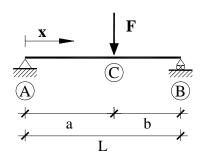
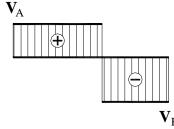
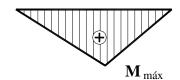
# VIGA SIMPLE APOYADA: carga puntual ${f F}$ genérica.







### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{Fb}}{\mathbf{L}}$$
  $\mathbf{R}_{B} = \frac{\mathbf{Fa}}{\mathbf{L}}$ 

Cortantes: 
$$V_{AC} = \frac{Fb}{L}$$
  $V_{CB} = -\frac{Fa}{L}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{\mathbf{F}\mathbf{b}}{\mathbf{I}}\mathbf{x}$$
  $\mathbf{M}_{CB} = \frac{\mathbf{F}\mathbf{a}}{\mathbf{I}}(\mathbf{L} - \mathbf{x})$ 

$$\mathbf{M}_{\text{máx}} = \mathbf{M}_{\text{C}} = \frac{Fab}{\tau}$$
 para  $\mathbf{x} = \mathbf{a}$ 

#### **Deformaciones**

Giros:

$$\phi_{A} = -\frac{Fab}{6EIL}(L+b)$$
 $\phi_{B} = \frac{Fab}{6EIL}(L+a)$ 
 $\phi_{C} = \frac{Fab}{3EIL}(a-b)$ 

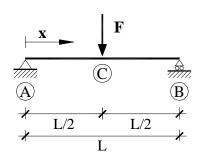
Elástica:

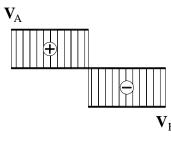
$$\mathbf{y}_{AC} = \frac{\mathbf{FLbx}}{6\mathbf{EI}} \left( 1 - \frac{\mathbf{b}^2}{\mathbf{L}^2} - \frac{\mathbf{x}^2}{\mathbf{L}^2} \right) \qquad \mathbf{y}_{CB} = \frac{\mathbf{FLa}(\mathbf{L} - \mathbf{x})}{6\mathbf{EI}} \left( 1 - \frac{\mathbf{a}^2}{\mathbf{L}^2} - \frac{(\mathbf{L} - \mathbf{x})^2}{\mathbf{L}^2} \right)$$

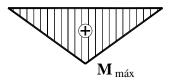
Flecha máxima (a>b):

$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{F} \mathbf{b}}{9 \mathbf{E} \mathbf{I} \mathbf{L} \sqrt{3}} (\mathbf{L}^2 - \mathbf{b}^2)^{3/2}$$
 para  $\mathbf{x} = \sqrt{\frac{\mathbf{L}^2 - \mathbf{b}^2}{3}}$ 

# VIGA SIMPLE APOYADA: carga puntual F centrada.







## Reacciones y solicitaciones

Reacciones:  $\mathbf{R}_{A} = \mathbf{R}_{B} = \frac{\mathbf{F}}{2}$ 

Cortantes: 
$$\mathbf{V}_{AC} = \frac{\mathbf{F}}{2}$$
  $\mathbf{V}_{CB} = -\frac{\mathbf{F}}{2}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{\mathbf{F}}{2}\mathbf{x}$$
  $\mathbf{M}_{CB} = \frac{\mathbf{F}}{2}(\mathbf{L} - \mathbf{x})$ 

$$\mathbf{M}_{\text{máx}} = \mathbf{M}_{\text{C}} = \frac{\mathbf{FL}}{4}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

#### **Deformaciones**

Giros:

$$\phi_A = -\frac{\mathbf{F}\mathbf{L}^2}{16\mathbf{E}\mathbf{I}} \qquad \qquad \phi_B = \frac{\mathbf{F}\mathbf{L}^2}{16\mathbf{E}\mathbf{I}} \qquad \qquad \phi_C = 0$$

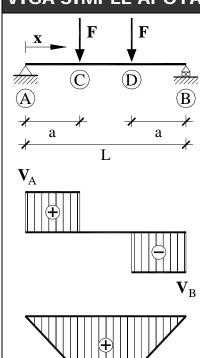
Elástica:

$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{L}^2 \mathbf{x}}{16 \,\mathbf{E}\mathbf{I}} \left( 1 - \frac{4}{3} \frac{\mathbf{x}^2}{\mathbf{L}^2} \right) \qquad \mathbf{y}_{CB} = \frac{\mathbf{F}\mathbf{L}^2 (\mathbf{L} - \mathbf{x})}{12 \,\mathbf{E}\mathbf{I}} \left( \frac{3}{4} - \frac{(\mathbf{L} - \mathbf{x})^2}{\mathbf{L}^2} \right)$$

Flecha máxima:

$$\mathbf{y}_{\text{máx}} = \mathbf{y}_{\text{C}} = \frac{\mathbf{F} \mathbf{L}^3}{48 \mathbf{E} \mathbf{I}}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

# VIGA SIMPLE APOYADA: cargas puntuales ${f F}$ simétricas.



### Reacciones y solicitaciones

 $\mathbf{V}_{\mathrm{DR}} = -\mathbf{F}$ 

Reacciones:  $\mathbf{R}_{A} = \mathbf{R}_{B} = \mathbf{F}$ 

Cortantes: 
$$\mathbf{V}_{AC} = \mathbf{F}$$
  $\mathbf{V}_{CD} = 0$ 

Flectores: 
$$\mathbf{M}_{AC} = \mathbf{F} \mathbf{x}$$
  $\mathbf{M}_{DB} = \mathbf{F} (\mathbf{L} - \mathbf{x})$   $\mathbf{M}_{CD} = \mathbf{M}_{m\acute{a}x} = \mathbf{F} \mathbf{a}$ 

#### **Deformaciones**

Giros:

$$-\phi_{A} = \phi_{B} = \frac{\mathbf{Fa}(\mathbf{L} - \mathbf{a})}{2\mathbf{EI}}$$
  $-\phi_{C} = \phi_{D} = \frac{\mathbf{Fa}(\mathbf{L} - 2\mathbf{a})}{2\mathbf{EI}}$ 

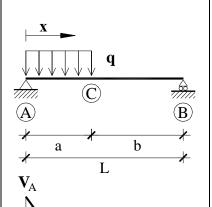
Elástica:

$$\mathbf{y}_{\mathrm{AC}} = \frac{\mathbf{F}\mathbf{x}}{6\mathbf{E}\mathbf{I}} \left( 3\mathbf{a}\mathbf{L} - 3\mathbf{a}^2 - \mathbf{x}^2 \right) \qquad \mathbf{y}_{\mathrm{CD}} = \frac{\mathbf{F}\mathbf{a}}{6\mathbf{E}\mathbf{I}} \left( 3\mathbf{L}\mathbf{x} - 3\mathbf{x}^2 - \mathbf{a}^2 \right)$$

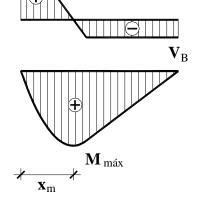
Flecha máxima:

$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{F} \mathbf{a}}{24 \mathbf{E} \mathbf{I}} (3 \mathbf{L}^2 - 4 \mathbf{a}^2)$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

# $f VIGA SIMPLE APOYADA: carga uniforme {f q} en un extremo.$



 $\mathbf{M}_{\text{máx}}$ 



### Reacciones y solicitaciones

Reacciones:  $\mathbf{R}_{A} = \frac{\mathbf{q}\mathbf{a}}{\mathbf{L}} \left( \mathbf{b} + \frac{\mathbf{a}}{2} \right)$   $\mathbf{R}_{B} = \frac{\mathbf{q}\mathbf{a}^{2}}{2\mathbf{L}}$ 

Cortantes:

$$\mathbf{V}_{A} = \frac{\mathbf{q}\mathbf{a}}{\mathbf{L}} \left( \mathbf{b} + \frac{\mathbf{a}}{2} \right)$$
  $\mathbf{V}_{AC} = \frac{\mathbf{q}\mathbf{a}}{\mathbf{L}} \left( \mathbf{b} + \frac{\mathbf{a}}{2} \right) - \mathbf{p}\mathbf{x}$   $\mathbf{V}_{CB} = -\frac{\mathbf{q}\mathbf{a}^{2}}{2\mathbf{L}}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{\mathbf{q}\mathbf{a}}{\mathbf{L}} \left( \mathbf{b} + \frac{\mathbf{a}}{2} \right) \mathbf{x} - \frac{\mathbf{p} \mathbf{x}^2}{2}$$
  $\mathbf{M}_{CB} = \frac{\mathbf{q}\mathbf{a}^2}{2\mathbf{L}} \left( \mathbf{L} - \mathbf{x} \right)$   $\mathbf{M}_{m\acute{a}x} = \frac{\mathbf{q}\mathbf{a}^2}{2} \left( 1 - \frac{\mathbf{a}}{2\mathbf{L}} \right)^2$  para  $\mathbf{x}_m = \mathbf{a} \left( 1 - \frac{\mathbf{a}}{2\mathbf{L}} \right)$ 

Giros: 
$$\varphi_{A} = -\frac{\mathbf{qa}^{2}}{24 \mathbf{EIL}} (\mathbf{L} + \mathbf{b})^{2}$$
  $\varphi_{B} = \frac{\mathbf{qa}^{2}}{12 \mathbf{EI}} \left(1 - \frac{\mathbf{a}^{2}}{2 \mathbf{L}^{2}}\right) \mathbf{L}$ 

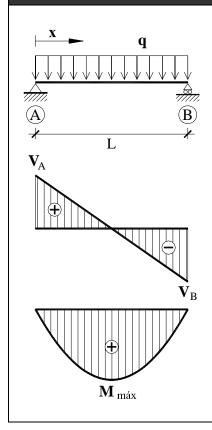
Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{q}\mathbf{x}}{24\mathbf{EIL}} \left[ \mathbf{L}\mathbf{x}^3 - 4\mathbf{a} \left( \mathbf{b} + \frac{\mathbf{a}}{2} \right) \mathbf{x}^2 + \mathbf{a}^2 \left( \mathbf{L} + \mathbf{b} \right)^2 \right]$$

$$\mathbf{y}_{CB} = -\frac{\mathbf{q}(\mathbf{L} - \mathbf{x})\mathbf{a}^2}{12\mathbf{EIL}} \left[ (\mathbf{L} - \mathbf{x})^2 - \mathbf{L}^2 \left( 1 - \frac{\mathbf{a}^2}{2\mathbf{L}^2} \right) \right]$$

Flecha máxima: 
$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{qa}^2}{216 \, \text{FU}} \left( 2\mathbf{L}^2 - \mathbf{a}^2 \right) \sqrt{6 \left( 2\mathbf{L}^2 - \mathbf{a}^2 \right)}$$

para **a**<0,4531**L** en **x** = **L** - 
$$\sqrt{\frac{\mathbf{L}^2}{3} - \frac{\mathbf{a}^2}{6}}$$

# VIGA SIMPLE APOYADA: carga uniforme q en todo el vano.



## Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \mathbf{R}_{B} = \frac{\mathbf{qL}}{2}$$

Cortantes: 
$$\mathbf{V}_{AB} = \mathbf{q} \left( \frac{\mathbf{L}}{2} - \mathbf{x} \right)$$
  $\mathbf{V}_{A} = -\mathbf{V}_{B} = \frac{\mathbf{q} \mathbf{L}}{2}$ 

Flectores: 
$$\mathbf{M}_{AB} = \frac{\mathbf{q}\mathbf{x}}{2}(\mathbf{L} - \mathbf{x})$$

$$\mathbf{M}_{\text{máx}} = \frac{\mathbf{qL}^2}{8}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

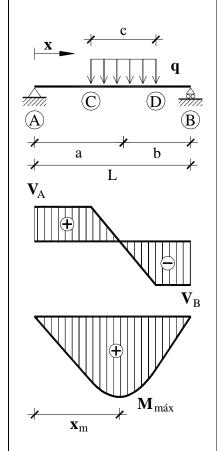
### **Deformaciones**

Giros: 
$$\varphi_{A} = -\frac{qL^{3}}{24 EI} \qquad \varphi_{B} = \frac{qL^{3}}{24 EI}$$

Elástica: 
$$\mathbf{y}_{AB} = \frac{\mathbf{q}\mathbf{x}}{24\mathbf{E}\mathbf{I}} \left( \mathbf{x}^3 - 2\mathbf{L}\mathbf{x}^2 + \mathbf{L}^3 \right)$$

Flecha máxima: 
$$\mathbf{y}_{\text{máx}} = \frac{5 \, \mathbf{q} \, \mathbf{L}^4}{384 \, \mathbf{FI}}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

# VIGA SIMPLE APOYADA: carga uniforme ${f q}$ en tramo intermedio.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{qbc}}{\mathbf{L}}$$
  $\mathbf{R}_{B} = \frac{\mathbf{qac}}{\mathbf{L}}$ 

Cortantes:

$$\mathbf{V}_{\mathrm{AC}} = \frac{\mathbf{qbc}}{\mathbf{L}}$$
  $\mathbf{V}_{\mathrm{CD}} = \frac{\mathbf{qbc}}{\mathbf{L}} - \mathbf{q} \left( \frac{\mathbf{c}}{2} - \mathbf{a} + \mathbf{x} \right)$   $\mathbf{V}_{\mathrm{DB}} = -\frac{\mathbf{qac}}{\mathbf{L}}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{\mathbf{qbc}}{\mathbf{L}}\mathbf{x}$$
  $\mathbf{M}_{CD} = \frac{\mathbf{qbc}}{\mathbf{L}}\mathbf{x} - \frac{\mathbf{q}}{2}\left[\mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2}\right]^2$ 

$$\mathbf{M}_{\mathrm{DB}} = \frac{\mathbf{qac}}{\mathbf{L}} (\mathbf{L} - \mathbf{x})$$
  $\mathbf{M}_{\mathrm{máx}} = \frac{\mathbf{qbc}}{2\mathbf{L}} \left( 2\mathbf{a} - \mathbf{c} + \frac{\mathbf{bc}}{\mathbf{L}} \right)$  para  $\mathbf{x}_{\mathrm{m}} = \mathbf{a} - \frac{\mathbf{c}}{2} + \frac{\mathbf{bc}}{\mathbf{L}}$ 

