172633408936
0.6	10	1.0	140.32222466601397
0.6	10	2.0	141.04939088266553
0.6	10	3.0	135.47926811628128

TW	N	P	MEAN FO
0.6	20	1.0	72.35493427527129
0.6	20	2.0	71.50623113689521
0.6	20	3.0	73.0918584785797
1.0	10	1.0	135.16286624737228
1.0	10	2.0	135.10776604353956
1.0	10	3.0	127.33376710426107
1.0	20	1.0	65.56949981696783
1.0	20	2.0	69.16079245860861
1.0	20	3.0	68.83284896882138

Table 8. Experiment number 1 with 50 dimensions and griewank as objective function.

Experiment #9

Number of executions → E = 30
Dimensions → D = 100
Objective Function = Griewank

TW	N	P	MEAN FO
0.2	10	1.0	262.96690501768364
0.2	10	2.0	267.21067999228313
0.2	10	3.0	276.21629433829855
0.2	20	1.0	143.2615649695297
0.2	20	2.0	139.61341067829207
0.2	20	3.0	142.128875222065
0.6	10	1.0	267.20685304221723
0.6	10	2.0	265.542566526189
0.6	10	3.0	271.5024782897072

TW	N	P	MEAN FO
0.6	20	1.0	136.1426125075685 1
0.6	20	2.0	140.0257112748249
0.6	20	3.0	137.39832071129518
1.0	10	1.0	262.3918071558688
1.0	10	2.0	265.8799163484977
1.0	10	3.0	272.23121820248804
1.0	20	1.0	143.81898796276127
1.0	20	2.0	140.09903679823225
1.0	20	3.0	142.44862532477524

Table 9. Experiment number 1 with 100 dimensions and griewank as objective function.

Experiment #10

Number of executions → E = 30
Dimensions → D = 20
Objective Function = Rastrigin

TW	N	P	MEAN FO
0.2	10	1.0	29.329799792702012
0.2	10	2.0	31.441248089186953
0.2	10	3.0	32.0239150182326
0.2	20	1.0	15.207110496840546
0.2	20	2.0	15.758102235894995
0.2	20	3.0	15.81574853694267
0.6	10	1.0	30.292228503866678
0.6	10	2.0	31.489835215029867
0.6	10	3.0	30.877141451304645

TW	N	P	MEAN FO
0.6	20	1.0	15.86628145252862
0.6	20	2.0	15.410416516778374
0.6	20	3.0	16.381099018180734
1.0	10	1.0	30.720660020848086
1.0	10	2.0	30.164301169059417
1.0	10	3.0	30.78748518016698
1.0	20	1.0	16.261753228239446
1.0	20	2.0	16.299890402882532
1.0	20	3.0	17.005593421092527

Table 10. Experiment number 1 with 20 dimensions and rastrigin as objective function.

Experiment #11

Number of executions → E = 30
Dimensions → D = 50
Objective Function = Rastrigin

TW	N	P	MEAN FO
0.2	10	1.0	78.10968711761767
0.2	10	2.0	79.97514281303451
0.2	10	3.0	83.1540219171008
0.2	20	1.0	42.085719386770684
0.2	20	2.0	43.485801936567896
0.2	20	3.0	43.071972464661414
0.6	10	1.0	79.80432344854583
0.6	10	2.0	81.57834273939365
0.6	10	3.0	80.60237281552803

TW	N	P	MEAN FO
0.6	20	1.0	42.77322873203786
0.6	20	2.0	44.38152620653586
0.6	20	3.0	44.40670716980537
1.0	10	1.0	84.05948360998114
1.0	10	2.0	80.70948376700123
1.0	10	3.0	84.2667067693743
1.0	20	1.0	43.292063841762015
1.0	20	2.0	43.79803936113085
1.0	20	3.0	43.48602221358634

Table 11. Experiment number 1 with 50 dimensions and rastrigin as objective function.

