Parcial # 2 Sistemas Dinamicos

Michael Andrés Vásquez Morales — COD: 20201005183

1. Para el sistema rotacional en la figura, determine:

   a. La respresentación en el espacio de estados (b) junto a su diagrama de bloques, (c) así como el diagrama de flijo de señal.

   b. La función de transferencia

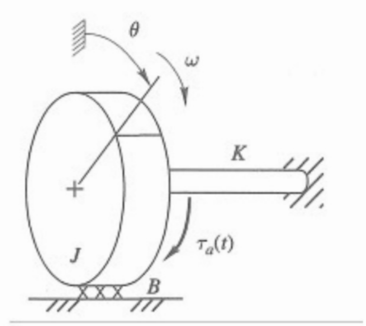
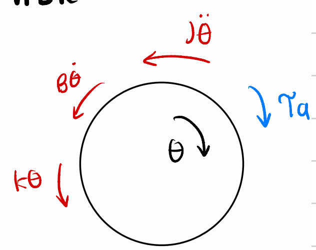


Diagrama de Cuerpo libre



Aplicando ley de Newton

$$\tau_a : \tau_n + \tau_B + \tau_K$$

$$\tau_n = J\ddot{\theta} \qquad \tau_B = B\dot{\theta} \qquad \tau_K = k\theta$$

Reemplazando:

$$\tau_a : J\ddot{\theta} + B\dot{\theta} + k\theta$$

$$\mathcal{L}^{-1} \downarrow$$

$$\tau_a : (Js^2 + Bs + k)\theta$$

Transfer Function:

$$\frac{\theta}{\tau_a} = \frac{\frac{1}{J}}{s^2 + \frac{B}{J}s + \frac{k}{J}} = \frac{\theta(s)}{\tau_a(s)} = \frac{1}{Js^2 + Bs + u}$$

State - space model.

- Variables de estado:

$$x_1 = \theta \qquad u = J\dot{x}_2 + Bx_2 + kx_1$$

$$x_2 = \dot{x}_1 = \dot{\theta} \qquad \text{reemplazando}$$

$$x_3 = \dot{x}_2 = \ddot{\theta} \qquad \dot{x}_2 = -\frac{k}{J}x_1 - \frac{B}{J}x_2 + \frac{1}{J}\tau_a$$

$$u = \tau_a$$

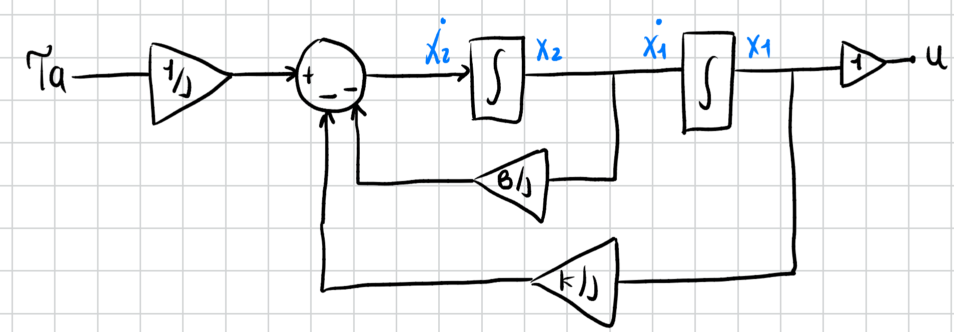
$$\dot{x} = Ax + Bx$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/J & -B/J \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} u$$

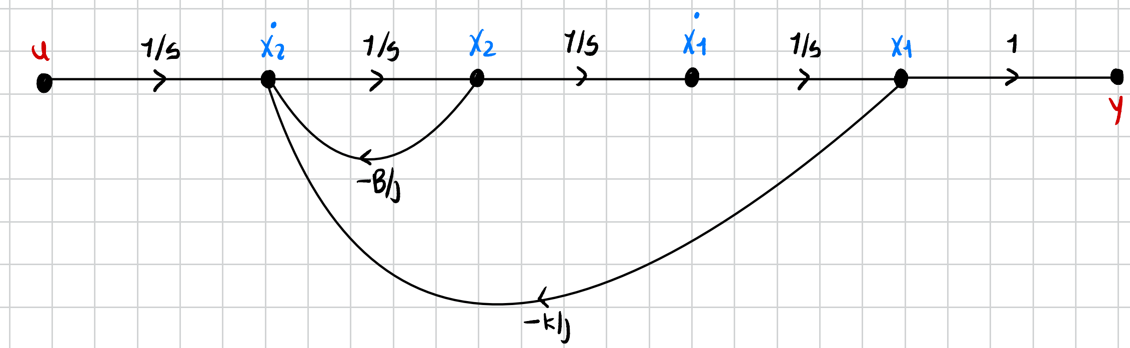
$$y = Cx + Du$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

