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A Data-based Model Predictive Decision Support System for Inventory Management in Hospitals

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***Abstract*—This paper presents experimental results from the application of a data-based model predictive decision support system to drug inventory management in the pharmacy of a mid-size hospital in Spain. The underlying objective is to improve the efficiency of their inventory policy by exploiting pharmacy historical data. To this end, the pharmacy staff was aided by a decision support system that provided them with quantities needed for the satisfaction of clinical needs and the risk of stockout in case no order is placed for different time horizons. With this information in mind, the pharmacy service takes the final order decisions. The results obtained during a test period of four months are provided and compared with those of a previous model predictive control approach, which was implemented in the same hospital in the past, and with the usual policy of the pharmacy department.**

***Index Terms*—Hospital pharmacy, Inventory management, Data-based decision support system**

I. Introduction

THE acquisition and storage of the medicines necessary to cover the clinical activities of a hospital are some of the primary management tasks performed by a Hospital Pharmacy Service. The societal relevance of the clinical needs and the frequent urgent requests coming from the rest of the hospital services (e.g., for inpatients, consultations, operating rooms, day hospital, outpatients, etc.) make extremely important to control the stock robustly so that stockouts are avoided, and the satisfaction of the demand can be guaranteed. Unfortunately, the high costs of many drugs –some items may cost several hundred or even thousands euros per unit– generates a substantial impact on the hospital budget, which is usually very constrained. Also, the staff working in the Pharmacy Service is limited, thus bounding the number of orders that can be placed and receptioned every day. As a result, a trade-off between demand satisfaction, inventory-related costs, and work burden must be attained, which requires to control the size of the orders and their frequency carefully and to avoid, as much as possible, expiration and unnecessary immobilization of resources. In addition, this problem also presents additional constraints and complicating issues. For example, it is important to take into account the space requirements for storage, especially for those that require refrigeration, given that the refrigerated storage rooms have much more restricted space and thermolabile medicines are becoming more numerous (in addition to the classic ones such as insulin, certain cytostatics, and the novel monoclonal antibodies). Likewise, orders may suffer transport delays and weekends and other holidays of the hospitals, labs, and distributors may impose additional constraints regarding order placement and delivery reception.

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In short, on the one hand, clinical needs must always be satisfied. However, the limitations of financial resources and logistical constraints require the use of acquisition strategies that minimize the amount of product to be stored, assuring, with a certain degree of certainty, that they respond to the clinical demand for a certain period [1], [2]. Hence, the relevance of applying a good management policy that meets these conflicting objectives. The implementation of efficient stock management strategies can lead to significant economic savings [3], [4], taking into account that up to 35% of purchases in goods and services in a hospital come from the Pharmacy Service [5], [6].

In general, stock management techniques are widely used in different businesses and organizations in order to optimize the use of resources, as the immobilized money due to the stock. With that objective, there are several common policies that decide when and how to place new orders [7], [8], [9]:

(1) A classic and very used approach is that of the reorder point (s, S), which consists of placing a new order to have S stocked items whenever the stock is below s. This strategy implies that the amount of items to order is almost fixed. These types of methods make different assumptions, e.g., constant transport delays and Gaussian distributions for the demand [10], [11]. For example, in [12], an (s, S) inventory optimization problem is solved and applied in a case study of a hospital in Turkey.

(2) The inventory policy (R, S) is applied in [13] for supply chains with four echelons. They consist of a manufacturing plant, a vendor distribution center, a retailer distribution center, and a retailer out-let store. This strategy assumes that R is the review interval, and S is the order-up-to-level. Also, demand and lead times are assumed to be stochastic variables in the model.

A noteworthy point is that stock does not need to be *locally* controlled. Indeed, in vendor-managed inventory policies vendors monitor their customers' stock, place orders and trigger deliveries. Following this idea, some works study the supply chain parameters and their effect on cost savings assuming a deterministic demand [14].

In general, simpler management policies are based on simplifications that result in a loss of performance. For example, since non-stationary policies increase the complexity of the problem, many studies consider time-varying deterministic

Fig. 1. Pharmacy Service at San Juan de Dios hospital in Spain. Monodose area.

demands, which is not realistic. Indeed, the costs of assuming a stationary demand are studied in [15]. Some works that deal with this problem are: [16], where an integer linear programming model for inventory lot-sizing and supplier selection problem is presented; [17], which models the non-stationary and stochastic demand by means a Markovian representation, also accounting for computation complexity and cost effectiveness of the policies; in [18], the demand is non-stationary over a finite set of periods and the inventory policy follows the FIFO strategy (first in, first out); a data-driven approach to model stock levels is presented in [19], which works with time-correlated and non-stationary demand; and [20], which develops a stochastic inventory model which combines FIFO and LIFO (Last In, First Out) policies under a non-stationary random demand.

Typically, model misspecifications and information losses derived from simplifying assumptions are mitigated by setting a safety stock to minimize stockouts at the expense of raising average inventory levels. Likewise, the safety stock also helps these methods deal with unexpected realizations of the demand and also with the different sampling times used for stock management in order to achieve a feasible work burden for the staff. In particular, Pareto principle is often applied in this context. As can be seen in Figure 1, the pharmacy department deals with hundreds of different products. For this reason, items are partitioned in several groups depending on their economic impact. In this way, the most expensive items are more frequently ordered than the inexpensive ones, leading to lower and higher average inventory levels, respectively.