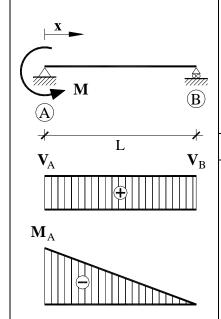
Giros: 
$$\phi_A = -\frac{\mathbf{qabc}}{6\mathbf{EIL}} \left( \mathbf{L} + \mathbf{b} - \frac{\mathbf{c}^2}{4\mathbf{a}} \right) \qquad \phi_B = \frac{\mathbf{qabc}}{6\mathbf{EIL}} \left( \mathbf{L} + \mathbf{a} - \frac{\mathbf{c}^2}{4\mathbf{b}} \right)$$

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{qbcx}}{6\mathbf{EIL}} \left| -\mathbf{x}^2 + \mathbf{a} \left( \mathbf{L} + \mathbf{b} - \frac{\mathbf{c}^2}{4\mathbf{a}} \right) \right|$$

$$\mathbf{y}_{CD} = \frac{\mathbf{q}}{24 \mathbf{EIL}} \left[ \mathbf{L} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)^4 - 4 \mathbf{b} \mathbf{c} \mathbf{x}^3 + 4 \mathbf{a} \mathbf{b} \mathbf{c} \left( \mathbf{L} + \mathbf{b} - \frac{\mathbf{c}^2}{4 \mathbf{a}} \right) \mathbf{x} \right]$$

$$\mathbf{y}_{\mathrm{DB}} = \frac{\mathbf{qac}(\mathbf{L} - \mathbf{x})}{6\mathbf{EIL}} \left[ -(\mathbf{L} - \mathbf{x})^2 + \mathbf{b} \left( \mathbf{L} + \mathbf{a} - \frac{\mathbf{c}^2}{4\mathbf{b}} \right) \right]$$

# VIGA SIMPLE APOYADA: momento puntual ${f M}$ en extremo.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{M}}{\mathbf{L}}$$
  $\mathbf{R}_{B} = -\frac{\mathbf{M}}{\mathbf{L}}$ 

Cortantes: 
$$V_{AB} = \frac{M}{L}$$

Flectores: 
$$\mathbf{M}_{AB} = -\mathbf{M} \left( 1 - \frac{\mathbf{x}}{\mathbf{L}} \right)$$
  $\mathbf{M}_{A} = -\mathbf{M}$   $\mathbf{M}_{B} = 0$ 

### **Deformaciones**

Giros: 
$$\varphi_{A} = \frac{ML}{3EI}$$
  $\varphi_{B} = -\frac{ML}{6EI}$ 

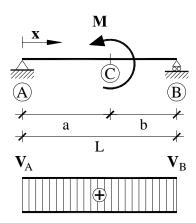
Elástica:

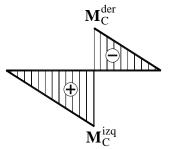
$$\mathbf{y}_{AB} = -\frac{\mathbf{ML}}{6\mathbf{EI}} \left( \mathbf{L} - \mathbf{x} \right) \left[ 1 - \left( \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}} \right)^{2} \right]$$

Flecha máxima:

$$\mathbf{y}_{\text{máx}} = -\frac{\mathbf{M} \mathbf{L}^2}{9 \mathbf{E} \mathbf{I} \sqrt{3}}$$
 para  $\mathbf{x} = \mathbf{L} \left( 1 - \frac{1}{\sqrt{3}} \right)$ 

# VIGA SIMPLE APOYADA: momento puntual ${f M}$ intermedio.





### Reacciones y solicitaciones

Reacciones:  $\mathbf{R}_{A} = \frac{\mathbf{M}}{\mathbf{L}}$   $\mathbf{R}_{B} = -\frac{\mathbf{M}}{\mathbf{L}}$ 

Cortantes:  $V_{AB} = \frac{M}{L}$ 

Flectores:  $\mathbf{M}_{AC} = \frac{\mathbf{M}}{\mathbf{L}} \mathbf{x}$   $\mathbf{M}_{CB} = -\frac{\mathbf{M}}{\mathbf{L}} (\mathbf{L} - \mathbf{x})$ 

 $\mathbf{M}_{\mathrm{C}}^{\mathrm{izq}} = \frac{\mathbf{M}}{\mathbf{L}}\mathbf{a}$   $\mathbf{M}_{\mathrm{C}}^{\mathrm{der}} = -\frac{\mathbf{M}}{\mathbf{L}}\mathbf{b}$   $\left|\mathbf{M}\right| = \left|\mathbf{M}_{\mathrm{C}}^{\mathrm{izq}}\right| + \left|\mathbf{M}_{\mathrm{C}}^{\mathrm{der}}\right|$ 

#### **Deformaciones**

Giros:

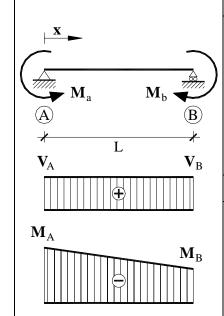
$$\phi_{A} = \frac{\mathbf{ML}}{6\mathbf{EI}} \left( 3\frac{\mathbf{b}^{2}}{\mathbf{L}^{2}} - 1 \right) \qquad \phi_{B} = \frac{\mathbf{ML}}{6\mathbf{EI}} \left( 3\frac{\mathbf{a}^{2}}{\mathbf{L}^{2}} - 1 \right) \qquad \phi_{C} = \frac{\mathbf{M}}{3\mathbf{EIL}^{2}} \left( \mathbf{a}^{3} + \mathbf{b}^{3} \right)$$

Elástica:

$$\mathbf{y}_{AC} = \frac{\mathbf{MLx}}{6\mathbf{EI}} \left( 1 - 3\frac{\mathbf{b}^2}{\mathbf{L}^2} - \frac{\mathbf{x}^2}{\mathbf{L}^2} \right) \qquad \mathbf{y}_{CB} = -\frac{\mathbf{ML}(\mathbf{L} - \mathbf{x})}{6\mathbf{EI}} \left[ 1 - 3\frac{\mathbf{a}^2}{\mathbf{L}^2} - \left(\frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}}\right)^2 \right]$$

Flecha en C:  $\mathbf{y}_{C} = -\frac{\mathbf{M} \mathbf{a} \mathbf{b}}{3 \mathbf{E} \mathbf{I} \mathbf{L}} (\mathbf{b} - \mathbf{a})$ 

# f VIGA SIMPLE APOYADA: momentos puntuales \$M\$ contrarios en extremos.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{M}_{a} - \mathbf{M}_{b}}{\mathbf{I}}$$
  $\mathbf{R}_{B} = -\frac{\mathbf{M}_{a} - \mathbf{M}_{b}}{\mathbf{I}}$ 

Cortantes: 
$$V_{AB} = \frac{M_a - M_b}{L}$$

Flectores:

$$\mathbf{M}_{AB} = -\frac{\mathbf{M}_a}{L}(\mathbf{L} - \mathbf{x}) - \frac{\mathbf{M}_b}{L}\mathbf{x}$$
  $\mathbf{M}_A = -\mathbf{M}_a$   $\mathbf{M}_B = -\mathbf{M}_b$ 

#### **Deformaciones**

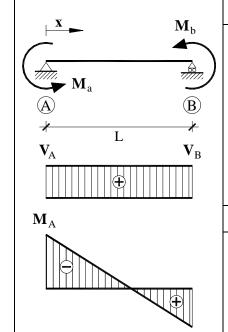
Giros:

$$\phi_A \, = \frac{L}{6EI} \big( 2 \boldsymbol{M}_a \, + \boldsymbol{M}_b \, \big) \qquad \quad \phi_B = -\frac{L}{6EI} \big( \boldsymbol{M}_a \, + 2 \boldsymbol{M}_b \, \big)$$

Elástica:

$$\mathbf{y}_{AB} = -\frac{\mathbf{M}_{a}\mathbf{x}}{6\mathbf{E}\mathbf{I}} \left( \mathbf{L} - \mathbf{x} \right) \left[ 1 + \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}} + \frac{\mathbf{M}_{b}}{\mathbf{M}_{a}} \left( 1 + \frac{\mathbf{x}}{\mathbf{L}} \right) \right]$$

# VIGA SIMPLE APOYADA: momentos puntuales ${f M}$ en extremos.



### Reacciones y solicitaciones

Reacciones:  $\mathbf{R}_{A} = \frac{\mathbf{M}_{a} + \mathbf{M}_{b}}{\mathbf{T}}$   $\mathbf{R}_{B} = -\frac{\mathbf{M}_{a} + \mathbf{M}_{b}}{\mathbf{T}}$ 

Cortantes: 
$$V_{AB} = \frac{M_a + M_b}{T}$$

Flectores:

$$\mathbf{M}_{AB} = -\frac{\mathbf{M}_{a}}{L}(\mathbf{L} - \mathbf{x}) + \frac{\mathbf{M}_{b}}{L}\mathbf{x}$$
  $\mathbf{M}_{A} = -\mathbf{M}_{a}$   $\mathbf{M}_{B} = \mathbf{M}_{b}$ 

#### **Deformaciones**

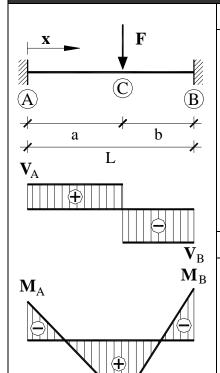
Giros:

$$\phi_{A} = -\frac{L}{6EI} (2M_{a} - M_{b}) \qquad \phi_{B} = -\frac{L}{6EI} (2M_{b} - M_{a})$$

Elástica:

$$\mathbf{y}_{AB} = \frac{\mathbf{x}(\mathbf{L} - \mathbf{x})}{6\mathbf{F}\mathbf{H}} \left[ (\mathbf{M}_{a} + \mathbf{M}_{b})\mathbf{x} - (2\mathbf{M}_{a} - \mathbf{M}_{b})\mathbf{L} \right]$$

# VIGA SIMPLE EMPOTRADA: carga puntual F genérica.



 $\mathbf{M}_{\mathbf{C}}$ 

### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{F}\mathbf{b}^{2}}{\mathbf{L}^{3}}(\mathbf{L} + 2\mathbf{a})$$
  $\mathbf{R}_{B} = \frac{\mathbf{F}\mathbf{a}^{2}}{\mathbf{L}^{3}}(\mathbf{L} + 2\mathbf{b})$ 

Cortantes: 
$$\mathbf{V}_{AC} = \frac{\mathbf{Fb}^2}{\mathbf{L}^3} (\mathbf{L} + 2\mathbf{a})$$
  $\mathbf{V}_{CB} = -\frac{\mathbf{Fa}^2}{\mathbf{L}^3} (\mathbf{L} + 2\mathbf{b})$ 

Flectores: 
$$\mathbf{M}_{A} = -\frac{\mathbf{F}\mathbf{a}\mathbf{b}^{2}}{\mathbf{L}^{2}}$$
  $\mathbf{M}_{B} = -\frac{\mathbf{F}\mathbf{a}^{2}\mathbf{b}}{\mathbf{L}^{2}}$   $\mathbf{M}_{C} = \frac{2\mathbf{F}\mathbf{a}^{2}\mathbf{b}^{2}}{\mathbf{L}^{3}}$ 

$$\mathbf{M}_{AC} = \frac{\mathbf{Fb}^2}{\mathbf{L}^3} (\mathbf{Lx} + 2\mathbf{ax} - \mathbf{aL})$$
  $\mathbf{M}_{CB} = \frac{\mathbf{Fa}^2}{\mathbf{L}^3} (\mathbf{Lb} + \mathbf{L}^2 - \mathbf{Lx} - 2\mathbf{bx})$ 

#### **Deformaciones**

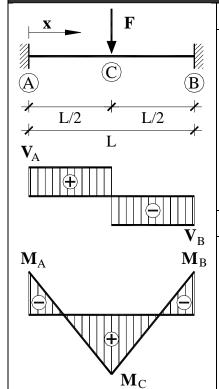
Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{Fb}^2}{6\mathbf{EI}} \left( 3\mathbf{a} - \mathbf{x} - \frac{2\mathbf{ax}}{\mathbf{L}} \right) \frac{\mathbf{x}^2}{\mathbf{L}^2}$$

$$\mathbf{y}_{CB} = \frac{\mathbf{Fa}^2}{6\mathbf{EI}} \left( 3\mathbf{b} - (\mathbf{L} - \mathbf{x}) - 2\mathbf{b} \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}} \right) \frac{(\mathbf{L} - \mathbf{x})^2}{\mathbf{L}^2}$$

Flechas:

$$\mathbf{y}_{\mathrm{C}} = \frac{\mathbf{F} \mathbf{b}^{3} \mathbf{a}^{3}}{3 \mathbf{E} \mathbf{I} \mathbf{L}^{3}}$$
  $\mathbf{y}_{\mathrm{máx}} = \frac{2 \mathbf{F} \mathbf{b}^{2} \mathbf{a}^{3}}{3 \mathbf{E} \mathbf{I} (\mathbf{L} + 2\mathbf{a})^{2}}$  para  $\mathbf{x} = \frac{2 \mathbf{a} \mathbf{L}}{\mathbf{L} + 2\mathbf{a}}$ 

# VIGA SIMPLE EMPOTRADA: carga puntual ${f F}$ centrada.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \mathbf{R}_{B} = \frac{\mathbf{F}}{2}$$

