A Novel Way of Updating Knowing How

Carlos Areces[1,2], Raul Fervari[1,2], Andrés R. Saravia[1,2], and Fernando R. Velázquez-Quesada[3]

[1] Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET), Argentina
[2] Universidad Nacional de Cordoba (UNC), Argentina
[3] Universitetet i Bergen, Norway

Abstract. This paper presents a novel way of updating what an agent knows how to do. We introduce an announcements-like modality that distinguishes the effect of a given action from the rest, raising new awareness for the agents about the effects of such an action. In this note we present a complete axiomatization of the resulting logic via reduction axioms.

Keywords: Knowing how · Updates · Axiomatization.

1 Introduction

In this work we investigate dynamic aspects of *knowing how logics*. This family of logics has received considerably attention lately (see e.g. [7,8,1,3]). In particular, we introduce a novel dynamic operator describing updates in the agents' knowledge about how to achieve a certain goal. We start from the results presented in [3], where a new semantics for knowing how epistemic operators is presented: Labeled Transition Systems (LTS) extended with an indistinguishability relation over plans. The indistinguishability relation characterizes each agent's uncertainty about the exact effects that actions have on the world.

We build on the work presented in [2] which is, to our knowledge, the first article studying the dynamics of knowing how. Therein, some dynamic operators are investigated, specially with respect to their expressive power. Axiomatizing the introduced operators is challenging. On the one hand the set of theorems of the resulting logics are usually not closed under uniform substitution. On the other hand, the dynamic operators increase the language's expressivity, so they cannot be axiomatized via reduction axioms. Here, we first extend the basic knowing how logic from [1,3], with a standard modality $[a]$ describing action execution. Then we define a dynamic modality $[!a]$ stating that the effect of action a must be distinguished from any other action, and that the agents become aware of that. We show that the new dynamic modality can be eliminated via reduction axioms, thus we obtain a complete axiomatization for the full logic.

2 Epistemic Updates over Knowing How

Syntax and Semantics. We start by introducing the syntax and semantics of the logic $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$, an extension of $\mathsf{L}_{\mathsf{Kh}_i}$, as presented in [1,3], with *(i)* the standard $[a]$ modality (see e.g., [5,4]) and ***(ii)*** a refinement modality $[!a]$, which publicly establishes (i.e., this information is known to all agents) that action a must be distinguished from any other. This operator models situations in which, previously uninformed agents receive concrete information about the actual effect of a particular action; as a result this action becomes distinguishable from any other. These ideas are formalized below.

Definition 1. *Let* Prop *be a countable set of propositional symbols,* Act *a denumerable set of action symbols, and* Agt *a non-empty finite set of agents. Formulas of the language* $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$ *are given by*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{Kh}_i(\varphi,\varphi) \mid [a]\varphi \mid [!a]\varphi,$$

with $p \in \mathsf{Prop}$, $a \in \mathsf{Act}$ *and* $i \in \mathsf{Agt}$. *Other Boolean connectives are defined as usual. Formulas of the form* $\mathsf{Kh}_i(\psi,\varphi)$, $[a]\varphi$ *and* $[!a]\varphi$ *are read as:* "when ψ is the case, the agent i knows how to make φ true", "every execution of action a leads always to situations in which φ holds", *and* "after announcing that a is distinguishable from any other plan, φ holds" *respectively. We define also* $\mathsf{A}\varphi := \bigvee_{i\in\mathsf{Agt}} \mathsf{Kh}_i(\neg\varphi,\bot)$ *and* $\mathsf{E}\varphi := \neg\mathsf{A}\neg\varphi$*; as established in [1,3], they behave exactly as the universal and existential modalities (see, e.g. [6]), respectively. Finally, the fragment without occurrences of the* $[!a]$ *modality is called* $\mathsf{L}_{\mathsf{Kh}_i,\Box}$.

In [7,8], formulas are interpreted over *labeled transition systems* (LTSs): relational models in which each (basic) relation indicates the source and target of a particular type of action the agent can perform. In the setting introduced in [1,3], LTSs are extended with a notion of *uncertainty* between plans.

