



TECNOLÓGICO NACIONAL DE MÉXICO



## Proyecto Final: Gemelo Digital

Departamento de Ingeniería Eléctrica y Electrónica, Ingeniería Biomédica

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B.C., México

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### Información general



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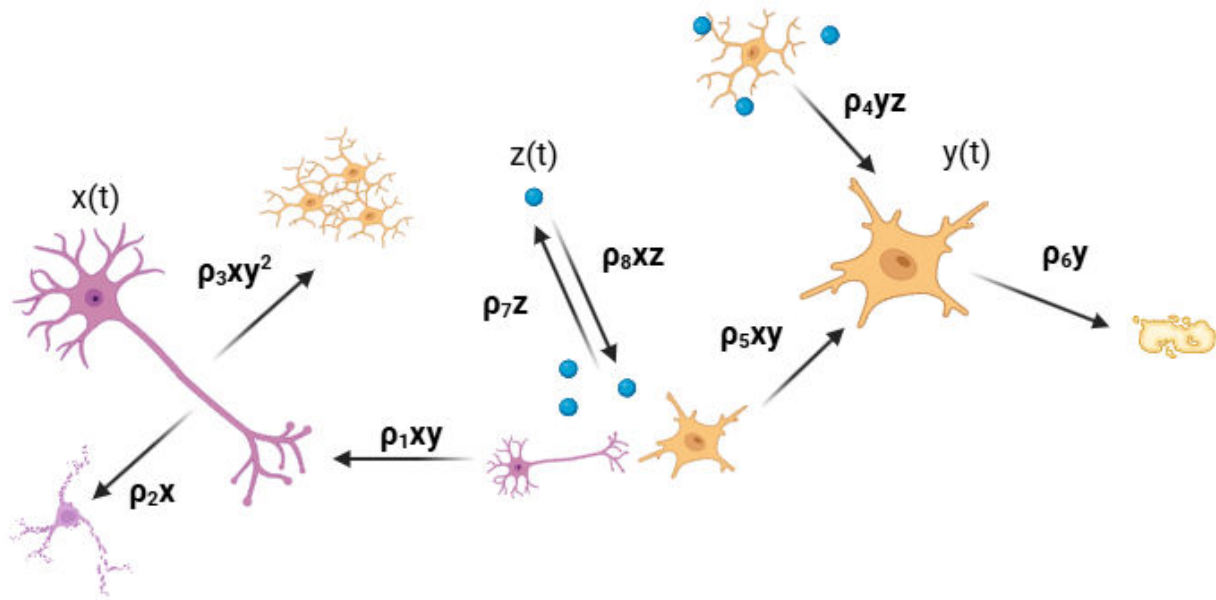
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## Data interpretations

[Figura representativa del sistema]



Created in BioRender.com bio

Figure. Glia ( $y(t)$ ) interaction with neurons ( $x(t)$ ) generates neurotransmitters ( $z(t)$ ) which are oftentimes eliminated by processes dependent on the same neural activity. Whereas both neurons and glia may suffer from fatigue and natural decay. Created in <https://BioRender.com>

$$\dot{x} = \rho_1xy - \rho_2x - \rho_3xy^2$$

The first equation describes the activity of a neuron, based on it's interaction with a glia ( $\rho_1xy$ ) which are subject to fatigue ( $-\rho_2x$ ), and how this one is inhibited more strongly when there's a higher level of activation ( $-\rho_3xy^2$ ).

$$\dot{y} = \rho_4yz + \rho_5xy - \rho_6y$$

The second equation models glia activation, showing how it's activity increases in response to the neurotransmitter concentration ( $\rho_4yz$ ) as well as it's neural activity ( $\rho_5xy$ ) while it presents a level of natural decay over time ( $-\rho_6y$ ).

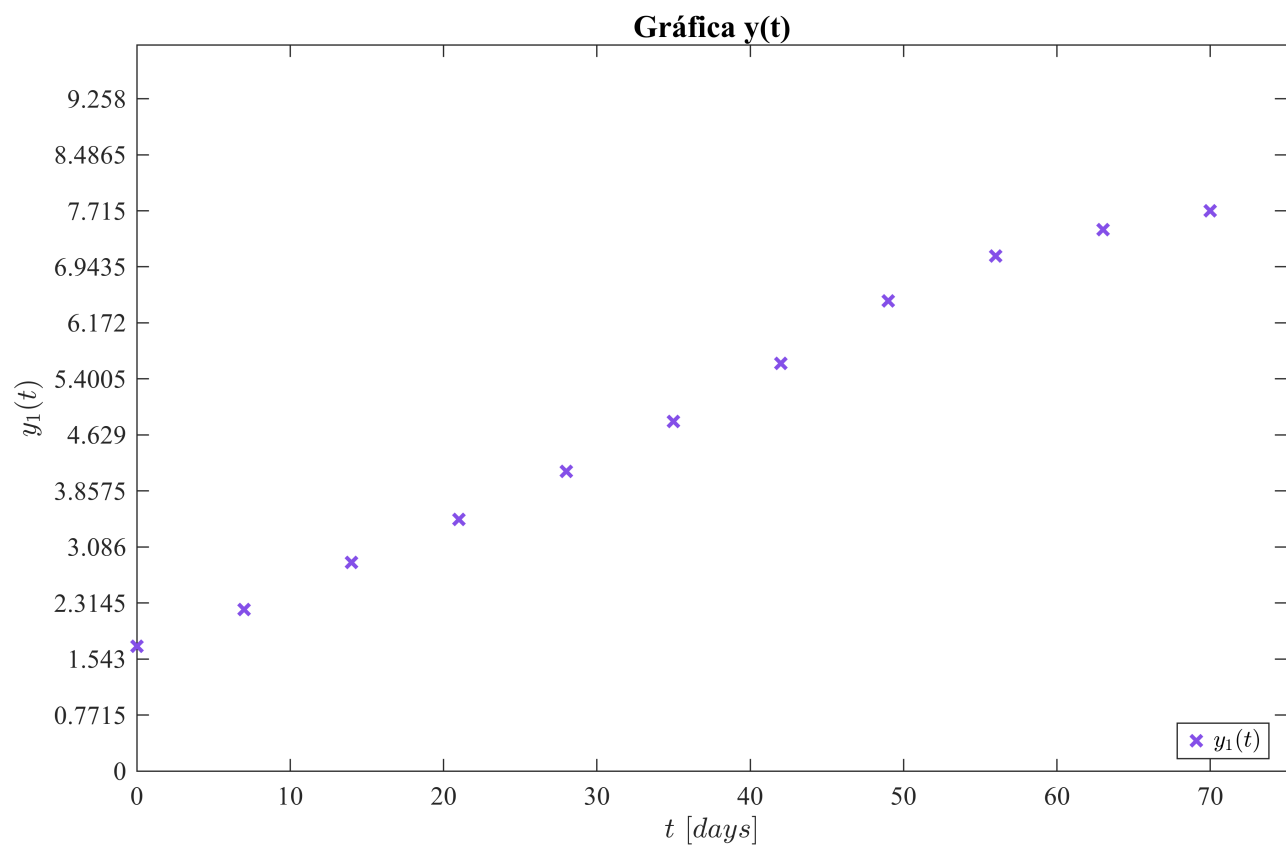
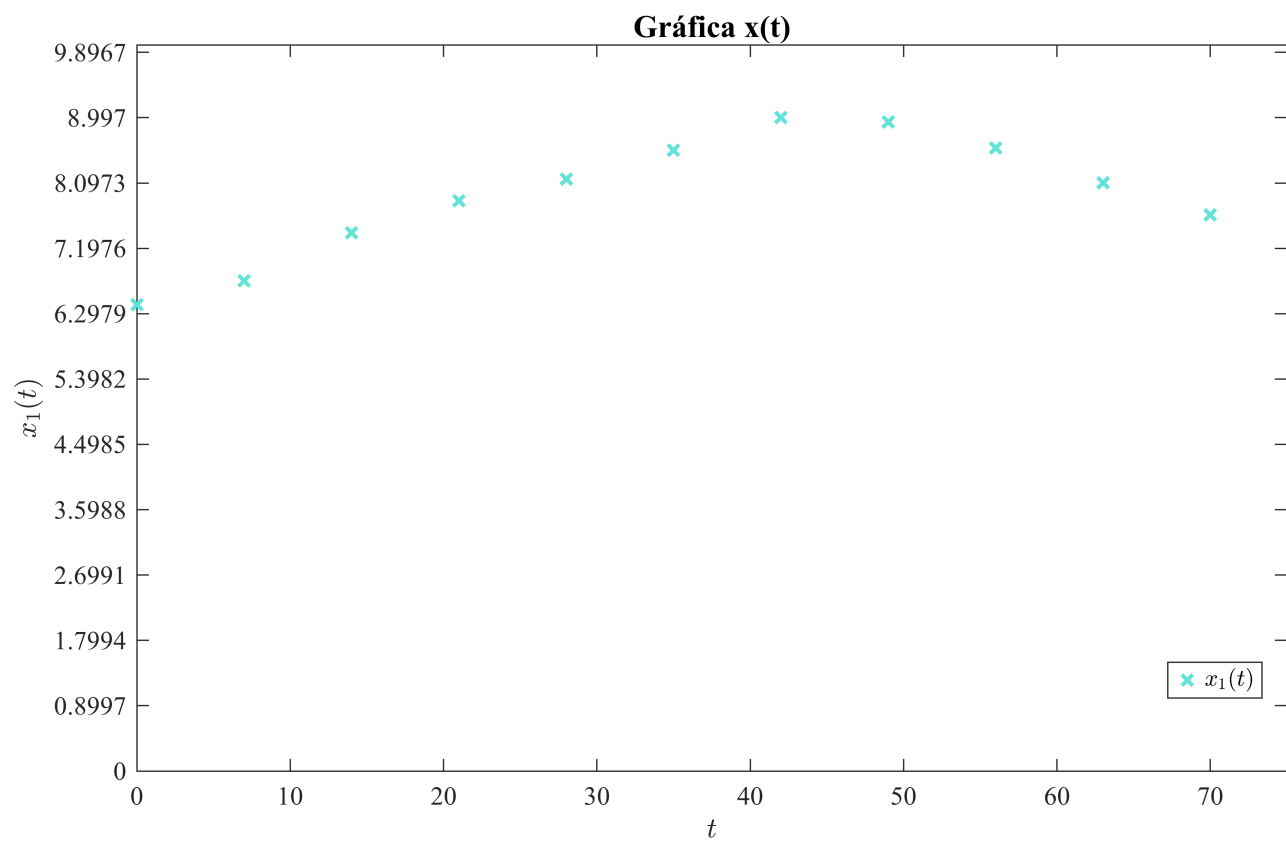
$$\dot{z} = \rho_7z - \rho_8xz$$