Block Diagram



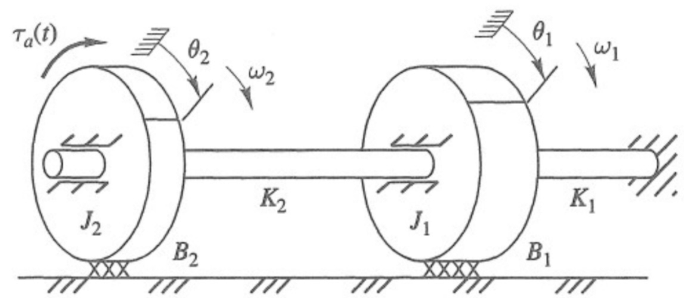
Signal flow diagram

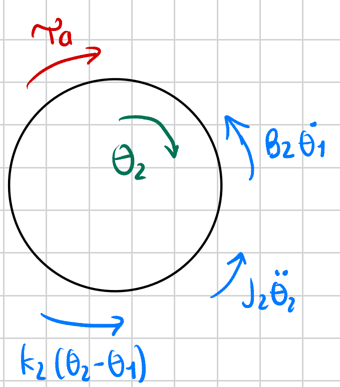


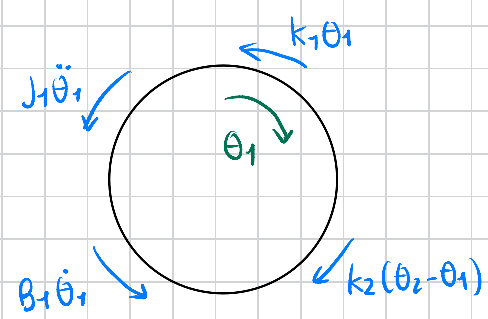
2. Para el sistema rotacional en la figura, asuma $\theta_2 > \theta_1$, y determine:

a. La función de transferencia relacionando θ_2 y τ_a.

b. La respresentación en el espacio de estados (b) junto a su diagrama de bloques, (c) así como el diagrama de flijo de señal. Todo en términos de θ_2







$$\Sigma F\theta_2 = \tau_a - k_2(\theta_2-\theta_1) - B_2\dot{\theta}_2 = J_2\ddot{\theta}_2$$

Reemplazando:

$$\tau_a = J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + k_2(\theta_2-\theta_1)$$

$$\mathcal{L}^{-1} \quad \downarrow$$

$$\tau_a(s) = \theta_2(s)\left(J_2 s^2 + B_1 s + k_2\right) - \theta_1(s)k_2$$

$$\Sigma F\theta_1 = k_2(\theta_2-\theta_1) - B_1\dot{\theta}_1 - k_1\theta_1 = J_1\ddot{\theta}_1$$

Reemplazando:

$$k_2\theta_2 - k_2\theta_1 - B_1\dot{\theta}_1 - k_1\theta_1 = J_1\dot{\theta}_1$$

$$k_2\theta_2 = J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + \theta_1(k_1+k_2)$$

$$\mathcal{L}^{-1} \quad \downarrow$$

$$\theta_2(s)k_2 = \theta_1(s)\left(J_1 s^2 + B_1 s + (k_1+k_2)\right)$$

$$\theta_1(s) = \frac{\theta_2(s)k_2}{J_1 s^2 + B_1 s + (k_1+k_2)}$$

Transfer Function:

$$\frac{\theta_2(s)}{\tau_a(s)} = \frac{J_1 s^2 + B_1 s + (k_1+k_2)}{(J_2 s^2 + B_2 s + k_2)(J_1 s^2 + B_1 s + (k_1+k_2)) - k^2}$$

