Ecuaciones Diferenciales - Variación de Parámetros

1. $y'' + y = \sec x$

Ec. característica: $r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x,\ y_2 = \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix} = -\sin x \sec x = -\tan x$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix} = \cos x \sec x = 1$$

$u_1 = \int \frac{W_1}{W} dx = \int (-\tan x) dx = \ln|\cos x|$

$u_2 = \int \frac{W_2}{W} dx = \int 1\, dx = x$

$y = c_1 \cos x + c_2 \sin x + \cos x \ln|\cos x| + x \sin x$

2. $y'' + y = \tan x$

Ec. característica: $r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x,\ y_2 = \sin x$

$W = 1$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix} = -\sin x \tan x = -\frac{\sin^2 x}{\cos x}$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix} = \cos x \tan x = \sin x$$

$u_1 = \int \frac{-\sin^2 x}{\cos x} dx = \int \frac{-(1-\cos^2 x)}{\cos x} dx = \int (-\sec x + \cos x) dx = -\ln|\sec x + \tan x| + \sin x$

$u_2 = \int \sin x\, dx = -\cos x$

$y = c_1 \cos x + c_2 \sin x + \cos x(-\ln|\sec x + \tan x| + \sin x) - \sin x \cos x$

$y = c_1 \cos x + c_2 \sin x - \cos x \ln|\sec x + \tan x|$

3. $y'' + y = \sec x$ (igual que 1)

$y = c_1 \cos x + c_2 \sin x + \cos x \ln|\cos x| + x \sin x$

4. $y'' + y = \sec\theta \tan\theta$

Ec. característica: $r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_c = c_1 \cos\theta + c_2 \sin\theta$

$y_1 = \cos\theta,\ y_2 = \sin\theta$

$W = 1$

$$W_1 = \begin{vmatrix} 0 & \sin\theta \\ \sec\theta\tan\theta & \cos\theta \end{vmatrix} = -\sin\theta\sec\theta\tan\theta = -\tan^2\theta$$

$$W_2 = \begin{vmatrix} \cos\theta & 0 \\ -\sin\theta & \sec\theta\tan\theta \end{vmatrix} = \cos\theta\sec\theta\tan\theta = \tan\theta$$

$u_1 = \int (-\tan^2\theta) d\theta = \int (-\sec^2\theta + 1) d\theta = -\tan\theta + \theta$

$u_2 = \int \tan\theta\, d\theta = -\ln|\cos\theta|$

$y = c_1 \cos\theta + c_2 \sin\theta + \cos\theta(-\tan\theta + \theta) - \sin\theta \ln|\cos\theta|$

$y = c_1 \cos\theta + c_2 \sin\theta - \sin\theta + \theta\cos\theta - \sin\theta \ln|\cos\theta|$

5. $y'' + y = \cos^2 x$

Ec. característica: $r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x,\ y_2 = \sin x$

$W = 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1+\cos 2x}{2} & \cos x \end{vmatrix} = -\sin x \frac{1+\cos 2x}{2}$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1+\cos 2x}{2} \end{vmatrix} = \cos x \frac{1+\cos 2x}{2}$$

$u_1 = \int \left(-\frac{\sin x}{2} - \frac{\sin x \cos 2x}{2}\right) dx = \frac{\cos x}{2} + \frac{1}{6}(2\sin x \sin 2x + \cos 2x)$

$u_2 = \int \left(\frac{\cos x}{2} + \frac{\cos x \cos 2x}{2}\right) dx = \frac{\sin x}{2} + \frac{1}{6}(-2\sin 2x \cos x + \sin 2x)$

$y_p = \frac{1}{2} + \frac{1}{2}\cos 2x$

$y = c_1 \cos x + c_2 \sin x + \frac{1}{2} + \frac{1}{2}\cos 2x$

6. $y'' + y = \sec^2 x$

Ec. característica: $r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x,\ y_2 = \sin x$

$W = 1$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \sec^2 x & \cos x \end{vmatrix} = -\sin x \sec^2 x = -\tan x \sec x$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec^2 x \end{vmatrix} = \cos x \sec^2 x = \sec x$$

