First Steps in Updating Knowing How

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Overview of the talk

- Background
- The Knowing How logic
- Dynamic modalities: Ontic & epistemic updates
- Conclusions and future work

An Epistemic Logic of Knowing How

- Knowing how: epistemic notion related to *the abilities of an agent has to achieve a goal*.
- Wang [2015,2018]: a framework for knowing how logics.
- Areces et al. [2021]: a generalized version by introducing epistemic indistinguishability.
  - Makes a distinction between ontic/factual information, and epistemic information.
- This work: a dynamic epistemic approach of knowing how.
  - Actions updating different kinds of information.

Dynamic Operators over Knowing How

- We introduce dynamic modalities of two types:
  1. Ontic updates: modify the ontic information of the models (announcements and arrow updates)
  2. Epistemic updates: modify the perception of the agent about her own abilities (refinements, learning how)

Knowing How: Models

Definition (Uncertainty-based LTS)

An LTS^{U} is a tuple $\mathcal{M} = \langle \mathsf{W}, \{\mathsf{R}_a\}_{a \in \mathsf{Act}}, \{\mathbb{S}_i\}_{i \in \mathsf{Agt}}, \mathsf{V} \rangle$ where:

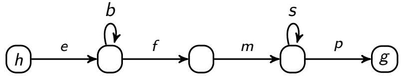
- $\langle \mathsf{W}, \{\mathsf{R}_a\}_{a \in \mathsf{Act}}, \mathsf{V} \rangle$ is an LTS and
- $\mathbb{S}_i \subseteq 2^{\mathsf{Act}^*} \setminus \{\emptyset\}$ s.t.
  - $\emptyset \notin \mathbb{S}_i$
  - $\pi_1, \pi_2 \in \mathbb{S}_i$ with $\pi_1 \neq \pi_2$ implies $\pi_1 \cap \pi_2 = \emptyset$

$\mathbb{S}_i$ represents the sets of plans agent i cannot distinguish between each other.

Baking a good cake (a simplified scenario)

- Two agents attempt to produce a good cake (a goal g), provided they have all the ingredientes (h).
- g is achieved via the following actions: adding eggs (e), beating the eggs (b), adding flour (f), adding milk (m), stir (s) and bake the preparation (p) (the plan *ebfmsp*).
- Agent i is aware of that is the way to get a good cake.
- Agent j considers that the order in the instructions do not matter (e.g., *ebfmsp* and *ebmfsp* are indistinguishable).

$\mathcal{M}$:



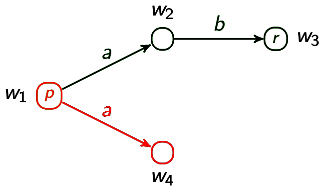
$$\mathbb{S}_i = \{\{ebfmsp\}\},$$
$$\mathbb{S}_j = \{\{ebfmsp, ebmfsp\}\}$$

Strong executability (SE)

A plan should be fail proof:

Every partial execution should be completed.

$\mathcal{M}$:



ab is not strongly executable at w_1

- $\sigma \in \text{Act}^*$ is SE at a state u iff

  every partial execution of σ from u can be completed.
- $\pi \subseteq \text{Act}^*$ is SE at a state u iff for *all* $\sigma \in \pi$, σ is SE at u.

Knowing How: Formulas and semantics

Definition ($\mathsf{L}_{\mathsf{Kh}_i}$-formulas)

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{Kh}_i(\varphi, \varphi)$$

$\mathsf{Kh}_i(\psi, \varphi)$: *"The agent i knows how to achieve φ given ψ."*

Definition

$\mathcal{M}, w \models \mathsf{Kh}_i(\psi, \varphi)$ *iff*$_{def}$ there is $\pi \in \mathbb{S}_i$ s.t.

1. π is SE at all ψ-states, and
2. from ψ-states π reaches only to φ-states.

Knowing How: Example

$\mathcal{M}, w \models \mathsf{Kh}_i(\psi, \varphi)$ *iff*$_{def}$ there is $\pi \in \mathbb{S}_i$ s.t.

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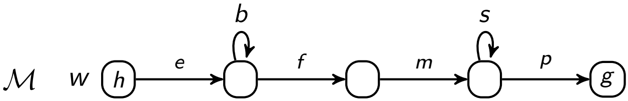


- $\mathbb{S}_i = \{\{ebfmsp\}\}, \qquad \mathbb{S}_j = \{\{ebfmsp, ebmfsp\}\}.$
- $\mathcal{M}, w \models \mathsf{Kh}_i(h, g) \wedge \neg\mathsf{Kh}_j(h, g)$

Ontic vs. Epistemic Information

Two distinct types of information in an $\mathsf{LTS}^{\mathsf{U}}$:

- Ontic information: provided by the graph part
  - the available states, the accessibility relations, etc.
- Epistemic information: given by the indistinguishability sets $\mathbb{S}_i$
  - the perception of each agent about her own abilities.