Nowadays, with the new technologies and the growing amount of information available due to the pervasive presence of computation is easier to use more sophisticated techniques for supply chain and inventory management problems. For example, in [21], [22], [23], [17], the inventory management problem is formulated in a Markovian decision process framework assuming stochastic demands. Recently, data-driven approaches are also gaining relevance, as shown in [24], [25], where different techniques and algorithms are reviewed, and [26], [27], which provide supporting arguments regarding the impact of data science and predictive analytics in supply chain management. Also, a data-driven approach that considers time-correlated and non-stationary demands is proposed in [19]. Likewise, the abundance of data can be directly exploited by methods such as model predictive control (MPC), which optimizes the sequence of future actions and states of a system along a given horizon while explicitly considering constraints on system variables and the uncertainty in exogenous inputs as the demand. An application of data-based stochastic MPC for this type of application can be found for example in [28], [29], where historical data were used to simulate a chance-constrained MPC.

The work presented in this article uses a different stochastic MPC strategy, namely, scenario-based MPC [30], [31], so that multiple previous realizations of the pharmacy demand are used in the computations. In particular, stochastic MPC is the core of a decision support system that presents order placement *suggestions* and assesses risks. Hence, a significant difference with other works in the literature is that the controller does not implement the actions computed. Instead, it is the pharmacist who chooses among the possibilities presented by the decision support system, which runs the data-based MPC policy for different horizons and risk levels. Hence, the orders finally implemented benefit from both the scenario-based MPC controller and the pharmacist knowledge. In this regard, this work aligns with other human-in-the-loop control methods such as [32] and, especially, [33], where operator selects actions within a set provided by the control system.

The proposed data-based decision support system is actually applied to the pharmacy of the hospital *San Juan de Dios* in Córdoba, Spain. The results obtained along a four month test period are assessed via simulation with a previous MPC controller implemented in the pharmacy [34] and a perfect forecast MPC. Likewise, the comparison is complemented with the results of the actual policy followed by the hospital when these controllers are not operating.

Finally, this work is embedded into a project named *Pharmacontrol*, where different hospitals in Andalusia cooperate with the Higher Technical School of Engineering of the University of Seville to assess and implement new strategies that improve the efficiency of Hospital Pharmacy Services. Before this project, the management policy used was based mainly on the previously mentioned technique of the reorder point. Nevertheless, the pharmacy staff used that only as a reference because other factors are also considered when placing a new order, e.g., fluctuations of the demand, the number of patients under treatment with highly controlled medicines, the proximity of the end of week and holidays in which orders are not served, and the need of other drugs from the same provider to reach the minimum amount established to place an order.

The remainder of the article is structured as follows: Section II presents the problem formulation. Section III explains the proposed methodology to improve the hospital drug inventory management policy. Section IV shows the experimental results obtained. Finally, conclusions are drawn in Section V.

Fig. 2. Hospital pharmacy storage area.

II. Problem Statement

The objective is to use historical pharmacy inventory records to support decisions regarding the size and timing of new orders. To this end, we consider a rolling horizon approach in which suggestions for inventory decisions are generated considering previous realizations of the demand to predict the stock evolution in the following days. The elements leading to the corresponding optimization problem are introduced in the next subsections, namely, the model describing the pharmacy inventory system, its constraints, and the main goals of the pharmacy service.

A. Pharmacy Inventory System

The hospital pharmacy manages a set $\mathcal{M} = \{1, 2, \ldots, M\}$ of medicines (hereinafter also referred to as *meds*) with time-varying inventory levels due to the deliveries coming from a set $\mathcal{P} = \{1, 2, \ldots, P\}$ of pharmacy distributors and the uncertainty of the demand from the different hospital services.

Let $s_i(t) \in \mathbb{Z}_{\geq 0}$ and $d_i(t) \in \mathbb{Z}$ respectively be the stock and demand of drug $i \in \mathcal{M}$ at day t, and $o_i^j(t - \tau_i^j) \in \mathbb{Z}_{\geq 0}$ represent the units of drug i ordered to provider $j \in \mathcal{P}$ τ_i^j days ago, with τ_i^j representing the transport delay.[1] Thus,

$$r_i(t) = \sum_{j \in \mathcal{P}} o_i^j(t - \tau_i^j). \tag{1}$$

units of drug i are received at day t, whereas

$$o_i(t) = \sum_{j \in \mathcal{P}} o_i^j(t) \tag{2}$$

represents the units of drug i ordered at day t. Given the demand, and the corresponding drug deliveries, the following discrete-time linear model can be used to represent the evolution of stock of drug i

$$s_i(t+1) = s_i(t) + r_i(t) - d_i(t), \tag{3}$$

from where it is clear that any constraint on the stock becomes stochastic due to the uncertainty of the demand.

[1] While the stock and the orders are integers greater or equal than 0, the demand can occasionally be *negative* due to items returned to the pharmacy.

Fig. 3. Hospital pharmacy storage refrigerator.