Cortantes: 
$$V_{AC} = \frac{F}{2}$$
  $V_{CB} = -\frac{F}{2}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{\mathbf{FL}}{8} \left( 4 \frac{\mathbf{x}}{\mathbf{L}} - 1 \right) \qquad \mathbf{M}_{CB} = \frac{\mathbf{FL}}{8} \left( 3 - 4 \frac{\mathbf{x}}{\mathbf{L}} \right)$$

$$\mathbf{M}_{A} = \mathbf{M}_{B} = -\frac{\mathbf{FL}}{8}$$
  $\mathbf{M}_{max} = \mathbf{M}_{C} = \frac{\mathbf{FL}}{8}$  para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

#### **Deformaciones**

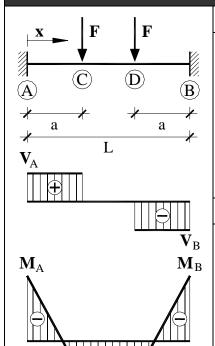
Elástica:

$$\mathbf{y}_{AC} = \frac{\mathbf{FLx}^2}{48\mathbf{EI}} \left( 3 - 4\frac{\mathbf{x}}{\mathbf{L}} \right)$$
  $\mathbf{y}_{CB} = \frac{\mathbf{F}(\mathbf{L} - \mathbf{x})^2}{48\mathbf{EI}} \left( 4\mathbf{x} - \mathbf{L} \right)$ 

Flecha máxima:

$$\mathbf{y}_{\text{máx}} = \mathbf{y}_{\text{C}} = \frac{\mathbf{F} \mathbf{L}^3}{192 \, \mathbf{EI}}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

# VIGA SIMPLE EMPOTRADA: cargas puntuales ${f F}$ simétricas.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \mathbf{R}_{B} = \mathbf{F}$$

Cortantes: 
$$\mathbf{V}_{AC} = \mathbf{F}$$
  $\mathbf{V}_{CD} = 0$   $\mathbf{V}_{DB} = -\mathbf{F}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{\mathbf{F}}{\mathbf{L}} (\mathbf{L}(\mathbf{x} - \mathbf{a}) + \mathbf{a}^2)$$
  $\mathbf{M}_{CD} = \frac{\mathbf{F} \mathbf{a}^2}{\mathbf{L}}$ 

$$\mathbf{M}_{\mathrm{DB}} = \frac{\mathbf{F}}{\mathbf{L}} \Big( \mathbf{L} \Big( \mathbf{L} - \mathbf{x} - \mathbf{a} \Big) + \mathbf{a}^2 \Big)$$
  $\mathbf{M}_{\mathrm{A}} = \mathbf{M}_{\mathrm{B}} = -\frac{\mathbf{F} \mathbf{a}}{\mathbf{L}} \Big( \mathbf{L} - \mathbf{a} \Big)$ 

#### **Deformaciones**

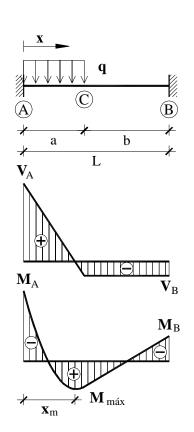
Elástica:

$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{x}^2}{6\mathbf{EIL}} \left( 3\mathbf{aL} - 3\mathbf{a}^2 - \mathbf{L}\mathbf{x} \right) \qquad \mathbf{y}_{CD} = \frac{\mathbf{F}\mathbf{a}^2}{6\mathbf{EIL}} \left( -\mathbf{aL} + 3\mathbf{L}\mathbf{x} - 3\mathbf{x}^2 \right)$$

Flecha máxima:

$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{F} \mathbf{a}^2}{24 \mathbf{EI}} (3\mathbf{L} - 4\mathbf{a})$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

# VIGA SIMPLE EMPOTRADA: carga uniforme q en un extremo.



### Reacciones y solicitaciones

Reacciones:

$$\mathbf{R}_{A} = \frac{\mathbf{q}\mathbf{a}}{2\mathbf{L}}(\mathbf{L} + \mathbf{b}) - \frac{\mathbf{M}_{A} - \mathbf{M}_{B}}{\mathbf{L}} \qquad \mathbf{R}_{B} = \frac{\mathbf{q}\mathbf{a}^{2}}{2\mathbf{L}} + \frac{\mathbf{M}_{A} - \mathbf{M}_{B}}{\mathbf{L}}$$

Cortantes:

$$\mathbf{V}_{\mathrm{A}} = \mathbf{R}_{\mathrm{A}}$$
  $\mathbf{V}_{\mathrm{AC}} = \mathbf{R}_{\mathrm{A}} - \mathbf{p}\mathbf{x}$   $\mathbf{V}_{\mathrm{CB}} = \mathbf{R}_{\mathrm{A}} - \mathbf{p}\mathbf{a}$   $\mathbf{V}_{\mathrm{B}} = -\mathbf{R}_{\mathrm{B}}$ 

Flectores:

$$\mathbf{M}_{AC} = \mathbf{R}_{A} \mathbf{x} + \mathbf{M}_{A} - \frac{\mathbf{p} \mathbf{x}^{2}}{2} \qquad \mathbf{M}_{CB} = \mathbf{R}_{B} (\mathbf{L} - \mathbf{x}) + \mathbf{M}_{B}$$

$$\mathbf{M}_{A} = -\frac{\mathbf{q} \mathbf{a}^{2}}{12\mathbf{I}^{2}} \left( 6\mathbf{L}^{2} - 8\mathbf{L}\mathbf{a} + 3\mathbf{a}^{2} \right) \qquad \mathbf{M}_{B} = -\frac{\mathbf{q} \mathbf{a}^{3}}{3\mathbf{I}} \left( 1 - \frac{3\mathbf{a}}{4\mathbf{I}} \right)$$

#### **Deformaciones**

Elástica:

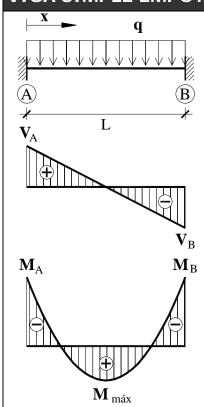
$$\mathbf{y}_{AC} = \frac{\mathbf{x}^2}{24\mathbf{E}\mathbf{I}} \left( \mathbf{q} \mathbf{x}^2 - 4\mathbf{R}_{A} \mathbf{x} - 12\mathbf{M}_{A} \right)$$

$$\mathbf{y}_{CB} = \frac{1}{6 \, \mathbf{EI}} \left[ \mathbf{R}_{B} \mathbf{x}^{3} - 3 \left( \mathbf{M}_{B} + \mathbf{L} \mathbf{R}_{B} \right) \mathbf{x}^{2} + 3 \left( 2 \mathbf{M}_{B} + \mathbf{L} \mathbf{R}_{B} \right) \mathbf{L} \mathbf{x} - \left( 3 \mathbf{M}_{B} + \mathbf{L} \mathbf{R}_{B} \right) \mathbf{L}^{2} \right]$$

Flecha máxima:

$$\mathbf{y}_{\text{máx}} = -\frac{2\mathbf{M}_{\text{B}}^3}{3\mathbf{R}_{\text{B}}^2\mathbf{EI}}$$
 para  $\mathbf{x} = \frac{2\mathbf{M}_{\text{B}} + \mathbf{L}\mathbf{R}_{\text{B}}}{\mathbf{R}_{\text{B}}}$ 

# VIGA SIMPLE EMPOTRADA: carga uniforme q en todo el vano.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \mathbf{R}_{B} = \frac{\mathbf{qL}}{2}$$

Cortantes: 
$$\mathbf{V}_{AB} = \mathbf{q} \left( \frac{\mathbf{L}}{2} - \mathbf{x} \right)$$
  $\mathbf{V}_{A} = -\mathbf{V}_{B} = \frac{\mathbf{q} \mathbf{L}}{2}$ 

Flectores: 
$$\mathbf{M}_{AB} = -\frac{\mathbf{q}}{12} (\mathbf{L}^2 - 6\mathbf{L}\mathbf{x} + 6\mathbf{x}^2)$$
  $\mathbf{M}_{A} = \mathbf{M}_{B} = -\frac{\mathbf{q}\mathbf{L}^2}{12}$ 

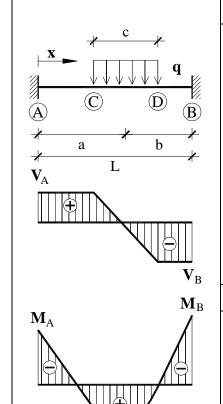
$$\mathbf{M}_{\text{máx}} = \frac{\mathbf{qL}^2}{24}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$   $\mathbf{M}_{\text{x}} = 0$  para  $\mathbf{x} = 0,2113\mathbf{L}$ 

#### **Deformaciones**

Elástica: 
$$\mathbf{y}_{AB} = \frac{\mathbf{q}\mathbf{L}^4}{24\mathbf{E}\mathbf{I}} \left(\frac{\mathbf{x}}{\mathbf{L}} - \frac{\mathbf{x}^2}{\mathbf{L}^2}\right)^2$$

Flecha máxima: 
$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{q} \mathbf{L}^4}{384 \, \text{FI}}$$
 para  $\mathbf{x} = \frac{\mathbf{L}}{2}$ 

# VIGA SIMPLE EMPOTRADA: carga uniforme ${f q}$ en tramo intermedio.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{qbc}}{\mathbf{L}} - \frac{\mathbf{M}_{A} - \mathbf{M}_{B}}{\mathbf{L}}$$
  $\mathbf{R}_{B} = \frac{\mathbf{qac}}{\mathbf{L}} + \frac{\mathbf{M}_{A} - \mathbf{M}_{B}}{\mathbf{L}}$ 

Cortantes:

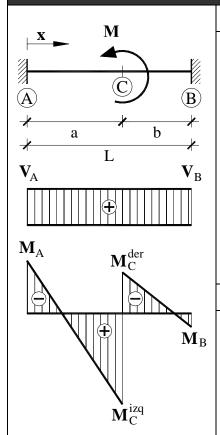
$$\mathbf{V}_{\mathrm{AC}} = \mathbf{R}_{\mathrm{A}}$$
  $\mathbf{V}_{\mathrm{CD}} = \mathbf{R}_{\mathrm{A}} - \mathbf{q} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)$   $\mathbf{V}_{\mathrm{DB}} = -\mathbf{R}_{\mathrm{B}}$ 

Flectores: 
$$\mathbf{M}_{AC} = \mathbf{R}_{A}\mathbf{x} + \mathbf{M}_{A}$$
  $\mathbf{M}_{CD} = \mathbf{R}_{A}\mathbf{x} + \mathbf{M}_{A} - \frac{\mathbf{q}}{2} \left[ \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right]^{2}$ 

$$\mathbf{M}_{\mathrm{DB}} = \mathbf{R}_{\mathrm{B}} (\mathbf{L} - \mathbf{x}) + \mathbf{M}_{\mathrm{B}} \qquad \mathbf{M}_{\mathrm{A}} = -\frac{\mathbf{q} \mathbf{c}^{3}}{12 \mathbf{L}^{2}} \left( \mathbf{L} - 3\mathbf{b} + \frac{12 \mathbf{a} \mathbf{b}^{2}}{\mathbf{c}^{2}} \right)$$
$$\mathbf{M}_{\mathrm{B}} = -\frac{\mathbf{q} \mathbf{c}^{3}}{12 \mathbf{L}^{2}} \left( \mathbf{L} - 3\mathbf{a} + \frac{12 \mathbf{a}^{2} \mathbf{b}}{\mathbf{c}^{2}} \right)$$

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{x}^2}{6\mathbf{E}\mathbf{I}} \left[ -3\mathbf{M}_A - \mathbf{R}_A \mathbf{x} \right]$$
$$\mathbf{y}_{CD} = \frac{1}{24\mathbf{E}\mathbf{I}} \left[ \mathbf{q} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)^4 - 4\mathbf{R}_A \mathbf{x}^3 - 12\mathbf{M}_A \mathbf{x}^2 \right]$$
$$\mathbf{y}_{DB} = \frac{1}{6\mathbf{E}\mathbf{I}} \left[ \mathbf{R}_B \mathbf{x}^3 - 3(\mathbf{M}_B + \mathbf{L}\mathbf{R}_B)\mathbf{x}^2 + 3(2\mathbf{M}_A + \mathbf{L}\mathbf{R}_B)\mathbf{L}\mathbf{x} - (3\mathbf{M}_B + \mathbf{L}\mathbf{R}_B)\mathbf{L}^2 \right]$$

# VIGA SIMPLE EMPOTRADA: momento puntual ${f M}$ intermedio.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{6\mathbf{M}}{\mathbf{L}^{3}}\mathbf{ab}$$
  $\mathbf{R}_{B} = -\frac{6\mathbf{M}}{\mathbf{L}^{3}}\mathbf{ab}$ 

Cortantes: 
$$V_{AB} = \frac{6M}{L^3}ab$$

Flectores:

$$\mathbf{M}_{AC} = \frac{\mathbf{Ma}}{\mathbf{L}} \left[ 1 - 3 \frac{\mathbf{a}}{\mathbf{L}} \left( 1 - 2 \frac{\mathbf{x}}{\mathbf{L}} \right) \right] \qquad \mathbf{M}_{CB} = -\frac{\mathbf{Mb}}{\mathbf{L}} \left[ 1 - 3 \frac{\mathbf{b}}{\mathbf{L}} \left( 1 - 2 \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}} \right) \right]$$

$$\mathbf{M}_{A} = \frac{\mathbf{Ma}}{\mathbf{L}} \left( 1 - 3 \frac{\mathbf{a}}{\mathbf{L}} \right) \qquad \mathbf{M}_{B} = -\frac{\mathbf{Mb}}{\mathbf{L}} \left( 1 - 3 \frac{\mathbf{b}}{\mathbf{L}} \right)$$

$$\mathbf{M}_{C}^{izq} = \mathbf{M}_{A} + \frac{6\mathbf{M}}{\mathbf{L}^{3}} \mathbf{a}^{2} \mathbf{b} \qquad \mathbf{M}_{C}^{der} = \mathbf{M}_{A} - \frac{\mathbf{M}}{\mathbf{L}^{3}} \left( \mathbf{L}^{3} - 6 \mathbf{a}^{2} \mathbf{b} \right)$$

