SAN DIEGO AND SACRAMENTO COUNTY HOUSING MARKET ANALYSIS

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A. Overview of the two counties' housing markets

San Diego County

The San Diego County housing market has been experiencing a significant increase in home prices in recent years, which has made it a challenging market for buyers. The median home price in San Diego County in 2022 was around $825,000, up by over 20% from 2 years ago.

A strong job market, low-interest rates, and limited housing supply have driven the demand for housing in San Diego County. The COVID-19 pandemic has also increased demand for housing as more people seek homes with more space and amenities for remote work and lifestyle changes. The low inventory of available homes in San Diego County has led to intense competition among buyers, with many homes receiving multiple offers and selling above the asking price. This has made it difficult for first-time buyers and those with lower incomes to enter the market. However, the San Diego County housing market is expected to continue to be strong, with low-interest rates and a robust job market continuing to drive demand. New construction of homes has been picking up, which could help alleviate some of the supply constraints in the market. The rental market in San Diego County has also been impacted by the high demand for housing. Rents have been increasing steadily in recent years, with the median rent for a one-bedroom apartment in San Diego County at around $1,700 per month. The COVID-19 pandemic has also impacted the rental market, with many renters seeking larger units or more desirable locations to accommodate remote work and lifestyle changes. This has further driven up demand for rental properties in San Diego County. Overall, the San Diego County housing market has been vital in recent years, with high demand for buying and renting properties. However, the high prices and low inventory have made it challenging for many buyers and renters to find affordable housing options.

Sacramento County

The Sacramento County housing market has been experiencing a strong seller's market in recent years, with high demand and low inventory leading to rising home prices. The median home price in Sacramento County in 2022 was around $550,000, up by over 20% from 2020. One of the driving factors behind the strong housing market in Sacramento County is the city's growing job market, which has attracted many new residents in recent years. In addition, the county has a diverse economy, with a strong presence in fields such as healthcare, education, and technology. The low inventory of available homes in Sacramento County has led to intense competition among buyers, with many homes receiving multiple offers and selling above the asking price. This has made it challenging for first-time homebuyers and those with lower incomes to enter the market. The rental market in Sacramento County has also been impacted by the strong housing market, with rents increasing steadily in recent years. The median rent for a one-bedroom apartment in Sacramento County was around $1,500 monthly. Overall, the Sacramento County housing market has been vital in recent years, driven by a growing economy and a strong demand for housing. However, the low inventory of available homes for sale and increasing rents have made it challenging for many residents to find affordable housing options. The Sacramento County housing market has seen an increase in new construction in recent years, which could help alleviate some of the supply constraints in the market. The county has also implemented various initiatives to increase the supply of affordable housing, including funding for new developments and incentives for developers to include affordable units in their projects. Another factor that has impacted the Sacramento County housing market is the COVID-19 pandemic, which has led to changes in housing preferences for many residents. More people are looking for homes with extra space for remote work and outdoor amenities, which has increased demand for single-family homes and properties with larger yards.

In summary, the Sacramento County housing market has been vital in recent years, driven by a growing economy and a strong demand for housing. However, the low inventory of available homes for sale and increasing rents have made it challenging for many residents to find affordable housing options. The county has taken steps to increase the supply of affordable housing, and the rising new construction could help alleviate some of the supply constraints in the market.

B. Application

Real estate investors and developers will benefit significantly from the econometric analysis we are building from housing data to identify trends in the market, evaluate potential investment opportunities, and make informed decisions about property acquisitions and development projects. This can help them to optimize their investment strategies and improve their returns. A few areas in which the econometric analysis can help are:

- Forecasting demand: Econometric analysis can help developers and investors forecast the market for different housing types in a particular area based on population growth, employment trends, and demographic changes. This can help them make more informed decisions about where to invest and what properties to build.

- Assessing risk: Econometric analysis can also evaluate the risk of investing in a particular area or property. Developers and investors can identify patterns and trends that may indicate potential risks or opportunities by analyzing historical data on housing prices, vacancy rates, and other factors.

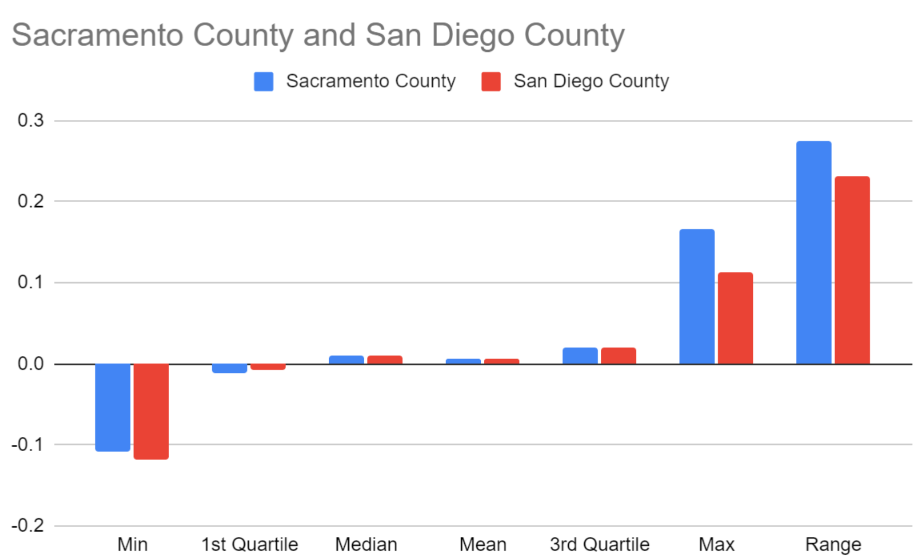
- Identifying market inefficiencies: Econometric analysis can help developers and investors identify market inefficiencies, such as areas where the housing supply is low but demand is high. By recognizing these inefficiencies, developers and investors can make strategic investments that exploit market imbalances.

Overall, econometric analysis can be a powerful tool for real estate developers and investors, enabling them to make more informed decisions and take advantage of opportunities in the local housing market.

C. Properties of the HPI Times Series

Descriptive Statistics

We focus on the percent returns of the San Diego and Sacramento HPI. In order to convert our data to percent returns we first take the log of both indices. This converts the data to percentages. We then apply differencing to the data by subtracting each observation with its observation from the previous period. This means that each observation in our data will now represent the percent change in the county's HPI. The rest of our analysis will be using this percentage return data.



The HPI is a measure of the change in single-family home prices based on a weighted average of price changes across different geographic regions. Here is a breakdown of the descriptive statistics:

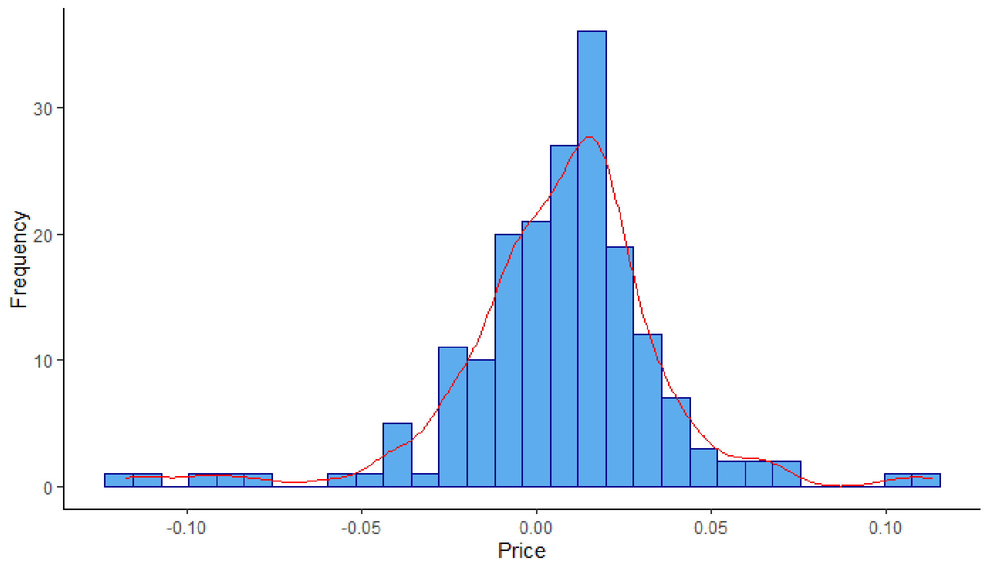
- The minimum HPI for San Diego County is 27.33, while the minimum HPI for Sacramento County is slightly higher at 28.08.
- The first quartile (25th percentile) HPI for Sacramento County is 74.18, while the first quartile HPI for San Diego County is higher at 79.28.
- The median HPI (50th percentile) for Sacramento County is 115.5, while the median HPI for San Diego County is higher at 126.48.
- The mean (average) HPI for Sacramento County is 152.51, while the mean HPI for San Diego County is higher at 178.03.

- The third quartile (75th percentile) HPI for Sacramento County is 214.76, while the third quartile HPI for San Diego County is much higher at 270.86.
- The maximum HPI for Sacramento County is 410.25, while the maximum HPI for San Diego County is significantly higher at 514.18.
- The range for Sacramento County is 382.17, while the range for San Diego County is higher at 486.85.