$u_1 = \int (-\tan x \sec x) dx = -\sec x$

$u_2 = \int \sec x\, dx = \ln|\sec x + \tan x|$

$y = c_1 \cos x + c_2 \sin x - \cos x \sec x + \sin x \ln|\sec x + \tan x|$

$y = c_1 \cos x + c_2 \sin x - 1 + \sin x \ln|\sec x + \tan x|$

7. $y'' - y = \cosh x$

Ec. característica: $r^2 - 1 = 0 \Rightarrow (r-1)(r+1) = 0 \Rightarrow r = 1, -1$

$y_c = c_1 e^x + c_2 e^{-x}$

$y_1 = e^x,\ y_2 = e^{-x}$

$$W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -e^x \cdot e^{-x} - e^x \cdot e^{-x} = -2$$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \cosh x & -e^{-x} \end{vmatrix} = -e^{-x}\cosh x = -\frac{1+e^{-2x}}{2}$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \cosh x \end{vmatrix} = e^x \cosh x = \frac{e^{2x}+1}{2}$$

$u_1 = \int \frac{-\frac{1+e^{-2x}}{2}}{-2} dx = \int \frac{1+e^{-2x}}{4} dx = \frac{x}{4} - \frac{e^{-2x}}{8}$

$u_2 = \int \frac{\frac{e^{2x}+1}{2}}{-2} dx = \int \frac{-(e^{2x}+1)}{4} dx = -\frac{e^{2x}}{8} - \frac{x}{4}$

$y_p = e^x\left(\frac{x}{4} - \frac{e^{-2x}}{8}\right) + e^{-x}\left(-\frac{e^{2x}}{8} - \frac{x}{4}\right) = \frac{xe^x}{4} - \frac{e^{-x}}{8} - \frac{e^x}{8} - \frac{xe^{-x}}{4}$

$y = c_1 e^x + c_2 e^{-x} + \frac{x \sinh x}{2} - \frac{\cosh x}{4}$

8. $y'' - y = \sinh 2x$

Ec. característica: $r^2 - 1 = 0 \Rightarrow r = 1, -1$

$y_c = c_1 e^x + c_2 e^{-x}$

$y_1 = e^x,\ y_2 = e^{-x}$

$W = -2$

$\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \sinh 2x & -e^{-x} \end{vmatrix} = -e^{-x}\sinh 2x = -\frac{e^x - e^{-3x}}{2}$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \sinh 2x \end{vmatrix} = e^x \sinh 2x = \frac{e^{3x} - e^{-x}}{2}$$

$u_1 = \int \frac{-\frac{e^x - e^{-3x}}{2}}{-2} dx = \int \frac{e^x - e^{-3x}}{4} dx = \frac{e^x}{4} + \frac{e^{-3x}}{12}$

$u_2 = \int \frac{\frac{e^{3x} - e^{-x}}{2}}{-2} dx = \int \frac{-(e^{3x} - e^{-x})}{4} dx = -\frac{e^{3x}}{12} - \frac{e^{-x}}{4}$

$y_p = e^x\left(\frac{e^x}{4} + \frac{e^{-3x}}{12}\right) + e^{-x}\left(-\frac{e^{3x}}{12} - \frac{e^{-x}}{4}\right) = \frac{e^{2x}}{4} + \frac{e^{-2x}}{12} - \frac{e^{2x}}{12} - \frac{e^{-2x}}{4} = \frac{\sinh 2x}{3}$

$y = c_1 e^x + c_2 e^{-x} + \frac{\sinh 2x}{3}$

9. $y'' - 4y = \frac{e^{2x}}{x}$

Ec. característica: $r^2 - 4 = 0 \Rightarrow (r-2)(r+2) = 0 \Rightarrow r = 2, -2$

$y_c = c_1 e^{2x} + c_2 e^{-2x}$

$y_1 = e^{2x},\ y_2 = e^{-2x}$

$$W = \begin{vmatrix} e^{2x} & e^{-2x} \\ 2e^{2x} & -2e^{-2x} \end{vmatrix} = -2e^{2x} \cdot e^{-2x} - 2e^{2x} \cdot e^{-2x} = -4$$

$$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{e^{2x}}{x} & -2e^{-2x} \end{vmatrix} = -\frac{e^{-2x} \cdot e^{2x}}{x} = -\frac{1}{x}$$

$$W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & \frac{e^{2x}}{x} \end{vmatrix} = \frac{e^{2x} \cdot e^{2x}}{x} = \frac{e^{4x}}{x}$$

