Epistemic logics based on abilities

PhD in Computer Science

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FaMAF, Universidad Nacional de Córdoba

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Organigrama

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- Modal Logics: Modes of truth about expressions

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  - Introduce multiple and less idealized agents

An example: Evacuation plan

EMERGENCY PROCEDURE

IN CASE OF FIRE

KEEP CALM

PULL FIRE ALARM

FROM A SAFE LOCATION

CALL 999 (FIRE BRIGADE)

Evacuation: use only stairs or ramps, avoid elevators.

If unsafe to evacuate: close door, block cracks, stay low near windows.

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Consider two agents i and j:

- i took a safety course and avoids using the elevator
- j considers all evacuation options as valid

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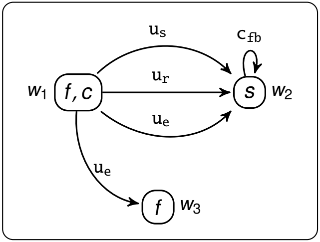

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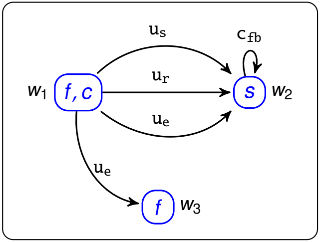

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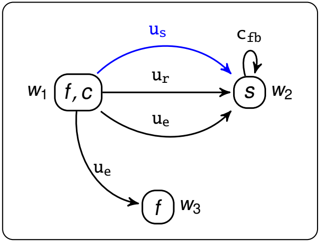

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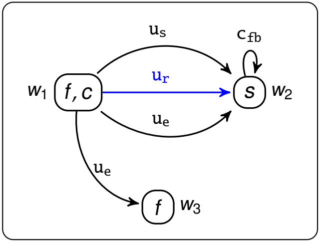

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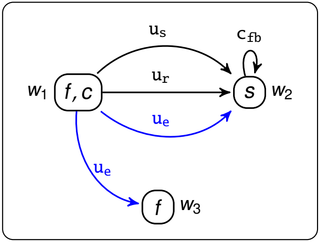

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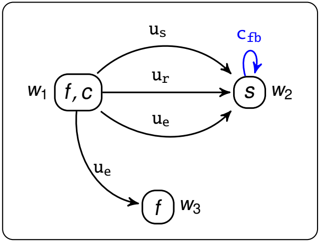

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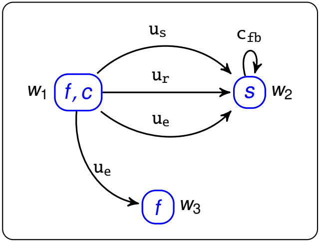

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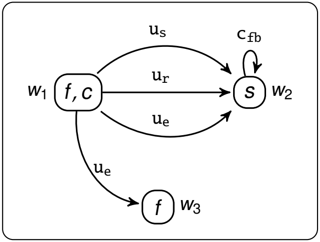
- $\mathrm{R}_{\mathtt{c_{fb}}} = \{(w_2, w_2)\}$
- $\mathrm{V}(w_1) = \{f, c\}, \mathrm{V}(w_2) = \{s\}, \mathrm{V}(w_3) = \{f\}$

Uncertainty-based LTSs (LTS^{U}s)

Definition (LTS^U)

Given Act, Agt and Prop. $\mathcal{M} = \langle \mathrm{S}, \{\mathrm{R}_a\}_{a \in \mathsf{Act}}, \{\mathrm{U}(i)\}_{i \in \mathsf{Agt}}, \mathrm{V} \rangle$:

- S is a non-empty set of states
- R_a a binary relation over S
- $\mathrm{V} : \mathrm{S} \to \mathcal{P}(\mathsf{Prop})$ is a valuation function
- $\mathrm{U}(i) \subseteq \mathcal{P}(\mathsf{Act}^*)$ s.t. if $\pi_1, \pi_2 \in \mathrm{U}(i)$ and $\pi_1 \neq \pi_2$, then $\pi_1 \cap \pi_2 = \emptyset$



- $\mathrm{S} = \{w_1, w_2, w_3\}$
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Uncertainty-based LTSs (LTS^Us)

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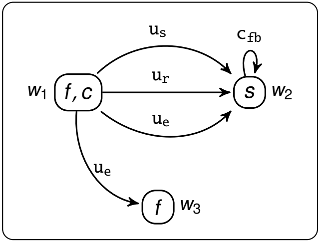

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Strong executability

Fail-proof plans:

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Fail-proof plans: Each partial execution has to be completed

