A Novel Way of Updating Knowing How

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ReacTS, Aveiro - Portugal

5/XI/2024

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- A framework for knowing how logics with a notion of epistemic indistinguishability (2021, 2023)

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  - sound and complete axiomatization for all models via reduction axioms

The models: $\mathsf{LTS^U}s$

Definition (Uncertainty-based LTS (2021, 2023))

An *uncertainty-based* LTS ($\mathsf{LTS^U}$) for Prop, Act and Agt is a tuple $\mathcal{M} = \langle \mathsf{W}, \{\mathsf{R}_a\}_{a \in \mathsf{Act}}, \{\mathsf{U}(i)\}_{i \in \mathsf{Agt}}, \mathsf{V} \rangle$ where:

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$\mathcal{M} = \langle \{w, v, u\}, \{\mathsf{R}_c, \mathsf{R}_d\}, \{\mathsf{U}(i)\}, \mathsf{V} \rangle$

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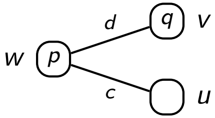

$\mathsf{U}(i) = \{\{c\}\}$

The language L_{Kh_i}

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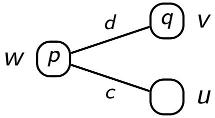

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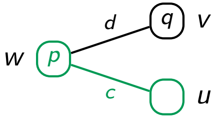
$\mathcal{M}:$ 

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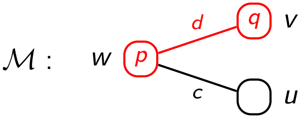

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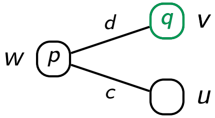
$\mathcal{M}$: 

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$\mathsf{A}\varphi := \mathsf{Kh}_i(\neg\varphi, \bot)$, $\mathsf{E}\varphi := \neg\mathsf{A}\neg\varphi$, $\mathsf{E}q$

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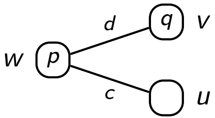

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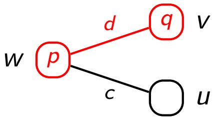

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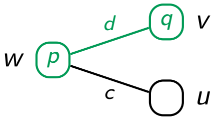

$\mathsf{U}'(i) = \{\{c\}, \{d\}\}$

$\mathcal{M}, w \models [!d]\mathsf{Kh}_i(p, q)$

Expressivity of $\mathsf{L}_{\mathsf{Kh}_i} + [!a]$

Kh_i is not sufficient to capture $[!a]$

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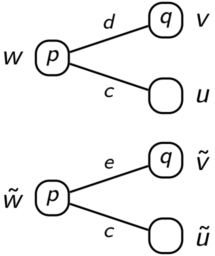

$U(i) = \{\{c\}\}$

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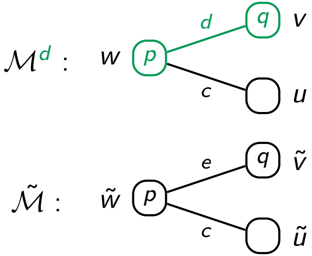

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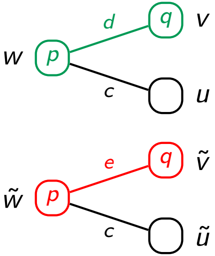
$\mathcal{M}^d$: ... $\tilde{\mathcal{M}}^d$:



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Expressivity of $L_{Kh_i} + [!a]$

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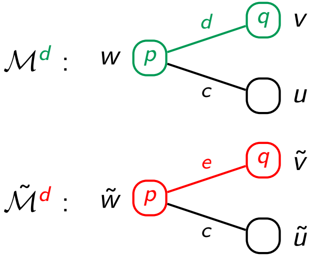

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Kh_i talks about plans implicitly

The language $L_{Kh_i,\Box,[!a]} = L_{Kh_i} + [a] + [!a]$

Definition

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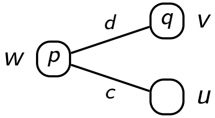

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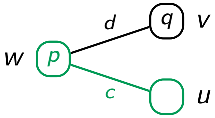

$U(i) = \{\{c\}\}$

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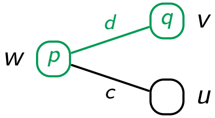

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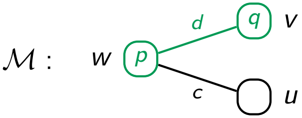

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KhA	$\vdash (\mathsf{A}(\chi \to \psi) \wedge \mathsf{Kh}_i(\psi, \varphi) \wedge \mathsf{A}(\varphi \to \theta)) \to \mathsf{Kh}_i(\chi, \theta)$
Dist$\Box$	$\vdash [a](\varphi \to \psi) \to ([a]\varphi \to [a]\psi)$
A$\Box$	$\vdash \mathsf{A}\varphi \to [a]\varphi$
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