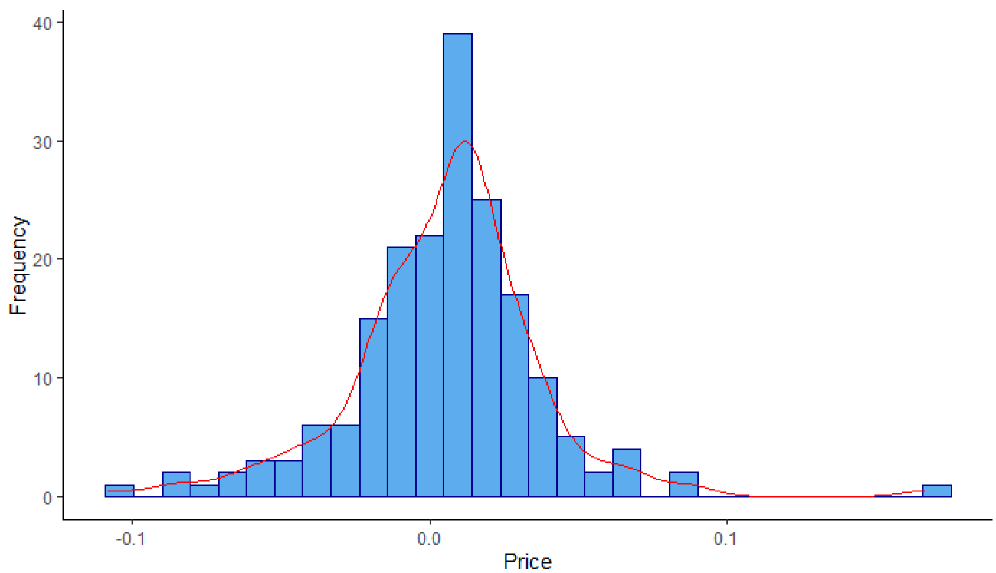
Visualizations

Histograms and Density Plots for return frequency

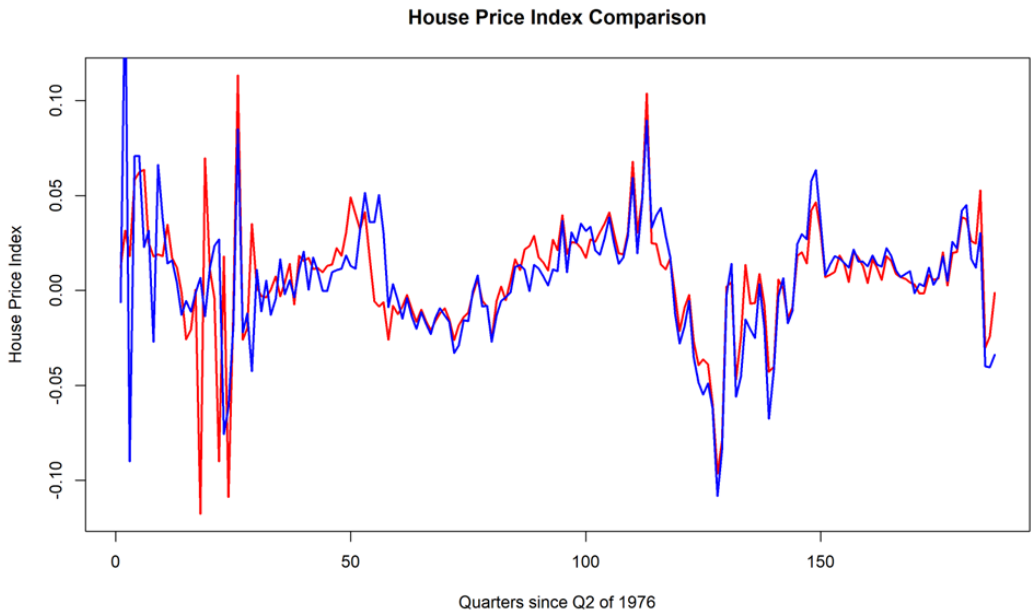
San Diego House Price Index



Sacramento House Price Index

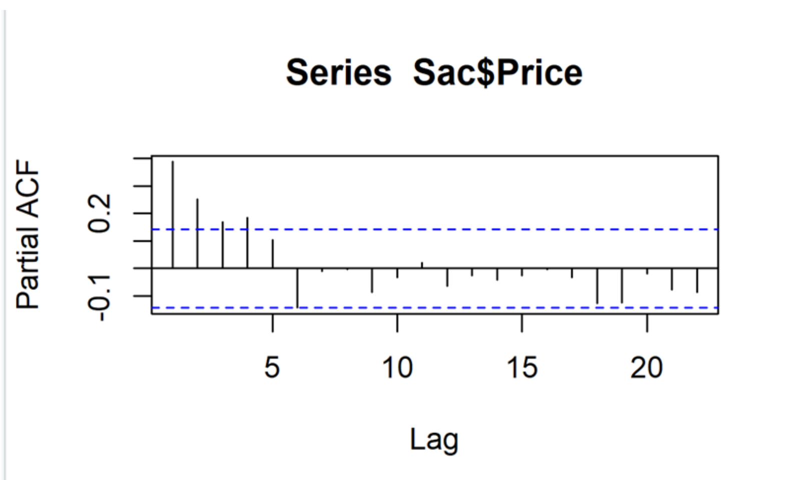


- The above visualizations depicts the frequency of the HPI returns for San Diego, and Sacramento.
- Generally, both counties have more positive than negative returns.



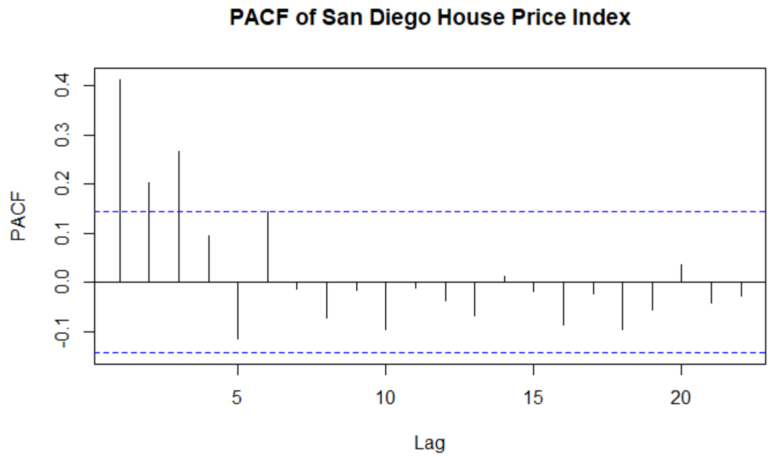
- This is a plotted line graph depicting the changes of house prices of both cities over time. San Diego is plotted in blue and Sacramento in red.

PACF for Sacramento



- Examining the PACF for Sacramento shows there could be 4 relevant lags to include in an AR model.

PACF for San Diego

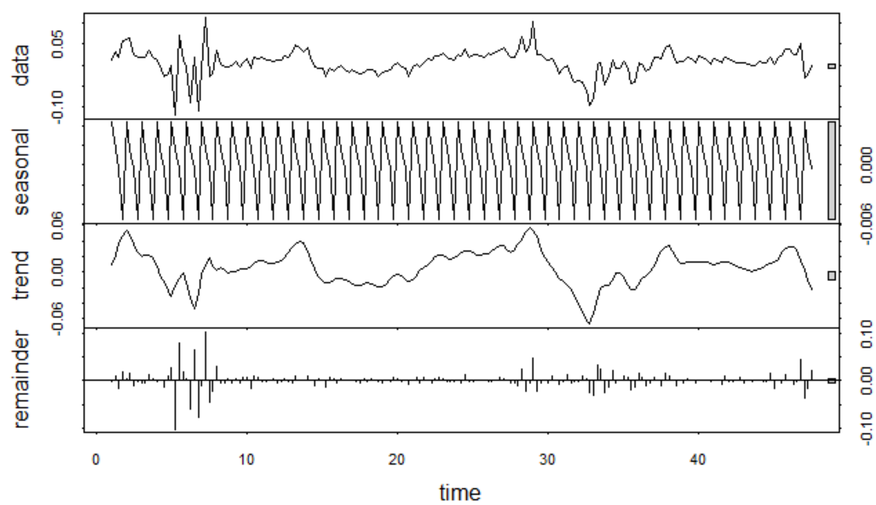


- The PACF for San Diego also shows there could be 3 or 4 relevant lags to include in an AR model of the series.