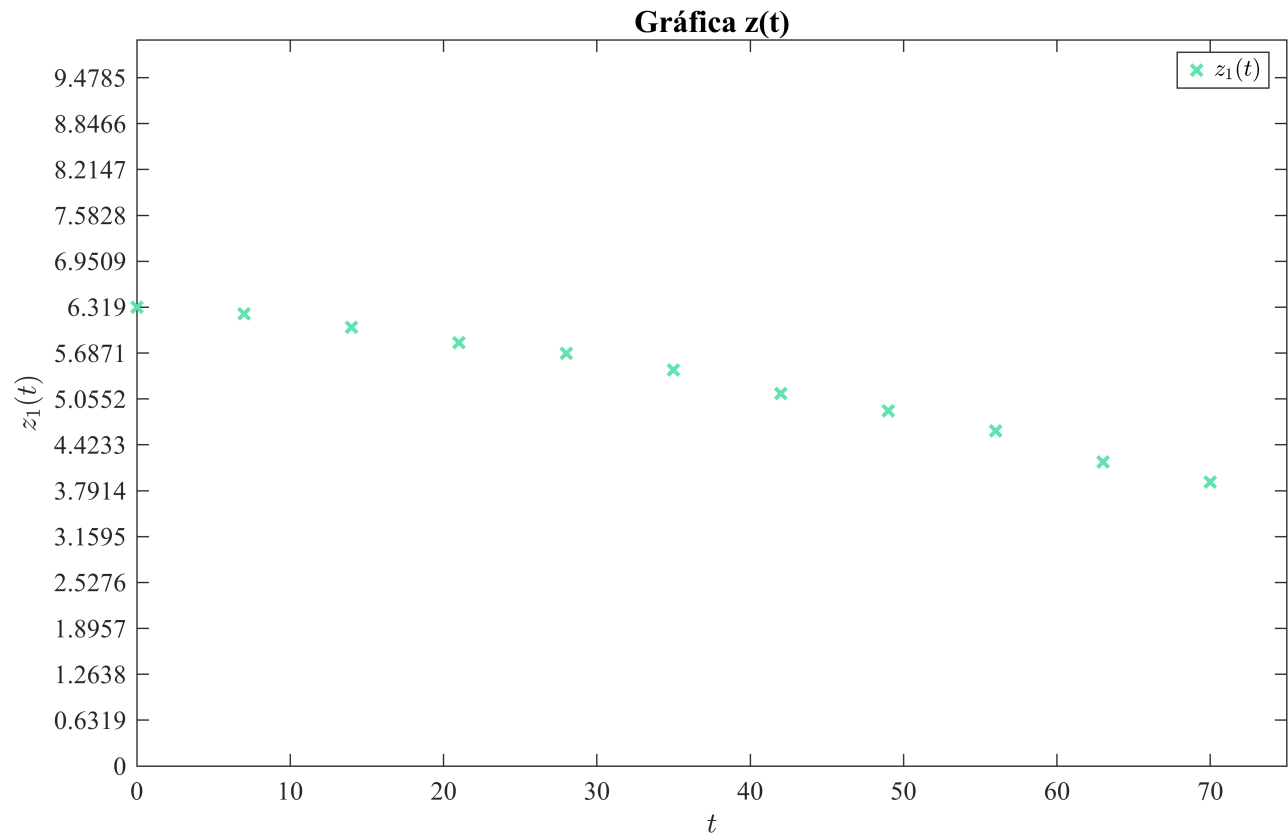
The third equation characterizes the neurotransmitter dynamics, how it can acumulate or undergo basal recycle ( $\rho_{7z}$ ) which is oftentimes eliminated by a multitude of processes dependent on neural activity ( $-\rho_{8xz}$ ) making it fall down as long as neural activity persists.

## Simulation data

```
clc; clear; close all; warning('off','all')
sys = readmatrix('data.csv');
to = sys(:,1);
x = sys(:,2);
y = sys(:,3);
z = sys(:,4);
T1 = array2table([to,x,y,z], 'VariableNames', {'Tiempo', 'x(t)', 'y(t)', 'z(t)'});
disp(T1); plotdata(to,x, y,z);
```

Tiempo	x(t)	y(t)	z(t)
0	6.423	1.72	6.319
7	6.752	2.224	6.228
14	7.414	2.873	6.043
21	7.852	3.467	5.832
28	8.15	4.126	5.682
35	8.548	4.815	5.456
42	8.997	5.615	5.13
49	8.94	6.476	4.892
56	8.581	7.091	4.615
63	8.099	7.457	4.188
70	7.661	7.715	3.911