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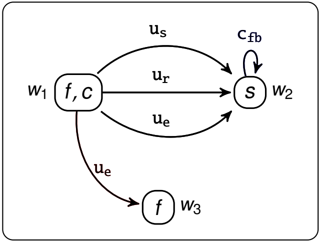

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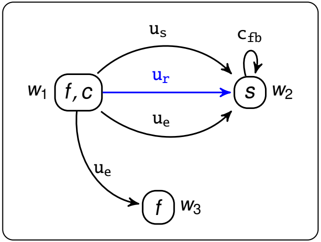

$\mathtt{u_r}$ is strongly executable at w_1

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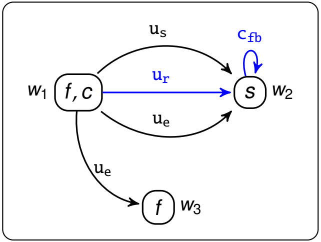

$\mathtt{u_r}$ is strongly executable at w_1

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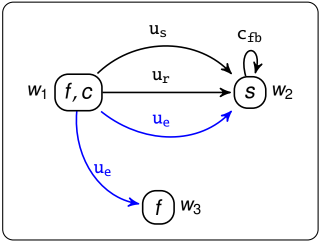

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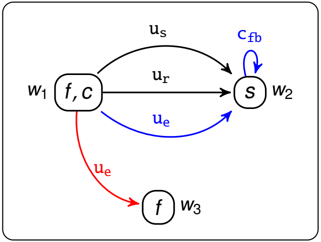

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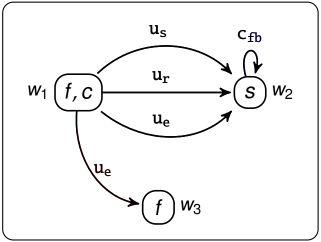

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Definition (Strong executability for a plan)

$\sigma = b_1 \ldots b_n \in \text{Act}^*$ is *strongly executable* (SE) at $u \in \mathrm{S}$ iff, for each $k = 1, \ldots, n-1$,

$$v \in \mathrm{R}_{b_1 \ldots b_k}(u) \quad \text{implies} \quad \mathrm{R}_{b_{k+1}}(v) \neq \emptyset.$$

Definition

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{Kh}_i(\varphi,\varphi)$$

$\mathsf{Kh}_i(\psi,\varphi)$: *"when ψ holds, agent i knows how to reach φ"*.

Syntax and semantics of $\mathsf{L}_{\mathsf{Kh}_i}$ over LTS^Us

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$\mathcal{M}, w \models p$	iff	$p \in \mathrm{V}(w)$
$\mathcal{M}, w \models \neg\varphi$	iff	$\mathcal{M}, w \not\models \varphi$

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Kh_i over LTS^Us

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$\mathrm{U}(i) = \{\{\mathtt{u_s c_{fb}}, \mathtt{u_r c_{fb}}\}, \{\mathtt{u_e c_{fb}}\}\}$
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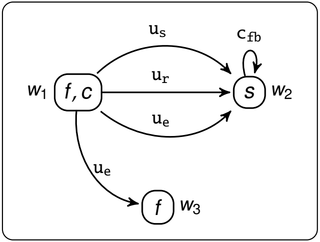

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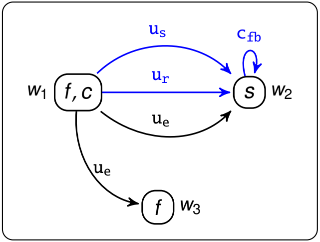

$\models \mathrm{Kh}_i(f \wedge c, s)$

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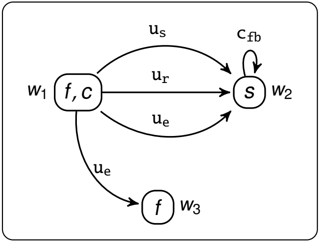

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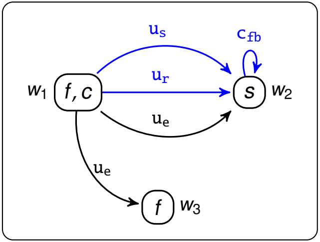

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- $\mathtt{u_s c_{fb}}$ and $\mathtt{u_r c_{fb}}$ take the agent from all $(f \wedge c)$-states and reaches only s-states;

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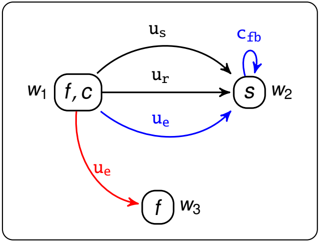