Definition 2 (Actions and plans). *Let* Act^* *be the set of finite sequences over* Act. *Elements of* Act^* *are called* plans*, with* ϵ *being the* empty plan. *Given* $\sigma \in \mathsf{Act}^*$, *we denote* $|\sigma|$ *the length of* σ *(note:* $|\epsilon| = 0$*). For* $0 \le k \le |\sigma|$, *the plan* σ_k *is* σ*'s initial segment up to (and including) the* k*th position (with* $\sigma_0 := \epsilon$*). For* $0 < k \le |\sigma|$, *the action* $\sigma[k]$ *is the one in* σ*'s* k*th position.*

Definition 3 (Uncertainty-based LTS). *An* uncertainty-based LTS ($\mathrm{LTS}^{\mathrm{U}}$) *for* Prop, Act *and* Agt *is a tuple* $\mathcal{M} = \langle \mathrm{W},\mathrm{R},\mathrm{U},\mathrm{V}\rangle$ *where:* $\langle \mathrm{W},\mathrm{R},\mathrm{V}\rangle$ *is an LTS, and* $\mathrm{U} = \{\mathrm{U}(i) \subseteq 2^{\mathsf{Act}^*} \setminus \{\emptyset\} \mid i \in \mathsf{Agt}\}$ *assigns to every agent a non-empty collection of pairwise disjoint non-empty sets of plans, i.e.:* ***(i)*** $\mathrm{U}(i) \neq \emptyset$, ***(ii)*** $\pi_1,\pi_2 \in \mathrm{U}(i)$ *with* $\pi_1 \neq \pi_2$ *implies* $\pi_1 \cap \pi_2 = \emptyset$, *and* ***(iii)*** $\emptyset \notin \mathrm{U}(i)$.

Definition 4. *Given* $\mathrm{R} = \{\mathrm{R}_a \subseteq \mathrm{W}\times\mathrm{W} \mid a \in \mathsf{Act}\}$ *and* $\sigma \in \mathsf{Act}^*$, *define* $\mathrm{R}_\sigma \subseteq \mathrm{W}\times\mathrm{W}$ *in the standard way. Then, for* $\pi \subseteq \mathsf{Act}^*$ *and* $U \cup \{u\} \subseteq \mathrm{W}$, *define* $\mathrm{R}_\pi := \bigcup_{\sigma\in\pi} \mathrm{R}_\sigma$, $\mathrm{R}_\pi(u) := \bigcup_{\sigma\in\pi} \mathrm{R}_\sigma(u)$, *and* $\mathrm{R}_\pi(U) := \bigcup_{u\in U} \mathrm{R}_\pi(u)$.

Definition 5 (Strong executability of plans). *Let* $\mathcal{M} = \langle \mathrm{W},\mathrm{R},\mathrm{U},\mathrm{V}\rangle$ *be an* $\mathrm{LTS}^{\mathrm{U}}$, *with* $\mathrm{R} = \{\mathrm{R}_a \subseteq \mathrm{W}\times\mathrm{W} \mid a \in \mathsf{Act}\}$. *A* plan $\sigma \in \mathsf{Act}^*$ *is* strongly executable *(SE) at* $u \in \mathrm{W}$ *if and only if* $v \in \mathrm{R}_{\sigma_k}(u)$ *implies* $\mathrm{R}_{\sigma[k+1]}(v) \neq \emptyset$ *for every* $k \in [0 \dots |\sigma|-1]$. *We define the set* $\mathrm{SE}^{\mathcal{M}}(\sigma) := \{w \in \mathrm{W} \mid \sigma$ *is SE at* $w\}$. *Then, a* set of plans $\pi \subseteq \mathsf{Act}^*$ *is* strongly executable *at* $u \in \mathrm{W}$ *if and only if* every *plan* $\sigma \in \pi$ *is* strongly executable *at* u. *Hence,* $\mathrm{SE}^{\mathcal{M}}(\pi) = \bigcap_{\sigma\in\pi} \mathrm{SE}^{\mathcal{M}}(\sigma)$ *is the set of the states in* W *where* π *is strongly executable.*

Thus, a plan is strongly executable (at a state) when *all* its partial executions can be completed. Then, a set of plans is strongly executable when *all* its plans are strongly executable. When the model is clear from the context, we will drop the superscript $\mathcal{M}$ and write simply $\mathrm{SE}(\sigma)$ and $\mathrm{SE}(\pi)$.