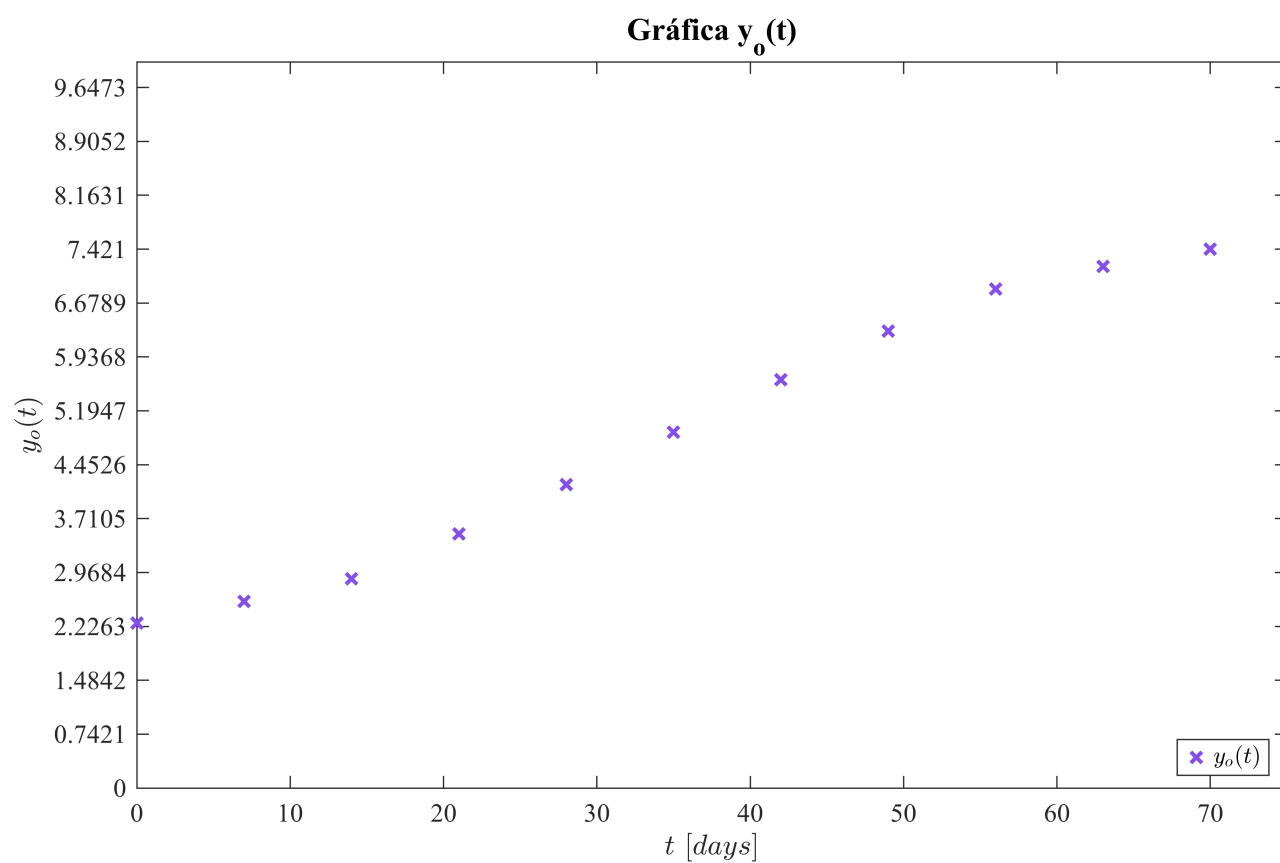
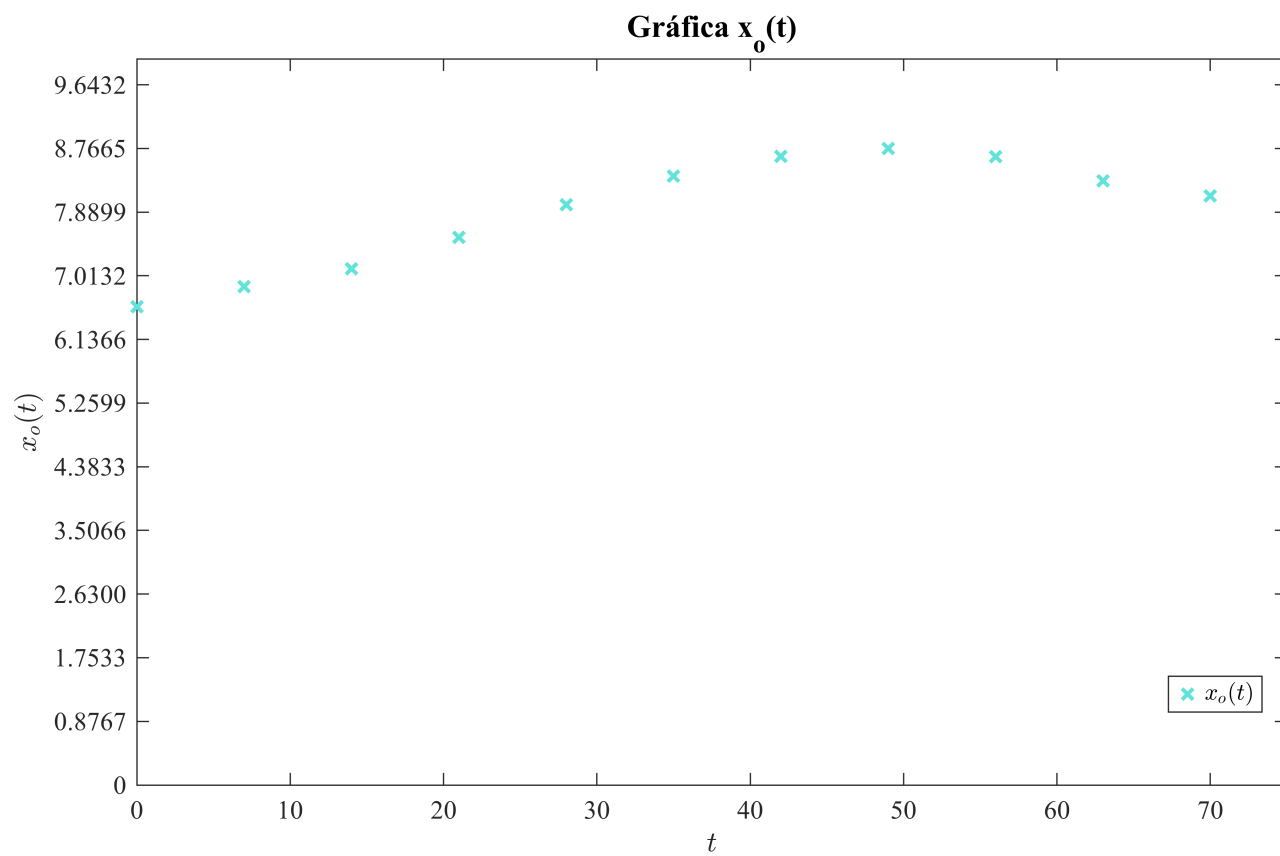
## Smooth data

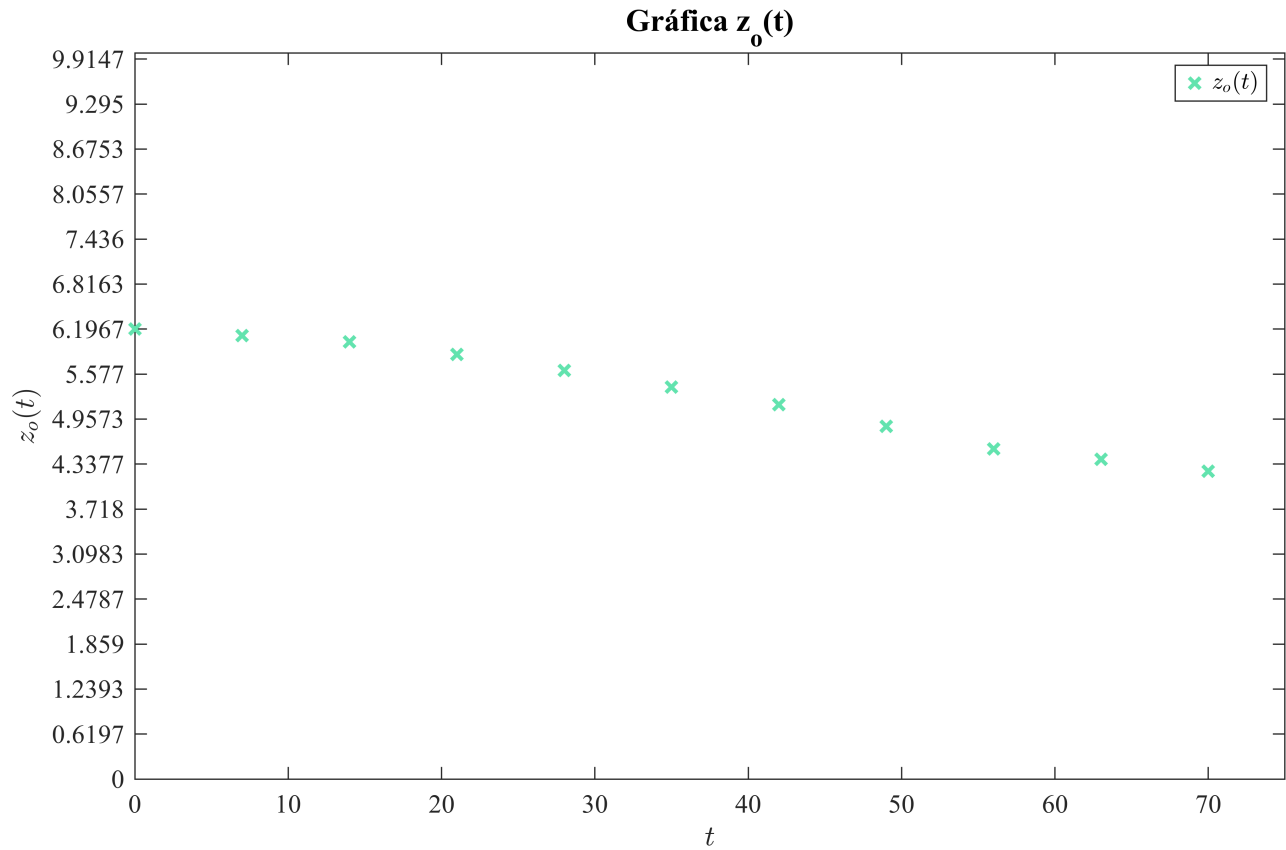
```

xo = smoothdata(x);
yo = smoothdata(y);
zo = smoothdata(z);
T2 = table(to, xo, yo, zo, 'VariableNames', {'Tiempo', 'x_smooth', 'y_smooth',
'z_smooth'});
writetable(T2, 'data_smooth.csv');
disp(T2); plotEDOSfit(to,xo,yo,zo);