$u_1 = \int \frac{-\frac{1}{x}}{-4} dx = \int \frac{1}{4x} dx = \frac{\ln|x|}{4}$

$u_2 = \int \frac{\frac{e^{4x}}{x}}{-4} dx = -\frac{1}{4}\int \frac{e^{4x}}{x} dx$

$y = c_1 e^{2x} + c_2 e^{-2x} + \frac{e^{2x}\ln|x|}{4} - \frac{e^{-2x}}{4}\int \frac{e^{4x}}{x} dx$

10. $y'' - 9y = \frac{9x}{e^{2x}}$

Ec. característica: $r^2 - 9 = 0 \Rightarrow (r-3)(r+3) = 0 \Rightarrow r = 3, -3$

$y_c = c_1 e^{3x} + c_2 e^{-3x}$

$y_1 = e^{3x},\ y_2 = e^{-3x}$

$$W = \begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix} = -3e^{3x} \cdot e^{-3x} - 3e^{3x} \cdot e^{-3x} = -6$$

$$W_1 = \begin{vmatrix} 0 & e^{-3x} \\ 9xe^{-2x} & -3e^{-3x} \end{vmatrix} = -9xe^{-2x} \cdot e^{-3x} = -9xe^{-5x}$$

$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & 9xe^{-2x} \end{vmatrix} = 9xe^{-2x} \cdot e^{3x} = 9xe^{x}$

$u_1 = \int \frac{-9xe^{-5x}}{-6} dx = \frac{3}{2} \int xe^{-5x} dx = \frac{3}{2}\left(-\frac{xe^{-5x}}{5} + \frac{e^{-5x}}{25}\right) = -\frac{3xe^{-5x}}{10} - \frac{3e^{-5x}}{50}$

$u_2 = \int \frac{9xe^{x}}{-6} dx = -\frac{3}{2} \int xe^{x} dx = -\frac{3}{2}(xe^{x} - e^{x}) = -\frac{3xe^{x}}{2} + \frac{3e^{x}}{2}$

$y_p = e^{3x}\left(-\frac{3xe^{-5x}}{10} - \frac{3e^{-5x}}{50}\right) + e^{-3x}\left(-\frac{3xe^{x}}{2} + \frac{3e^{x}}{2}\right)$

$y_p = -\frac{3xe^{-2x}}{10} - \frac{3e^{-2x}}{50} - \frac{3xe^{-2x}}{2} + \frac{3e^{-2x}}{2} = -\frac{8xe^{-2x}}{5} + \frac{37e^{-2x}}{25}$

$y = c_1 e^{3x} + c_2 e^{-3x} - \frac{8xe^{-2x}}{5} + \frac{37e^{-2x}}{25}$

11. $y'' + 3y' + 2y = \frac{1}{1+e^{x}}$

Ec. característica: $r^2 + 3r + 2 = 0 \Rightarrow (r+1)(r+2) = 0 \Rightarrow r = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

$y_1 = e^{-x}$, $y_2 = e^{-2x}$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-x} \cdot e^{-2x} + e^{-x} \cdot e^{-2x} = -e^{-3x}$

$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{1+e^{x}} & -2e^{-2x} \end{vmatrix} = -\frac{e^{-2x}}{1+e^{x}}$

$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{1+e^{x}} \end{vmatrix} = \frac{e^{-x}}{1+e^{x}}$

$u_1 = \int \frac{-\frac{e^{-2x}}{1+e^{x}}}{-e^{-3x}} dx = \int \frac{e^{x}}{1+e^{x}} dx = \ln(1+e^{x})$

$u_2 = \int \frac{\frac{e^{-x}}{1+e^{x}}}{-e^{-3x}} dx = -\int \frac{e^{2x}}{1+e^{x}} dx = -e^{x} + \ln(1+e^{x})$

$y = c_1 e^{-x} + c_2 e^{-2x} + e^{-x}\ln(1+e^{x}) + e^{-2x}(-e^{x} + \ln(1+e^{x}))$

12. $y'' - 2y' + y = \frac{e^{x}}{1+x^2}$

Ec. característica: $r^2 - 2r + 1 = 0 \Rightarrow (r-1)^2 = 0 \Rightarrow r = 1$ (doble)