This enables us to define ways of updating these two types of information.

Ontic updates: Announcement

Definition ($\mathsf{PAL}_{\mathsf{Kh}_i}$ formulas)

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{Kh}_i(\varphi, \varphi) \mid [!\varphi]\varphi$$

$[!\chi]\varphi$: *"After announcing χ, φ holds."*

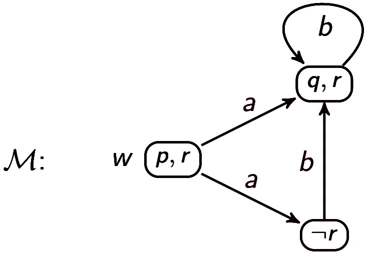
Definition ($\mathcal{M}_{!\chi}$)

$\mathcal{M}, w \models [!\chi]\varphi$ *iff* $\mathcal{M}, w \models \chi$ implies $\mathcal{M}_{!\chi}, w \models \varphi$; where $\mathcal{M}_{!\chi} = \langle \mathsf{W}_{!\chi}, \mathsf{R}_{!\chi}, \mathbb{S}, \mathsf{V}_{!\chi} \rangle$:

- $\mathsf{W}_{!\chi} = [\![\chi]\!]^{\mathcal{M}}$,
- $(\mathsf{R}_{!\chi})_a = \{(w, v) \in \mathsf{R}_a \mid w \in [\![\chi]\!]^{\mathcal{M}}, \mathsf{R}_a(w) \subseteq [\![\chi]\!]^{\mathcal{M}}\}$, and
- $\mathsf{V}_{!\chi}(w) = \mathsf{V}(w)$.

Announcement: Example

$$\mathcal{M}, w \models \mathsf{Kh}_i(p, q),\ \mathbb{S}_i = \{\{ab\}\}$$



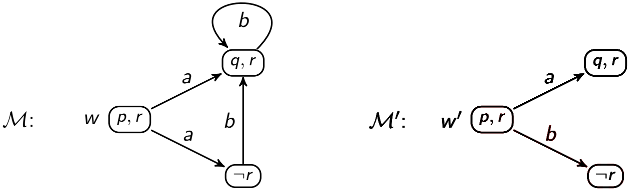
$$\mathcal{M}, w \models [!r]\mathsf{Kh}_i(p, q)$$

$$\mathcal{M}|_r, w \not\models \mathsf{Kh}_i(p, q)$$

Theorem

$\mathsf{PAL}_{\mathsf{Kh}_i}$ *is more expresive than* $\mathsf{L}_{\mathsf{Kh}_i}$ *over arbitrary* $\mathsf{LTS}^{\mathsf{U}}$*s.*

Let $\mathcal{M}$ and $\mathcal{M}'$ two indistinguishable $\mathsf{LTS}^{\mathsf{U}}$s for $\mathsf{L}_{\mathsf{Kh}_i}$:



$\mathcal{M}, w \not\models [!r]\mathrm{Kh}_i(p, q)$ $\quad$ $\mathcal{M}', w' \models [!r]\mathrm{Kh}_i(p, q)$

$\mathbb{S}_i := \{\{ab\}\}$ $\quad$ $\mathbb{S}'_i := \{\{a\}\}$

$\mathsf{PAL}_{\mathsf{Kh}_i}$ can distinguish between the *class of arbitrary models* and the class of models s.t. for all $\pi \in \mathbb{S}_i$, $\pi \subseteq \mathsf{Act}$.

Reduction axioms

$\mathsf{PAL}_{\mathsf{Kh}_i}$ can distinguish between the *class of arbitrary models* and the class of models s.t. for all $\pi \in \mathbb{S}_i$, $\pi \subseteq \mathsf{Act}$.

- This cannot be done in $\mathsf{L}_{\mathsf{Kh}_i}$ [Areces et al. (TARK 2021)].
- In these models (where $\pi \subseteq \mathsf{Act}$), every $\mathsf{PAL}_{\mathsf{Kh}_i}$ formula can be reduced to a $\mathsf{L}_{\mathsf{Kh}_i}$ formula:

$$[!\chi]\mathsf{Kh}_i(\varphi, \psi) \leftrightarrow (\chi \rightarrow \mathsf{Kh}_i(\chi \wedge [!\chi]\varphi, \chi \wedge [!\chi]\psi)).$$

Other kinds of updates

- $\mathsf{PAL}_{\mathsf{Kh}_i}$ is not the only way of updating ontic information.
- We applied similar ideas using an Arrow Update Logic [Kooi and Renne (RSL 2011)].
- We can also perform epistemic updates (affecting directly the "knowing how").
- Proposal: refining the indistinguishability between plans, i.e., making plans distinguishable for the agent.
  1. Explicit refinement for two given plans.
  2. Arbitrary refinements.
  3. "Learning how".