```

Tiempo	x_smooth	y_smooth	z_smooth
0	6.5875	2.2723	6.1967
7	6.863	2.571	6.1055
14	7.1102	2.882	6.0208
21	7.542	3.501	5.8482
28	7.991	4.1792	5.6286
35	8.3867	4.8998	5.3984
42	8.6587	5.6246	5.155
49	8.7665	6.2908	4.8562
56	8.6542	6.8708	4.5472
63	8.3202	7.1848	4.4015
70	8.1137	7.421	4.238





## Nonlinear Algorithms

$$\dot{x} = \rho_1 xy - \rho_2 x - \rho_3 xy^2$$

$$\dot{y} = \rho_4 yz + \rho_5 xy - \rho_6 y$$

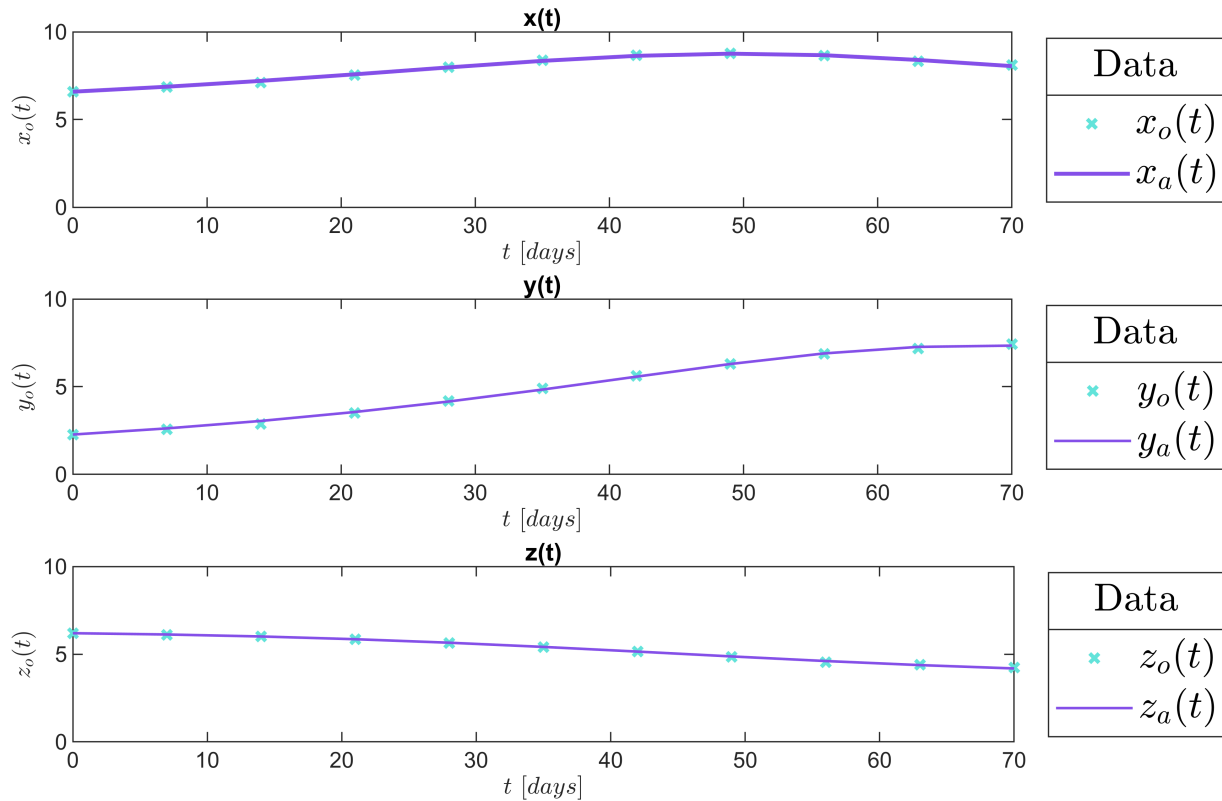
$$\dot{z} = \rho_7 z - \rho_8 xz$$

```
P0 = [0.00696, 0.005197, 0.000965, 0.0165, 0.008033, 0.13523, 0.019183, 0.003106];
[mdl,xa,ya,za] = Varied(to,xo,yo,zo,P0); plotreqs(to,[xo,xa],[yo,ya],[zo,za])
```

Sample size (n): 11  
Parameters to be estimated (pars): 8  
Degrees of freedom: 25  
Significance level (alpha): 0.05  
t-Student value: 2.0595  
R-squared: 0.99919  
Corrected AIC (n/pars < 40): -80.4396

Parameters	Estimate	SE	MoE	CI95		pvalue
p1	0.0069597	0.0010741	0.0022121	0.0047477	0.0091718	8.7138e-07
p2	0.0051957	0.0023098	0.0047572	0.00043851	0.0099528	0.033541
p3	0.00096592	0.00011209	0.00023086	0.00073506	0.0011968	5.9035e-09
p4	0.0165	0.0011757	0.0024215	0.014079	0.018921	2.3369e-13
p5	0.0080334	0.0010499	0.0021623	0.0058711	0.010196	5.2389e-08
p6	0.13523	0.014162	0.029166	0.10606	0.1644	8.0683e-10

p7	0.019184	0.002447	0.0050396	0.014144	0.024223	3.3917e-08
p8	0.003106	0.00031612	0.00065105	0.002455	0.0037571	4.5676e-10



## Equilibrium Points and Jacobian matrix

```
clear; close all; clc;
syms x y z rho1 rho2 rho3 rho4 rho5 rho6 rho7 rho8
dx = rho1*x*y - rho2*x - rho3*x*y^2;
dy = rho4*y*z + rho5*x*y - rho6*y;
dz = rho7*z - rho8*x*z;
J = jacobian([dx,dy,dz],[x,y,z]);
fprintf('Jacobian matrix of the Lotka-Volterra system:'); disp(J)
```

Jacobian matrix of the Lotka-Volterra system:

$$\begin{pmatrix} -\rho_3 y^2 + \rho_1 y - \rho_2 & \rho_1 x - 2\rho_3 x y & 0 \\ \rho_5 y & \rho_5 x - \rho_6 + \rho_4 z & \rho_4 y \\ -\rho_8 z & 0 & \rho_7 - \rho_8 x \end{pmatrix}$$

```
dx = rho1*x*y - rho2*x - rho3*x*y^2 == 0;
dy = rho4*y*z + rho5*x*y - rho6*y == 0;
dz = rho7*z - rho8*x*z == 0;
edos = solve([dx,dy,dz],[x,y,z]);
fprintf(['The Lotka-Volterra system has ', num2str(length(edos.x)), ' equilibrium points.'])
```



The Lotka-Volterra system has 5 equilibrium points.

```
X0 = edos.x(1); Y0 = edos.y(1); Z0 = edos.z(1);
X1 = edos.x(2); Y1 = edos.y(2); Z1 = edos.z(2);
X2 = edos.x(3); Y2 = edos.y(3); Z2 = edos.z(3);
X3 = edos.x(4); Y3 = edos.y(4); Z3 = edos.z(4);
X4 = edos.x(5); Y4 = edos.y(5); Z4 = edos.z(5);
syms x0 y0 z0 x1 y1 z1 x2 y2 z2 x3 y3 z3 x4 y4 z4
fprintf('Equilibrium points of the Lotka-Volterra system:');
disp([x0,y0,z0,X0,Y0,Z0]); disp([x1,y1,z1,X1,Y1,Z1]); disp([x2,y2,z2,X2,Y2,Z2]);
disp([x3,y3,z3,X3,Y3,Z3]); disp([x4,y4,z4,X4,Y4,Z4]);
```

Equilibrium points of the Lotka-Volterra system:

$(x_0 \ y_0 \ z_0 \ 0 \ 0 \ 0)$

$$\left( x_1 \ y_1 \ z_1 \ \frac{\rho_7}{\rho_8} \ \frac{\rho_1 + \sqrt{\rho_1^2 - 4\rho_2\rho_3}}{2\rho_3} \ -\frac{\rho_5\rho_7 - \rho_6\rho_8}{\rho_4\rho_8} \right)$$

$$\left( x_2 \ y_2 \ z_2 \ \frac{\rho_6}{\rho_5} \ \frac{\rho_1 + \sqrt{\rho_1^2 - 4\rho_2\rho_3}}{2\rho_3} \ 0 \right)$$

$$\left( x_3 \ y_3 \ z_3 \ \frac{\rho_7}{\rho_8} \ \frac{\rho_1 - \sqrt{\rho_1^2 - 4\rho_2\rho_3}}{2\rho_3} \ -\frac{\rho_5\rho_7 - \rho_6\rho_8}{\rho_4\rho_8} \right)$$

$$\left( x_4 \ y_4 \ z_4 \ \frac{\rho_6}{\rho_5} \ \frac{\rho_1 - \sqrt{\rho_1^2 - 4\rho_2\rho_3}}{2\rho_3} \ 0 \right)$$

```
clear rho1 rho2 rho3 rho4 rho5 rho6 rho7 rho8
%p = [0.00696, 0.005197, 0.000965, 0.0165, 0.008033, 0.13523, 0.019183, 0.003106];
rho1 = 0.00696; rho2 = 0.005197; rho3 = 0.000965; rho4 = 0.0165;
rho5 = 0.008033; rho6 = 0.13523; rho7 = 0.019183; rho8 = 0.003106;
eq1 = '(x0,y0,z0) = (0,0,0)';
eq2 = ['(x1,y1,z1) = (' num2str(rho7/rho8) ', ' num2str(rho1 + sqrt(rho1^2 -
4*rho2*rho3)/(2*rho3)) ', ' num2str(-(rho5*rho7 - rho6*rho8)/(rho4*rho8)) ')'];
eq3 = ['(x2,y2,z2) = (' num2str(rho6/rho5) ', ' num2str(rho1 + sqrt(rho1^2 -
4*rho2*rho3)/(2*rho3)) ', '0)'];
eq4 = ['(x3,y3,z3) = (' num2str(rho7/rho8) ', ' num2str(rho1 - sqrt(rho1^2 -
4*rho2*rho3)/(2*rho3)) ', ' num2str(-(rho5*rho7 - rho6*rho8)/(rho4*rho8)) ')'];
eq5 = ['(x4,y4,z4) = (' num2str(rho6/rho5) ', ' num2str(rho1 - sqrt(rho1^2 -
4*rho2*rho3)/(2*rho3)) ', '0)'];
disp(eq1); disp(eq2); disp(eq3); disp(eq4); disp(eq5)
```

```
(x0,y0,z0) = (0,0,0)
(x1,y1,z1) = (6.1761,2.7673,5.1889)
(x2,y2,z2) = (16.8343,2.7673,0)
(x3,y3,z3) = (6.1761,-2.7534,5.1889)
(x4,y4,z4) = (16.8343,-2.7534,0)
```

## Local stability

```

clear; close all; clc;
syms x y z
rho1 = 0.00696; rho2 = 0.005197; rho3 = 0.000965; rho4 = 0.0165;
rho5 = 0.008033; rho6 = 0.13523; rho7 = 0.019183; rho8 = 0.003106;
dx = rho1*x*y - rho2*x - rho3*x*y^2 == 0;
dy = rho4*y*z + rho5*x*y - rho6*y == 0;
dz = rho7*z - rho8*x*z == 0;
edos = solve([dx,dy,dz],[x,y,z]);
%fprintf(['The Lotka-Volterra system has ',num2str(length(edos.x)), ' equilibrium
points.'])
x0 = double(edos.x(1)); y0 = double(edos.y(1)); z0 = double(edos.z(1));
x1 = double(edos.x(2)); y1 = double(edos.y(2)); z1 = double(edos.z(2));
x2 = double(edos.x(3)); y2 = double(edos.y(3)); z2 = double(edos.z(3));
x3 = double(edos.x(4)); y3 = double(edos.y(4)); z3 = double(edos.z(4));
x4 = double(edos.x(5)); y4 = double(edos.y(5)); z4 = double(edos.z(5));
clear x y z
x = [x0; x1; x2; x3; x4]; y = [y0; y1; y2; y3; y4]; z = [z0; z1; z2; z3; z4];
var = {'(x0,y0,z0)'; '(x1,y1,z1)'; '(x2,y2,z2)'; '(x3,y3,z3)'; '(x4,y4,z4)'};
Equilibria = table(x,y,z,'RowNames',var);
Equilibria.Properties.VariableNames = {'xe','ye','ze'};
fprintf('Equilibrium points of the Lotka-Volterra system:\n'); disp(Equilibria)

```

Equilibrium points of the Lotka-Volterra system:

	xe	ye	ze
(x0,y0,z0)	0	0	0
(x1,y1,z1)	6.1761	0.84591	5.1889
(x2,y2,z2)	6.1761	6.3665	5.1889
(x3,y3,z3)	16.834	0.84591	0
(x4,y4,z4)	16.834	6.3665	0

```

L = zeros(length(x),3);
for i = 1:length(x)
    J = [- rho3*y(i)^2 + rho1*y(i) - rho2, rho1*x(i) - 2*rho3*x(i)*y(i), 0;
        rho5*y(i), rho5*x(i) - rho6 + rho4*z(i), rho4*y(i);
        -rho8*z(i), 0, rho7 - rho8*x(i)];
    L(i,:) = double(eig(J));
end
L1 = L(:,1); L2 = L(:,2); L3 = L(:,3);
var = {'(x0,y0,z0)'; '(x1,y1,z1)'; '(x2,y2,z2)'; '(x3,y3,z3)'; '(x4,y4,z4)'};
Lambdas = table(L1,L2,L3,'RowNames',var);
disp('Eigenvalues of the Jacobian matrix evaluated at each equilibrium point:');
disp(Lambdas)

```

Eigenvalues of the Jacobian matrix evaluated at each equilibrium point:

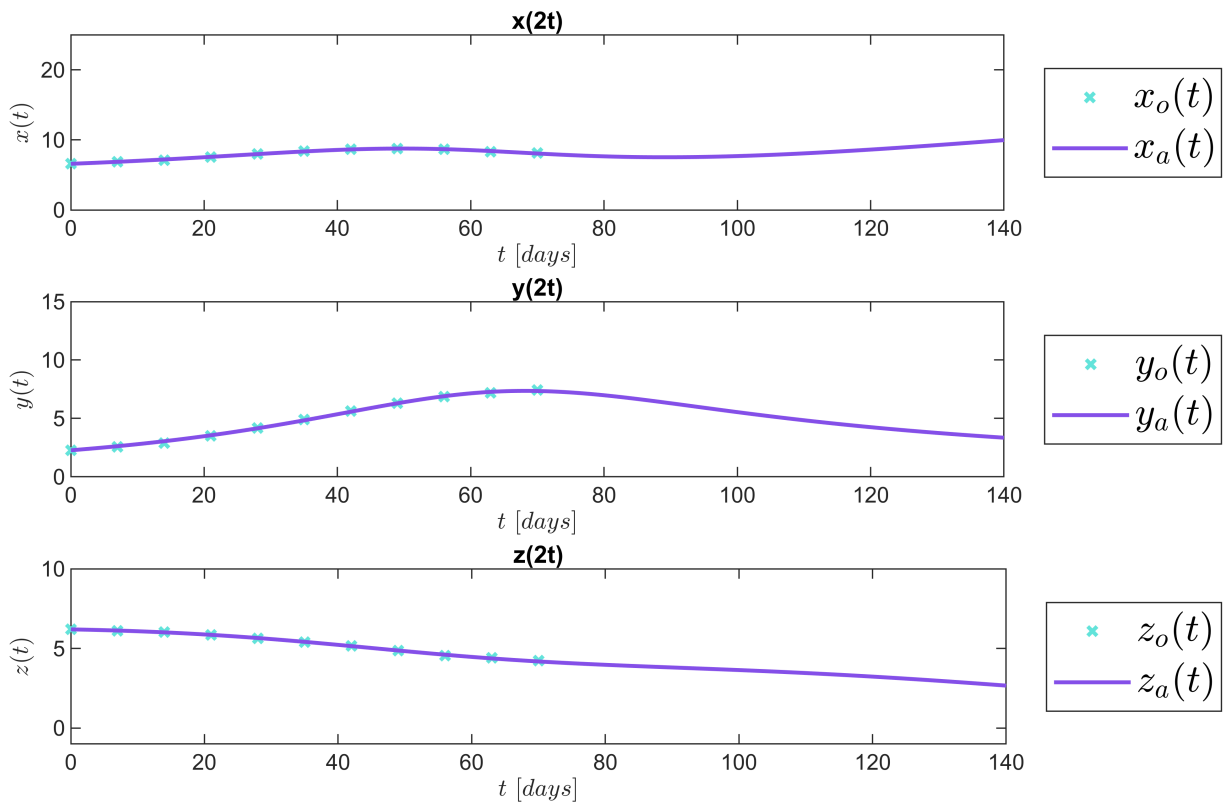
	L1	L2	L3
(x0,y0,z0)	-0.13523+0i	-0.005197+0i	0.019183+0i
(x1,y1,z1)	-0.023273+0i	0.011636+0.013514i	0.011636-0.013514i
(x2,y2,z2)	-0.012217+0.046157i	-0.012217-0.046157i	0.024435+0i
(x3,y3,z3)	0.024686+0i	-0.024686+0i	-0.033104+0i
(x4,y4,z4)	0+0.067724i	0-0.067724i	-0.033104+0i