$y_c = c_1 e^{x} + c_2 xe^{x}$

$y_1 = e^{x}$, $y_2 = xe^{x}$

$W = \begin{vmatrix} e^{x} & xe^{x} \\ e^{x} & e^{x} + xe^{x} \end{vmatrix} = e^{x}(e^{x} + xe^{x}) - xe^{x} \cdot e^{x} = e^{2x}$

$W_1 = \begin{vmatrix} 0 & xe^{x} \\ \frac{e^{x}}{1+x^2} & e^{x} + xe^{x} \end{vmatrix} = -\frac{xe^{x} \cdot e^{x}}{1+x^2} = -\frac{xe^{2x}}{1+x^2}$

$W_2 = \begin{vmatrix} e^{x} & 0 \\ e^{x} & \frac{e^{x}}{1+x^2} \end{vmatrix} = \frac{e^{x} \cdot e^{x}}{1+x^2} = \frac{e^{2x}}{1+x^2}$

$u_1 = \int \frac{-\frac{xe^{2x}}{1+x^2}}{e^{2x}} dx = \int \frac{-x}{1+x^2} dx = -\frac{1}{2}\ln(1+x^2)$

$u_2 = \int \frac{\frac{e^{2x}}{1+x^2}}{e^{2x}} dx = \int \frac{1}{1+x^2} dx = \arctan x$

$y = c_1 e^{x} + c_2 xe^{x} - \frac{e^{x}}{2}\ln(1+x^2) + xe^{x}\arctan x$

13. $y'' + 3y' + 2y = \sec e^{x}$

Ec. característica: $r^2 + 3r + 2 = 0 \Rightarrow (r+1)(r+2) = 0 \Rightarrow r = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

$y_1 = e^{-x}$, $y_2 = e^{-2x}$

$W = -e^{-3x}$

$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \sec e^{x} & -2e^{-2x} \end{vmatrix} = -e^{-2x}\sec e^{x}$

$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \sec e^{x} \end{vmatrix} = e^{-x}\sec e^{x}$

$u_1 = \int \frac{-e^{-2x}\sec e^{x}}{-e^{-3x}} dx = \int e^{x}\sec e^{x} dx = \ln|\sec e^{x} + \tan e^{x}|$

$u_2 = \int \frac{e^{-x}\sec e^{x}}{-e^{-3x}} dx = -\int e^{2x}\sec e^{x} dx$

$y = c_1 e^{-x} + c_2 e^{-2x} + e^{-x}\ln|\sec e^{x} + \tan e^{x}| + e^{-2x}u_2(x)$

14. $y'' - 2y' + y = e^{t}\arctan t$

Ec. característica: $r^2 - 2r + 1 = 0 \Rightarrow (r-1)^2 = 0 \Rightarrow r = 1$ (doble)

$y_c = c_1 e^{t} + c_2 te^{t}$

$y_1 = e^{t}$, $y_2 = te^{t}$

$W = e^{2t}$

$W_1 = \begin{vmatrix} 0 & te^{t} \\ e^{t}\arctan t & e^{t} + te^{t} \end{vmatrix} = -te^{t} \cdot e^{t}\arctan t = -te^{2t}\arctan t$

$W_2 = \begin{vmatrix} e^{t} & 0 \\ e^{t} & e^{t}\arctan t \end{vmatrix} = e^{t} \cdot e^{t}\arctan t = e^{2t}\arctan t$

$u_1 = \int \frac{-te^{2t}\arctan t}{e^{2t}} dt = -\int t\arctan t\, dt = -\frac{t^2\arctan t}{2} + \frac{t}{2} - \frac{\arctan t}{2}$

$u_2 = \int \frac{e^{2t}\arctan t}{e^{2t}} dt = \int \arctan t\, dt = t\arctan t - \frac{1}{2}\ln(1+t^2)$

$y = c_1 e^{t} + c_2 te^{t} + e^{t}\left(-\frac{t^2\arctan t}{2} + \frac{t}{2} - \frac{\arctan t}{2}\right) + te^{t}\left(t\arctan t - \frac{1}{2}\ln(1+t^2)\right)$