Definition 6. *Let $\mathcal{M} = \langle \mathrm{W}, \mathrm{R}, \{\mathrm{U}(i)\}_{i \in \mathsf{Agt}}, \mathrm{V}\rangle$ be an $\mathrm{LTS}^{\mathrm{U}}$, with $w \in \mathrm{W}$. The satisfiability relation $\models$ for $\mathsf{L}_{\mathsf{Kh}_i}$ is defined as follows (Boolean cases are omitted):*

$$\begin{array}{lll} \mathcal{M}, w \models [a]\varphi & \textit{iff}_{def} & \mathcal{M}, v \models \varphi \text{ for all } v \in \mathrm{R}_a(w), \\ \mathcal{M}, w \models \mathsf{Kh}_i(\psi, \varphi) & \textit{iff}_{def} & \text{there is } \pi \in \mathrm{U}(i) \text{ such that:} \\ & & \textbf{(i)}\ [\![\psi]\!]^{\mathcal{M}} \subseteq \mathrm{SE}(\pi), \text{ and} \\ & & \textbf{(ii)}\ \mathrm{R}_\pi([\![\psi]\!]^{\mathcal{M}}) \subseteq [\![\varphi]\!]^{\mathcal{M}}, \end{array}$$

where $[\![\chi]\!]^{\mathcal{M}} := \{w \in \mathrm{W} \mid \mathcal{M}, w \models \chi\}$. A set $\pi \in \mathrm{U}(i)$ satisfying the conditions that make $\mathsf{Kh}_i(\psi, \varphi)$ true is called a witness *for $\mathsf{Kh}_i(\psi, \varphi)$. Moreover,*

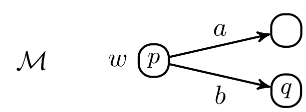
$$\mathcal{M}, w \models [!a]\varphi \quad \textit{iff}_{def} \quad \mathcal{M}^a, w \models \varphi,$$

where $\mathcal{M}^a = \langle \mathrm{W}, \mathrm{R}, \mathrm{U}', \mathrm{V}\rangle$, with:

- $\mathrm{U}'(i) = (\mathrm{U}(i) \setminus \{\pi\}) \cup \{\{a\}\}$ *if there is $\pi \in \mathrm{U}(i)$ such that $a \in \pi$,*
- $\mathrm{U}'(i) = \mathrm{U}(i) \cup \{\{a\}\}$ *otherwise.*

In short, $[!a]$ is an announcement that action a has a behaviour that can be uniquely determined. In case a is not a part of the ones under the consideration of a certain agent, it is added to it, making the agent aware of its existence.

Example 1. Let $\mathcal{M}$ be the single agent model depicted below, with $\mathrm{U}(i) := \{\{a\}\}$. It is clear that $\mathcal{M}, w \not\models \mathsf{Kh}_i(p, q)$. But we have that $\mathcal{M}, w \models [!b]\mathsf{Kh}_i(p, q)$, as after announcing b, the agent becomes aware of the existence of that plan, and she can use it to guarantee q given p.



Axiom System. The axiom system for $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$ consists of two parts. The first one is an extension of the axiomatization for $\mathsf{L}_{\mathsf{Kh}_i}$ from [1,3], with the addition of the axioms $\mathsf{Dist}\Box$ and $\mathsf{A}\Box$. This works as an axiomatization for the fragment $\mathsf{L}_{\mathsf{Kh}_i,\Box}$. The second part provides reduction axioms that enable us to translate $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$-formulas into $\mathsf{L}_{\mathsf{Kh}_i,\Box}$-formulas by induction on the structure of $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$ formulas. This is possible since the expressive power of the underlying static logic ($\mathsf{L}_{\mathsf{Kh}_i,\Box}$) is sufficient to capture the behaviour of $[!a]$, hence obtaining an axiomatization via reduction axioms. For each case, the ocurrences of a $[!a]$ are either eliminated or pushed deep into the formula. Since $[!a]$ modifies U only, the challenge is to eliminate the occurence of that modality in presence of Kh_i. Consider a formula $[!a]\mathsf{Kh}_i(\psi, \varphi)$. After the “announcement” of action a being different to any other plan, there are two possible reasons of why