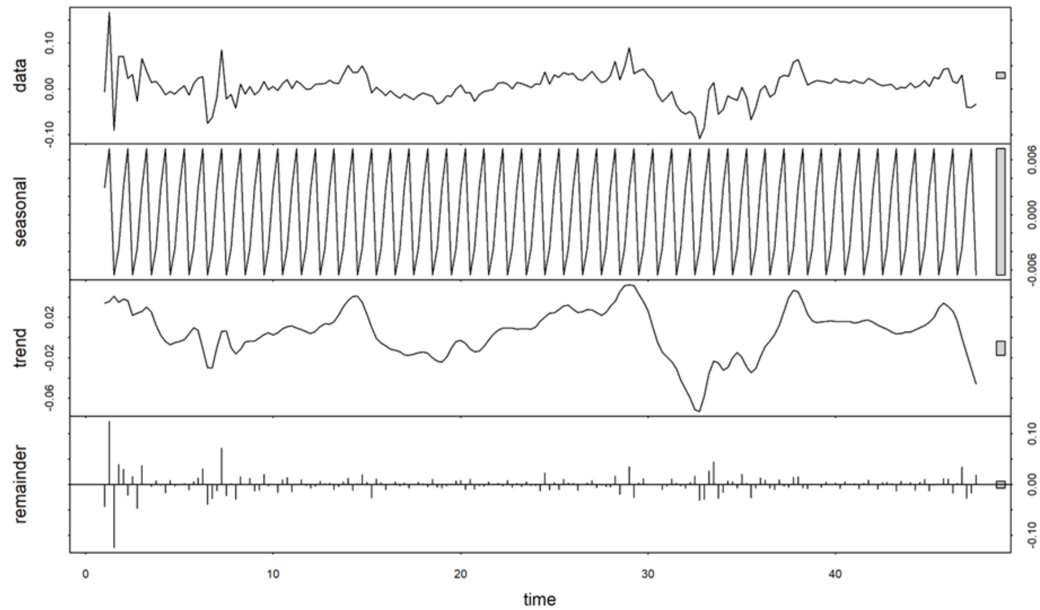
Structural Decomposition

We conducted an STL decomposition to visually confirm the existence of seasonality and trend. We also conducted the Shapiro test to determine whether the residuals are normally distributed or not. The p-values from the Shapiro tests were both near zero, thus we reject the null hypothesis and conclude that the residuals are not normally distributed for San Diego house prices and Sacramento house prices.

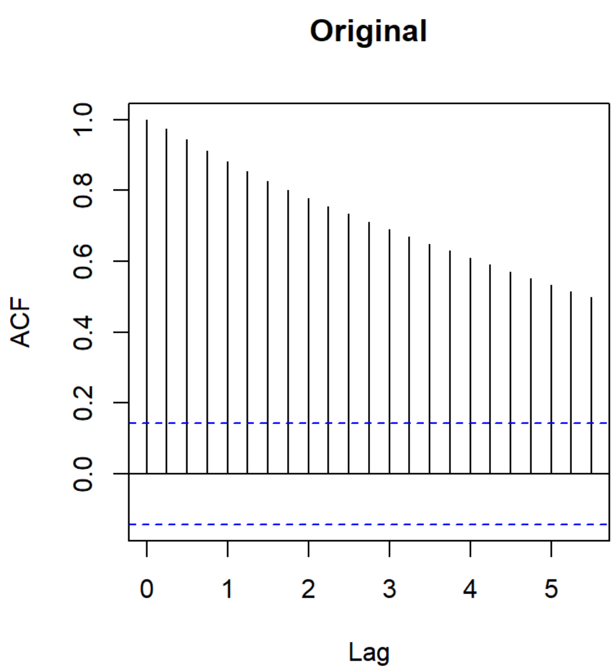
Decomposition for San Diego

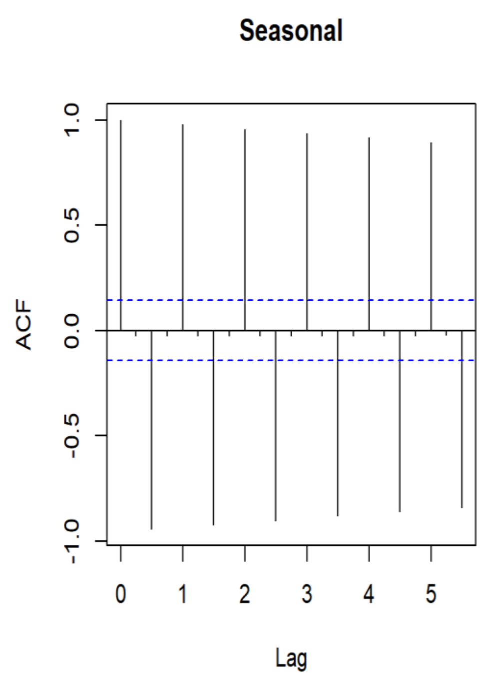


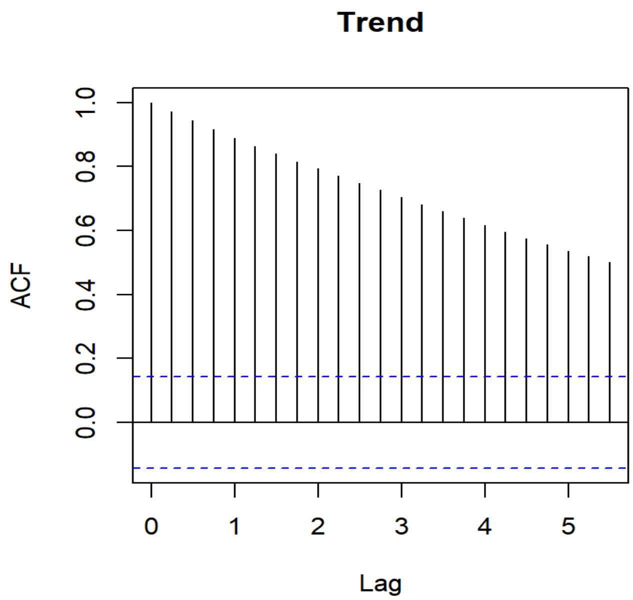
Decomposition for Sacramento

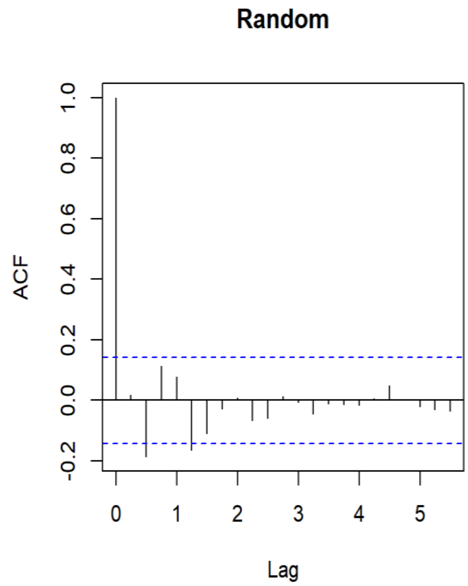


1) San Diego Decomposition Figures:





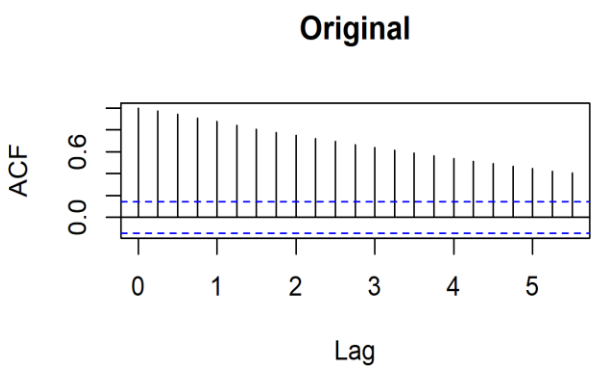


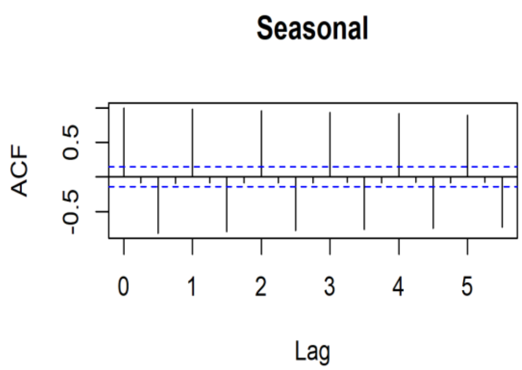


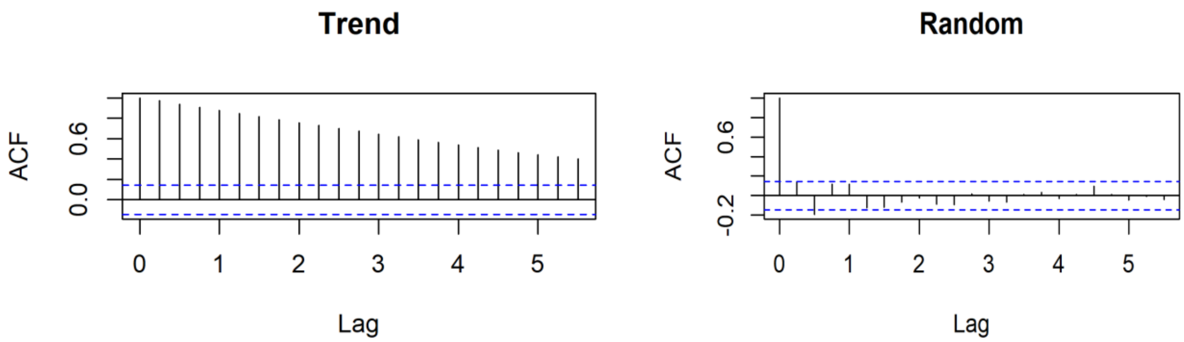
The San Diego output indicates:

- Original: Descending shows that the data are stationary.
- Seasonal: The HHI price is strongly negatively correlated with the price two quarters ago. It is strongly positively correlated with the price one year ago.
- Trend: The ACF confirms the existence of a stationary trend.
- Random: The spike at 0.5 and 1.2 indicates that additional structures may be in the data not captured by our decomposition.