15. $y'' + 2y' + y = e^{-t}\ln t$

Ec. característica: $r^2 + 2r + 1 = 0 \Rightarrow (r+1)^2 = 0 \Rightarrow r = -1$ (doble)

$y_c = c_1 e^{-t} + c_2 te^{-t}$

$y_1 = e^{-t}$, $y_2 = te^{-t}$

$W = e^{-2t}$

$W_1 = \begin{vmatrix} 0 & te^{-t} \\ e^{-t}\ln t & e^{-t} - te^{-t} \end{vmatrix} = -te^{-t} \cdot e^{-t}\ln t = -te^{-2t}\ln t$

$W_2 = \begin{vmatrix} e^{-t} & 0 \\ -e^{-t} & e^{-t}\ln t \end{vmatrix} = e^{-t} \cdot e^{-t}\ln t = e^{-2t}\ln t$

$u_1 = \int \frac{-te^{-2t}\ln t}{e^{-2t}} dt = -\int t\ln t\, dt = -\frac{t^2\ln t}{2} + \frac{t^2}{4}$

$u_2 = \int \frac{e^{-2t}\ln t}{e^{-2t}} dt = \int \ln t\, dt = t\ln t - t$

$y = c_1 e^{-t} + c_2 te^{-t} + e^{-t}\left(-\frac{t^2\ln t}{2} + \frac{t^2}{4}\right) + te^{-t}(t\ln t - t)$

$y = c_1 e^{-t} + c_2 te^{-t} + \frac{t^2 e^{-t}}{2}\ln t + \frac{t^2 e^{-t}}{4} - t^2 e^{-t}$

16. $2y'' + 2y' + y = \frac{4}{\sqrt{x}}$

Ec. característica: $2r^2 + 2r + 1 = 0 \Rightarrow r = \frac{-2\pm\sqrt{4-8}}{4} = \frac{-2\pm 2i}{4} = \frac{-1\pm i}{2}$

$y_c = e^{-x/2}\left(c_1\cos\frac{x}{2} + c_2\sin\frac{x}{2}\right)$

$y_1 = e^{-x/2}\cos\frac{x}{2}$, $y_2 = e^{-x/2}\sin\frac{x}{2}$

$y_1' = e^{-x/2}\left(-\frac{1}{2}\cos\frac{x}{2} - \frac{1}{2}\sin\frac{x}{2}\right)$

$y_2' = e^{-x/2}\left(-\frac{1}{2}\sin\frac{x}{2} + \frac{1}{2}\cos\frac{x}{2}\right)$

$W = y_1y_2' - y_2y_1' = e^{-x}\left(\cos\frac{x}{2}\left(-\frac{1}{2}\sin\frac{x}{2} + \frac{1}{2}\cos\frac{x}{2}\right) - \sin\frac{x}{2}\left(-\frac{1}{2}\cos\frac{x}{2} - \frac{1}{2}\right.\right.$

$\frac{e^{-x}}{2}$

$W_1 = \begin{vmatrix} 0 & e^{-x/2}\sin\frac{x}{2} \\ \frac{2}{\sqrt{x}} & e^{-x/2}(-\frac{1}{2}\sin\frac{x}{2} + \frac{1}{2}\cos\frac{x}{2}) \end{vmatrix} = -\frac{2e^{-x/2}\sin\frac{x}{2}}{\sqrt{x}}$

$W_2 = \begin{vmatrix} e^{-x/2}\cos\frac{x}{2} & 0 \\ e^{-x/2}(-\frac{1}{2}\cos\frac{x}{2} - \frac{1}{2}\sin\frac{x}{2}) & \frac{2}{\sqrt{x}} \end{vmatrix} = \frac{2e^{-x/2}\cos\frac{x}{2}}{\sqrt{x}}$

$u_1 = \int \frac{-\frac{2e^{-x/2}\sin\frac{x}{2}}{\sqrt{x}}}{\frac{e^{-x}}{2}} dx = -\int \frac{4e^{x/2}\sin\frac{x}{2}}{\sqrt{x}} dx$

$u_2 = \int \frac{\frac{2e^{-x/2}\cos\frac{x}{2}}{\sqrt{x}}}{\frac{e^{-x}}{2}} dx = \int \frac{4e^{x/2}\cos\frac{x}{2}}{\sqrt{x}} dx$