Epistemic updates: Refinement ($\mathsf{L}_{\mathsf{Ref}}$)

Definition ($\mathsf{L}_{\mathsf{Ref}}$ formulas)

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{Kh}_i(\varphi, \varphi) \mid \langle \sigma_1 \not\sim \sigma_2 \rangle \varphi$$

$\langle \sigma_1 \not\sim \sigma_2 \rangle \varphi$: *"After it is stated that plans σ_1 and σ_2 are distinguishable, φ holds."*

Epistemic updates: Refinement ($\mathsf{L}_{\mathsf{Ref}}$) (cont.)

$$\mathcal{M}: \begin{tikzcd} h \arrow[r, "e"] & \circ \arrow[loop above, "b"] \arrow[r, "f"] & \circ \arrow[r, "m"] & \circ \arrow[loop above, "s"] \arrow[r, "p"] & g \end{tikzcd}$$

$$\mathbb{S}_i = \{\{ebfmsp\}\}, \qquad \mathbb{S}_j = \{\{ebfmsp, ebmfsp\}\}$$

$$\mathbb{S}_j = \{\{ebfmsp\}, \{ebmfsp\}\}$$

- $\mathcal{M}, w \not\models \mathsf{Kh}_j(h, g)$ but $\mathcal{M}, w \models \langle ebfmsp \not\sim ebmfsp \rangle \mathsf{Kh}_j(h, g)$;
  - **generates new knowledge**
- $\mathcal{M}, w \models \mathsf{Kh}_i(h, g)$ and $\mathcal{M}, w \models \langle ebfmsp \not\sim ebmfsp \rangle \mathsf{Kh}_i(h, g)$.
  - **preserves knowledge**

Property:

$\mathsf{L}_{\mathsf{Ref}}$ is more expressive than $\mathsf{L}_{\mathsf{Kh}_i}$.

Arbitrary Refinement ($\mathsf{L}_{\mathsf{ARef}}$)

Definition ($\mathsf{L}_{\mathsf{ARef}}$)

$\mathcal{M}, w \models \langle \not\sim \rangle \varphi$ *iff*$_{def}$
there are $\sigma_1, \sigma_2 \in \mathsf{Act}^*$ s.t. $\mathcal{M}, w \models \langle \sigma_1 \not\sim \sigma_2 \rangle \varphi$.

$\langle \not\sim \rangle \varphi$: *"After it is stated that some pair of plans are distinguishable, φ holds."*

Property:

$\mathsf{L}_{\mathsf{ARef}}$ is more expressive than $\mathsf{L}_{\mathsf{Kh}_i}$.

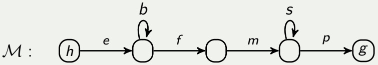
Learning How (L_{Lh})

These new modalities enable us to define a goal-oriented learning modality:

$$\langle \psi, \varphi \rangle_i \chi := \langle \not\sim \rangle (\mathsf{Kh}_i(\psi, \varphi) \wedge \chi)$$

"The agent i can learn how to achieve φ given ψ and after this learning operation takes place, χ holds."

$\mathsf{L}_i(\psi, \varphi) := \langle \psi, \varphi \rangle_i \top$: **learnability test**



$\mathbb{S}_i = \{\{ebfmsp\}\}$,
$\mathbb{S}_j = \{\{ebfmsp, ebmfsp\}\}$

$$\mathcal{M}, w \models \mathsf{L}_j(h, g)$$

Property:

L_{Lh} is more expressive than $\mathsf{L}_{\mathsf{Kh}_i}$.

Conclusions

Dynamic modalities in the context of *knowing how* logics.

- Ontic updates:
  - Announcement-like and arrow-update-like modalities
  - Axiomatizations over a particular class of models
- Epistemic updates:
  - Refining the perception of an agent regarding her own abilities.
  - Preliminary thoughts and some semantic properties.

Future work

- Study other dynamic operators in this context.
- Explore alternative techniques for obtaining proof systems without a general rule of substitution.
- Find fragments that are axiomatizable via reduction axioms by studying the operators' expressivity.