2) Sacramento Decomposition Figures:







The Sacramento output indicates:

- Original: Descending shows that the data are stationary.
- Seasonal: The HHI price is strongly negatively correlated with the price two quarters ago. It is strongly positively correlated with the price one year ago.
- Trend: The ACF confirms the existence of a stationary trend.
- Random: The spike at 0.5 and 1.2 indicates that additional structures may be in the data not captured by our decomposition.

Autocorrelation

We also conducted a Ljung box test to determine if the time series is autocorrelated, which means there is a relationship between its values at different time points. The Ljung-Box test compares the time series's observed autocorrelations to the expected autocorrelations that would be seen in a series of uncorrelated random data points. If the observed autocorrelations are significantly different from the expected ones, then the time series is considered to be autocorrelated. The p-value turned out to be ~0; thus, we reject the null hypothesis and can infer that there is autocorrelation in the time series for San Diego's house prices. We conducted the same tests for Sacramento house prices as well. The p-value was ~0, indicating strong evidence to reject the null hypothesis that the residuals are uncorrelated and hence white noise. In other words, the test suggests significant evidence of autocorrelation in the residuals of our time series model at one or more lags.

Stationarity

1) San Diego Augmented Dicky-Fuller Test:

```
############################################################################
At the 5pct level:
The model is of type drift
tau2: The first null hypothesis is rejected, unit root is not present
phi1: The second null hypothesis is rejected, unit root is not present
      and there is drift.
############################################################################
```

- The first null hypothesis, represented by tau3, is that there is a unit root present. This null hypothesis is rejected, indicating no evidence of a unit root stationarity in the San Diego County data.
- The second null hypothesis we tested, represented by phi1, is that no drift is present. This null hypothesis is also rejected, indicating that there is no evidence of a unit root and there is drift in the data.
- We also tested for stationarity with drift and trend, but the test was inconclusive.

2) Sacramento Augmented Dicky-Fuller Test:

```
########################################################################
> interp_urdf(ur.df(Sac_ts, type = "trend"))
########################################################################
At the 5pct level:
The model is of type trend
tau3: The first null hypothesis is not rejected, unit root is present
phi3: The second null hypothesis is not rejected, unit root is present
      and there is no trend
phi2: The third null hypothesis is not rejected, unit root is present
      there is no trend, and there is no drift
########################################################################
```

- All three of the Sacramento County hypotheses were identical to San Diego, and they exhibited that unit root is also present. The Sacramento data exhibits a non-stationary behavior with a unit root present and no evidence of a trend or drift.

Outliers in the Data

We also wanted to check for any outliers in our data set, so we conducted the Grubbs test, which is used to identify outliers in a univariate time series data set like ours. The Grubbs test on the San Diego HPI data resulted in a p-value that was effectively zero (less than any standard significance level like 0.05), so we can reject the null hypothesis and conclude that the highest value (24.7912) in the residuals is an outlier. We ran the same test for the Sacramento HPI data, which again returned a p-value which was functionally zero. Thus, we can conclude that there are outlier residuals present in the Sacramento data as well. This means we will need to conduct further analysis to account for these outliers in order to improve the accuracy and efficiency of our HPI model.

D) Arima and ARIMA-X Models:

1) San Diego Auto-ARIMA:

```
Series: AllSan_ts
ARIMA(4,0,1)(2,0,0)[12] with zero mean

Coefficients:
         ar1      ar2     ar3      ar4      ma1      sar1    sar2
      1.2481  -0.1307  0.1626  -0.3047  -0.9615  -0.0135  0.1520
s.e.  0.0741   0.1148  0.1141   0.0707   0.0281   0.0750  0.1007

sigma^2 = 0.0006696:  log likelihood = 420.98
AIC=-825.96   AICc=-825.15   BIC=-800.11
```

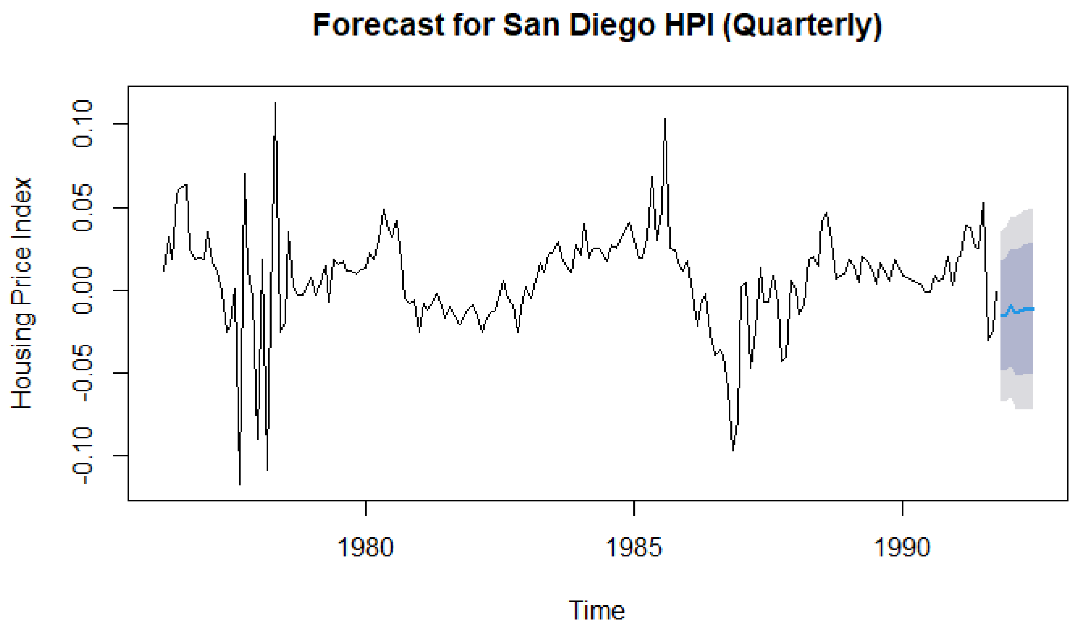
2) Sacramento Auto-ARIMA

```
> summary(best_model_no_drift)
Series: tstest
ARIMA(5,0,4) with zero mean

Coefficients:
          ar1      ar2     ar3     ar4     ar5     ma1     ma2     ma3     ma4
      -0.6002  -0.1517  0.3212  0.3411  0.4594  0.8208  0.5854  0.0252  0.0245
s.e.   0.3536   0.2026  0.1522  0.3933  0.1106  0.3681  0.2746  0.2282  0.3407

sigma^2 = 0.0007389:  log likelihood = 412.58
AIC=-805.15   AICc=-803.9   BIC=-772.84

Training set error measures:
                       ME       RMSE        MAE      MPE     MAPE      MASE       ACF1
Training set 0.0009735514 0.02652038 0.01725974 107.6074 216.8894 0.9032093 0.01447316
```