Taut	$\vdash \varphi$ for φ a propositional tautology
DistA	$\vdash \mathsf{A}(\varphi \to \psi) \to (\mathsf{A}\varphi \to \mathsf{A}\psi)$
TA	$\vdash \mathsf{A}\varphi \to \varphi$
4KhA	$\vdash \mathsf{Kh}_i(\psi, \varphi) \to \mathsf{A}\mathsf{Kh}_i(\psi, \varphi)$
5KhA	$\vdash \neg\mathsf{Kh}_i(\psi, \varphi) \to \mathsf{A}\neg\mathsf{Kh}_i(\psi, \varphi)$
KhA	$\vdash (\mathsf{A}(\chi \to \psi) \land \mathsf{Kh}_i(\psi, \varphi) \land \mathsf{A}(\varphi \to \theta)) \to \mathsf{Kh}_i(\chi, \theta)$
Dist□	$\vdash [a](\varphi \to \psi) \to ([a]\varphi \to [a]\psi)$
A□	$\vdash \mathsf{A}\varphi \to [a]\varphi$
MP	From $\vdash \varphi$ and $\vdash \varphi \to \psi$ infer $\vdash \psi$
NecA	From $\vdash \varphi$ infer $\vdash \mathsf{A}\varphi$
RAtom	$\vdash [!a]p \leftrightarrow p$
R¬	$\vdash [!a]\neg\varphi_1 \leftrightarrow \neg[!a]\varphi_1$
R∨	$\vdash [!a](\varphi_1 \lor \varphi_2) \leftrightarrow ([!a]\varphi_1 \lor [!a]\varphi_2)$
R□	$\vdash [!a][a]\varphi_1 \leftrightarrow [a][!a]\varphi_1$
RKh	$\vdash [!a]\mathsf{Kh}_i(\varphi_1, \varphi_2) \leftrightarrow (\mathsf{Kh}_i([!a]\varphi_1, [!a]\varphi_2) \lor \mathsf{A}([!a]\varphi_1 \to (\langle a\rangle\top \land [a][!a]\varphi_2)))$
$\mathsf{RE}_{[!]}$	From $\vdash \varphi \leftrightarrow \psi$ derive $\vdash [!a]\varphi \leftrightarrow [!a]\psi$

Table 1: Axiomatization for $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$.

$\mathsf{Kh}_i(\psi, \varphi)$ holds. The first possibility, is that action a plays no role in the truth of $\mathsf{Kh}_i(\psi, \varphi)$. In this case, we can push the dynamic modality into the front of the pre and post-conditions of the Kh_i modality, i.e., we obtain $\mathsf{Kh}_i([!a]\psi, [!a]\varphi)$. The second possibility is that a is crucial in order to raise new knowledge for the agent. If the singleton $\{a\}$ is the witness for $\mathsf{Kh}_i(\psi, \varphi)$ is because: ***(i)*** a is SE at every state satisfying ψ after the annoucement of a (notice that as a is a single action, SE is equivalent to executability); and ***(ii)*** from every ψ-state, after every execution of a we always get that φ holds (in the model updated by $[!a]$). The former is captured by $[!a]\psi \to \langle a\rangle\top$ being true in the whole model, while the latter is captured by ensuring $[!a]\psi \to [a][!a]\varphi$, also globally. Putting all together, this case is reflected by $\mathsf{A}([!a]\psi \to (\langle a\rangle\top \land [a][!a]\varphi))$. The other cases are standard reductions. All axioms are shown in Table 1.

Theorem 1. *The axiom system from Table 1 is sound and strongly complete for* $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$ *w.r.t. the class of all models.*

3 Conclusions

In this paper we investigated a new dynamic modality in the context of *knowing how* logics. The approached just introduced paves the way to many others. In particular, the same idea can be used for modalities whose announcement is a sequence of actions instead of a single action. Thanks to the expressivity given by the modality $[a]$, we can also characterize the semantics conditions for $[!\sigma]$, with $\sigma \in \mathsf{Act}^*$, generalizing what we did for $[!a]$. Moreover, we can get decidability for the satisfiability problem for $\mathsf{L}_{\mathsf{Kh}_i,\Box}$ using a filtration argument similar to the one used in [3] and thus for $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!a]}$ and its plan extension $\mathsf{L}_{\mathsf{Kh}_i,\Box,[!\sigma]}$.

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