B. Pharmacy Constraints

In order to obtain valid results, it is necessary to take into account the following constraints:

(i) **Storage constraints.** The storage space in the pharmacy is limited as can be seen in Figure 2, very particularly R2.4. for cold storage drugs, which are shown in Figure 3. Thus, the stock levels should remain below admissible values, especially for those drugs that must be stored in refrigerators. Likewise, stocks are required to be either 0 or positive. These constraints are translated into a minimum 0 and maximum number $\mathrm{s}_i^{\max}$ of units for each drug $i \in \mathcal{M}$, i.e.,

$$s_i(t) \in [0, \mathrm{s}_i^{\max}]. \tag{4}$$

Note that the minimum could also be imposed by the safety stock, in case it is used.

(ii) **Order placement constraints.** The number of units ordered for each med $i \in \mathcal{M}$ can either be zero or be bounded by a minimum and a maximum value, i.e.,

$$o_i(t) \in \{0\} \cup [\mathrm{o}_i^{\min}, \mathrm{o}_i^{\max}]. \tag{5}$$

Since this constraint set is not convex, a binary variable $\delta_i(t)$ modeling the action of order placement can be introduced to deal with this issue. In this way, $\delta_i(t) o_i(t)$ represents the number of ordered units, so that $o_i(t) \in [\mathrm{o}_i^{\min}, \mathrm{o}_i^{\max}]$ iff $\delta_i(t) = 1$.
Also, pharmaceutical laboratories do not provide drugs unless a minimum amount of money is spent. This constraint can be posed as

$$\sum_{i \in \mathcal{M}} c_i^j o_i^j(t) \geq \mathrm{o}_{\$}^j, \tag{6}$$

where $o_{\$}^{j}$ represents the minimum amount of money to be spent when placing an order to supplier $j \in \mathcal{P}$ and c_i^j is the unitary cost of drug i.

(iii) **Non-working days.** Another issue to take into account is that labs, distributors, and pharmacies have *non-working days* (e.g., Sundays, holidays), which leads to

$$o_i(t) = 0, \quad \forall t \notin \{\text{working days}\} \tag{7}$$

(iv) **Work burden.** Due to the limited staff at the pharmacy, a constraint has to be imposed on the orders that can be daily handled, which also limits the corresponding deliveries, leading to

$$\sum_{i\in\mathcal{M}} \delta_i(t) \le \delta_{\max}. \tag{8}$$

C. Goals of the Hospital Pharmacy Service

Given the critical nature of the application we deal with, the primary concern is to ensure that the required meds are available at all times to the patients. Nevertheless, a further objective is to increase inventory efficiency by reducing ordering and holding inventory expenses, i.e., lowering the number of orders placed and stock levels. From an MPC viewpoint, at day t, the pharmacy managers deal with the following optimization problem

$$\begin{aligned}\min_{[O_i,\Delta_i]_{i\in\mathcal{M}}} \sum_{l=t}^{t+N_\mathrm{p}-1} \textstyle\sum_{i\in\mathcal{M}} \quad & \big(\alpha C_{o,i} s_i(l+1) \\ & + \beta c_i \delta_i(l) o_i(l) + \gamma\delta_i(l)\big),\end{aligned} \tag{9}$$

where α, β and γ weight respectively the stage inventory costs, the acquisition of new drugs to replenish the stock levels and the effort of the pharmacy service derived from the order placement and deliveries. Here, N_p represents the prediction horizon for the planning, $C_{o,i}$ can be interpreted as a cost of opportunity or inventory cost, c_i represents the average cost of drug i, $O_i = [o_i(t), o_i(t+1), \ldots, o_i(t+N_\mathrm{p}-1)]$ is the sequence of orders for drug i for the current and the next $N_\mathrm{p}-1$ days, and $\Delta_i = [\delta_i(t), \delta_i(t+1), \ldots, \delta_i(t+N_\mathrm{p}-1)]$ is defined analogously for the order placement binary variable. The optimization problem (9) has to be solved every day t subject to the model equations (1) to (3), and the constraints (4) to (8). The first component of the optimal sequence O_i represent the units of drug i to be ordered on day t, while the rest of the sequence is discarded. This planning has to be repeated every day t in a receding horizon fashion.

As can be seen, the optimization problem *merges* different objectives into a single performance index so as to obtain a trade-off between the goals considered. The problem presents several challenging issues, e.g., the bilinear term $\delta_i(t)o_i(t)$, the binary nature of $\delta_i(t)$, the uncertainty derived from the stochasticity of the demand, binding constraints (equations (2), (6), and (8)), and also the size of the problem, for hospitals may work with more than one thousand different drugs. A general strategy to solve this type of problem might be based in the use of branch-and-bound methods, but it is not surprising that simplifications are performed and heuristics are used to simplify the management problem. For example, one could separate the problem in item's sub-problems, solve them in a decentralized fashion, and check afterwards if binding constraints are violated. In any case, the role of the pharmacist is essential to check and guarantee the feasibility of the implemented solution.