State - space model:

- Variables de estado:

$$x_1 = \theta_2 \qquad u = \tau_a$$

$$x_2 = \dot{x}_1 = \dot{\theta}_2 \qquad y = x_1$$

$$x_3 = \theta_1$$

$$x_4 = \dot{x}_3 = \dot{\theta}_1$$

$$u = J_2\dot{x}_2 + B_2x_2 + k_2x_1 - k_2x_3$$

reemplazando

$$\dot{x}_2 = -\frac{k_2}{J_2}x_1 - \frac{B_2}{J_2}x_2 + \frac{k_2}{J_2}x_3 + \frac{1}{J_2}u$$

$$k_2x_1 - k_2x_3 = J_1\dot{x}_4 + B_1x_4 + k_1k_3$$

reemplazando

$$\dot{x}_4 = \frac{k_2}{J_1}x_1 - \frac{(k_1+k_2)}{J_1}x_3 - \frac{B_1}{J_1}x_4$$

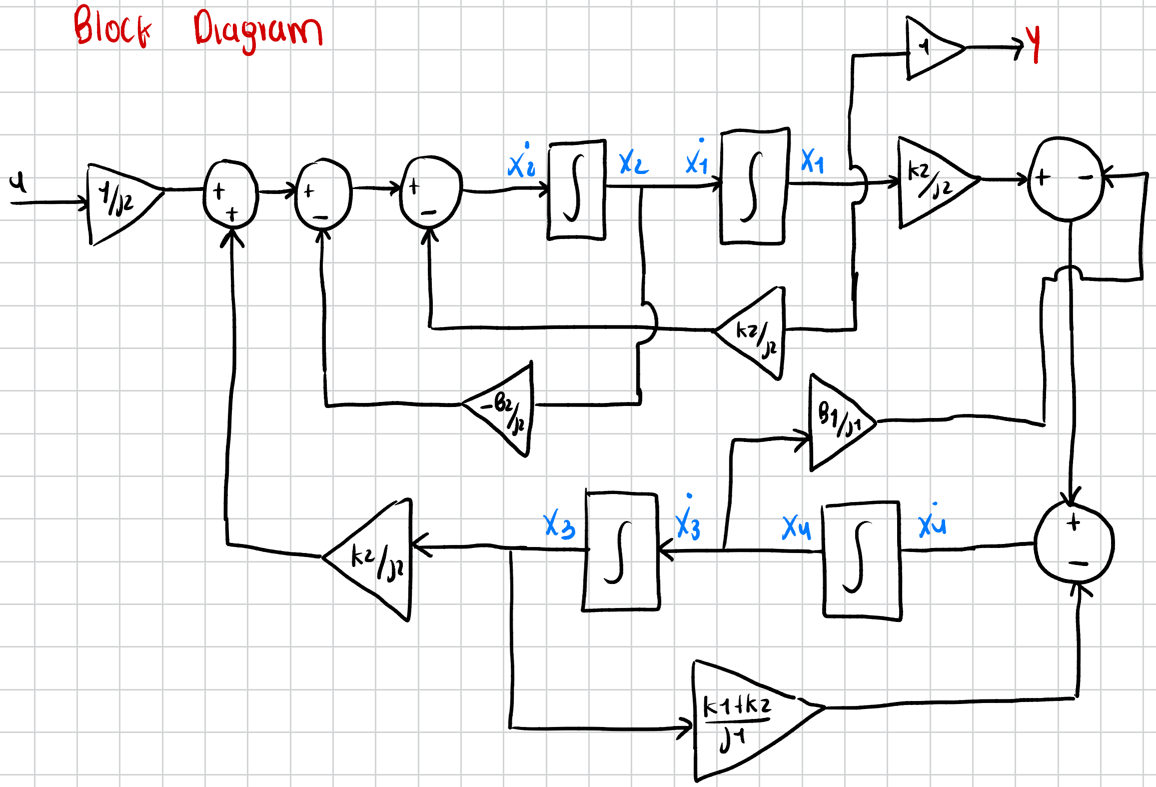
$$\dot{x} = Ax + Bx$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_2}{J_2} & -\frac{B_2}{J_2} & \frac{k_2}{J_2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{J_1} & 0 & -\frac{(k_1+k_2)}{J_1} & -\frac{B_1}{J_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_2 \\ 0 \\ 0 \end{bmatrix} u$$