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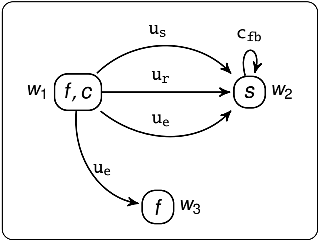

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Universal modality A

$\mathsf{A}\varphi := \bigvee_{i \in \mathsf{Agt}} \mathsf{Kh}_i(\neg\varphi, \bot)$: *"at all states φ holds"*

$\mathcal{M}, w \models \mathsf{A}\varphi$ iff for all $u \in \mathrm{S}$, $\mathcal{M}, u \models \varphi$

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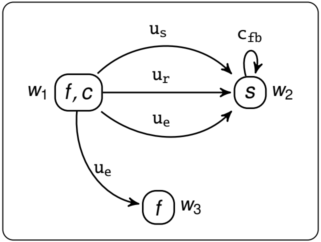

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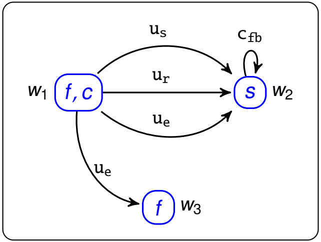

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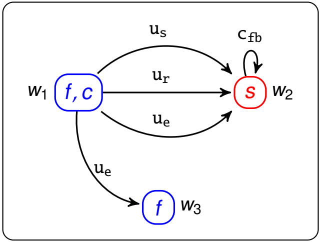

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Axiomatization $\mathcal{L}_{\mathsf{Kh}_i}^{\mathrm{LTS}^U}$ de $\mathsf{L}_{\mathsf{Kh}_i}$

$\underline{\mathcal{L}}$:	TAUT	$\vdash \varphi$ for propositional tautologies
	DISTA	$\vdash \mathsf{A}(\varphi \to \psi) \to (\mathsf{A}\varphi \to \mathsf{A}\psi)$
	TA	$\vdash \mathsf{A}\varphi \to \varphi$
	4KhA	$\vdash \mathsf{Kh}_i(\psi, \varphi) \to \mathsf{A}\mathsf{Kh}_i(\psi, \varphi)$
	5KhA	$\vdash \neg\mathsf{Kh}_i(\psi, \varphi) \to \mathsf{A}\neg\mathsf{Kh}_i(\psi, \varphi)$
$\underline{\mathcal{L}_{\mathrm{LTS}^U}}$:	KhA	$\vdash (\mathsf{A}(\chi \to \psi) \land \mathsf{Kh}_i(\psi, \varphi) \land \mathsf{A}(\varphi \to \theta)) \to \mathsf{Kh}_i(\chi, \theta)$
<u>Rules:</u>		$\dfrac{\varphi \quad (\varphi \to \psi)}{\psi}$ MP $\qquad \dfrac{\varphi}{\mathsf{A}\varphi}$ NECA

Representation benefits

- Improvement from a previous proposal ($\mathsf{L_{Kh}}$) that used LTSs without uncertainty ($\mathcal{L} = \langle \mathrm{S}, \{\mathrm{R}_a\}_{a \in \mathsf{Act}}, \mathrm{V} \rangle$)

  $\mathcal{L}, w \models \mathsf{Kh}(\psi, \varphi)$ iff there exists $\sigma \in \mathsf{Act}^*$ s.t.

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Reasons for not knowing how that are not represented.

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Representation benefits

- Improvement from a previous proposal ($\mathsf{L_{Kh}}$) that used LTSs without uncertainty ($\mathcal{L} = \langle \mathrm{S}, \{\mathrm{R}_a\}_{a \in \mathsf{Act}}, \mathrm{V} \rangle$)

  $\mathcal{L}, w \models \mathsf{Kh}(\psi, \varphi)$ iff there exists $\sigma \in \mathsf{Act}^*$ s.t.

  1. is SE at all ψ-states and
  2. its executions from ψ-states always end in φ-states

Reasons for not knowing how that are not represented.

$\{\mathrm{U}(i)\}_{i \in \mathsf{Agt}}$ solved two major issues:

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- Issue 2: The agent distinguishes each plan from all the others
  - For the agent the effects of two plans can be indistinct ($\mathrm{U}(j) = \{\{\mathtt{u_s c_{fb}}, \mathtt{u_r c_{fb}}, \mathtt{u_e c_{fb}}\}\}$)

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- We can define ontic and epistemic dynamic operators that modifiy each type of information

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Deontic operators (abilities, norms and compliance) [Ch. 10]

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- Consider other dynamic operators such as “learning how” and “forgetting how”

Referencias

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