## Algorithm in 2t

```
clc; clear; close all;
sys = readmatrix('data_smooth.csv');
to = sys(:,1);
xo = sys(:,2);
yo = sys(:,3);
zo = sys(:,4);
T2 = array2table([to,xo,yo,zo], 'VariableNames', {'Tiempo', 'xo(t)', 'yo(t)',
'zo(t)'});
disp(T2);
```

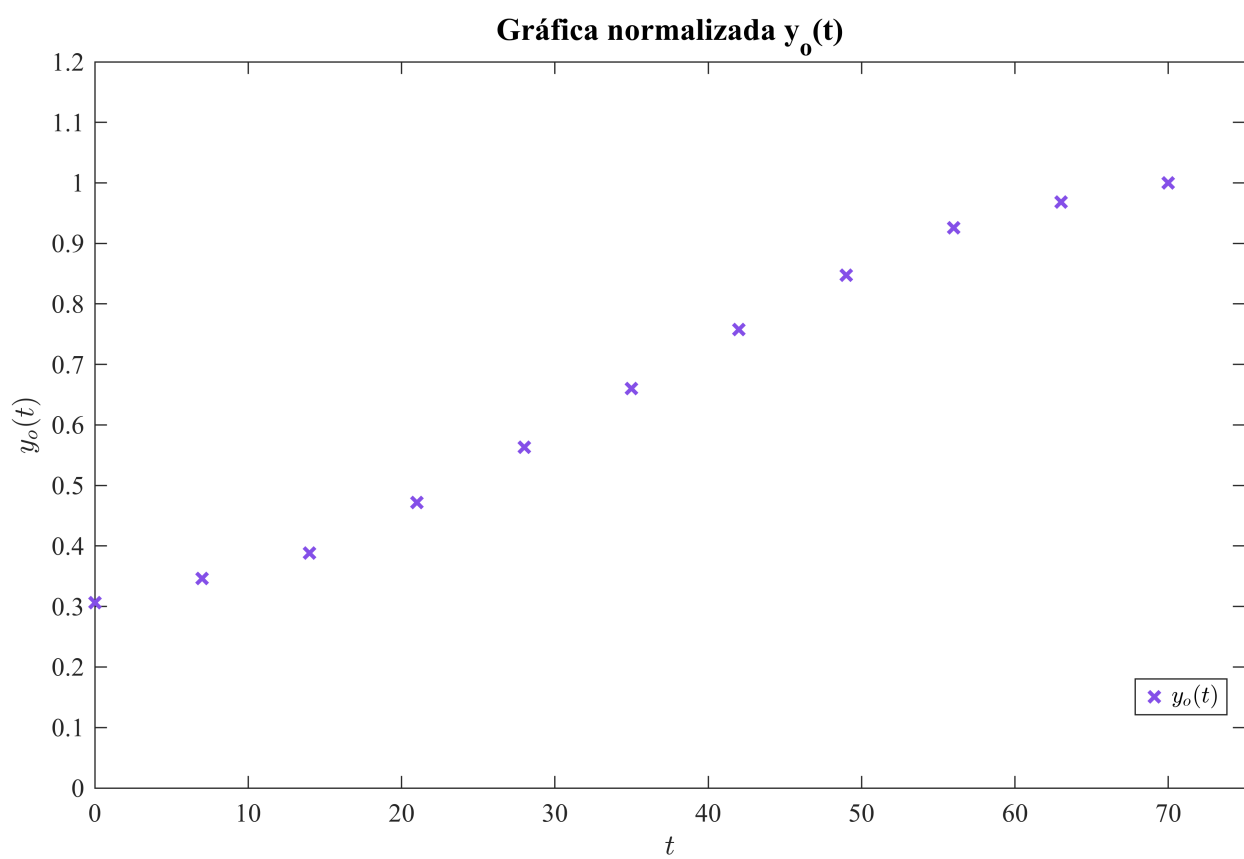
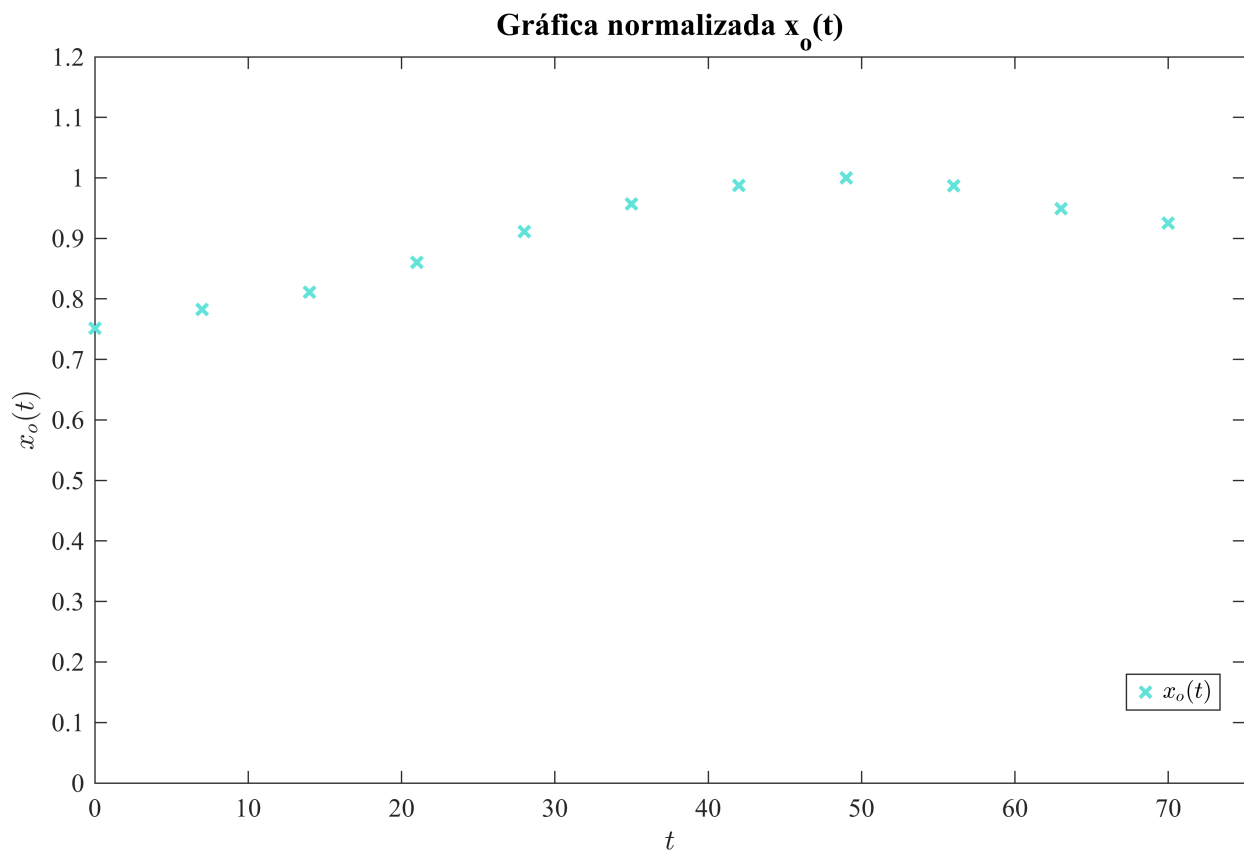
Tiempo	xo(t)	yo(t)	zo(t)
0	6.5875	2.2723	6.1967
7	6.863	2.571	6.1055
14	7.1102	2.882	6.0208
21	7.542	3.501	5.8482
28	7.991	4.1792	5.6286
35	8.3867	4.8998	5.3984
42	8.6587	5.6246	5.155
49	8.7665	6.2908	4.8562
56	8.6542	6.8708	4.5472
63	8.3202	7.1848	4.4015
70	8.1137	7.421	4.238