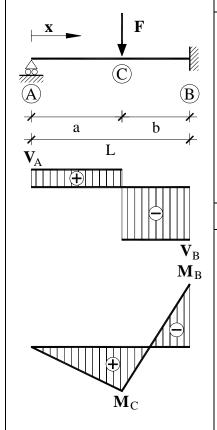
### **Deformaciones**

Elástica:

$$\mathbf{y}_{\mathrm{AC}} = \frac{\mathbf{Mbx}^2}{2\,\mathbf{EIL}} \left( 2\mathbf{a} \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}^2} - \frac{\mathbf{b}}{\mathbf{L}} \right) \qquad \mathbf{y}_{\mathrm{CB}} = -\frac{\mathbf{Ma}(\mathbf{L} - \mathbf{x})^2}{2\,\mathbf{EIL}} \left( 2\frac{\mathbf{bx}}{\mathbf{L}^2} - \frac{\mathbf{a}}{\mathbf{L}} \right)$$

Flecha en C: 
$$\mathbf{y}_{C} = -\frac{\mathbf{M} \mathbf{a}^{2} \mathbf{b}^{2}}{2 \mathbf{E} \mathbf{I} \mathbf{L}^{3}} (\mathbf{a} - \mathbf{b})$$

# $oldsymbol{\mathsf{VIGA}}$ $oldsymbol{\mathsf{SIMPLE}}$ $oldsymbol{\mathsf{APOYADA-EMPOTRADA:}}$ carga puntual $oldsymbol{\mathsf{F}}$ genérica.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{F}\mathbf{b}^{2}}{2\mathbf{I}^{3}}(3\mathbf{L} - \mathbf{b})$$
  $\mathbf{R}_{B} = \frac{\mathbf{F}\mathbf{a}}{2\mathbf{I}^{3}}(3\mathbf{L}^{2} - \mathbf{a}^{2})$ 

Cortantes: 
$$\mathbf{V}_{AC} = \frac{\mathbf{F}\mathbf{b}^2}{2\mathbf{I}^3} (3\mathbf{L} - \mathbf{b})$$
  $\mathbf{V}_{CB} = -\frac{\mathbf{F}\mathbf{a}}{2\mathbf{I}^3} (3\mathbf{L}^2 - \mathbf{a}^2)$ 

Flectores: 
$$\mathbf{M}_{\mathrm{B}} = -\frac{\mathbf{F}\mathbf{a}}{2\mathbf{I}^2} (\mathbf{L}^2 - \mathbf{a}^2)$$
  $\mathbf{M}_{\mathrm{C}} = \frac{\mathbf{F}\mathbf{a}}{2\mathbf{I}^3} \mathbf{b}^2 (3\mathbf{a} + 2\mathbf{b})$ 

$$\mathbf{M}_{AC} = \frac{\mathbf{F}\mathbf{x}\mathbf{b}^2}{2\mathbf{L}^3} (3\mathbf{a} + 2\mathbf{b}) \qquad \mathbf{M}_{CB} = \frac{\mathbf{F}\mathbf{a}}{2\mathbf{L}^3} (2\mathbf{L}^3 - 3\mathbf{L}^2\mathbf{x} + \mathbf{a}^2\mathbf{x})$$

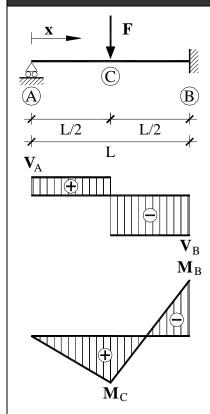
Giros: 
$$\varphi_{A} = -\frac{\mathbf{Fa}}{4\mathbf{EIL}}(\mathbf{L} - \mathbf{a})^{2}$$
  $\varphi_{C} = -\frac{\mathbf{Fa}(\mathbf{L} - \mathbf{a})^{2}}{4\mathbf{EIL}^{3}}(\mathbf{L}^{2} - 2\mathbf{aL} - \mathbf{a}^{2})$ 

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{b}^2\mathbf{x}}{12\mathbf{E}\mathbf{I}\mathbf{L}^3} \left[ 3\mathbf{a}\mathbf{L}^2 - \mathbf{x}^2 (2\mathbf{L} + \mathbf{a}) \right]$$

$$\mathbf{y}_{CB} = \frac{\mathbf{Fa}(\mathbf{L} - \mathbf{x})^2}{12\mathbf{EI}} \left[ 3\left(1 - \frac{\mathbf{a}^2}{\mathbf{L}^2}\right) - \left(3 - \frac{\mathbf{a}^2}{\mathbf{L}^2}\right) \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}} \right]$$

Flechas: 
$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{F} \mathbf{b}^2 \mathbf{a}}{6\mathbf{E} \mathbf{I}} \sqrt{\frac{\mathbf{a}}{2\mathbf{L} + \mathbf{a}}}$$
 para  $\mathbf{x} = \mathbf{L} \sqrt{\frac{\mathbf{a}}{2\mathbf{L} + \mathbf{a}}}$ 

# $\overline{ extsf{VIGA SIMPLE APOYADA-EMPOTRADA:}}$ carga puntual $\overline{ extsf{F}}$ centrada.



## Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{5}{16}\mathbf{F}$$
  $\mathbf{R}_{B} = \frac{11}{16}\mathbf{F}$ 

Cortantes: 
$$\mathbf{V}_{AC} = \frac{5}{16}\mathbf{F}$$
  $\mathbf{V}_{CB} = -\frac{11}{16}\mathbf{F}$ 

Flectores: 
$$\mathbf{M}_{AC} = \frac{5}{16} \mathbf{F} \mathbf{x}$$
  $\mathbf{M}_{CB} = \frac{\mathbf{F} \mathbf{L}}{16} \left( 11 \frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}} - 3 \right)$ 

$$\mathbf{M}_{\mathrm{B}} = -\frac{3}{16}\mathbf{FL} \qquad \mathbf{M}_{\mathrm{C}} = \frac{5}{32}\mathbf{FL}$$

Giros: 
$$\phi_{A} = -\frac{\mathbf{F}\mathbf{a}^{2}}{32\mathbf{E}\mathbf{I}} \qquad \qquad \phi_{C} = \frac{\mathbf{F}\mathbf{L}^{2}}{128\mathbf{E}\mathbf{I}}$$

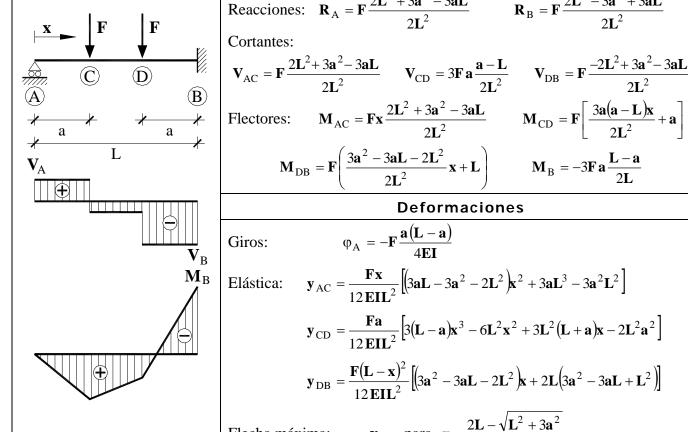
Elástica:

$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{L}^2}{96\mathbf{E}\mathbf{I}}\mathbf{x} \left(3 - 5\frac{\mathbf{x}^2}{\mathbf{L}^2}\right) \qquad \mathbf{y}_{CB} = \frac{\mathbf{F}\mathbf{L}}{96\mathbf{E}\mathbf{I}}(\mathbf{L} - \mathbf{x})^2 \left(9 - 11\frac{\mathbf{L} - \mathbf{x}}{\mathbf{L}}\right)$$

Flechas:

$$\mathbf{y}_{\mathrm{C}} = \frac{7\mathbf{F}\mathbf{L}^{3}}{768\mathbf{E}\mathbf{I}}$$
  $\mathbf{y}_{\mathrm{máx}} = \frac{\mathbf{F}\mathbf{L}^{3}}{48\sqrt{5}\mathbf{E}\mathbf{I}}$  para  $\mathbf{x} = \frac{\mathbf{L}}{\sqrt{5}}$ 

# VIGA SIMPLE APOYADA-EMPOTRADA: cargas puntuales ${f F}$ simétricas.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \mathbf{F} \frac{2\mathbf{L}^2 + 3\mathbf{a}^2 - 3\mathbf{a}\mathbf{L}}{2\mathbf{L}^2}$$
  $\mathbf{R}_{B} = \mathbf{F} \frac{2\mathbf{L}^2 - 3\mathbf{a}^2 + 3\mathbf{a}\mathbf{L}}{2\mathbf{L}^2}$ 

$$\mathbf{V}_{AC} = \mathbf{F} \frac{2\mathbf{L}^2 + 3\mathbf{a}^2 - 3\mathbf{a}\mathbf{L}}{2\mathbf{L}^2}$$
  $\mathbf{V}_{CD} = 3\mathbf{F} \mathbf{a} \frac{\mathbf{a} - \mathbf{L}}{2\mathbf{L}^2}$   $\mathbf{V}_{DB} = \mathbf{F} \frac{-2\mathbf{L}^2 + 3\mathbf{a}^2 - 3\mathbf{a}\mathbf{L}}{2\mathbf{L}^2}$ 

lectores: 
$$\mathbf{M}_{AC} = \mathbf{F} \mathbf{x} \frac{2\mathbf{L}^2 + 3\mathbf{a}^2 - 3\mathbf{a}\mathbf{L}}{2\mathbf{L}^2}$$
  $\mathbf{M}_{CD} = \mathbf{F} \left[ \frac{3\mathbf{a}(\mathbf{a} - \mathbf{L})\mathbf{x}}{2\mathbf{L}^2} + \mathbf{a} \right]$ 

$$\mathbf{M}_{\mathrm{DB}} = \mathbf{F} \left( \frac{3\mathbf{a}^2 - 3\mathbf{a}\mathbf{L} - 2\mathbf{L}^2}{2\mathbf{L}^2} \mathbf{x} + \mathbf{L} \right) \qquad \mathbf{M}_{\mathrm{B}} = -3\mathbf{F} \mathbf{a} \frac{\mathbf{L} - \mathbf{a}}{2\mathbf{L}}$$

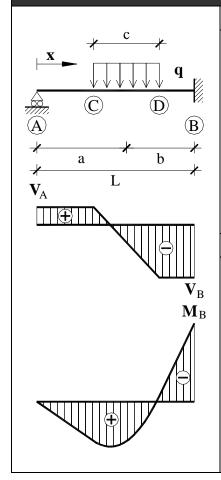
Giros: 
$$\phi_A = -F \frac{a(L-a)}{4EI}$$

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{x}}{12\mathbf{E}\mathbf{I}\mathbf{L}^2} \left[ \left( 3\mathbf{a}\mathbf{L} - 3\mathbf{a}^2 - 2\mathbf{L}^2 \right) \mathbf{x}^2 + 3\mathbf{a}\mathbf{L}^3 - 3\mathbf{a}^2\mathbf{L}^2 \right]$$
$$\mathbf{y}_{CD} = \frac{\mathbf{F}\mathbf{a}}{12\mathbf{E}\mathbf{I}\mathbf{L}^2} \left[ 3(\mathbf{L} - \mathbf{a})\mathbf{x}^3 - 6\mathbf{L}^2\mathbf{x}^2 + 3\mathbf{L}^2(\mathbf{L} + \mathbf{a})\mathbf{x} - 2\mathbf{L}^2\mathbf{a}^2 \right]$$

$$\mathbf{y}_{\mathrm{DB}} = \frac{\mathbf{F}(\mathbf{L} - \mathbf{x})^{2}}{12\mathbf{E}\mathbf{I}\mathbf{L}^{2}} \left[ \left( 3\mathbf{a}^{2} - 3\mathbf{a}\mathbf{L} - 2\mathbf{L}^{2} \right) \mathbf{x} + 2\mathbf{L} \left( 3\mathbf{a}^{2} - 3\mathbf{a}\mathbf{L} + \mathbf{L}^{2} \right) \right]$$

Flecha máxima: 
$$\mathbf{y}_{\text{máx}}$$
 para  $\mathbf{x} = \frac{2\mathbf{L} - \sqrt{\mathbf{L}^2 + 3\mathbf{a}^2}}{3(\mathbf{L} - \mathbf{a})\mathbf{L}}$ 

# VIGA SIMPLE APOYADA-EMPOTRADA: carga uniforme q intermedia.



# Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{\mathbf{qbc}}{\mathbf{L}} + \frac{\mathbf{M}_{B}}{\mathbf{L}} \qquad \qquad \mathbf{R}_{B} = \frac{\mathbf{qac}}{\mathbf{L}} - \frac{\mathbf{M}_{B}}{\mathbf{L}}$$

Cortantes: 
$$\mathbf{V}_{AC} = \mathbf{R}_{A}$$
  $\mathbf{V}_{CD} = \mathbf{R}_{A} - \mathbf{q} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)$   $\mathbf{V}_{DB} = -\mathbf{R}_{B}$ 

Flectores: 
$$\mathbf{M}_{AC} = \mathbf{R}_{A} \mathbf{x}$$
  $\mathbf{M}_{CD} = \mathbf{R}_{A} \mathbf{x} - \frac{\mathbf{q}}{2} \left[ \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right]^{2}$ 

$$\mathbf{M}_{\mathrm{DB}} = \mathbf{R}_{\mathrm{B}} (\mathbf{L} - \mathbf{x}) + \mathbf{M}_{\mathrm{B}}$$
  $\mathbf{M}_{\mathrm{B}} = -\frac{\mathbf{qabc}}{2\mathbf{L}^2} (\mathbf{L} + \mathbf{a} - \frac{\mathbf{c}^2}{4\mathbf{b}})$ 