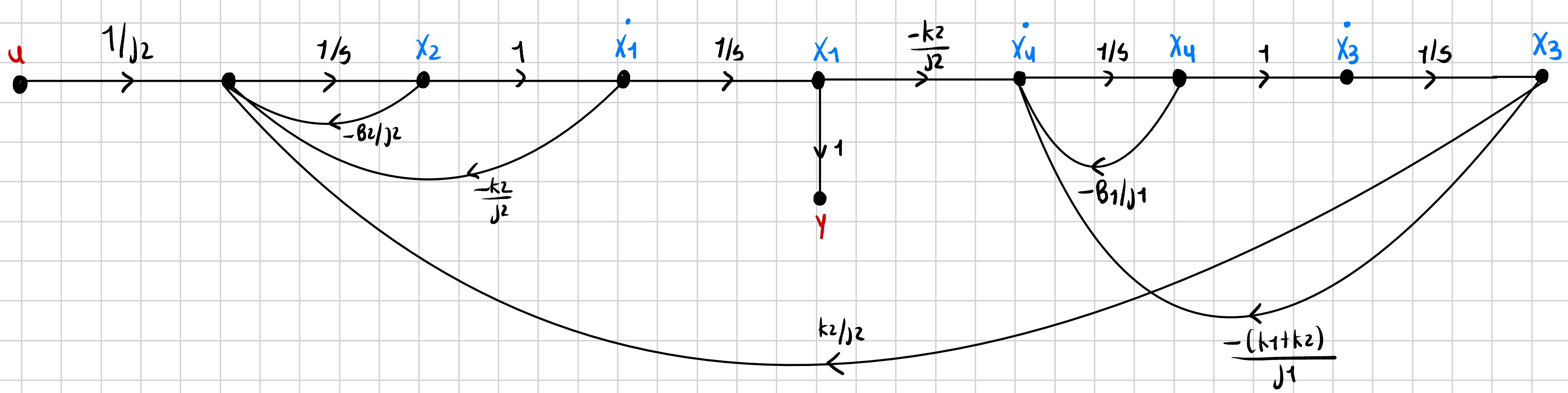
$$Y = Cx + Du$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$

Block Diagram



Signal flow diagram



3. Para el sistema del ítem anterior, mismos requerimientos considerando $K_1 = 0$

Haciendo $k_1 = 0$

Transfer Function:

$$\frac{\Theta(s)}{T_a(s)} = \frac{J_1 s^2 + B_1 s + k_2}{(J_2 s^2 + B_2 s)(J_1 s^2 + B_1 s + k_2) + k_2 (J_1 s^2 + B_1 s)}$$