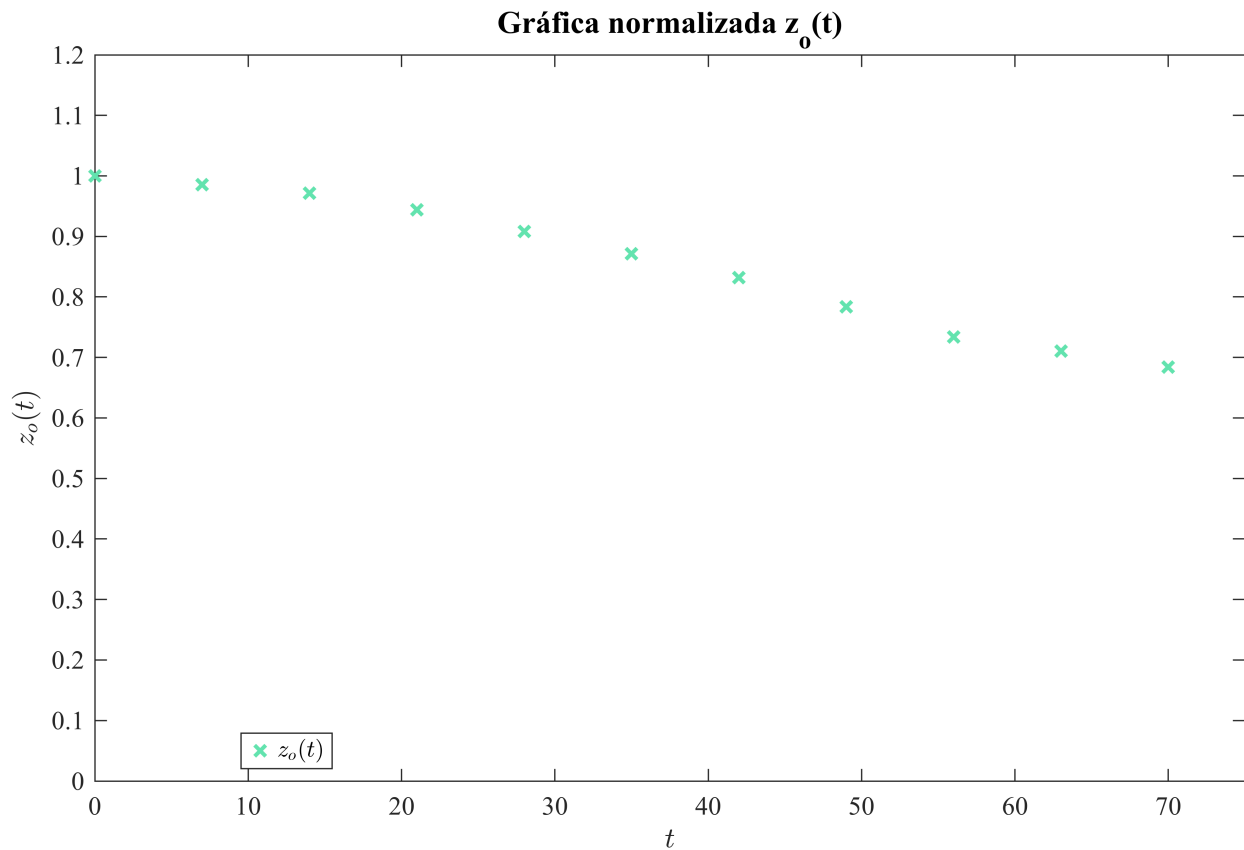
```
tend = 2*max(to); dt = 1E-2;
p = [0.00696, 0.005197, 0.000965, 0.0165, 0.008033, 0.13523, 0.019183, 0.003106];
[t,x,y,z] = Variante(xo(1),yo(1),zo(1),dt,tend,p); plotgreys(to, xo, yo, zo, t, x,
y, z)
```



## Normalized plot

```
plotnorm(to,xo,yo,zo)
```





## Conclusions

This project has allowed us to explore the behaviour of a set of biological variables and analyze their evolution over time. Through this interpretation, we designed a model that can simulate its dynamics in a way while also determining the statistical significance and local stability of the model. Overall the model captured a system where there exists oscillations inside of variables  $x(t)$  and  $y(t)$  meanwhile  $z(t)$  shows a convergence to 0 where  $t \rightarrow \infty$ .

## Functions

### Plotting

```
function plotdata(t,x,y,z)

    c1 = [98,227,218]/255;
    c2 = [132,79,232]/255;
    c3 = [98,227,174]/255;

    % Gráfica para x(t)
    figure('Color', 'w', 'Units', 'Centimeters', 'Position', [2, 2, 20, 12])
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    hold on; box on; grid off;
    plot(t, x, 'x', 'LineWidth', 1.5, 'Color', c1, 'DisplayName', '$x_1(t)$')
```

```

xlabel('$t$', 'Interpreter', 'latex')
ylabel('$x_1(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 10]); yticks(0:max(x)/10:10)
title('Gráfica x(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

% Gráfica para y(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [24, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, y, 'x', 'LineWidth', 1.5, 'Color', c2, 'DisplayName', '$y_1(t)$')
xlabel('$t$ $[days]$', 'Interpreter', 'latex')
ylabel('$y_1(t)$', 'Interpreter', 'latex')
xlim([min(t), max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 10]); yticks(0:max(y)/10:10)
title('Gráfica y(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

% Gráfica para z(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [46, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, z, 'x', 'LineWidth', 1.5, 'Color', c3, 'DisplayName', '$z_1(t)$')
xlabel('$t$', 'Interpreter', 'latex')
ylabel('$z_1(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 10]); yticks(0:max(z)/10:10)
title('Gráfica z(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')
end

function plotEDOSfit(t,x,y,z)

c1 = [98,227,218]/255;
c2 = [132,79,232]/255;
c3 = [98,227,174]/255;

% Gráfica para x(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [2, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, x, 'x', 'LineWidth', 1.5, 'Color', c1, 'DisplayName', '$x_o(t)$')
xlabel('$t$', 'Interpreter', 'latex')
ylabel('$x_o(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 10]); yticks(0:max(x)/10:10)
title('Gráfica x_o(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

```

```

% Gráfica para y(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [24, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, y, 'x', 'LineWidth', 1.5, 'Color', c2, 'DisplayName', '$y_o(t)$')
xlabel('$t$ $[days]$', 'Interpreter', 'latex')
ylabel('$y_o(t)$', 'Interpreter', 'latex')
xlim([min(t), max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 10]); yticks(0:max(y)/10:10)
title('Gráfica y_o(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

% Gráfica para z(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [46, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, z, 'x', 'LineWidth', 1.5, 'Color', c3, 'DisplayName', '$z_o(t)$')
xlabel('$t$', 'Interpreter', 'latex')
ylabel('$z_o(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 10]); yticks(0:max(z)/10:10)
title('Gráfica z_o(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')
end

function plotreqs(t,x,y,z)
    set(gcf, 'Color', 'w')
    set(gcf, 'Units', 'Centimeters', 'Position', [2,2,20,12])
    set(gca, 'FontName', 'Times New Roman')
    fontsize(12, 'points')
    c1 = [98,227,218]/255;
    c2 = [132,79,232]/255;

    subplot(3,1,1)
    hold on; box on; grid off;
    plot(t,x(:,1), 'x', 'MarkerSize', 5, 'LineWidth', 1.5, 'Color', c1)
    plot(t,x(:,2), '-', 'LineWidth', 1.5, 'Color', c2)
    xlabel('$t$ $[days]$', 'Interpreter', 'latex')
    ylabel('$x_o(t)$', 'Interpreter', 'latex')
    L = legend('$x_o(t)$', '$x_a(t)$');
    set(L, 'Interpreter', 'latex', 'FontSize', 15, 'Location', 'EastOutside', 'Box', 'On')
    title(L, 'Data')
    title('x(t)')
    xlim([min(t) max(t)])
    ylim([0 10])

    subplot(3,1,2)
    hold on; box on; grid off;
    plot(t,y(:,1), 'x', 'MarkerSize', 5, 'LineWidth', 1.5, 'Color', c1)
    plot(t,y(:,2), '-', 'LineWidth', 1, 'Color', c2)

```



```

xlabel('$t$ $[days]$', 'Interpreter', 'latex')
ylabel('$y_o(t)$', 'Interpreter', 'latex')
L = legend ('$y_o(t)$', '$y_a(t)$');
set(L, 'Interpreter', 'latex', 'FontSize', 15, 'Location', 'EastOutside', 'Box', 'On')
title(L, 'Data')
title('y(t)')
xlim([min(t) max(t)])
ylim([0 10])