#### **Deformaciones**

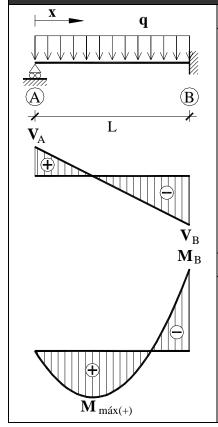
Giros: 
$$\varphi_{A} = -\frac{\mathbf{q} \mathbf{c}^{3}}{48 \mathbf{EIL}} \left( \mathbf{L} - 3\mathbf{b} + \frac{12 \mathbf{a} \mathbf{b}^{2}}{\mathbf{c}^{2}} \right)$$

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{x}}{48\mathbf{EIL}} \left[ -8\mathbf{R}_{A}\mathbf{L}\mathbf{x}^{2} + \mathbf{q}\mathbf{c}^{3} \left(\mathbf{L} - 3\mathbf{b} + \frac{12\mathbf{a}\mathbf{b}^{2}}{\mathbf{c}^{2}}\right) \right]$$

$$\mathbf{y}_{\text{CD}} = \frac{1}{48 \, \text{EIL}} \left[ -8 \mathbf{R}_{\text{A}} \mathbf{L} \mathbf{x}^3 + 2 \mathbf{q} \mathbf{L} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)^4 + \mathbf{q} \mathbf{c}^3 \left( \mathbf{L} - 3 \mathbf{b} + \frac{12 \mathbf{a} \mathbf{b}^2}{\mathbf{c}^2} \right) \mathbf{x} \right]$$

$$\mathbf{y}_{\mathrm{DB}} = -\frac{(\mathbf{L} - \mathbf{x})^{2}}{6\mathbf{E}\mathbf{I}} \left[ \mathbf{R}_{\mathrm{B}} (\mathbf{L} - \mathbf{x}) + 3\mathbf{M}_{\mathrm{B}} \right]$$

# VIGA SIMPLE APOYADA-EMPOTRADA: carga uniforme ${f q}$ en todo el vano.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = \frac{3}{8}\mathbf{q}\mathbf{L}$$

$$\mathbf{R}_{\mathrm{B}} = \frac{5}{8}\mathbf{q}\mathbf{L}$$

Cortantes: 
$$\mathbf{V}_{AB} = \mathbf{q}\mathbf{L} \left( \frac{3}{8} - \frac{\mathbf{x}}{\mathbf{L}} \right)$$
  $\mathbf{V}_{A} = \frac{3}{8}\mathbf{q}\mathbf{L}$   $\mathbf{V}_{B} = -\frac{5}{8}\mathbf{q}\mathbf{L}$ 

$$V_A = \frac{3}{9} qL$$

$$\mathbf{V}_{\mathrm{B}} = -\frac{5}{8}\mathbf{q}\mathbf{L}$$

Flectores: 
$$\mathbf{M}_{AB} = \frac{\mathbf{q} \mathbf{x}}{8} (3\mathbf{L} - 4\mathbf{x})$$
  $\mathbf{M}_{B} = -\frac{\mathbf{q} \mathbf{L}^{2}}{8}$ 

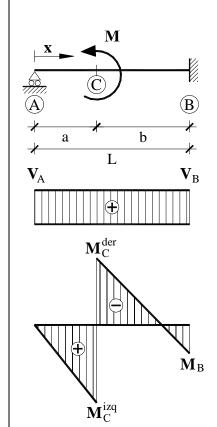
$$\mathbf{M}_{\text{máx}(+)} = \frac{9}{128} \mathbf{q} \mathbf{L}^2$$
 para  $\mathbf{x} = \frac{3}{8} \mathbf{L}$   $\mathbf{M}_{x} = 0$  para  $\mathbf{x} = \frac{3}{4} \mathbf{L}$ 

Giros: 
$$\varphi_{A} = -\frac{qL^{3}}{48EI}$$

Elástica: 
$$\mathbf{y}_{AB} = \frac{\mathbf{q}\mathbf{x}}{48\,\mathbf{FI}}(\mathbf{L} + 2\mathbf{x})(\mathbf{L} - \mathbf{x})^2$$

Flecha máxima: 
$$\mathbf{y}_{\text{máx}} = \frac{\mathbf{qL}^4}{185 \, \text{FI}}$$
 para  $\mathbf{x} = \frac{1 + \sqrt{33}}{16} \mathbf{L}$ 

# VIGA SIMPLE APOYADA-EMPOTRADA: momento puntual ${f M}$ intermedio.



### Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{A} = -\mathbf{R}_{B} = \frac{3}{2} \frac{\mathbf{M}}{\mathbf{L}^{3}} (\mathbf{L}^{2} - \mathbf{a}^{2})$$

Cortantes: 
$$V_{AB} = R_A$$

Flectores:

$$\mathbf{M}_{AC} = \frac{3}{2} \frac{\mathbf{M} \mathbf{x}}{\mathbf{L}^3} \left( \mathbf{L}^2 - \mathbf{a}^2 \right) \qquad \mathbf{M}_{CB} = \frac{\mathbf{M}}{2} \left[ 3 \frac{\mathbf{x}}{\mathbf{L}} \left( 1 - \frac{\mathbf{a}^2}{\mathbf{L}^2} \right) - 2 \right]$$

$$\mathbf{M}_{\mathrm{B}} = \frac{\mathbf{M}}{2\mathbf{L}^2} \left( \mathbf{L}^2 - 3\mathbf{a}^2 \right) \qquad \mathbf{M}_{\mathrm{C}}^{\mathrm{izq}} = \mathbf{R}_{\mathrm{A}} \mathbf{a} \qquad \mathbf{M}_{\mathrm{C}}^{\mathrm{der}} = \mathbf{R}_{\mathrm{A}} \mathbf{a} - \mathbf{M}$$

#### **Deformaciones**

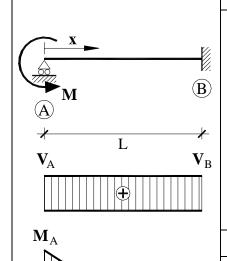
Giros: 
$$\varphi_{A} = -\frac{M}{4EU}(L-a)(3a-L)$$

$$\phi_{C} = \frac{\mathbf{M}\mathbf{b}}{4\mathbf{E}\mathbf{I}} \left[ 4 - 3\frac{\mathbf{b}}{\mathbf{L}} \left( 1 + \frac{\mathbf{a}}{\mathbf{L}} \right)^{2} \right]$$

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{Mbx}}{4\mathbf{EU}^3} \left[ -4\mathbf{L}^3 - (\mathbf{x}^2 - 3\mathbf{L}^2)(\mathbf{a} + \mathbf{L}) \right]$$

$$\mathbf{y}_{CB} = \frac{\mathbf{M} (\mathbf{L} - \mathbf{x})^2}{4 \mathbf{E} \mathbf{H}^3} \left[ 2\mathbf{a}^2 \mathbf{L} - \mathbf{x} (\mathbf{L}^2 - \mathbf{a}^2) \right]$$

# $f VIGA\ SIMPLE\ APOYADA-EMPOTRADA:\ momento\ puntual\ M\ extremo.$



# Reacciones y solicitaciones

Reacciones:

$$\mathbf{R}_{\mathrm{A}} = -\mathbf{R}_{\mathrm{B}} = \frac{3}{2} \frac{\mathbf{M}}{\mathbf{L}}$$

Cortantes:

$$\mathbf{V}_{\Delta \mathbf{R}} = \mathbf{R}_{\Delta}$$

Flectores:

$$\mathbf{M}_{AB} = \frac{\mathbf{M}}{2} \left( 3 \frac{\mathbf{x}}{\mathbf{L}} - 2 \right) \qquad \mathbf{M}_{B} = \frac{\mathbf{M}}{2}$$

### Deformaciones

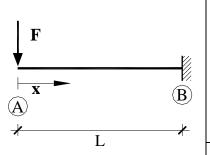
Giros:

$$\varphi_{A} = \frac{ML}{4EI}$$

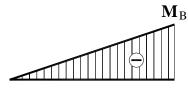
Elástica:

$$\mathbf{y}_{AB} = -\frac{\mathbf{M}\,\mathbf{x}}{4\,\mathbf{EHL}}(\mathbf{L} - \mathbf{x})^2$$

# VIGA SIMPLE EN VOLADIZO: carga puntual $\mathbf{F}$ en extremo.







### Reacciones y solicitaciones

Reacciones:

$$\mathbf{R}_{\mathrm{B}} = \mathbf{F}$$

Cortantes:

$$\mathbf{V}_{\mathrm{AB}} = -\mathbf{F}$$

Flectores:

$$\mathbf{M}_{\mathrm{CB}} = -\mathbf{F}\mathbf{x}$$
  $\mathbf{M}_{\mathrm{B}} = -\mathbf{F}\mathbf{L}$ 

$$\mathbf{M}_{\mathrm{B}} = -\mathbf{F}\mathbf{L}$$

### **Deformaciones**

Giros:

$$\varphi_{A} = \frac{\mathbf{FL}^{2}}{2\mathbf{EI}}$$

Elástica:

$$\mathbf{y}_{AB} = \frac{\mathbf{F}}{6\mathbf{EI}}(\mathbf{L} - \mathbf{x})^2(2\mathbf{L} + \mathbf{x})$$

Flechas:

$$\mathbf{y}_{\mathbf{A}} = \frac{\mathbf{F}\mathbf{L}^3}{3\mathbf{E}\mathbf{I}}$$

# VIGA SIMPLE EN VOLADIZO: carga puntual F genérica.





Reacciones y solicitaciones

Cortantes:

$$\mathbf{V}_{\mathrm{AC}} = 0 \qquad \qquad \mathbf{V}_{\mathrm{CB}} = -\mathbf{F}$$

Flectores:

$$\mathbf{M}_{\Delta C} = 0$$

$$\mathbf{M}_{AC} = 0$$
  $\mathbf{M}_{CB} = -\mathbf{F}(\mathbf{x} - \mathbf{a})$   $\mathbf{M}_{B} = -\mathbf{F}\mathbf{b}$ 

$$\mathbf{M}_{\mathbf{R}} = -\mathbf{F}\mathbf{b}$$

### **Deformaciones**



 $\mathbf{M}_{\mathrm{B}}$ 

$$\varphi_{\rm C} = \varphi_{\rm A} = \frac{\mathbf{Fb}^2}{2\mathbf{EI}}$$

Elástica:

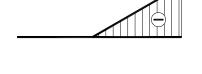
$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{b}^2}{6\mathbf{E}\mathbf{I}} [3(\mathbf{L} - \mathbf{x}) - \mathbf{b}]$$

$$\mathbf{y}_{AC} = \frac{\mathbf{F}\mathbf{b}^2}{6\mathbf{E}\mathbf{I}} [3(\mathbf{L} - \mathbf{x}) - \mathbf{b}]$$
  $\mathbf{y}_{CB} = \frac{\mathbf{F}}{6\mathbf{E}\mathbf{I}} (\mathbf{L} - \mathbf{x})^2 (2\mathbf{b} - \mathbf{a} + \mathbf{x})$ 

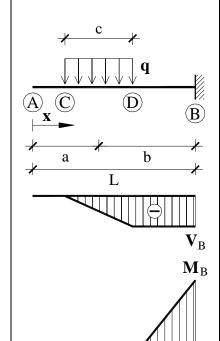
Flechas:

$$\mathbf{y}_{\mathrm{C}} = \frac{\mathbf{F} \mathbf{b}^{3}}{3\mathbf{E}\mathbf{I}}$$

$$\mathbf{y}_{\mathrm{C}} = \frac{\mathbf{F}\mathbf{b}^{3}}{3\mathbf{E}\mathbf{I}}$$
  $\mathbf{y}_{\mathrm{A}} = \frac{\mathbf{F}\mathbf{b}^{2}}{6\mathbf{E}\mathbf{I}}(2\mathbf{b} + 3\mathbf{a})$ 



# VIGA SIMPLE EN VOLADIZO: carga uniforme q intermedia.



# Reacciones y solicitaciones

Reacciones: 
$$\mathbf{R}_{\mathrm{B}} = \mathbf{qc}$$

Cortantes: 
$$\mathbf{V}_{AC} = 0$$
  $\mathbf{V}_{CD} = -\mathbf{q} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)$   $\mathbf{V}_{B} = -\mathbf{q} \mathbf{c}$ 

Flectores: 
$$\mathbf{M}_{AC} = 0$$
  $\mathbf{M}_{CD} = -\frac{\mathbf{q}}{2} \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)^2$ 

$$\mathbf{M}_{\mathrm{DB}} = -\mathbf{q}\mathbf{c}(\mathbf{x} - \mathbf{a})$$
  $\mathbf{M}_{\mathrm{B}} = -\mathbf{q}\mathbf{c}\mathbf{b}$ 

#### **Deformaciones**

Giros: 
$$\varphi_{A} = \varphi_{C} = \frac{\mathbf{qc}}{2\mathbf{EI}} \left( \mathbf{b}^{2} + \frac{\mathbf{c}^{2}}{12} \right) \varphi_{D} = \frac{\mathbf{qc}}{2\mathbf{EI}} \left( \mathbf{b}^{2} - \frac{\mathbf{c}^{2}}{4} \right)$$

Elástica: 
$$\mathbf{y}_{AC} = \frac{\mathbf{qc}}{6\mathbf{EI}} \left[ \left( \mathbf{a} - \mathbf{x} \right) \left( 3\mathbf{b}^2 + \frac{\mathbf{c}^2}{4} \right) + 2\mathbf{b}^3 \right]$$