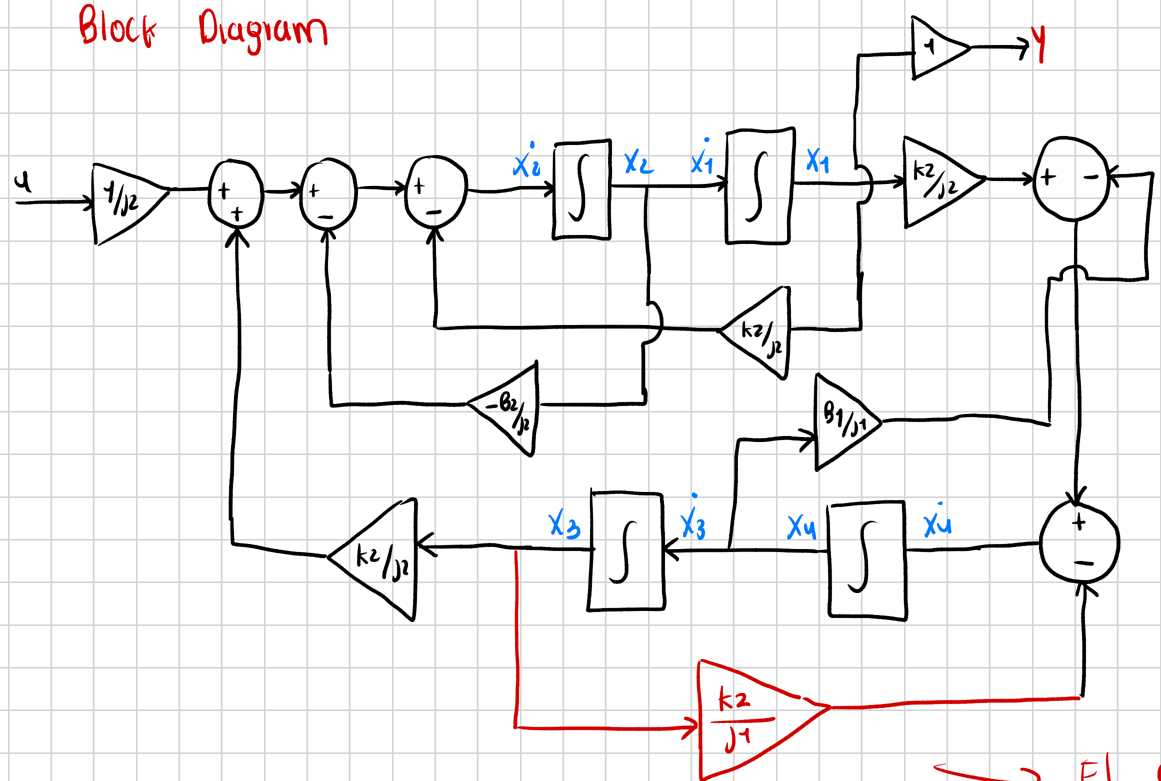
$$\dot{x} = Ax + Bx$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_2}{J_2} & -\frac{B_2}{J_2} & \frac{k_2}{J_2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{J_1} & 0 & -\frac{k_2}{J_1} & -\frac{B_1}{J_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_2 \\ 0 \\ 0 \end{bmatrix} u$$

$$Y = Cx + Du$$

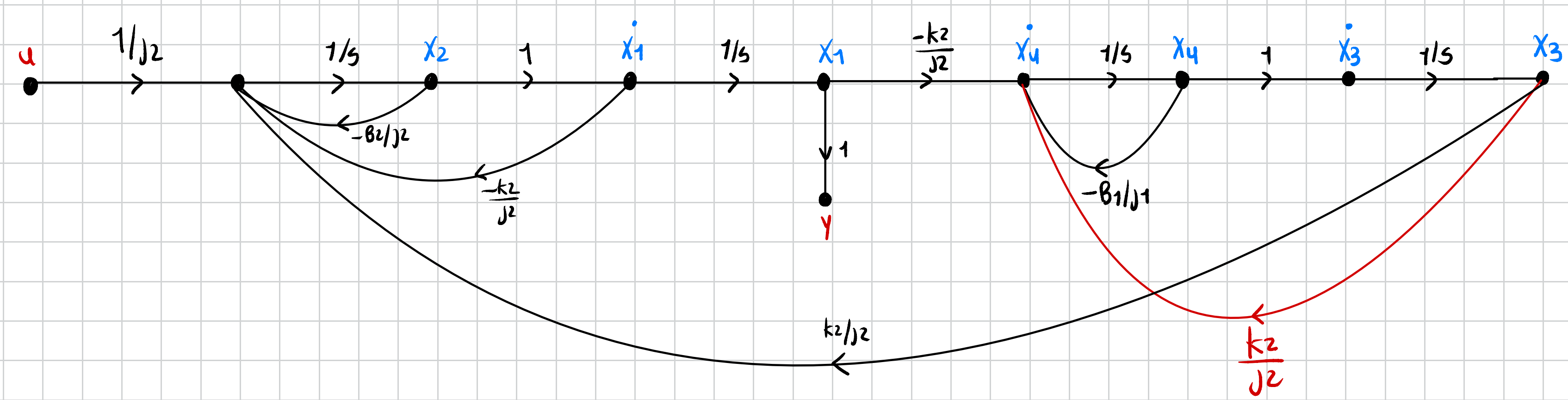
$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$

Block Diagram



El diagrama de bloques es igual, solo cambia esta ganancia

Signal flow diagram



4. Para el sistema rotacional en la figura, determine:

a. La función de transferencia relacionando θ y τ_a.

b. La respresentación en el espacio de estados (c) junto a su diagrama de bloques, (d) así como el diagrama de flujo de señal.