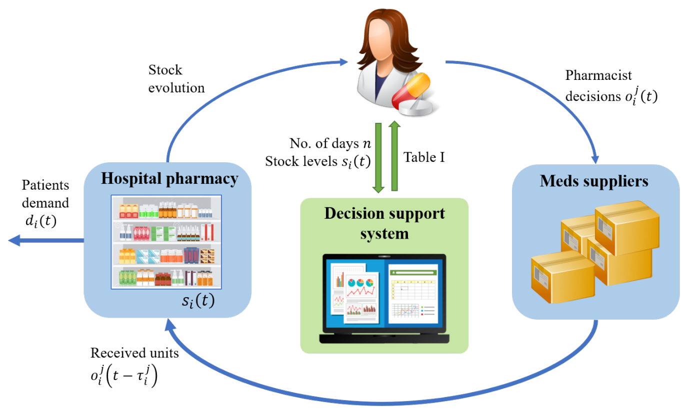

Fig. 4. Steps and components of the proposed DB-MPDSS: 1) at the beginning of day t, the pharmacist observes the stock levels of each med; 2) this information is introduced into the MPDSS, together with the desired value(s) for n (from 2 to 8 in the performed test); 3) the DB-MPDSS processes the data and provides the pharmacist with Table I and the orders for each case considered; 4) the pharmacist evaluates this information and places the corresponding orders; 5) meds arrive to the pharmacy after the corresponding transport delay.

III. Data-based Decision Support System

In this section, we describe a data-based model predictive decision support system (hereafter, DB-MPDSS) that we have implemented in a real hospital. In particular, this DB-MPDSS exploits historical records of the drugs and provides useful results to set the size and timing of the orders according to the quantities of units in stock. Also, it learns daily as new data become available in the database. In a broad sense, it can be considered from the viewpoint of human-in-the-loop applications (HIL) within the context of cyber-physical systems, because humans make the final decisions, interact with a real-world process, and provide information to the MPDSS. Figure 4 illustrates the structure of the HIL-MPDSS tested in this paper. Four main interacting components constitute it: the pharmacist, i.e., the person responsible for the pharmacy storage; the meds suppliers; the hospital pharmacy; and the MPDSS, which exploits recorded data to help the human decision-making process.

Hereon, we adopt the following notation: $\mathcal{D}$ is the database containing the historical of demanded and ordered quantities for all $i \in \mathcal{M}$, and $\mathcal{T} = \{1, 2, \ldots, t-1\}$ is the set of days for which there exist data in $\mathcal{D}$, i.e.,

$$\mathcal{D} = \{d_i(\hat{t}), o_i(\hat{t})\}_{\forall i\in\mathcal{M}, \hat{t}\in\mathcal{T}}. \tag{10}$$

Although not explicitly specified, through this paper all probabilities calculations will be based on the data provided by $\mathcal{D}$.

Each day t, the pharmacist provides the DSS with the currently on-hand quantities of each of the meds under study, i.e., $s_i(t)$ for all $i \in \mathcal{M}$, and sets a number of days for which future information is required. Considering the latter, and the information in $\mathcal{D}$, the MPDSS automatically provides the following results:

- Minimum quantity of each med i that should be ordered to obtain 0% probability of stockout during the next n days according to historical data, calculated as

$$o_{i,n}(t) = \max\left(0, \max_{\hat{t}\in\hat{\mathcal{T}}} \sum_{k=\hat{t}}^{\hat{t}+n} d_i(k) - s_i(t)\right), \quad (11)$$

  where $\hat{\mathcal{T}} = \{1, 2, \ldots, t-n-1\}$.
- Total cost of the order that ensures a 0% probability of stockout for all meds during the same period, i.e.,

$$C_n(t) = \sum_{i\in\mathcal{M}} c_i o_{i,n}(t). \quad (12)$$

  Correspondingly, it could be determined the total amount of items that the pharmacy must order $\sum_i o_{i,n}(t)$.
- Probability of stockout for each med $i \in \mathcal{M}$, assuming that no order is placed, for a different number n of days ahead, i.e.,

$$p_{i,n}(t) = p\left(s_i(t+n) \leq 0\right). \quad (13)$$

  where $p(s_i(t+n) \leq 0)$ denotes the probability of $s_i(t+n)$
R2.5. being lower or equal to 0. Notice that $p_{i,n}(t) = 0$ entails that there is no period of length n stored in $\mathcal{D}$ in which the cumulative demand of i surpassed the amount of units in stock. Additionally, note that the stock and demand are related through (3). Thus, it is possible to consider the stochasticity from the viewpoint of the stock.
- Probability of not having stockout for all meds, assuming that no order is placed, i.e.,

$$P_n(t) = \prod_{i\in\mathcal{M}} \left(1 - p_{i,n}(t)\right). \quad (14)$$

That is, on the one hand, the DSS provides the quantities of the minimum drugs that should arrive at the pharmacy to cover the patients' demands according to historical records, and, on the other hand, it assesses the risk of this not occurring. The analysis of data allows a more accurate calculation of these values, which conventionally are estimated based on the pharmacist's experience and intuition. In view of the information contained in Table I and the constraints described in Section II, it is the pharmacist who decides when and how many units of each drug should be ordered to the laboratories. Hence, the MPDSS does not undermine the human decision-making power, but augments the information available for managing the inventory, thus facilitating current and future decisions.

Finally, it must be remarked that the use of the proposed DB-MPDSS policy also involves some risks. To begin with, it is assumed that future demand realizations are contained in the database, which may bias suggested orders due to issues such as extreme demand peaks in the past, abrupt demand increments, and seasonal behaviour, to name a few potential problems. Another simplification is the probability of constraint satisfaction provided in Table I, which is calculated empirically using the available data. To have robust statistical guarantees such as those given in [31], a very large number of scenarios are required and the database may not contain enough data. Even when there are some possibilities to mitigate these issues, e.g., filtering data to remove outliers, generate additional scenarios using resampling methods, etc., their impact must be evaluated.