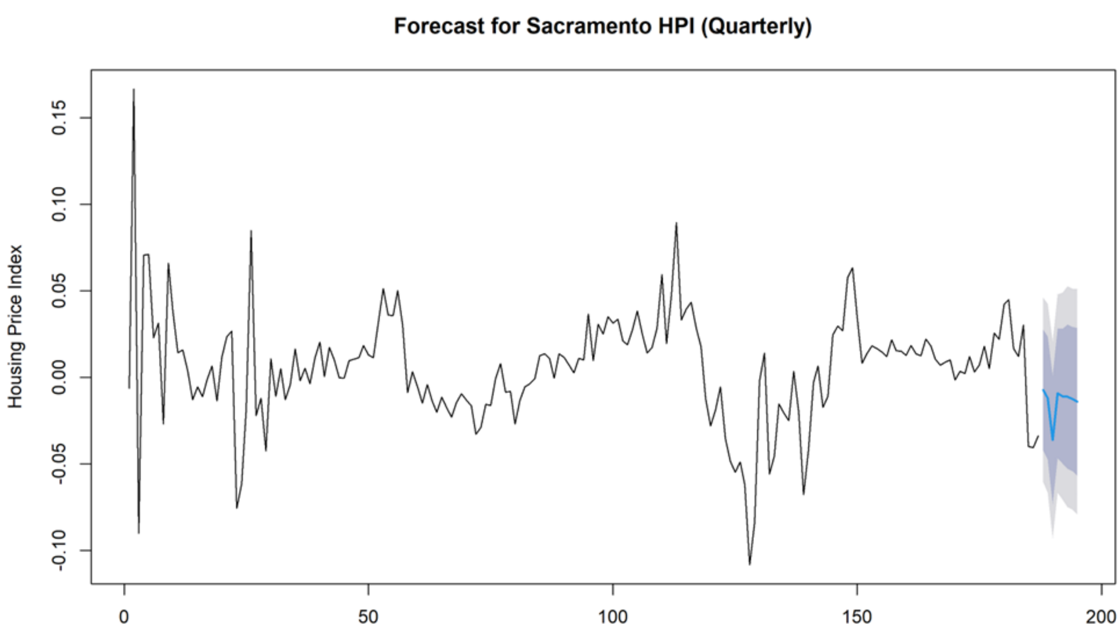
The first value represents the autoregressive (AR) component (p). Autoregression refers to the use of past observations in the model. An AR(p) model uses p-lagged observations to predict the current value. In this case, it is 4 for San Diego and 5 for Sacramento. That means the auto ARIMA model determined four relevant lags to include in San Diego's model and 5 for Sacramento. The second value, "0" for both, represents the differencing (I) component (d). Differencing removes trend and seasonality from the time series data, making it stationary. In an ARIMA model, the "d" value indicates the number of times differencing has been applied to achieve stationarity. Since it is zero (0), no differencing has been performed by the auto-ARIMA function. The third value, "1", represents the moving average (MA) component (q). The MA component uses the error terms of past observations to predict the current value. An MA(q) model uses q-lagged error terms. Here, we have one lagged error term for San Diego and 4 for Sacramento.

E) Forecasting power of the ARIMA model(s):

1) Forecast for San Diego HPI returns:



2) Forecast for Sacramento HPI returns:



Coefficient Estimates for San Diego

	Intercept	Mortgage Rate	Population	Sentiment	Unemployment Rate
Coefficient	-427.021	-0.021	0.003	-0.385	-3.927
Std Err.	34.906	0.066	0.0001	0.070	0.686

Coefficient Estimates for Sacramento

	Intercept	Mortgage Rate	Population	Sentiment	Unemployment Rate
Coefficient	-454.736	0.003	0.002	0.001	-0.0195
Std Err.	76.600	0.005	0.0003	0.0082	0.1818

*Table is constructed from nominal HPI and not from real returns

Covariates:

- ➔ The above forecasts include the following covariates:
  - I. Mortgage Rate
  - II. Population
  - III. Sentiment
  - IV. Unemployment Rate
  - V. GDP
- ➔ All covariates are not seasonally adjusted and are quarterly data.

F. Analyzing HPI Returns

Granger Causality

The Granger causality test is a statistical test used to determine whether one time series can predict or "Granger-cause" another time series. It helps to assess the causal relationship between variables in an econometric model. The test is based on the idea that if variable X Grangercauses variable Y, then the past values of X should provide information to predict Y's current and future values beyond what can be predicted using only the past values of Y itself.

To conduct this test, we use the following ARIMA-X variables: Sentiment, Mortgage rate, Unemployment rate, and National population.
We used the “Consumer Price Index for All Urban Consumers: All Items in U.S. City Average” (CPIAUCNS) to go from nominal to real.
We are indexed to 1995 = 100

Output for Sacramento and San Diego:

```
> causality(var_model, cause = "AllSan")
$Granger

	Granger causality H0: AllSan do not Granger-cause Sac

data:  VAR object var_model
F-Test = 4.9515, df1 = 5, df2 = 342, p-value = 0.0002165


$Instant

	H0: No instantaneous causality between: AllSan and Sac

data:  VAR object var_model
Chi-squared = 49.453, df = 1, p-value = 2.032e-12
```

```
> causality(var_model, cause = "Sac")
$Granger

	Granger causality H0: Sac do not Granger-cause AllSan

data:  VAR object var_model
F-Test = 6.0712, df1 = 5, df2 = 342, p-value = 2.109e-05


$Instant

	H0: No instantaneous causality between: Sac and AllSan

data:  VAR object var_model
Chi-squared = 49.453, df = 1, p-value = 2.032e-12
```

The results of the Granger Causality Test indicate that San Diego HPI returns Granger cause Sacramento HPI returns and that Sacramento HPI returns also Granger cause San Diego HPI

VAR Model

Because our data are stationary we apply a VAR model. The outputs are as follows:

```
VAR Estimation Results:
=========================
Endogenous variables: AllSan, Sac
Deterministic variables: const
Sample size: 182
Log Likelihood: 921.744
Roots of the characteristic polynomial:
0.8873 0.8403 0.8403 0.6993 0.6986 0.6986 0.6799 0.6799 0.4464 0.2549
Call:
VAR(y = data, p = 5)


Estimation results for equation AllSan:
=======================================
AllSan = AllSan.l1 + Sac.l1 + AllSan.l2 + Sac.l2 + AllSan.l3 + Sac.l3 + AllSan.l4 + Sac.l4 + AllSan.l5 + Sac.l5 + const

           Estimate Std. Error t value Pr(>|t|)
AllSan.l1  0.051884   0.092579   0.560 0.575920
Sac.l1     0.434332   0.106981   4.060 7.46e-05 ***
AllSan.l2  0.160398   0.094288   1.701 0.090733 .
Sac.l2    -0.268068   0.109359  -2.451 0.015242 *
AllSan.l3  0.325127   0.091544   3.552 0.000495 ***
Sac.l3    -0.078921   0.098622  -0.800 0.424681
AllSan.l4 -0.008977   0.095346  -0.094 0.925095
Sac.l4     0.183366   0.085684   2.140 0.033771 *
AllSan.l5 -0.167138   0.093535  -1.787 0.075726 .
Sac.l5     0.052127   0.082475   0.632 0.528212
const      0.001700   0.001869   0.910 0.364157
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.02398 on 171 degrees of freedom
Multiple R-Squared: 0.3834,     Adjusted R-squared: 0.3474
F-statistic: 10.63 on 10 and 171 DF,  p-value: 6.437e-14



Estimation results for equation Sac:
====================================
Sac = AllSan.l1 + Sac.l1 + AllSan.l2 + Sac.l2 + AllSan.l3 + Sac.l3 + AllSan.l4 + Sac.l4 + AllSan.l5 + Sac.l5 + const

            Estimate Std. Error t value Pr(>|t|)
AllSan.l1  0.2422974  0.0800668   3.026  0.00286 **
Sac.l1     0.4168503  0.0925219   4.505 1.22e-05 ***
AllSan.l2 -0.0007800  0.0815450  -0.010  0.99238
Sac.l2    -0.1816299  0.0945789  -1.920  0.05647 .
AllSan.l3  0.1499158  0.0791718   1.894  0.05997 .
Sac.l3     0.0984374  0.0852930   1.154  0.25007
AllSan.l4 -0.2206787  0.0824596  -2.676  0.00817 **
Sac.l4     0.1913915  0.0741036   2.583  0.01064 *
AllSan.l5  0.1265126  0.0808936   1.564  0.11968
Sac.l5     0.0278283  0.0713282   0.390  0.69692
const     -0.0003901  0.0016161  -0.241  0.80953
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.02074 on 171 degrees of freedom
Multiple R-Squared: 0.511,      Adjusted R-squared: 0.4824
F-statistic: 17.87 on 10 and 171 DF,  p-value: < 2.2e-16


Covariance matrix of residuals:
          AllSan       Sac
AllSan 0.0005749 0.0003037
Sac    0.0003037 0.0004300

Correlation matrix of residuals:
       AllSan    Sac
AllSan 1.0000 0.6108
Sac    0.6108 1.0000
```

We also conducted the Johansen Procedure as well:

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.3990047 0.1499946

Values of teststatistic and critical values of test:

            test 10pct  5pct  1pct
r <= 1 |  30.06  6.50  8.18 11.65
r = 0  | 124.26 15.66 17.95 23.52

Eigenvectors, normalised to first column:
(These are the cointegration relations)

            AllSan.l2    Sac.l2
AllSan.l2  1.0000000 1.0000000
Sac.l2    -0.9513085 0.8072617

Weights W:
(This is the loading matrix)