```

```

subplot(3,1,3)
hold on; box on; grid off;
plot(t,z(:,1), 'x', 'MarkerSize', 5, 'LineWidth', 1.5, 'Color', c1)
plot(t,z(:,2), '-', 'LineWidth', 1, 'Color', c2)
xlabel('$t$ $[days]$', 'Interpreter', 'latex')
ylabel('$z_o(t)$', 'Interpreter', 'latex')
L = legend ('$z_o(t)$', '$z_a(t)$');
set(L, 'Interpreter', 'latex', 'FontSize', 15, 'Location', 'EastOutside', 'Box', 'On')
title(L, 'Data')
title('z(t)')
xlim([min(t) max(t)])
ylim([0 10])

```

end

```

function plotgreqs(to, xo, yo, zo, t, xa, ya, za)
    set(gcf, 'Color', 'w')
    set(gcf, 'Units', 'Centimeters', 'Position', [2,2,20,12])
    set(gca, 'FontName', 'Times New Roman')
    fontsize(12, 'points')

    c1 = [98,227,218]/255;
    c2 = [132,79,232]/255;

    subplot(3,1,1)
    hold on; box on; grid off;
    plot(to, xo, 'x', 'MarkerSize', 5, 'LineWidth', 1.5, 'Color', c1)
    plot(t, xa, '-', 'LineWidth', 1.5, 'Color', c2)
    xlabel('$t$ $[days]$', 'Interpreter', 'latex')
    ylabel('$x_o(t)$', 'Interpreter', 'latex')
    legend({'$x_o(t)$', '$x_a(t)$'}, 'Interpreter', 'latex', 'FontSize', 15, ...
        'Location', 'EastOutside', 'Box', 'on')
    title('x(2t)')
    xlim([min(t) max(t)])
    ylim([0 25])

    subplot(3,1,2)
    hold on; box on; grid off;
    plot(to, yo, 'x', 'MarkerSize', 5, 'LineWidth', 1.5, 'Color', c1)
    plot(t, ya, '-', 'LineWidth', 1.5, 'Color', c2)
    xlabel('$t$ $[days]$', 'Interpreter', 'latex')
    ylabel('$y_o(t)$', 'Interpreter', 'latex')

```

```

legend({'$y_o(t)$', '$y_a(t)$'}, 'Interpreter', 'latex', 'FontSize', 15, ...
       'Location', 'EastOutside', 'Box', 'on')
title('y(2t)')
xlim([min(t) max(t)])
ylim([0 15])

subplot(3,1,3)
hold on; box on; grid off;
plot(to, zo, 'x', 'MarkerSize', 5, 'LineWidth', 1.5, 'Color', c1)
plot(t, za, '-', 'LineWidth', 1.5, 'Color', c2)
xlabel('$t$ $[days]$', 'Interpreter', 'latex')
ylabel('$z(t)$', 'Interpreter', 'latex')
legend({'$z_o(t)$', '$z_a(t)$'}, 'Interpreter', 'latex', 'FontSize', 15, ...
       'Location', 'EastOutside', 'Box', 'on')
title('z(2t)')
xlim([min(t) max(t)])
ylim([-1 10])
end

function plotnorm(t,x,y,z)

c1 = [98,227,218]/255;
c2 = [132,79,232]/255;
c3 = [98,227,174]/255;

x_norm = x / max(x);
y_norm = y / max(y);
z_norm = z / max(z);

% Gráfica para x(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [2, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, x_norm, 'x', 'LineWidth', 1.5, 'Color', c1, 'DisplayName', '$x_o(t)$')
xlabel('$t$', 'Interpreter', 'latex')
ylabel('$x_o(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 1.2]); yticks(0:0.1:1.2)
title('Gráfica normalizada x_o(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

% Gráfica para y(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [24, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, y_norm, 'x', 'LineWidth', 1.5, 'Color', c2, 'DisplayName', '$y_o(t)$')
xlabel('$t$', 'Interpreter', 'latex')
ylabel('$y_o(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 1.2]); yticks(0:0.1:1.2)

```

```

title('Gráfica normalizada y_o(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

% Gráfica para z(t)
figure('Color', 'w', 'Units', 'Centimeters', 'Position', [46, 2, 20, 12])
set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
hold on; box on; grid off;
plot(t, z_norm, 'x', 'LineWidth', 1.5, 'Color', c3, 'DisplayName', '$z_o(t)$')
xlabel('$t$', 'Interpreter', 'latex')
ylabel('$z_o(t)$', 'Interpreter', 'latex')
xlim([0 max(t)+5]); xticks(0:10:max(t)+5)
ylim([0 1.2]); yticks(0:0.1:1.2)
title('Gráfica normalizada z_o(t)', 'FontSize', 12)
legend('Interpreter', 'latex', 'Location', 'best')

```

end

## Nonlinear Modelling

```

function [mdl,xa,ya,za] = Varied(to,xo,yo,zo,P0)
    x0 = xo(1); y0 = yo(1); z0 = zo(1);
    to = [to;to;to];
    fo = [xo;yo;z0];

function fi = model(p,t)
    dt = 1E-2;
    t = reshape (t,[],3); t = t(:,1);
    time = (0:dt:max(t));
    n = round(max(t)/dt);
    x = zeros(n+1,1); x(1) = x0;
    y = zeros(n+1,1); y(1) = y0;
    z = zeros(n+1,1); z(1) = z0;

    for i = 1:n
        [fx,fy,fz] = f(x(i),y(i),z(i));
        xn = x(i) + fx*dt;
        yn = y(i) + fy*dt;
        zn = z(i) + fz*dt;
        [fxn,fyn,fzn] = f(xn,yn,zn);

        x(i+1) = x(i) + (fx + fxn)*dt/2;
        y(i+1) = y(i) + (fy + fyn)*dt/2;
        z(i+1) = z(i) + (fz + fzn)*dt/2;
    end

function [dx,dy,dz] = f(x,y,z)
    dx = p(1)*x*y - p(2)*x - p(3)*x*y^2;
    dy = p(4)*y*z + p(5)*x*y - p(6)*y;
    dz = p(7)*z - p(8)*x*z;
end

```

```

        xi = interp1(time,x,t);
        yi = interp1(time,y,t);
        zi = interp1(time,z,t);

        fi = [xi;yi;zi];
end

mdl = fitnlm(to,fo,@model,P0);

fa = mdl.Fitted;
fn = reshape(fa,[],3);
xa = fn(:,1); ya = fn(:,2); za = fn(:,3);

Estimate = table2array(mdl.Coefficients(:,1));
SE = table2array(mdl.Coefficients(:,2));
pvalue = table2array(mdl.Coefficients(:,4));
alpha = 0.05;
CI95 = coefCI(mdl,alpha);
dof = mdl.DFE;
tval = tinv(1-alpha/2,dof);
MoE = SE*tval;
Parameters = ['p1';'p2';'p3';'p4';'p5';'p6';'p7';'p8'];
Results = table(Parameters,Estimate,SE,MoE,CI95,pvalue);

fprintf(['\nSample size (n): ', num2str(numel(xo))])
fprintf(['\nParameters to be estimated (pars): ', num2str(numel(P0))])
fprintf(['\nDegrees of freedom: ', num2str(dof)])
fprintf(['\nSignificance level (alpha): ', num2str(alpha)])
fprintf(['\nt-Student value: ', num2str(tval)])
fprintf(['\nR-squared: ', num2str(mdl.Rsquared.Ordinary)])
fprintf(['\nCorrected AIC (n/pars < 40): ',
num2str(mdl.ModelCriterion.AICc),'\n\n'])
disp(Results)
end

function [t,x,y,z] = Variante(xo,yo,zo,dt,tend,p)
    t = (0:dt:tend)';
    N = length(t);
    x = zeros(N,1); x(1) = xo;
    y = zeros(N,1); y(1) = yo;
    z = zeros(N,1); z(1) = zo;

    for i = 1:N-1
        [fx, fy, fz] = f(x(i), y(i), z(i), p);
        xn = x(i) + fx * dt;
        yn = y(i) + fy * dt;
        zn = z(i) + fz * dt;

        [fxn, fyn, fzn] = f(xn, yn, zn, p);
    end
end

```

```

        x(i+1) = x(i) + (fx + fxn) * dt / 2;
        y(i+1) = y(i) + (fy + fyn) * dt / 2;
        z(i+1) = z(i) + (fz + fzn) * dt / 2;
    end
end

function [dx, dy, dz] = f(x, y, z, p)
    dx = p(1)*x*y - p(2)*x - p(3)*x*y^2;
    dy = p(4)*y*z + p(5)*x*y - p(6)*y;
    dz = p(7)*z - p(8)*x*z;
end

```

## Bibliografia

[1] Paul. A. Valle, Syllabus de Biomatemáticas para la asignatura de Gemelos Digitales, Tecnológico Nacional de México/IT Tijuana, Tijuana, B.C., México, 2025. Permalink: <https://www.dropbox.com/s/6yf9afxzih9y458/Biomatematicas.pdf>