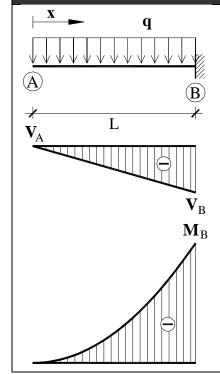
$$\mathbf{y}_{\mathrm{CD}} = \frac{\mathbf{q}}{24\,\mathbf{EI}} \left[ \left( \mathbf{x} - \mathbf{a} + \frac{\mathbf{c}}{2} \right)^4 + 4\mathbf{c} \left( \mathbf{a} - \mathbf{x} \right) \left( 3\mathbf{b}^2 + \frac{\mathbf{c}^2}{4} \right) + 8\mathbf{b}^3 \mathbf{c} \right]$$

$$\mathbf{y}_{\mathrm{DB}} = \frac{\mathbf{q} \, \mathbf{c}}{6 \mathbf{E} \mathbf{I}} (\mathbf{L} - \mathbf{x})^2 (2\mathbf{b} - \mathbf{a} + \mathbf{x})$$

Elástica: 
$$\mathbf{y}_{A} = \frac{\mathbf{q} \mathbf{c}}{6\mathbf{E}\mathbf{I}} \left[ \mathbf{a} \left( 3\mathbf{b}^{2} + \frac{\mathbf{c}^{2}}{4} \right) + 2\mathbf{b}^{3} \right]$$

$$\mathbf{y}_{\mathrm{C}} = \frac{\mathbf{q} \mathbf{c}}{12\mathbf{E}\mathbf{I}} \left[ \left( \mathbf{b} + \frac{\mathbf{c}}{2} \right)^{2} \left( 4\mathbf{b} - \mathbf{c} \right) + \mathbf{c}^{3} \right] \qquad \mathbf{y}_{\mathrm{D}} = \frac{\mathbf{q} \mathbf{c}}{\mathbf{E}\mathbf{I}} \left( \mathbf{b} - \frac{\mathbf{c}}{2} \right)^{2} \left( \frac{\mathbf{b}}{3} + \frac{\mathbf{c}}{12} \right)$$

# VIGA SIMPLE EN VOLADIZO: carga uniforme $\mathbf{q}$ en todo el vano.



# Reacciones y solicitaciones

Reacciones:  $R_B = qL$ 

Cortantes:  $V_{AB} = -q x$   $V_{B} = -qL$ 

Flectores:  $\mathbf{M}_{AB} = -\frac{\mathbf{q} \mathbf{x}^2}{2}$   $\mathbf{M}_{B} = -\frac{\mathbf{q} \mathbf{L}^2}{2}$ 

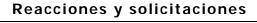
#### **Deformaciones**

Giros:  $\varphi_A = \frac{\mathbf{q} \mathbf{L}^3}{6\mathbf{E}\mathbf{I}}$ 

Elástica:  $\mathbf{y}_{AB} = \frac{\mathbf{q}}{24 \, \text{FJ}} (\mathbf{L} - \mathbf{x})^2 (3\mathbf{L}^2 + 2\mathbf{L}\mathbf{x} + \mathbf{x}^2)$ 

Flecha máxima:  $\mathbf{y}_{A} = \frac{\mathbf{q} \mathbf{L}^{4}}{8 \mathbf{E} \mathbf{I}}$ 

# VIGA SIMPLE EN VOLADIZO: momento puntual ${f M}$ extremo.



Reacciones:

$$\mathbf{R}_{\mathrm{B}} = 0$$

Cortantes:

$$\mathbf{V}_{AB} = 0$$

Flectores:

$$\mathbf{M}_{AB} = -\mathbf{M}$$

### **Deformaciones**

Giros:

$$\phi_{A} = -\frac{\mathbf{M}\,\mathbf{L}}{\mathbf{E}\mathbf{I}}$$

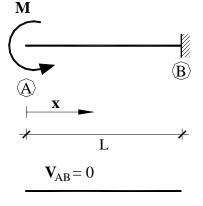
Elástica:

 $\mathbf{M}_{\mathrm{B}}$ 

$$\mathbf{y}_{AB} = \frac{\mathbf{M}}{2\mathbf{E}\mathbf{I}} \left( \mathbf{x}^2 - 2\mathbf{L}\mathbf{x} + \mathbf{L}^2 \right)$$

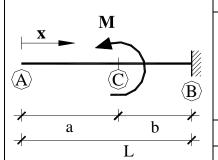
Flecha:

$$\mathbf{y}_{\mathrm{A}} = \frac{\mathbf{M}\mathbf{L}^2}{2\mathbf{E}\mathbf{I}}$$

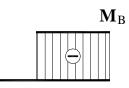




# VIGA SIMPLE EN VOLADIZO: momento puntual M intermedio.



$$\mathbf{V}_{\mathrm{AB}} = 0$$



Reacciones:  $\mathbf{R}_{\mathrm{B}} = 0$ 

Cortantes:  $\mathbf{V}_{AB} = 0$ 

Flectores:

$$\mathbf{M}_{\mathrm{AC}} = 0 \qquad \qquad \mathbf{M}_{\mathrm{CB}} = -\mathbf{M}$$

$$\mathbf{M}_{\mathrm{B}} = -\mathbf{M}$$

#### **Deformaciones**

Giros:

$$\phi_A = \phi_C = \frac{Mb}{EI}$$

Elástica:

$$\mathbf{y}_{AC} = \frac{\mathbf{M}}{2\mathbf{EI}} \mathbf{b} (2\mathbf{L} - 2\mathbf{x} - \mathbf{b})$$
  $\mathbf{y}_{CB} = \frac{\mathbf{M}}{2\mathbf{EI}} (\mathbf{L} - \mathbf{x})^2$ 

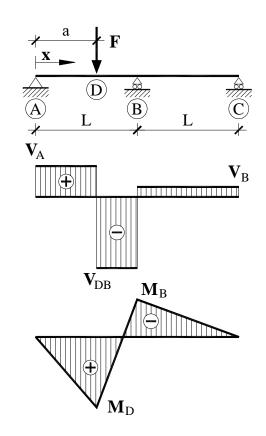
$$\mathbf{y}_{\mathrm{CB}} = \frac{\mathbf{M}}{2\mathbf{E}\mathbf{I}} (\mathbf{L} - \mathbf{x})^2$$

Flecha:

$$\mathbf{y}_{\mathrm{A}} = \frac{\mathbf{M}}{2\mathbf{E}\mathbf{I}}\mathbf{b}(2\mathbf{L} - \mathbf{b})$$
  $\mathbf{y}_{\mathrm{C}} = \frac{\mathbf{M}\mathbf{b}^{2}}{2\mathbf{E}\mathbf{I}}$ 

$$\mathbf{y}_{\mathrm{C}} = \frac{\mathbf{M}\mathbf{b}^2}{2\mathbf{E}\mathbf{I}}$$

# VIGA 2 VANOS IGUALES: carga puntual ${f F}$ en 1 $^{ m er}$ vano.



$$\mathbf{R}_{A} = \frac{\mathbf{F}}{4\mathbf{L}^{3}} \cdot \left(4\mathbf{L}^{3} - 5\mathbf{L}^{2}\mathbf{a} + \mathbf{a}^{3}\right)$$

$$\mathbf{R}_{\mathrm{B}} = \frac{\mathbf{F}\mathbf{a}}{2\mathbf{L}^{3}} \cdot \left(3\mathbf{L}^{2} - \mathbf{a}^{2}\right)$$

$$\mathbf{R}_{\mathrm{C}} = -\frac{\mathbf{F}\mathbf{a}}{4\mathbf{L}^{3}} \cdot \left(\mathbf{L}^{2} - \mathbf{a}^{2}\right)$$

## Cortantes

$$\mathbf{V}_{\mathrm{AD}} = \mathbf{R}_{\mathrm{A}}$$

$$\mathbf{V}_{\mathrm{DB}} = \frac{\mathbf{F}\mathbf{a}}{4\mathbf{L}^{3}} \cdot \left(\mathbf{a}^{2} - 5\mathbf{L}^{2}\right)$$

$$\mathbf{V}_{\mathrm{BC}} = -\mathbf{R}_{\mathrm{C}}$$

$$\mathbf{V}_{BC} = -\mathbf{R}_{C}$$

$$\mathbf{M}_{AD} = \frac{\mathbf{F}}{4\mathbf{L}^{3}} \cdot \left(4\mathbf{L}^{3} - 5\mathbf{L}^{2}\mathbf{a} + \mathbf{a}^{3}\right) \cdot \mathbf{x}$$

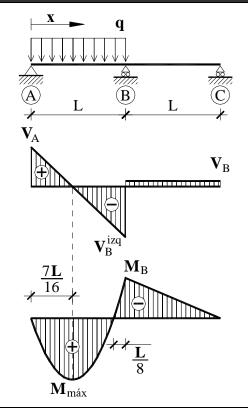
$$\mathbf{M}_{\mathrm{DB}} = \frac{\mathbf{F}\mathbf{a}}{4\mathbf{I}^{3}} \cdot \left(\mathbf{a}^{2} - 5\mathbf{L}^{2}\right) \cdot \mathbf{x} + \mathbf{P}\mathbf{a}$$

$$\mathbf{M}_{\mathrm{BC}} = \frac{\mathbf{F}\mathbf{a}}{4\mathbf{I}^{3}} \cdot \left(\mathbf{L}^{2} - \mathbf{a}^{2}\right) \cdot \left(\mathbf{x} - 2\mathbf{L}\right)$$

$$\mathbf{M}_{\mathrm{D}} = \frac{\mathbf{F}\mathbf{a}}{4\mathbf{L}^{3}} \cdot \left(4\mathbf{L}^{3} - 5\mathbf{L}^{2}\mathbf{a} + \mathbf{a}^{3}\right)$$

$$\mathbf{M}_{\mathrm{B}} = -\frac{\mathbf{F}\mathbf{a}}{4\mathbf{L}^2} \cdot \left(\mathbf{L}^2 - \mathbf{a}^2\right)$$

# VIGA 2 VANOS IGUALES: carga repartida en 1er vano.



#### Reacciones

$$\mathbf{R}_{A} = \frac{7 \, \mathbf{q} \mathbf{L}}{16}$$

$$\mathbf{R}_{\mathrm{B}} = \frac{5\,\mathrm{qL}}{8}$$

$$\mathbf{R}_{\mathrm{C}} = -\frac{\mathbf{q}\mathbf{L}}{16}$$

## Cortantes

$$\mathbf{V}_{AB} = \frac{\mathbf{q}}{16} \cdot (7\mathbf{L} - 16\mathbf{x})$$

$$V_{\rm B}^{\rm izq} = -\frac{9\,\rm qL}{16}$$

$$\mathbf{V}_{\mathrm{BC}} = \frac{\mathbf{q} \, \mathbf{L}}{16}$$

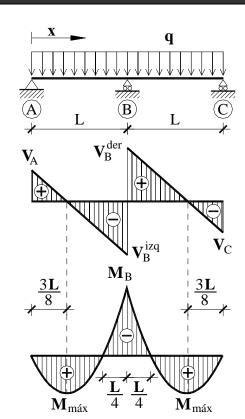
### Flectores

$$\mathbf{M}_{AB} = \frac{\mathbf{q} \, \mathbf{x}}{16} \cdot \left( 7\mathbf{L} - 8\mathbf{x} \right)$$

$$\mathbf{M}_{\mathrm{BC}} = -\frac{\mathbf{q}\,\mathbf{L}}{16} \cdot (2\mathbf{L} - \mathbf{x})$$

$$\mathbf{M}_{\text{máx}} = \frac{49 \,\mathbf{q} \,\mathbf{L}^2}{512} \qquad \text{para } \mathbf{x} = \frac{7 \,\mathbf{L}}{16}$$

# VIGA 2 VANOS IGUALES: carga repartida en los 2 vanos.



#### Reacciones

$$\mathbf{R}_{\mathbf{A}} = \mathbf{R}_{\mathbf{C}} = \frac{3\,\mathbf{q}\mathbf{L}}{8}$$

$$\mathbf{R}_{\mathrm{B}} = \frac{5\,\mathbf{q}\mathbf{L}}{4}$$

### Cortantes

$$\mathbf{V}_{AB} = \frac{\mathbf{q}}{8} \cdot \left( 3\mathbf{L} - 8\mathbf{x} \right)$$

$$\mathbf{V}_{\mathrm{B}}^{\mathrm{izq}} = -\mathbf{V}_{\mathrm{B}}^{\mathrm{der}} = -\frac{5\,\mathbf{q}\mathbf{L}}{8}$$

$$\mathbf{V}_{\mathrm{BC}} = \frac{\mathbf{q}}{8} \cdot \left( 13\mathbf{L} - 8\mathbf{x} \right)$$

### **Flectores**

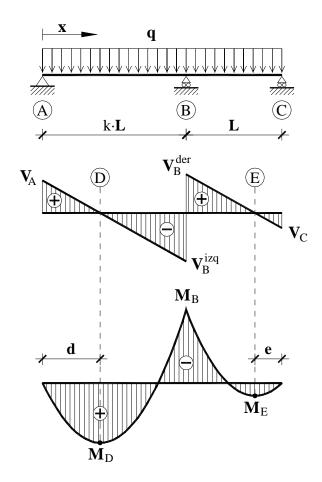
$$\mathbf{M}_{AB} = \frac{\mathbf{q} \, \mathbf{x}}{8} \cdot \left( 3\mathbf{L} - 4\mathbf{x} \right)$$

$$\mathbf{M}_{BC} = \frac{\mathbf{q}}{8} \cdot (2\mathbf{L} - \mathbf{x}) \cdot (4\mathbf{x} - 5\mathbf{L})$$

$$\mathbf{M}_{\mathrm{B}} = -\frac{\mathbf{q}\mathbf{L}^2}{8}$$

$$\mathbf{M}_{\text{máx}} = \frac{9 \,\mathbf{q} \,\mathbf{L}^2}{128} \qquad \text{para} \quad \mathbf{x} = \frac{3 \,\mathbf{L}}{8} \quad \text{y} \quad \mathbf{x} = \frac{13 \,\mathbf{L}}{8}$$