           AllSan.l2     Sac.l2
AllSan.d -0.5314340 -0.2266696
Sac.d     0.6810322 -0.2012124
```

G. Conditional Variance Analysis: Testing Various GARCH models

Because the returns data for both counties were stationary, we ran a VAR analysis instead of a VECM. The GJR-GARCH model, also known as the Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity model, is a type of GARCH model that is able to account for an empirical phenomenon known as the leverage effect by incorporating an additional parameter. The leverage effect refers to the phenomenon where negative shocks or returns have a stronger impact on volatility compared to positive shocks or returns of the same magnitude. In traditional GARCH models, the conditional variance is modeled as a function of lagged squared returns and lagged conditional variances. However, the GJR-GARCH model introduces an extra term that considers the lagged squared returns with a specific condition related to the sign of the return. This additional term allows for capturing the asymmetry in the response of volatility to positive and negative shocks.

The leverage effect is observed in various financial markets, such as stocks, where negative news or events tend to have a more pronounced impact on market volatility compared to positive news or events. By incorporating the leverage effect, the GJR-GARCH model provides a more accurate representation of the volatility dynamics and improves the model's ability to capture the asymmetry in financial data.

Functional Form:
$\sigma^2(t) = \omega + \alpha\varepsilon^2(t-1) + \gamma\varepsilon^2(t-1)I[\varepsilon(t-1) < 0] + \beta\sigma^2(t-1)$

Where:
- $\sigma^2(t)$ represents the conditional variance at time t.
- ω is the constant term, which represents the baseline level of volatility.
- α is the coefficient associated with the lagged squared residuals, representing the impact of past volatility on current volatility.
- γ is the coefficient associated with the term capturing the leverage effect. It quantifies the additional impact of negative returns on volatility.
- $\varepsilon(t-1)$ is the lagged standardized residual, representing the deviation of the return from its expected value.
- $I[\varepsilon(t-1) < 0]$ is an indicator function that takes the value 1 if the lagged standardized residual $\varepsilon(t-1)$ is negative, and 0 otherwise.
- β is the coefficient associated with the lagged conditional variance, representing the persistence of volatility.

This simplified GJR-GARCH(1,1) model allows you to estimate the conditional variance based on the lagged squared residuals, capturing both the general autoregressive behavior of volatility (with α) and the leverage effect (with γ). It is a popular choice for modeling and forecasting financial time series data due to its ability to capture volatility clustering and asymmetric responses to market shocks.

In addition to the GJR-GARCH model, we also run sGARCH, iGARCH and eGARCH models to determine which performs the best. The results are as follows:

Sacramento County sGARCH (1,1)

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(5,0,4)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.003799    0.000002  2113.1362 0.000000
ar1     0.760264    0.000203  3750.0876 0.000000
ar2     0.858015    0.000213  4035.3876 0.000000
ar3    -0.042974    0.000027 -1601.1478 0.000000
ar4    -0.874731    0.000209 -4176.0512 0.000000
ar5     0.275277    0.000078  3516.7309 0.000000
ma1    -0.182755    0.000082 -2223.6526 0.000000
ma2    -0.924656    0.000240 -3855.7171 0.000000
ma3    -0.541660    0.000274 -1979.3207 0.000000
ma4     0.572202    0.000149  3833.8967 0.000000
omega   0.000031    0.000013     2.3795 0.017336
alpha1  0.498572    0.121907     4.0898 0.000043
beta1   0.500369    0.079813     6.2692 0.000000

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.003799    0.000365   10.40130  0.00000
ar1     0.760264    0.012632   60.18612  0.00000
ar2     0.858015    0.006518  131.63190  0.00000
ar3    -0.042974    0.005086   -8.45003  0.00000
ar4    -0.874731    0.014493  -60.35560  0.00000
ar5     0.275277    0.008968   30.69512  0.00000
ma1    -0.182755    0.016722  -10.92870  0.00000
ma2    -0.924656    0.008821 -104.82559  0.00000
ma3    -0.541660    0.055649   -9.73357  0.00000
ma4     0.572202    0.006117   93.54401  0.00000
omega   0.000031    0.000220    0.13951  0.88904
alpha1  0.498572    2.744701    0.18165  0.85586
beta1   0.500369    2.410948    0.20754  0.83559

LogLikelihood : 466.514

Information Criteria
------------------------------------

Akaike       -4.8504
Bayes        -4.6258
Shibata      -4.8593
Hannan-Quinn -4.7594
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      2.316 0.1280160
Lag[2*(p+q)+(p+q)-1][26]   15.564 0.0003918
Lag[4*(p+q)+(p+q)-1][44]   22.769 0.4489726
d.o.f=9
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      7.917 0.004898
Lag[2*(p+q)+(p+q)-1][5]    11.347 0.004127
Lag[4*(p+q)+(p+q)-1][9]    12.177 0.016529
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     2.698 0.500 2.000  0.1004
ARCH Lag[5]     2.799 1.440 1.667  0.3203
ARCH Lag[7]     2.902 2.315 1.543  0.5322
```

```
Nyblom stability test
------------------------------------
Joint Statistic:  2.9255
Individual Statistics:
mu     0.019068
ar1    0.009590
ar2    0.009484
ar3    0.019058
ar4    0.011303
ar5    0.012950
ma1    0.019054
ma2    0.009210
ma3    0.019013
ma4    0.009229
omega  0.093730
alpha1 0.090051
beta1  0.082040

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.89 3.15 3.69
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias          1.43045 0.154302
Negative Sign Bias 0.03176 0.974700
Positive Sign Bias 2.63829 0.009054 ***
Joint Effect       7.06776 0.069769   *
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     33.43      0.02145
2    30     43.53      0.04060
3    40     54.50      0.05070
4    50     62.47      0.09368


Elapsed time : 0.589426
```

Sacramento County iGARCH

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(5,0,4)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.014853    0.009523  1.55958 0.118858
ar1    -0.432205    0.468296 -0.92293 0.356042
ar2     0.371043    0.370388  1.00177 0.316455
ar3     0.559565    0.175416  3.18993 0.001423
ar4    -0.068140    0.242346 -0.28117 0.778583
ar5     0.121780    0.297354  0.40954 0.682140
ma1     1.191246    0.499590  2.38445 0.017105
ma2     0.599276    0.379501  1.57911 0.114310
ma3    -0.066734    0.566483 -0.11780 0.906224
ma4     0.104580    0.320238  0.32657 0.743993
omega   0.000030    0.000012  2.44961 0.014301
alpha1  0.474354    0.100314  4.72869 0.000002
beta1   0.525646          NA       NA       NA
```

```
Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.014853    0.030151  0.492614 0.622286
ar1    -0.432205    0.372659 -1.159787 0.246136
ar2     0.371043    0.904565  0.410189 0.681667
ar3     0.559565    0.598837  0.934419 0.350088
ar4    -0.068140    0.424508 -0.160514 0.872476
ar5     0.121780    0.891193  0.136648 0.891309
ma1     1.191246    0.300164  3.968656 0.000072
ma2     0.599276    0.871898  0.687323 0.491879
ma3    -0.066734    1.488773 -0.044825 0.964247
ma4     0.104580    1.002536  0.104316 0.916919
omega   0.000030    0.000017  1.838974 0.065919
alpha1  0.474354    0.141508  3.352134 0.000802
beta1   0.525646          NA        NA       NA

LogLikelihood : 456.9579

Information Criteria
------------------------------------

Akaike       -4.7589
Bayes        -4.5516
Shibata      -4.7665
Hannan-Quinn -4.6749
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                       4.60 3.196e-02
Lag[2*(p+q)+(p+q)-1][26]    18.48 1.255e-14
Lag[4*(p+q)+(p+q)-1][44]    26.11 1.518e-01
d.o.f=9
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      9.972 0.001589
Lag[2*(p+q)+(p+q)-1][5]    11.143 0.004663
Lag[4*(p+q)+(p+q)-1][9]    11.639 0.021897
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.193 0.500 2.000  0.2747
ARCH Lag[5]     1.227 1.440 1.667  0.6670
ARCH Lag[7]     1.408 2.315 1.543  0.8396
```

```
Nyblom stability test
------------------------------------
Joint Statistic:  1.9313
Individual Statistics:
mu     0.14979
ar1    0.17165
ar2    0.16353
ar3    0.07696
ar4    0.03812
ar5    0.02345
ma1    0.08112
ma2    0.14706
ma3    0.18911
ma4    0.18570
omega  0.08888
alpha1 0.14808

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.69 2.96 3.51
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           0.8028 0.423118
Negative Sign Bias  0.2136 0.831068
Positive Sign Bias  3.2288 0.001475 ***
Joint Effect       10.8306 0.012678  **
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     15.67      0.67892
2    30     40.01      0.08385
3    40     40.38      0.40914
4    50     51.77      0.36625


Elapsed time : 0.2632942
```

Sacramento County eGARCH

```
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(5,0,4)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.004465    0.000004  1205.2258 0.000000
ar1     0.740321    0.000272  2722.9206 0.000000
ar2     0.849692    0.000260  3272.0506 0.000000
ar3     0.010295    0.000031   328.2335 0.000000
ar4    -0.886455    0.000249 -3561.9476 0.000000
ar5     0.258568    0.000104  2478.9554 0.000000
ma1    -0.198525    0.000143 -1392.0723 0.000000
ma2    -0.959399    0.000342 -2808.3341 0.000000
ma3    -0.518080    0.000365 -1420.8184 0.000000
ma4     0.624430    0.000205  3040.2287 0.000000
omega  -0.796144    0.324638    -2.4524 0.014190
alpha1  0.038405    0.049472     0.7763 0.437570
beta1   0.898254    0.039436    22.7777 0.000000
gamma1  0.698361    0.145147     4.8114 0.000001
```