TABLE I
RESULTS PROVIDED DAILY TO THE PHARMACY SERVICE BY THE DECISION-SUPPORT SYSTEM.

		Numbers of days (n)			
		1	2	$\cdots$	N
Meds (i)	1	$o_{1,1}$	$o_{1,2}$	$\cdots$	$o_{1,N}$
	2	$o_{2,1}$	$o_{2,2}$	$\cdots$	$o_{2,N}$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	M	$o_{M,1}$	$o_{M,2}$	$\cdots$	$o_{M,N}$
C_n		$\sum_i c_i o_{i,1}$	$\sum_i c_i o_{i,2}$	$\cdots$	$\sum_i c_i o_{i,N}$
Meds (i)	1	$p_{1,1}$	$p_{1,2}$	$\cdots$	$p_{1,N}$
	2	$p_{2,1}$	$p_{2,2}$	$\cdots$	$p_{2,N}$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	M	$p_{M,1}$	$p_{M,2}$	$\cdots$	$p_{M,N}$
P_n		$\prod_i(1-p_{i,1})$	$\prod_i(1-p_{i,2})$	$\cdots$	$\prod_i(1-p_{i,N})$

IV. Case Study and Results

The proposed DB-MPDSS has been tested during a four-month period in the hospital San Juan de Dios, which is located in the Spanish city of Córdoba. To this end, a group of 11 drugs was selected for the study; all of them supplied by the same laboratory. The hospital pharmacy provided the historical data of demand and orders for these 11 drugs during the last previous two years, which allowed us to calculate the historical evolution of the stock levels. For confidentiality reasons, the name, prices, and further specific information about the drugs are not disclosed in this paper.

During the test period, the stock levels of the 11 drugs under study were introduced daily into the system by the pharmacy service. Considering this, the DB-MPDSS automatically calculated the values described in Section III for periods of 2 to 8 days, that is: minimum order quantities to prevent stockouts during the corresponding number of days (see Table II) and probabilities of this occurring in case no order is placed (see Table III). In the same way, the DB-MPDSS provided the total costs of the orders to assure 0% probability of stockout (also in Table II), which, for the reasons mentioned above, are calculated considering unitary prices $c_i = 1$ for all drugs $i = 1, \ldots, 11$. It can be seen that the number of items to order increases with the number of days without placing any order due to the progressive increase in the cumulative patients'

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TABLE II
ADDITIONAL UNITS OF EACH DRUG NEEDED TO AVOID STOCKOUT FOR 2 TO 8 DAYS, AND TOTAL COST WITH $c_i = 1$, FOR ALL $i = 1, ..., 11$.

		Numbers of days (n)						
		2	3	4	5	6	7	8
Meds (i)	1	0	50	50	50	100	100	150
	2	0	4	8	8	12	16	16
	3	0	0	0	0	0	0	4
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	10	160	240	250	270	350
	7	0	0	0	0	0	0	40
	8	0	0	0	0	20	20	120
	9	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0
C_n		0	64	218	298	382	406	680

TABLE III
INDIVIDUAL AND JOINT PROBABILITY OF STOCKOUT IN 2 TO 8 DAYS WITHOUT ORDERS.

		Numbers of days (n)						
		2	3	4	5	6	7	8
Meds (i)	1	0	0.09	0.28	1.31	2.34	4.40	9.75
	2	0	0.09	0.65	1.59	4.12	7.77	12.93
	3	0	0	0	0	0	0	0.09
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0.09	0.47	0.84	1.31	2.53	7.59
	7	0	0	0	0	0	0	0.09
	8	0	0	0	0	0.09	0.28	0.75
	9	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0
P_n		1	99,73	98,61	96,31	92,33	85,70	71,94

demands. Accordingly, the joint probability of preventing any stockout decreases with the number of days. In particular, the values given in Tables II and III correspond to a day of the test period in which the stock levels of the drugs where respectively $80, 10, 24, 12, 33, 495, 119, 114, 80, 59$ and 52 units. The reason for setting the minimum period to 2 days is the laboratory transport delay, i.e., all drugs ordered on the day t arrive on the day $t + 2$ in the worst case.

Such tabular data were calculated by the DB-MPDSS and provided to the pharmacy staff, who made the final decisions taking into account all the constraints of the problem (maximum storable stock, minimum order at provider, etc.). During the experiment, the pharmacy service decided when to order depending on the stockout probability they were willing to assume, but the recommendations of the DSS about the quantities were followed faithfully. That is, if the pharmacy placed an order of drug i, then the ordered quantity was that recommended by the DSS to avoid stockout during the number of days considered by the pharmacy manager. Following these recommendations, it has been seen that it is possible to reduce both the number of drugs stored and the number of orders without jeopardizing the satisfaction of the demand.

A. Performance Assessment

To assess the performance of the proposed method during the entire test period, we compare the results with the model predictive control approach proposed in [34], which was previously implemented and tested in this hospital for the same drugs. In particular, figures 5 and 6 show the evolution of the stock levels and the orders placed for the test period during which the data-based decision support system was applied, and the simulation results with the model predictive control (MPC) technique described in [34].