# VIGA 2 VANOS DESIGUALES: carga repartida en los 2 vanos.



### Reacciones:

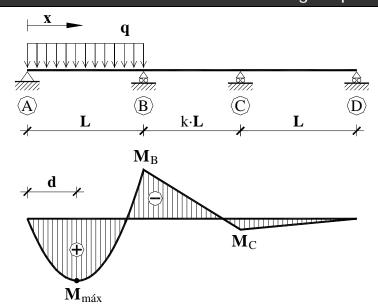
$$\mathbf{R}_{\mathrm{A}} = \mathbf{V}_{\mathrm{A}}$$

$$\mathbf{R}_{\mathrm{B}} = -\mathbf{V_{\mathrm{B}}}^{\mathrm{izq}} + \mathbf{V_{\mathrm{B}}}^{\mathrm{der}}$$

$$\mathbf{R}_{\mathrm{C}} = -\mathbf{V}_{\mathrm{C}}$$

		Corta	antes		Flectores					
k	$\mathbf{V}_{\mathrm{A}}$	$\mathbf{V}_{\mathrm{B}}^{\mathrm{izq}}$	${f V_B}^{ m der}$	$\mathbf{V}_{\mathrm{C}}$	$\mathbf{M}_{\mathrm{B}}^{(-)}$	$\mathbf{M_D}^{(+)}$	d	$\mathbf{M}_{\mathrm{E}}^{(+)}$	e	
	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L^2})$	$(\times \mathbf{q} \cdot \mathbf{L^2})$	$(\times \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L^2})$	$(\times \mathbf{L})$	
1,1	0,424	-0,676	0,639	-0,361	-0,139	0,090	0,424	0,065	0,361	
1,2	0,471	-0,729	0,655	-0,345	-0,155	0,111	0,471	0,060	0,345	
1,3	0,516	-0,784	0,674	-0,326	-0,174	0,133	0,516	0,053	0,326	
1,4	0,560	-0,840	0,695	-0,305	-0,195	0,157	0,560	0,047	0,305	
1,5	0,604	-0,896	0,719	-0,281	-0,219	0,183	0,604	0,040	0,281	
1,6	0,647	-0,953	0,745	-0,255	-0,245	0,209	0,647	0,033	0,255	
1,7	0,689	-1,011	0,774	-0,226	-0,274	0,237	0,689	0,026	0,226	
1,8	0,730	-1,070	0,805	-0,195	-0,305	0,267	0,730	0,019	0,195	
1,9	0,772	-1,128	0,839	-0,161	-0,339	0,298	0,772	0,013	0,161	
2	0,812	-1,188	0,875	-0,125	-0,375	0,330	0,812	0,008	0,125	
2,1	0,853	-1,247	0,914	-0,086	-0,414	0,364	0,853	0,004	0,086	
2,2	0,892	-1,308	0,954	-0,045	-0,455	0,399	0,892	0,001	0,045	
2,3	0,933	-1,367	0,999	-0,001	-0,499	0,435	0,933	~ 0	0,001	
2,4	0,973	-1,427	1,045	0,045	-0,545	0,473	0,973			
2,5	1,013	-1,487	1,094	0,094	-0,594	0,513	1,013			

# VIGA 3 VANOS DESIGUALES: carga repartida en vano extremo.



$$\mathbf{M}_{\text{máx}} = \frac{1}{8} \cdot \left( \frac{3 + 7k + 3k^2}{4 + 8k + 3k^2} \right)^2 \cdot \mathbf{qL}^2$$
 en  $\mathbf{x} = \mathbf{d} = \frac{3 + 7k + 3k^2}{4 + 8k + 3k^2} \cdot \frac{\mathbf{L}}{2}$ 

#### Reacciones:

$$\mathbf{R}_{A} = \frac{3 + 7k + 3k^{2}}{4 + 8k + 3k^{2}} \cdot \frac{1}{2} \cdot \mathbf{qL}$$

$$\mathbf{R}_{B} = \frac{2 + 13k + 18k^{2} + 6k^{3}}{4 + 8k + 3k^{2}} \cdot \frac{1}{4k} \cdot \mathbf{qL}$$

$$\mathbf{R}_{C} = \frac{-(1+k) \cdot (2+k)}{4 + 8k + 3k^{2}} \cdot \frac{1}{4k} \cdot \mathbf{qL}$$

$$\mathbf{R}_{D} = \frac{k}{4 + 8k + 3k^{2}} \cdot \frac{1}{4} \cdot \mathbf{qL}$$

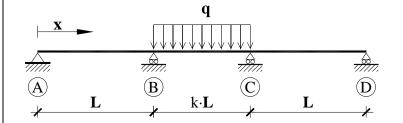
#### Flectores:

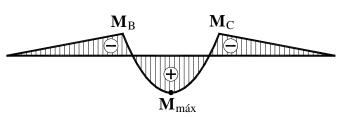
$$\mathbf{M}_{\mathrm{B}} = \frac{-(1+k)}{4+8k+3k^{2}} \cdot \frac{1}{2} \cdot \mathbf{q} \mathbf{L}^{2}$$
$$\mathbf{M}_{\mathrm{C}} = \frac{k}{4+8k+3k^{2}} \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^{2}$$

$$\mathbf{n} \quad \mathbf{x} = \mathbf{d} = \frac{3 + 7k + 3k^2}{3} \cdot \frac{\mathbf{L}}{3}$$

		Reaco	iones			Flect	tores	
k	$\mathbf{R}_{A}$	$\mathbf{R}_{\mathrm{B}}$	$\mathbf{R}_{\mathrm{C}}$	$\mathbf{R}_{\mathrm{D}}$	$\mathbf{M}_{\mathrm{B}}$	$\mathbf{M}_{\mathrm{C}}$	$\mathbf{M}_{ ext{máx}}$	d
	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times \mathbf{L})$			
0,5	0,414	0,786	-0,214	0,014	-0,086	0,014	0,086	0,414
0,6	0,419	0,741	-0,175	0,015	-0,081	0,015	0,088	0,419
0,7	0,423	0,709	-0,148	0,016	-0,077	0,016	0,090	0,423
0,8	0,427	0,685	-0,128	0,016	-0,073	0,016	0,091	0,427
0,9	0,430	0,665	-0,112	0,017	-0,070	0,017	0,093	0,430
1	0,433	0,650	-0,100	0,017	-0,067	0,017	0,094	0,433
1,1	0,436	0,637	-0,090	0,017	-0,064	0,017	0,095	0,436
1,2	0,439	0,626	-0,082	0,017	-0,061	0,017	0,096	0,439
1,3	0,441	0,617	-0,075	0,017	-0,059	0,017	0,097	0,441
1,4	0,443	0,609	-0,069	0,017	-0,057	0,017	0,098	0,443
1,5	0,445	0,603	-0,064	0,016	-0,055	0,016	0,099	0,445
1,6	0,447	0,597	-0,060	0,016	-0,053	0,016	0,100	0,447
1,7	0,449	0,591	-0,056	0,016	-0,051	0,016	0,101	0,449
1,8	0,450	0,586	-0,053	0,016	-0,050	0,016	0,101	0,450
1,9	0,452	0,582	-0,050	0,016	-0,048	0,016	0,102	0,452
2	0,453	0,578	-0,047	0,016	-0,047	0,016	0,103	0,453
2,1	0,454	0,575	-0,044	0,015	-0,046	0,015	0,103	0,454
2,2	0,456	0,571	-0,042	0,015	-0,044	0,015	0,104	0,456
2,3	0,457	0,568	-0,040	0,015	-0,043	0,015	0,104	0,457
2,4	0,458	0,566	-0,038	0,015	-0,042	0,015	0,105	0,458
2,5	0,459	0,563	-0,037	0,015	-0,041	0,015	0,105	0,459
2,6	0,460	0,561	-0,035	0,014	-0,040	0,014	0,106	0,460
2,7	0,461	0,559	-0,034	0,014	-0,039	0,014	0,106	0,461
2,8	0,462	0,557	-0,033	0,014	-0,038	0,014	0,107	0,462
2,9	0,463	0,555	-0,031	0,014	-0,037	0,014	0,107	0,463
3	0,464	0,553	-0,030	0,014	-0,036	0,014	0,107	0,464

# VIGA 3 VANOS DESIGUALES: carga repartida en vano central.





### Reacciones

$$\mathbf{R}_{A} = \mathbf{R}_{D} = \frac{-\mathbf{k}^{3}}{2+3\mathbf{k}} \cdot \frac{1}{4} \cdot \mathbf{qL}$$

$$\mathbf{R}_{\mathrm{B}} = \mathbf{R}_{\mathrm{C}} = \frac{4 + 6k + k^2}{2 + 3k} \cdot \frac{k}{4} \cdot \mathbf{qL}$$

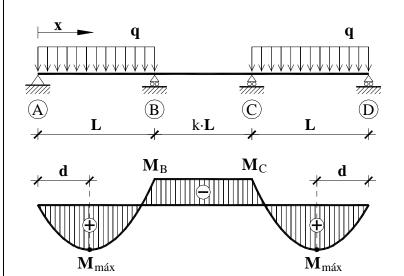
#### **Flectores**

$$\mathbf{M}_{\mathrm{B}} = \mathbf{M}_{\mathrm{C}} = \frac{-\mathbf{k}^3}{2+3\mathbf{k}} \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^2$$

$$\mathbf{M}_{\text{máx}} = \frac{2+k}{2+3k} \cdot \frac{k^2}{8} \cdot \mathbf{qL}^2$$

			2   3K	O		
	Reacci	iones	Flectores			
k	$\mathbf{R}_{\mathbf{A}} = \mathbf{R}_{\mathbf{D}}$ $(\times \mathbf{q} \cdot \mathbf{L})$	$\mathbf{R}_{\mathrm{B}} = \mathbf{R}_{\mathrm{C}}$ $(\times \mathbf{q} \cdot \mathbf{L})$	$\mathbf{M}_{\mathrm{B}} = \mathbf{M}_{\mathrm{C}}$ $(\times \mathbf{q} \cdot \mathbf{L}^{2})$	$\mathbf{M}_{ ext{m\'ax}} \ ( imes \mathbf{q} \cdot \mathbf{L^2})$		
0,5	-0,009	0,259	-0,009	0,022		
0,6	-0,014	0,314	-0,014	0,031		
0,7	-0,021	0,371	-0,021	0,040		
0,8	-0,029	0,429	-0,029	0,051		
0,9	-0,039	0,489	-0,039	0,062		
1	-0,050	0,550	-0,050	0,075		
1,1	-0,063	0,613	-0,063	0,088		
1,2	-0,077	0,677	-0,077	0,103		
1,3	-0,093	0,743	-0,093	0,118		
1,4	-0,111	0,811	-0,111	0,134		
1,5	-0,130	0,880	-0,130	0,151		
1,6	-0,151	0,951	-0,151	0,169		
1,7	-0,173	1,023	-0,173	0,188		
1,8	-0,197	1,097	-0,197	0,208		
1,9	-0,223	1,173	-0,223	0,229		
2	-0,250	1,250	-0,250	0,250		
2,1	-0,279	1,329	-0,279	0,272		
2,2	-0,310	1,410	-0,310	0,295		
2,3	-0,342	1,492	-0,342	0,319		
2,4	-0,376	1,576	-0,376	0,344		
2,5	-0,411	1,661	-0,411	0,370		
2,6	-0,448	1,748	-0,448	0,397		
2,7	-0,487	1,837	-0,487	0,424		
2,8	-0,528	1,928	-0,528	0,452		
2,9	-0,570	2,020	-0,570	0,481		
3	-0,614	2,114	-0,614	0,511		

# VIGA 3 VANOS DESIGUALES: carga repartida en vanos extremos.



### Reacciones

$$\mathbf{R}_{A} = \mathbf{R}_{D} = \frac{2 + 5k + 2k^{2}}{4 + 8k + 3k^{2}} \cdot \frac{3}{4} \cdot \mathbf{qL}$$

$$\mathbf{R}_{\rm B} = \mathbf{R}_{\rm C} = \frac{10 + 17k + 6k^2}{4 + 8k + 3k^2} \cdot \frac{1}{4} \cdot \mathbf{qL}$$

### **Flectores**

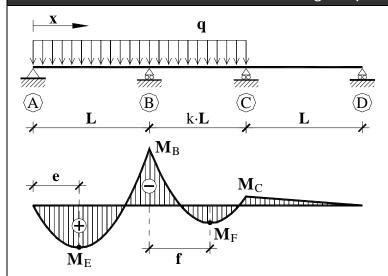
$$\mathbf{M}_{\mathrm{B}} = \mathbf{M}_{\mathrm{C}} = \frac{-\left(2+\mathrm{k}\right)}{4+8\mathrm{k}+3\mathrm{k}^{2}} \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^{2}$$

$$\mathbf{M}_{\text{máx}} = \frac{9}{32} \cdot \left( \frac{2 + 5k + 2k^2}{4 + 8k + 3k^2} \right)^2 \cdot \mathbf{qL}^2$$

con 
$$\mathbf{d} = \frac{2 + 5k + 2k^2}{4 + 8k + 3k^2} \cdot \frac{3L}{4}$$

	Reaco	iones	Flectores				
k	$\mathbf{R}_{\mathrm{A}} = \mathbf{R}_{\mathrm{D}}$	$\mathbf{R}_{\mathrm{B}} = \mathbf{R}_{\mathrm{C}}$	$\mathbf{M}_{\mathrm{B}} = \mathbf{M}_{\mathrm{C}}$	$\mathbf{M}_{ ext{m\'ax}}$	d		
	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$( imes {f q}{\cdot}{f L}^2)$	$(\times \mathbf{L})$		
0,5	0,429	0,571	-0,071	0,092	0,429		
0,6	0,434	0,566	-0,066	0,094	0,434		
0,7	0,439	0,561	-0,061	0,096	0,439		
0,8	0,443	0,557	-0,057	0,098	0,443		
0,9	0,447	0,553	-0,053	0,100	0,447		
1	0,450	0,550	-0,050	0,101	0,450		
1,1	0,453	0,547	-0,047	0,103	0,453		
1,2	0,455	0,545	-0,045	0,104	0,455		
1,3	0,458	0,542	-0,042	0,105	0,458		
1,4	0,460	0,540	-0,040	0,106	0,460		
1,5	0,462	0,538	-0,038	0,107	0,462		
1,6	0,463	0,537	-0,037	0,107	0,463		
1,7	0,465	0,535	-0,035	0,108	0,465		
1,8	0,466	0,534	-0,034	0,109	0,466		
1,9	0,468	0,532	-0,032	0,109	0,468		
2	0,469	0,531	-0,031	0,110	0,469		
2,1	0,470	0,530	-0,030	0,110	0,470		
2,2	0,471	0,529	-0,029	0,111	0,471		
2,3	0,472	0,528	-0,028	0,111	0,472		
2,4	0,473	0,527	-0,027	0,112	0,473		
2,5	0,474	0,526	-0,026	0,112	0,474		
2,6	0,474	0,526	-0,026	0,113	0,474		
2,7	0,475	0,525	-0,025	0,113	0,475		
2,8	0,476	0,524	-0,024	0,113	0,476		
2,9	0,477	0,523	-0,023	0,114	0,477		
3	0,477	0,523	-0,023	0,114	0,477		