```
Robust Standard Errors:
        Estimate  Std. Error     t value Pr(>|t|)
mu      0.004465    0.000073  6.1275e+01 0.000000
ar1     0.740321    0.005532  1.3382e+02 0.000000
ar2     0.849692    0.002174  3.9081e+02 0.000000
ar3     0.010295    0.000016  6.3334e+02 0.000000
ar4    -0.886455    0.000742 -1.1941e+03 0.000000
ar5     0.258568    0.002426  1.0657e+02 0.000000
ma1    -0.198525    0.002948 -6.7337e+01 0.000000
ma2    -0.959399    0.000259 -3.7056e+03 0.000000
ma3    -0.518080    0.007351 -7.0481e+01 0.000000
ma4     0.624430    0.000895  6.9757e+02 0.000000
omega  -0.796144    5.973520 -1.3328e-01 0.893973
alpha1  0.038405    2.128653  1.8042e-02 0.985605
beta1   0.898254    0.710026  1.2651e+00 0.205835
gamma1  0.698361    0.176440  3.9581e+00 0.000076

LogLikelihood : 467.8996

Information Criteria
------------------------------------

Akaike       -4.8545
Bayes        -4.6126
Shibata      -4.8647
Hannan-Quinn -4.7565
```

```
Nyblom stability test
------------------------------------
Joint Statistic:  3.6077
Individual Statistics:
mu     0.01491
ar1    0.01443
ar2    0.01268
ar3    0.01494
ar4    0.01322
ar5    0.01488
ma1    0.01482
ma2    0.01285
ma3    0.01475
ma4    0.01288
omega  0.05697
alpha1 0.07275
beta1  0.05664
gamma1 0.05286

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         3.08 3.34 3.9
Individual Statistic:    0.35 0.47 0.75
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                       1.81  0.1785
Lag[2*(p+q)+(p+q)-1][26]    12.73  0.9073
Lag[4*(p+q)+(p+q)-1][44]    19.91  0.7630
d.o.f=9
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      8.387 0.003779
Lag[2*(p+q)+(p+q)-1][5]    11.741 0.003257
Lag[4*(p+q)+(p+q)-1][9]    12.523 0.013767
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     2.495 0.500 2.000  0.1142
ARCH Lag[5]     2.643 1.440 1.667  0.3460
ARCH Lag[7]     2.724 2.315 1.543  0.5672
```

```
Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.3722 0.17169
Negative Sign Bias  0.1263 0.89963
Positive Sign Bias  2.5718 0.01092  **
Joint Effect        6.7408 0.08064   *
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     55.46    1.976e-05
2    30     57.33    1.311e-03
3    40     76.74    2.920e-04
4    50     82.79    1.817e-03


Elapsed time : 1.221735
```

Sacramento County GJR-GARCH

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(5,0,4)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.050913    0.025173  2.02254 0.043120
ar1    -0.039129    0.150830 -0.25942 0.795310
ar2     0.085516    0.347369  0.24618 0.805541
ar3     0.122810    0.025575  4.80197 0.000002
ar4     0.109741    0.207901  0.52785 0.597601
ar5     0.629847    0.191522  3.28865 0.001007
ma1     0.973185    0.221552  4.39257 0.000011
ma2     0.909654    0.540452  1.68314 0.092349
ma3     0.637066    0.523216  1.21760 0.223377
ma4     0.648220    0.197406  3.28369 0.001025
omega   0.000051    0.000015  3.31284 0.000924
alpha1  1.000000    0.236622  4.22615 0.000024
beta1   0.396016    0.024916 15.89436 0.000000
gamma1 -0.794032    0.295827 -2.68411 0.007272
```

```
Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.050913    0.148392  0.343099 0.731524
ar1    -0.039129    0.935169 -0.041841 0.966625
ar2     0.085516    2.033562  0.042052 0.966457
ar3     0.122810    0.341677  0.359435 0.719270
ar4     0.109741    1.218411  0.090069 0.928232
ar5     0.629847    1.120901  0.561912 0.574176
ma1     0.973185    1.323859  0.735112 0.462271
ma2     0.909654    3.142880  0.289433 0.772250
ma3     0.637066    3.034994  0.209907 0.833740
ma4     0.648220    1.181006  0.548871 0.583094
omega   0.000051    0.000055  0.923441 0.355778
alpha1  1.000000    0.575459  1.737744 0.082256
beta1   0.396016    0.513133  0.771761 0.440256
gamma1 -0.794032    0.470770 -1.686664 0.091668

LogLikelihood : 462.386

Information Criteria
------------------------------------

Akaike       -4.7956
Bayes        -4.5537
Shibata      -4.8058
Hannan-Quinn -4.6976
```

```
Nyblom stability test
------------------------------------
Joint Statistic:  2.4936
Individual Statistics:
mu     0.005933
ar1    0.109067
ar2    0.082604
ar3    0.127370
ar4    0.020798
ar5    0.017135
ma1    0.162528
ma2    0.112348
ma3    0.129177
ma4    0.114351
omega  0.144816
alpha1 0.239816
beta1  0.148726
gamma1 0.169677

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         3.08 3.34 3.9
Individual Statistic:    0.35 0.47 0.75
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      10.61 0.0011271
Lag[2*(p+q)+(p+q)-1][26]    29.62 0.0000000
Lag[4*(p+q)+(p+q)-1][44]    37.65 0.0001277
d.o.f=9
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      14.61 0.0001322
Lag[2*(p+q)+(p+q)-1][5]     15.31 0.0003672
Lag[4*(p+q)+(p+q)-1][9]     15.91 0.0021511
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3] 0.0002983 0.500 2.000  0.9862
ARCH Lag[5] 0.1227163 1.440 1.667  0.9825
ARCH Lag[7] 0.4095681 2.315 1.543  0.9858
```

```
Sign Bias Test
------------------------------------
                   t-value      prob sig
Sign Bias           1.5022 1.348e-01
Negative Sign Bias  0.4892 6.253e-01
Positive Sign Bias  5.2284 4.661e-07 ***
Joint Effect       28.1727 3.341e-06 ***
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     12.68       0.8546
2    30     28.45       0.4937
3    40     39.95       0.4277
4    50     72.63       0.0158


Elapsed time : 1.434166
```

The eGARCH model has the best performance among all four models for Sacramento HPI returns. The eGARCH model has a slightly higher log-likelihood (467.90) than the sGARCH model (466.51), while also having a slightly lower AIC, (-4.8545) as compared to (-4.8504). This would suggest that the eGARCH model might fit the data better than sGARCH. But both models have similar p-values for the Ljung-Box test on standardized residuals, which suggests that they both fit the data quite well. A notable difference between the models is that the parameter estimates of the sGARCH model seem to be more significant compared to the eGARCH model, but this might not translate to better prediction accuracy.

San Diego County sGARCH (1,1)

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.009476    0.011463  0.82666 0.408428
ar1     0.938372    0.040021 23.44723 0.000000
ma1    -0.321229    0.098323 -3.26709 0.001087
omega   0.000035    0.000022  1.61103 0.107173
alpha1  0.521609    0.111850  4.66348 0.000003
beta1   0.477391    0.095981  4.97379 0.000001

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.009476    0.014592   0.6494 0.516078
ar1     0.938372    0.079415  11.8160 0.000000
ma1    -0.321229    0.140557  -2.2854 0.022290
omega   0.000035    0.000047   0.7362 0.461608
alpha1  0.521609    0.162314   3.2136 0.001311
beta1   0.477391    0.172495   2.7676 0.005648

LogLikelihood : 480.4385

Information Criteria
------------------------------------

Akaike       -5.0742
Bayes        -4.9705
Shibata      -5.0762
Hannan-Quinn -5.0322
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2456 0.62016
Lag[2*(p+q)+(p+q)-1][5]    3.9785 0.07027
Lag[4*(p+q)+(p+q)-1][9]    6.6663 0.16345
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      0.326  0.5680
Lag[2*(p+q)+(p+q)-1][5]     1.825  0.6602
Lag[4*(p+q)+(p+q)-1][9]     2.322  0.8632
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.151 0.500 2.000  0.2833
ARCH Lag[5]     1.619 1.440 1.667  0.5615
ARCH Lag[7]     1.691 2.315 1.543  0.7823

Nyblom stability test
------------------------------------
Joint Statistic:  2.1641
Individual Statistics:
mu     0.1239
ar1    0.1972
ma1    0.1107
omega  0.1350
alpha1 0.4470
beta1  0.2154