$y = c_1 e^{-x/2}\cos\frac{x}{2} + c_2 e^{-x/2}\sin\frac{x}{2} + y_p$

17. $3y'' - 6y' + 6y = e^{x}\sec x$

Ec. característica: $3r^2 - 6r + 6 = 0 \Rightarrow r = \frac{6\pm\sqrt{36-72}}{6} = \frac{6\pm 6i}{6} = 1 \pm i$

$y_c = e^{x}(c_1\cos x + c_2\sin x)$

$y_1 = e^{x}\cos x$, $y_2 = e^{x}\sin x$

$y_1' = e^{x}(\cos x - \sin x)$, $y_2' = e^{x}(\sin x + \cos x)$

$W = e^{x}\cos x \cdot e^{x}(\sin x + \cos x) - e^{x}\sin x \cdot e^{x}(\cos x - \sin x) = e^{2x}(\cos x\sin x + \cos^2 x - \sin x\cos x + \sin^2 x) = e^{2x}$

$W_1 = \begin{vmatrix} 0 & e^{x}\sin x \\ \frac{e^{x}\sec x}{3} & e^{x}(\sin x + \cos x) \end{vmatrix} = -\frac{e^{x}\sin x \cdot e^{x}\sec x}{3} = -\frac{e^{2x}\tan x}{3}$

$W_2 = \begin{vmatrix} e^{x}\cos x & 0 \\ e^{x}(\cos x - \sin x) & \frac{e^{x}\sec x}{3} \end{vmatrix} = \frac{e^{x}\cos x \cdot e^{x}\sec x}{3} = \frac{e^{2x}}{3}$

$u_1 = \int \frac{-\frac{e^{2x}\tan x}{3}}{e^{2x}} dx = -\frac{1}{3}\int \tan x\, dx = \frac{\ln|\cos x|}{3}$

$u_2 = \int \frac{\frac{e^{2x}}{3}}{e^{2x}} dx = \frac{1}{3}\int dx = \frac{x}{3}$

$y = c_1 e^{x}\cos x + c_2 e^{x}\sin x + \frac{e^{x}\cos x}{3}\ln|\cos x| + \frac{xe^{x}\sin x}{3}$

18. $4y'' - 4y' + y = \frac{e^{x/2}}{\sqrt{1-x^2}}$

Ec. característica: $4r^2 - 4r + 1 = 0 \Rightarrow (2r-1)^2 = 0 \Rightarrow r = \frac{1}{2}$ (doble)

$y_c = c_1 e^{x/2} + c_2 xe^{x/2}$

$y_1 = e^{x/2}$, $y_2 = xe^{x/2}$

$W = \begin{vmatrix} e^{x/2} & xe^{x/2} \\ \frac{1}{2}e^{x/2} & e^{x/2} + \frac{x}{2}e^{x/2} \end{vmatrix} = e^{x/2}\left(e^{x/2} + \frac{x}{2}e^{x/2}\right) - xe^{x/2} \cdot \frac{1}{2}e^{x/2} = e^{x}$

$W_1 = \begin{vmatrix} 0 & xe^{x/2} \\ \frac{e^{x/2}}{\sqrt{1-x^2}} & e^{x/2} + \frac{x}{2}e^{x/2} \end{vmatrix} = -\frac{xe^{x/2} \cdot e^{x/2}}{\sqrt{1-x^2}} = -\frac{xe^{x}}{\sqrt{1-x^2}}$

$W_2 = \begin{vmatrix} e^{x/2} & 0 \\ \frac{1}{2}e^{x/2} & \frac{e^{x/2}}{\sqrt{1-x^2}} \end{vmatrix} = \frac{e^{x/2} \cdot e^{x/2}}{\sqrt{1-x^2}} = \frac{e^{x}}{\sqrt{1-x^2}}$

$u_1 = \int \frac{-\frac{xe^{x}}{\sqrt{1-x^2}}}{e^{x}} dx = -\int \frac{x}{\sqrt{1-x^2}} dx = \sqrt{1-x^2}$

$u_2 = \int \frac{\frac{e^{x}}{\sqrt{1-x^2}}}{e^{x}} dx = \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x$

$y = c_1 e^{x/2} + c_2 xe^{x/2} + e^{x/2}\sqrt{1-x^2} + xe^{x/2}\arcsin x$