# VIGA 3 VANOS DESIGUALES: carga repartida en 2 vanos contiguos.



#### Reacciones

$$\mathbf{R}_{A} = \left(\frac{3 + 7k + 3k^{2}}{4 + 8k + 3k^{2}} - \frac{k^{3}}{2 \cdot (2 + 3k)}\right) \cdot \frac{1}{4} \cdot \mathbf{qL}$$

$$\mathbf{R}_{A} = \left[ \frac{3 + 7k + 3k}{4 + 8k + 3k^{2}} - \frac{k^{2}}{2 \cdot (2 + 3k)} \right] \cdot \frac{1}{4} \cdot \mathbf{qL}$$

$$\mathbf{R}_{B} = \left( \frac{2 + 13k + 18k^{2} + 6k^{3}}{4 + 8k + 3k^{2}} \cdot \frac{1}{k} + \frac{4 + 6k + k^{2}}{2 + 3k} \cdot k \right) \cdot \frac{\mathbf{qL}}{4}$$

$$\mathbf{R}_{C} = \left(\frac{4+6k+k^{2}}{2+3k} \cdot k - \frac{(1+k)(2+k)}{4+8k+3k^{2}} \cdot \frac{1}{k}\right) \cdot \frac{\mathbf{qL}}{4}$$

$$\mathbf{R}_{\mathrm{D}} = \left(\frac{1}{4 + 8k + 3k^{2}} - \frac{k^{2}}{2 + 3k}\right) \cdot \frac{k}{4} \cdot \mathbf{qL}$$

Flectores:

$$\mathbf{M}_{\rm B} = -\left(\frac{k^3}{2+3k} + \frac{2(1+k)}{4+8k+3k^2}\right) \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^2 \; ; \qquad \qquad \mathbf{M}_{\rm C} = \left(\frac{k}{4+8k+3k^2} - \frac{k^3}{2+3k}\right) \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^2$$

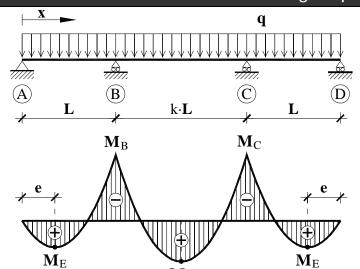
$$\mathbf{M}_{\mathrm{E}} = \frac{\mathbf{R}_{\mathrm{A}}^{2}}{2\pi} \quad \text{con } \mathbf{e} = \frac{\mathbf{R}_{\mathrm{A}}}{2\pi}$$

$$\mathbf{M}_{\mathrm{C}} = \left(\frac{\mathbf{k}}{4 + 8\mathbf{k} + 3\mathbf{k}^2} - \frac{\mathbf{k}^3}{2 + 3\mathbf{k}}\right) \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^2$$

$$\mathbf{M}_{\mathrm{E}} = \frac{\mathbf{R}_{\mathrm{A}}^{2}}{2 \mathbf{q}} \quad \text{con} \quad \mathbf{e} = \frac{\mathbf{R}_{\mathrm{A}}}{\mathbf{q}} \quad ; \qquad \qquad \mathbf{M}_{\mathrm{F}} = \frac{\left(\mathbf{R}_{\mathrm{A}} + \mathbf{R}_{\mathrm{B}}\right)^{2}}{2 \mathbf{q}} - \mathbf{L} \mathbf{R}_{\mathrm{B}} \quad \text{con} \quad \mathbf{f} = \frac{\mathbf{R}_{\mathrm{A}} + \mathbf{R}_{\mathrm{B}}}{\mathbf{q}} - \mathbf{L}$$

	2 \	<b>1</b>	4		- 4 Y					
		Reacc	iones		Flectores					
k	$\mathbf{R}_{A}$	$\mathbf{R}_{\mathrm{B}}$	$\mathbf{R}_{\mathrm{C}}$	$\mathbf{R}_{\mathrm{D}}$	$\mathbf{M}_{\mathrm{B}}$	$\mathbf{M}_{\mathrm{C}}$	$\mathbf{M}_{\mathrm{E}}$	e	$\mathbf{M}_{\mathrm{F}}$	f
	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times L)$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$	$(\times \mathbf{L})$			
0,5	0,405	1,045	0,045	0,005	-0,095	0,005	0,082	0,405	0,007	0,450
0,6	0,405	1,055	0,139	0,001	-0,095	0,001	0,082	0,405	0,011	0,460
0,7	0,402	1,080	0,223	-0,005	-0,098	-0,005	0,081	0,402	0,019	0,482
0,8	0,398	1,114	0,301	-0,013	-0,102	-0,013	0,079	0,398	0,029	0,512
0,9	0,392	1,154	0,376	-0,022	-0,108	-0,022	0,077	0,392	0,040	0,546
1	0,383	1,200	0,450	-0,033	-0,117	-0,033	0,073	0,383	0,053	0,583
1,1	0,373	1,250	0,523	-0,046	-0,127	-0,046	0,070	0,373	0,068	0,623
1,2	0,361	1,304	0,595	-0,060	-0,139	-0,060	0,065	0,361	0,083	0,665
1,3	0,348	1,360	0,668	-0,076	-0,152	-0,076	0,060	0,348	0,099	0,708
1,4	0,332	1,420	0,742	-0,094	-0,168	-0,094	0,055	0,332	0,116	0,753
1,5	0,315	1,482	0,816	-0,113	-0,185	-0,113	0,050	0,315	0,133	0,798
1,6	0,296	1,547	0,891	-0,134	-0,204	-0,134	0,044	0,296	0,152	0,843
1,7	0,276	1,614	0,967	-0,157	-0,224	-0,157	0,038	0,276	0,171	0,890
1,8	0,253	1,683	1,044	-0,181	-0,247	-0,181	0,032	0,253	0,192	0,937
1,9	0,229	1,755	1,123	-0,207	-0,271	-0,207	0,026	0,229	0,213	0,984
2	0,203	1,828	1,203	-0,234	-0,297	-0,234	0,021	0,203	0,235	1,031
2,1	0,176	1,904	1,284	-0,264	-0,324	-0,264	0,015	0,176	0,258	1,079
2,2	0,146	1,981	1,367	-0,294	-0,354	-0,294	0,011	0,146	0,281	1,127
2,3	0,115	2,060	1,451	-0,327	-0,385	-0,327	0,007	0,115	0,306	1,175
2,4	0,082	2,141	1,537	-0,361	-0,418	-0,361	0,003	0,082	0,331	1,224
2,5	0,048	2,224	1,624	-0,397	-0,452	-0,397	0,001	0,048	0,357	1,272
2,6	0,012	2,309	1,713	-0,434	-0,488	-0,434	0,000	0,012	0,384	1,321
2,7	-0,026	2,396	1,803	-0,473	-0,526	-0,473	0,000	-0,026	0,412	1,370
2,8	-0,066	2,484	1,895	-0,514	-0,566	-0,514	0,002	-0,066	0,440	1,419
2,9	-0,107	2,575	1,988	-0,556	-0,607	-0,556	0,006	-0,107	0,470	1,468
3	-0,150	2,667	2,083	-0,600	-0,650	-0,600	0,011	-0,150	0,500	1,517

# VIGA 3 VANOS DESIGUALES: carga repartida en todos los vanos.



#### Reacciones

$$\mathbf{R}_{A} = \mathbf{R}_{D} = \left(\frac{3 \cdot (2 + 5k + 2k^{2})}{4 + 8k + 3k^{2}} - \frac{k^{3}}{2 + 3k}\right) \cdot \frac{\mathbf{qL}}{4}$$

$$\mathbf{R}_{\rm B} = \mathbf{R}_{\rm C} = \left(\frac{10 + 17k + 6k^2}{4 + 8k + 3k^2} + \frac{k \cdot (4 + 6k + k^2)}{2 + 3k}\right) \cdot \frac{\mathbf{qL}}{4}$$

Flectores
$$\mathbf{M}_{\mathrm{B}} = \mathbf{M}_{\mathrm{C}} = -\left(\frac{\mathbf{k}^{3}}{2+3\mathbf{k}} + \frac{2+\mathbf{k}}{4+8\mathbf{k}+3\mathbf{k}^{2}}\right) \cdot \frac{1}{4} \cdot \mathbf{q} \mathbf{L}^{2}$$

$$\mathbf{M}_{\mathrm{E}}$$

$$\mathbf{M}_{E} = \left(\frac{3 \cdot \left(2 + 5k + 2k^{2}\right)}{4 + 8k + 3k^{2}} - \frac{k^{3}}{2 + 3k}\right)^{2} \cdot \frac{\mathbf{qL}^{2}}{32} \quad \text{con} \quad \mathbf{e} = \left(\frac{3 \cdot \left(2 + 5k + 2k^{2}\right)}{4 + 8k + 3k^{2}} - \frac{k^{3}}{2 + 3k}\right) \cdot \frac{\mathbf{L}}{4} \quad ; \quad \mathbf{e} = 0 \text{ con } k = 2,669$$

$$\mathbf{M}_{F} = \left(\frac{(2+k)^{2}}{2} - \frac{10 + 17k + 6k^{2}}{4 + 8k + 3k^{2}} - \frac{k \cdot (4 + 6k + k^{2})}{2 + 3k}\right) \cdot \frac{1}{4} \cdot \mathbf{qL}^{2} ; \qquad \mathbf{M}_{F} = 0 \text{ con } k = 0,840$$

			•		/ Floatowe					
		Reacc			3.6		Flectores		3.6	
k	$\mathbf{R}_{A}$	$\mathbf{R}_{\mathbf{B}}$	$\mathbf{R}_{\mathbf{C}}$	$\mathbf{R}_{\mathrm{D}}$	$\mathbf{M}_{\mathrm{B}}$	$\mathbf{M}_{\mathbf{C}}$	$\mathbf{M}_{\mathrm{E}}$	e	$\mathbf{M}_{\mathrm{F}}$	
	$(\times \mathbf{q} \cdot \mathbf{L})$	$(\times \mathbf{q} \cdot \mathbf{L^2})$	$(\times \mathbf{q} \cdot \mathbf{L^2})$	$(\times \mathbf{q} \cdot \mathbf{L^2})$	$(\times L)$	$(\times \mathbf{q} \cdot \mathbf{L}^2)$				
0,5	0,420	0,830	0,830	0,420	-0,080	-0,080	0,088	0,420	-0,049	
0,6	0,420	0,880	0,880	0,420	-0,080	-0,080	0,088	0,420	-0,035	
0,7	0,418	0,932	0,932	0,418	-0,082	-0,082	0,087	0,418	-0,021	
0,8	0,414	0,986	0,986	0,414	-0,086	-0,086	0,086	0,414	-0,006	
0,9	0,408	1,042	1,042	0,408	-0,092	-0,092	0,083	0,408	0,009	
1	0,400	1,100	1,100	0,400	-0,100	-0,100	0,080	0,400	0,025	
1,1	0,390	1,160	1,160	0,390	-0,110	-0,110	0,076	0,390	0,041	
1,2	0,378	1,222	1,222	0,378	-0,122	-0,122	0,072	0,378	0,058	
1,3	0,365	1,285	1,285	0,365	-0,135	-0,135	0,066	0,365	0,076	
1,4	0,349	1,351	1,351	0,349	-0,151	-0,151	0,061	0,349	0,094	
1,5	0,332	1,418	1,418	0,332	-0,168	-0,168	0,055	0,332	0,113	
1,6	0,313	1,487	1,487	0,313	-0,187	-0,187	0,049	0,313	0,133	
1,7	0,292	1,558	1,558	0,292	-0,208	-0,208	0,043	0,292	0,153	
1,8	0,269	1,631	1,631	0,269	-0,231	-0,231	0,036	0,269	0,174	
1,9	0,245	1,705	1,705	0,245	-0,255	-0,255	0,030	0,245	0,196	
2	0,219	1,781	1,781	0,219	-0,281	-0,281	0,024	0,219	0,219	
2,1	0,191	1,859	1,859	0,191	-0,309	-0,309	0,018	0,191	0,242	
2,2	0,161	1,939	1,939	0,161	-0,339	-0,339	0,013	0,161	0,266	
2,3	0,130	2,020	2,020	0,130	-0,370	-0,370	0,008	0,130	0,291	
2,4	0,097	2,103	2,103	0,097	-0,403	-0,403	0,005	0,097	0,317	
2,5	0,063	2,188	2,188	0,063	-0,438	-0,438	0,002	0,063	0,344	
2,6	0,026	2,274	2,274	0,026	-0,474	-0,474	0,000	0,026	0,371	
2,7	-0,012	2,362	2,362	-0,012	-0,512	-0,512			0,399	
2,8	-0,052	2,452	2,452	-0,052	-0,552	-0,552			0,428	
2,9	-0,093	2,543	2,543	-0,093	-0,593	-0,593			0,458	
3	-0,136	2,636	2,636	-0,136	-0,636	-0,636			0,489	