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.49 1.68 2.12
Individual Statistic:    0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     24.02       0.1955
2    30     38.08       0.1206
3    40     37.39       0.5436
4    50     59.79       0.1389
```

San Diego County iGARCH

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.009495    0.011245  0.84441 0.398441
ar1     0.938481    0.038429 24.42105 0.000000
ma1    -0.321393    0.097210 -3.30617 0.000946
omega   0.000034    0.000014  2.43428 0.014921
alpha1  0.522240    0.089295  5.84849 0.000000
beta1   0.477760          NA       NA       NA

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.009495    0.013692  0.69348 0.488010
ar1     0.938481    0.064021 14.65904 0.000000
ma1    -0.321393    0.130027 -2.47175 0.013445
omega   0.000034    0.000021  1.67376 0.094177
alpha1  0.522240    0.119390  4.37423 0.000012
beta1   0.477760          NA       NA       NA

LogLikelihood : 480.4662

Information Criteria
------------------------------------

Akaike       -5.0852
Bayes        -4.9988
Shibata      -5.0866
Hannan-Quinn -5.0502
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2439 0.62138
Lag[2*(p+q)+(p+q)-1][5]    3.9697 0.07185
Lag[4*(p+q)+(p+q)-1][9]    6.6564 0.16461
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.3251  0.5686
Lag[2*(p+q)+(p+q)-1][5]    1.8263  0.6600
Lag[4*(p+q)+(p+q)-1][9]    2.3236  0.8631
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.152 0.500 2.000  0.2832
ARCH Lag[5]     1.618 1.440 1.667  0.5618
ARCH Lag[7]     1.690 2.315 1.543  0.7825

Nyblom stability test
------------------------------------
Joint Statistic:  0.9898
Individual Statistics:
mu     0.1235
ar1    0.1971
ma1    0.1110
omega  0.1345
alpha1 0.1716

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.28 1.47 1.88
Individual Statistic:    0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     24.02       0.1955
2    30     38.08       0.1206
3    40     37.39       0.5436
4    50     59.79       0.1389
```

San Diego County eGARCH

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.019703    0.010979   1.7947 0.072705
ar1     0.931559    0.037531  24.8209 0.000000
ma1    -0.281084    0.095383  -2.9469 0.003210
omega  -0.803317    0.336264  -2.3889 0.016897
alpha1  0.110213    0.103325   1.0667 0.286125
beta1   0.890237    0.041843  21.2756 0.000000
gamma1  1.022967    0.168715   6.0633 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.019703    0.013043  1.51066 0.130876
ar1     0.931559    0.054997 16.93845 0.000000
ma1    -0.281084    0.118683 -2.36837 0.017867
omega  -0.803317    0.424518 -1.89230 0.058450
alpha1  0.110213    0.156561  0.70396 0.481458
beta1   0.890237    0.050639 17.58001 0.000000
gamma1  1.022967    0.219582  4.65870 0.000003

LogLikelihood : 487.2268

Information Criteria
------------------------------------

Akaike       -5.1361
Bayes        -5.0152
Shibata      -5.1388
Hannan-Quinn -5.0871
```


```
Information Criteria
------------------------------------

Akaike       -5.1361
Bayes        -5.0152
Shibata      -5.1388
Hannan-Quinn -5.0871
```


```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.06195 0.80344
Lag[2*(p+q)+(p+q)-1][5]   3.87598 0.09042
Lag[4*(p+q)+(p+q)-1][9]   6.93337 0.13455
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.07638  0.7823
Lag[2*(p+q)+(p+q)-1][5]   1.63678  0.7063
Lag[4*(p+q)+(p+q)-1][9]   2.29811  0.8666
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.6259 0.500 2.000  0.4289
ARCH Lag[5]    1.5045 1.440 1.667  0.5911
ARCH Lag[7]    1.6262 2.315 1.543  0.7957

Nyblom stability test
------------------------------------
Joint Statistic:  0.9968
Individual Statistics:
mu     0.16074
ar1    0.33506
ma1    0.06913
omega  0.11857
alpha1 0.18264
beta1  0.12367
gamma1 0.07414

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     30.01      0.05173
2    30     32.95      0.27990
3    40     40.81      0.39094
4    50     51.24      0.38606
```

San Diego County GJR-GARCH

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.009167    0.011054  0.82930 0.406935
ar1     0.935789    0.041022 22.81167 0.000000
ma1    -0.295883    0.104922 -2.82002 0.004802
omega   0.000037    0.000026  1.44047 0.149735
alpha1  0.606940    0.207601  2.92359 0.003460
beta1   0.450855    0.114531  3.93653 0.000083
gamma1 -0.117591    0.249458 -0.47139 0.637364

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.009167    0.014409  0.63622  0.52463
ar1     0.935789    0.086364 10.83546  0.00000
ma1    -0.295883    0.201994 -1.46482  0.14297
omega   0.000037    0.000064  0.57344  0.56635
alpha1  0.606940    0.430165  1.41095  0.15826
beta1   0.450855    0.276170  1.63253  0.10257
gamma1 -0.117591    0.533627 -0.22036  0.82559

LogLikelihood : 480.5473

Information Criteria
------------------------------------

Akaike       -5.0647
Bayes        -4.9437
Shibata      -5.0673
Hannan-Quinn -5.0157
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.3859 0.53444
Lag[2*(p+q)+(p+q)-1][5]    4.4284 0.02051
Lag[4*(p+q)+(p+q)-1][9]    7.2693 0.10425
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.4194  0.5173
Lag[2*(p+q)+(p+q)-1][5]    1.8868  0.6454
Lag[4*(p+q)+(p+q)-1][9]    2.4154  0.8501
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.192 0.500 2.000  0.2749
ARCH Lag[5]     1.717 1.440 1.667  0.5372
ARCH Lag[7]     1.811 2.315 1.543  0.7573

Nyblom stability test
------------------------------------
Joint Statistic:  2.3644
Individual Statistics:
mu     0.1737
ar1    0.2070
ma1    0.1066
omega  0.1447
alpha1 0.5134
beta1  0.2202
gamma1 0.5764

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     22.95       0.2397
2    30     36.16       0.1691
3    40     43.80       0.2750
4    50     61.93       0.1016
```

The GJR-GARCH model seems well specified after examining the Ljung-Box and ARCH tests. However, the asymmetry parameter ‘gamma1’ is not statistically significant, suggesting that the leverage effect may not be present in the data. We will examine how it compares to the other

GARCH models regarding information criteria, residuals, and out-of-sample forecasting performance.

Looking at the optimal parameters and robust standard errors across the models, we find that they are fairly similar among the four. In terms of goodness-of-fit, the eGARCH model also has lowest AIC and highest log-likelihood, which indicates that this model performs better than the others. All models seem to have no significant serial correlation in the residuals, as indicated by the high p-values in the Ljung-Box tests. This would indicate that all three models have captured most of the dependencies in the data. The Nyblom stability test results indicate that all three models appear relatively stable, with their joint statistics being smaller than their critical values.

Overall, the eGARCH model for San Diego County performs slightly better than the other two in terms of fitting the data, as indicated by higher log-likelihood and lower information criteria. Even when comparing to GJR-GARCH, the eGARCH model is a better choice when taking into account AIC and BIC, while having parameters that are generally more statistically significant. It also doesn’t appear to have autocorrelation in the residuals.

H. Value-at-Risk Analysis

San Diego GARCH		
Probability	VaR	ES
0.95	0.0744	0.0901
0.99	0.1	0.1128
0.999	0.1287	0.1391
0.9999	0.1523	0.1613

San Diego 1 Year Forcast		
Probability	VaR	ES
0.95	0.1469	0.1809
0.99	0.2025	0.2301
0.999	0.2648	0.2874
0.9999	0.3161	0.3357

Sacramento GARCH		
Probability	VaR	ES
0.95	0.0964	0.1186
0.99	0.1326	0.1506
0.999	0.1731	0.1878
0.9999	0.2065	0.2192

Sacramento 1 Year Forcast		
Probability	VaR	ES
0.95	0.2203	0.2739
0.99	0.3077	0.3512
0.999	0.4058	0.4413
0.9999	0.4865	0.5172

San Diego Empirical Quantile VaR		
0.95	0.99	0.999
0.0611	0.0974	0.1142

Sacramento Empirical Quantile VaR		
0.95	0.99	0.999
0.0511	0.0856	0.1653

J. Managerial Implications of VaR

We can see from the value-at-risk analysis that the Sacramento housing market tends to carry more risk compared to San Diego, at all confidence levels. For San Diego, there is a 5% chance of experiencing a loss greater than about 7.4% and a 1% chance of a loss greater than 10%. Looking at the one year (4 period) forecast for San Diego, there is a 5% chance of experiencing a loss greater than 14.7% and a 1% chance of incurring a loss greater than 20%. Alternatively, when investing in Sacramento's housing market, there is a 5% chance of a loss greater than 9.6% in one period (1 quarter) and a 1% chance of taking a loss greater than 13%. If the investment is held for 4 periods (1 year) then there is a 5% chance of seeing a loss greater than 22% of your principle and a 1% chance of a loss greater than 30%. Additionally, the worst case scenario (about a 0.00001% chance) for a one year investment in Sacramento is a 48% loss, whereas in San Diego your worst case scenario is about a 31% loss. The expected shortfall in this worst case scenario is 51% and 34% for Sacramento and San Diego respectively. The Empirical Quantile value at risk suggests that, historically, San Diego has carried more risk at the lower probability levels and that it's only in the worst case scenario that the Sacramento investment market carries more risk.

Seeing as how real estate requires a large principal investment, minimizing loss should be a priority. Taking these risk metrics into consideration, all else equal, San Diego would likely be a more favorable investment environment than Sacramento. This is particularly true for any investor who is highly risk averse, as the losses in a worst case scenario situation could be substantially larger in Sacramento county compared to San Diego county.