Experiment #12

Number of executions → E = 30
Dimensions → D = 100
Objective Function = Rastrigin

TW	N	P	MEAN FO
0.2	10	1.0	163.08315343335596
0.2	10	2.0	164.87668243036083
0.2	10	3.0	165.58154239759187
0.2	20	1.0	84.24242704889905
0.2	20	2.0	87.20186870847853
0.2	20	3.0	86.79307435766961
0.6	10	1.0	165.8111940904125
0.6	10	2.0	168.98950391802143
0.6	10	3.0	165.8837473101491

TW	N	P	MEAN FO
0.6	20	1.0	88.25154191442401
0.6	20	2.0	86.42143742817763
0.6	20	3.0	87.03983084815242
1.0	10	1.0	167.47309099209608
1.0	10	2.0	166.31704857848757
1.0	10	3.0	168.74431795596604
1.0	20	1.0	88.45677623911698
1.0	20	2.0	90.1083947463037
1.0	20	3.0	88.87585895288707

Table 12. Experiment number 1 with 100 dimensions and rastrigin as objective function.

ANALISIS

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- The possibility of changing the local minimum through a disturbance meant better results in al possible dimensionalities and objective functions.
- Good results are found and some not so much depending on the algorithm's randomness.
- The best results of the 12 experiments were with a number of neighbors equal to 20, meaning that the greater the neighbors have the evaluation of the algorithm, the better results will be obtained.
- For the objective function **rastrigin**, the results with the best solution had the same parameter settings in 20, 50 and 100 dimensions: tw = 0.2, n = 20, p = 1.0.

Best sphere results

	Random Search			Local Iterated search		
Tweak				1.0	1.0	0.2
Number of neighbors				20	20	20
Perturbation				3.0	2.0	3.0
Dimensions	20	50	100	20	50	100
Total Average	23347.7178	93736.8297	225179.7916	2528.6371	7361.9331	15274.5594

Table 13. Best sphere results: Random search vs Local iterated search.

Best schwefel results

	Random Search			Local Iterated search		
Tweak				1.0	1.0	0.2
Number of neighbors				20	20	20
Perturbation				3.0	1.0	1.0
Dimensions	20	50	100	20	50	100
Total Average	25317.7757	143917.3663	327051.2013	15961.9737	91400.1831	402427.2636

Table 14. Best schwefel results : Random search vs Local iterated search.

Best griewank results

	Random Search			Local Iterated search		
Tweak				0.6	1.0	0.6
Number of neighbors				20	20	20
Perturbation				1.0	1.0	1.0
Dimensions	20	50	100	20	50	100
Total Average	209.7369	838.8434	2032.4766	25.8761	65.5694	136.1426

Table 15. Best griewank results : Random search vs Local iterated search.

Best rastrigin results

	Random Search			Local Iterated search		
Tweak				0.2	0.2	0.2
Number of neighbors				20	20	20
Perturbation				1.0	1.0	1.0
Dimensions	20	50	100	20	50	100
Total Average	178.1266	435.2144	870.5943	15.2071	42.0857	84.2424

Table 16. Best rastrigin results : Random search vs Local iterated search.

Iterated local search comparison with 20 dimensions

	Sphere	Schwefel	Griewank	Rastrigin
Tweak	1.0	1.0	0.6	0.2
Number of neighbors	20	20	20	20
Perturbation	3.0	3.0	1.0	1.0
Total Average	2528.6371	15961.9737	25.8761	15.2071

Table 17. Iterated local search comparison with 20 dimensions and the four objective functions.

Iterated local search comparison with 50 dimensions

	Sphere	Schwefel	Griewank	Rastrigin
Tweak	1.0	1.0	1.0	0.2
Number of neighbors	20	20	20	20
Perturbation	2.0	1.0	1.0	1.0
Total Average	7361.9331	91400.1831	65.5694	42.0857

Table 18. Iterated local search comparison with 50 dimensions and the four objective functions.

Iterated local search comparison with 100 dimensions

	Sphere	Schwefel	Griewank	Rastrigin
Tweak	0.2	0.2	0.6	0.2
Number of neighbors	20	20	20	20
Perturbation	3.0	1.0	1.0	1.0
Total Average	15274.5594	402427.2636	136.1426	84.2424

Table 19. Iterated local search comparison with 100 dimensions and the four objective functions.

CONCLUSIONS

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- Iterated local search is simple, easy to implement, robust, and highly effective.
- The results provided by the iterated local search algorithm performed much better than the random search algorithm.
- The number of neighbors defines the performance of the algorithm.
- Proper parameter settings will allow minimization of an objective function to be effective.
- How effective this approach turns out to be depends mainly on the choice of the local search, the perturbations, and the acceptance criterion.

¡MANY THANKS!

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