In the simulations shown here, the MPC controller, based on the daily stock data and taking into account the mean demand of the same database, optimizes the number of items to be ordered by minimizing the objective function (9) with $\alpha = \beta = 1$ and $\gamma = 0$ subject to the constraint set (4)-(8). In particular, we have used $N_p = 20$ for the horizon. Also, we have considered a control horizon $N_c = 2$, i.e., the MPC controller decides whether drugs should be ordered the current or the following day. The rest of alternatives are not considered for simplicity. Note that this decision does not affect the performance of the controller when new items are to be ordered in the current day.

For the sake of comparison, we assume that the pharmacy service would follow faithfully the size and timing of the orders recommended by the controller. For a more detailed comparison between the MPC simulation results and the performance of the DB-MPDSS proposed in this paper, Table IV shows the following key performance indicators (KPI) for each of the meds i:

- Mean value (μ_i) and standard deviation (σ_i) of the items in stock.
- Maximum (M_i) and minimum (m_i) registered number of items in stock.
- Number of stockouts (SO_i) registered in the period of study.
- Order rate (OR_i), i.e., the number of days an order for drug i was placed divided by the total number of days in the test period.
- Average size of the orders (AO_i).

Our results show that the MPC controller would have outperformed the DB-MPDSS results in some KPI. As shown in Table V, the average stock of the drugs was reduced with the MPC, saving 649.5 euros, and the number of orders was also

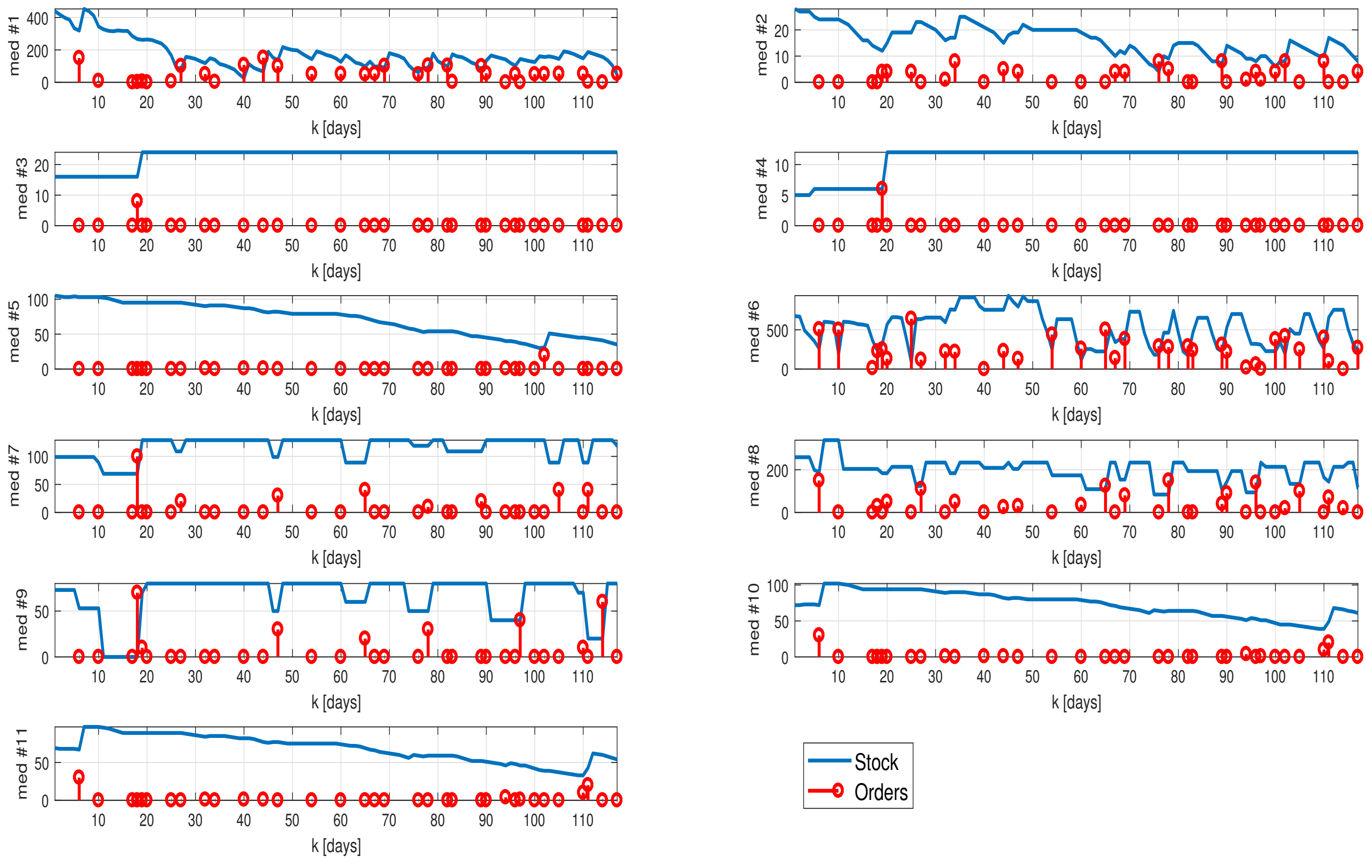

Fig. 5. Evolution of stocks and orders during the test period.

reduced, being 31 days with this technique, compared to 34 with the DB-DSS[2]. However, the improvement comes at a cost in the satisfaction of clinical demands: the MPC method ended up with four stockouts, caused by the differences between the real patients' demands and the average considered for the predictions, hence compromising the patients' safety.