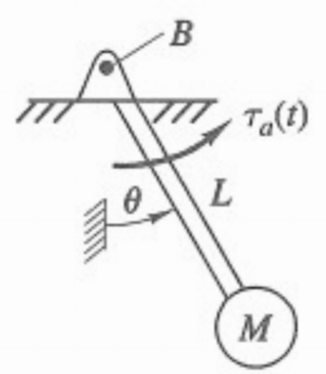
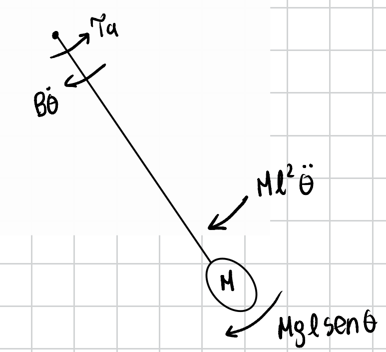


Diagrama de Cuerpo libre



$$\Sigma F = m\ell^2\ddot{\theta}$$

$$\Upsilon - mg\ell\, sen(\theta) - B\dot{\theta} = m\ell^2\ddot{\theta}$$

$$\ddot{\theta} = \frac{\Upsilon}{m\ell^2} - \frac{g\, sen(\theta)}{\ell} - \frac{B\dot{\theta}}{m\ell^2}$$

linealización en $sen\theta = \theta$

$$\ddot{\theta} = \frac{\Upsilon}{m\ell^2} - \frac{g(\theta)}{\ell} - \frac{B\dot{\theta}}{m\ell^2}$$

$$\tau_a = \ddot{\theta} m\ell^2 + mg\ell\theta + B\dot{\theta}$$

$\mathcal{L}^{-1}$ ↓

$$\tau_a(s) = (m\ell^2 s^2 + mg\ell + Bs)\,\theta(s)$$

Transfer function

$$\frac{\theta(s)}{\tau_a(s)} = \frac{1}{m\ell^2 s^2 + mg\ell + Bs}$$

State - space model:

- Variables de estado:

$x_1 = \theta$

$x_2 = \dot{x}_1 = \dot{\theta}$

$x_3 = \dot{x}_2 = \ddot{\theta}$

reemplazando

$$\dot{x}_2 = -\frac{g}{\ell}x_1 - \frac{B}{m\ell^2}x_2 + \frac{\tau_a}{m\ell^2}$$

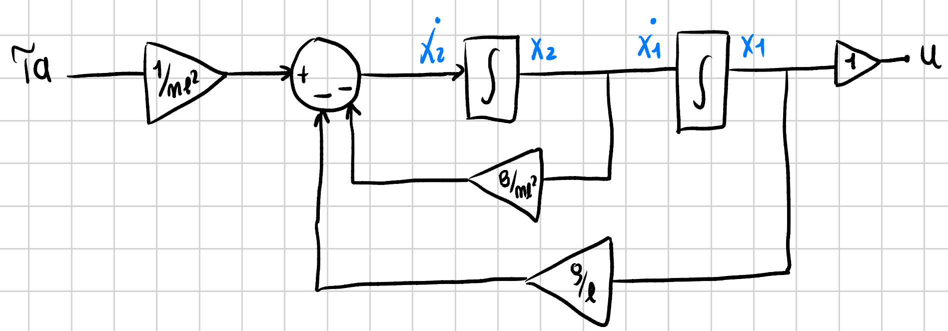
$$\dot{x} = Ax + Bx$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{g}{\ell} & -\frac{B}{m\ell^2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m\ell^2}\end{bmatrix}u$$

$$Y = Cx + Du$$

$$Y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0]\,u$$

Block Diagram



Signal flow diagram