Additionally, in the period before this experiment, the pharmacy was controlling these drugs without the aid of the MPC controller. In Table VI, we show the KPIs obtained during the four months previous to the experiment for the sake of comparison. It can be seen that the overall average of stored items was reduced during the test period of the DB-DSS, and yet the number of stockouts was 0. Also, the maximums reached by the stock levels of each med were reduced or maintained around a similar value. Regarding the number of orders, it decreased from 42 to 34 when implementing the DB-DSS, i.e., a reduction of 19%. However, the average money immobilized in the pharmacy increased in 366.73 € due to a slight increase of the mean stock levels of some of the most expensive drugs, possibly derived from some peaks in the historical demand and the imposed risk stockout events, which was 0%. In this regard, the extra expenditures aim to minimize the risks of stockouts, and indeed, the quantities stored are determined to satisfy past real demands. Accordingly, the lowest values of the most expensive drugs were observed when using the MPC policy as might be expected from (9). Hence, the implementation of the DB-DSS improved the management in most of the KPI with respect to the usual methodology, guaranteeing the satisfaction of the demand, and also providing important advantages such as the reduction of the storage space required and the workload by reducing the number of orders.

Finally, in order to assess the upper bound on performance of MPC-based strategies, it is considered the ideal case of a MPC controller with perfect demand forecast and prediction horizon $N_{\mathrm{p}} = 5$, which leads to the same number of orders as in the experiment, i.e., 34. In this way, it is possible to focus on the comparison of the management of stock levels. The simulation of this strategy leads to no stockouts with pharmacy savings of 1346.5€. Hence, the performed test with the DB-MPDSS obtained savings that are slightly below the half of those of the theoretical optimum. While this optimal result is not achievable due to the unrealistic assumption of perfect forecast, it suggests that there may be room for further performance enhancements with the proposed method.

[2]The economic savings given were calculated considering the mean stocks and the original unitary prices of the drugs

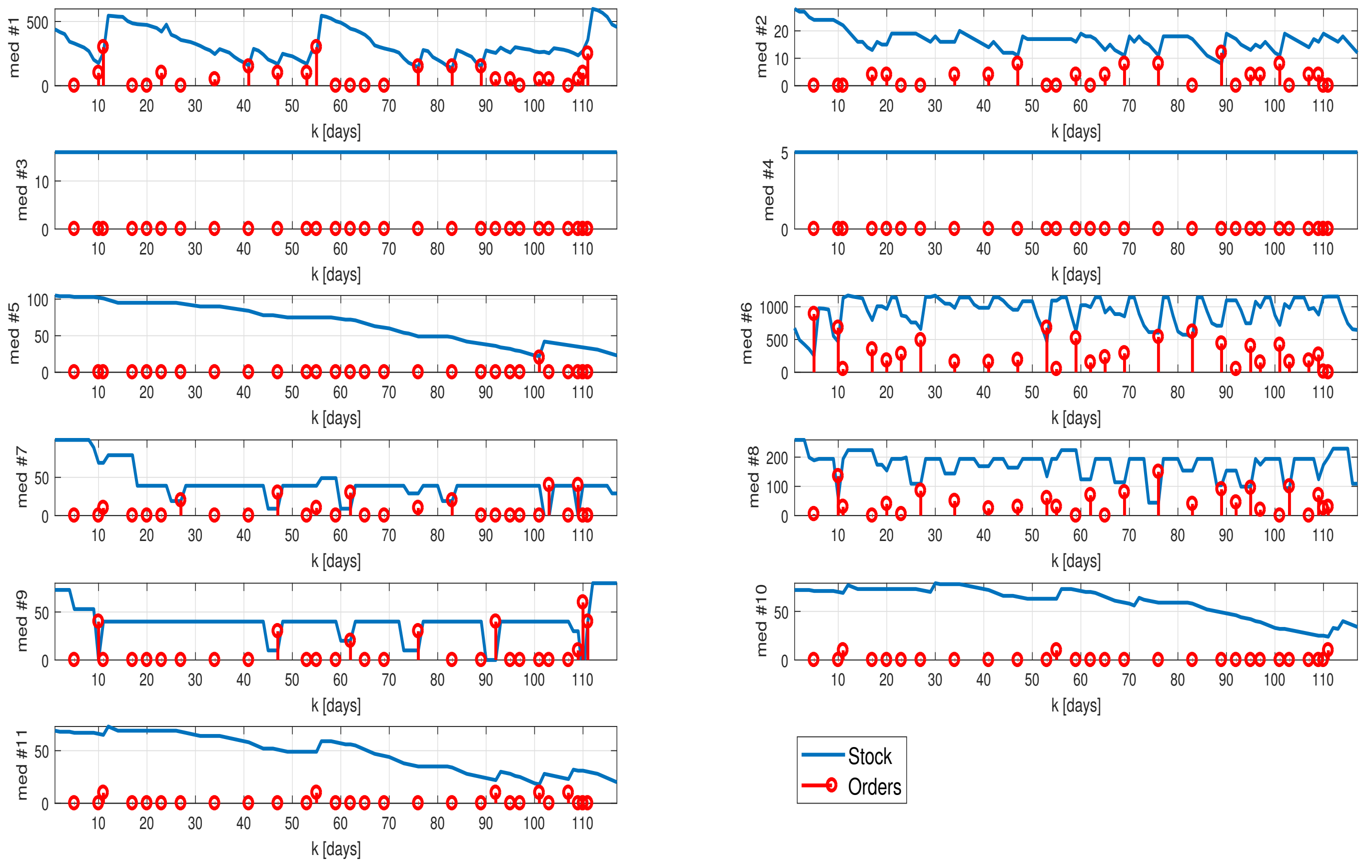

Fig. 6. Evolution of stocks and orders in the MPC simulation [34].

V. Conclusions

In this work, a data-based model predictive decision support system was developed and applied to improve the stock management policy in the pharmacy of the hospital *San Juan de Dios*, located in Córdoba, Spain.

From the viewpoint of the pharmacy staff, the new tool developed increases their security and confidence when placing orders, making them more aware of the risks assumed.In this regard, the risk and orders tables provided by the DB-MPDSS are much richer than information given by the current information system used at the hospital, allowing pharmacists to take better decisions. Likewise, the proposed method reinforces the role of the pharmacist in the inventory management loop and its implementation is simple, i.e., it does not alter the workflows in the pharmacy department, which are relevant aspects to facilitate its adoption.

Also, this contribution has shown improvement with respect to other methods applied, namely, a simple MPC controller and the usual policy of the pharmacy service. In particular, the stock levels have been lowered, and the number of orders has also been reduced. It must be noticed that our test was performed with a small subset of economically inexpensive drugs. Considering that the hospital manages more than 1300 different drugs and that the unit cost of some of them is beyond one thousand euros, any performance improvement that maintains the quality of service level results in significant savings. Moreover, our assessment with respect to a perfect forecast MPC also indicates that there may be room for improving performance, e.g., by filtering extreme past demand peaks from the data.

Therefore, it can be considered that the application of data-based policies in this context is easy to implement and promising due to its positive impact in the pharmacy department, reducing both stock levels and staff workload. This method was considered as a high-value support system by the pharmacy service. This aid is essential in a context where stock management is surrounded by a high degree of uncertainty. For this reason, it is expected to expand the number of drugs controlled in this way by incorporating other providers into the software. In addition, for safety, the software developed was independent of the hospital information system, thus requiring the pharmacy staff to enter the stock data each day manually. This task is tedious and, therefore, another expected improvement is to establish a direct connection with the hospital database. In this way, it will be easier to use the DB-MPDSS in a generalized manner with the rest of the drugs and suppliers.

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TABLE IV
KPI COMPARISON BETWEEN THE DB-DSS IMPLEMENTED IN THE EXPERIMENT AND THE OBTAINED SIMULATION RESULTS WITH THE MPC CONTROLLER IN [34].

		μ_i		σ_i		M_i		m_i		SO_i		OR_i		AO_i	
		DB	MPC	DB	MPC	DB	MPC	DB	MPC	DB	MPC	DB	MPC	DB	MPC
Meds (i)	1	174.29	320.13	91.37	117.87	453	598	26	137	0	0	0.250	0.154	59.48	125.00
	2	16.07	16.68	5.56	3.61	28	28	4	8	0	0	0.162	0.154	4.68	5.33
	3	22.78	16.00	2.88	0	24	16	16	16	0	0	0.009	0	8.00	0
	4	11.00	5.00	2.30	0	12	5	5	5	0	0	0.009	0	6.00	0
	5	71.09	66.97	22.94	25.85	105	105	30	22	0	0	0.043	0.009	4.80	20.00
	6	531.17	934.35	218.33	218.10	935	1172	86	253	0	0	0.274	0.256	264.25	288.33
	7	116.20	42.42	18.85	21.74	129	99	69	-1	0	3	0.068	0.077	37.50	23.33
	8	198.32	174.47	53.02	44.96	339	259	84	44	0	0	0.154	0.197	73.06	56.96
	9	65.85	39.37	23.70	16.73	80	80	0	-20	0	1	0.068	0.068	33.75	33.75
	10	73.37	59.69	17.25	15.73	102	79	39	24	0	0	0.068	0.051	8.50	10.00
	11	68.23	47.75	17.46	17.21	97	73	33	18	0	0	0.068	0.043	8.50	10.00

TABLE V
OVERALL PERFORMANCE COMPARISON OF THE DB-DSS AND THE MPC APPROACH IN [34]

	#Orders	Savings [€]	#Stockouts
DB (Experimental results)	34	-	0
MPC (Simulation results)	31	649.5	4

TABLE VI
KPI DURING THE FOUR-MONTHS PERIOD PREVIOUS TO THE EXPERIMENT.

		μ_i	σ_i	M_i	m_i	SO_i	OR_i	AO_i
Meds (i)	1	375.75	87.90	568	183	0	0.265	56.03
	2	13.73	6.43	33	0	0	0.179	10.86
	3	11.94	5.14	21	0	0	0.094	7.27
	4	8.84	1.67	11	6	0	0	0
	5	79.81	31.10	138	3	0	0.137	22.19
	6	828.31	220.95	1355	212	0	0.248	236.45
	7	77.09	23.61	109	9	0	0.068	44.13
	8	292.08	94.35	504	124	0	0.077	144.67
	9	107.03	26.15	173	33	0	0.034	45.50
	10	71.53	17.27	102	40	0	0.171	13.50
	11	75.76	20.07	111	34	0	0.171	13.35

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