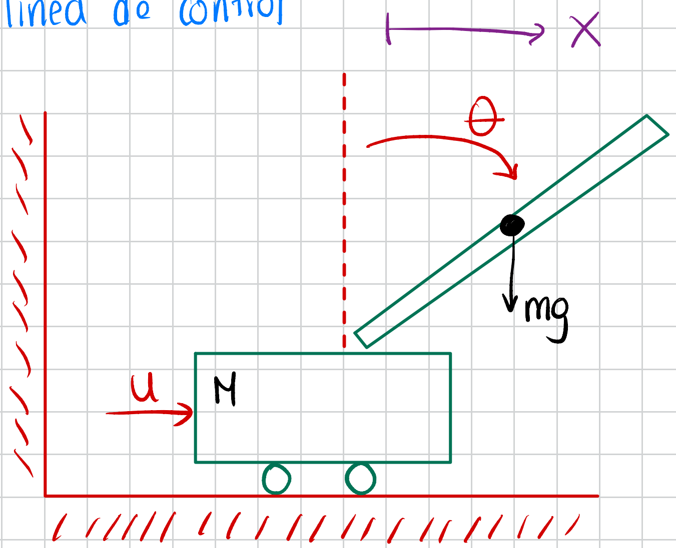
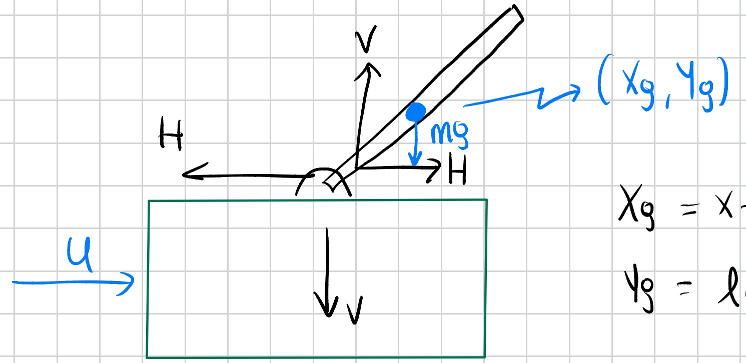
Clase 22 abril

libro ogata 5ta edición

linea de control





$X_g = x + l\,sen\theta$

$Y_g = l\cos\theta$

Movimiento Rotacional

$$I\ddot{\theta} = Vl\,sen\theta - Hl\,sen\theta \qquad (3,9)$$

Movimiento Horizontal

$$H = \frac{md^2(x + l\,sen\theta)}{dt^2}$$

Derivando:

$$H = m\ddot{x} + \frac{md^2(l\,sen\,\theta)}{dt^2}$$

$$H = m\ddot{x} + \frac{md(l\cos(\theta)\dot{\theta})}{dt}$$

$$H = m\ddot{x} + ml\frac{d\cos(\theta)\dot{\theta}}{dt}$$

$$H = m\ddot{x} + ml[-sen(\theta)\dot{\theta}\dot{\theta} + \cos(\theta)\ddot{\theta}]$$

$$H = m\ddot{x} - ml\,sen(\theta)\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta} \qquad (3,10)$$

Movimiento Vertical

$$V - mg = \frac{md^2(l\cos(\theta))}{dt^2} \qquad (3,11)$$

Movimiento Horizontal del carro:

$$M\ddot{x} = u - H \qquad (3,12)$$

→ Problema: alinealidad

Tratamos con problemas LTI

→ Cumple con la
- Homogeneidad
- Superposición

- Se debe linealizar

Angulos de desviación pequeños

$\theta \to 0$

Poder controlar

→ Teniendo en cuenta las ecuaciones

$$\theta \to 0 \begin{cases} sen\,\theta \approx \theta \\ \cos\theta \approx 1 \\ \theta\dot{\theta}^2 \approx 0 \end{cases}$$

→ Teniendo en cuenta las ecuaciones

$$\theta \to 0 \begin{cases} \operatorname{sen}\theta \approx \theta \\ \cos\theta \approx 1 \\ \theta\dot{\theta}^2 \approx 0 \end{cases}$$

From (3-9): $I\ddot{\theta} = Vl\cancel{\operatorname{sen}\theta}^{\,\theta} - Hl\cancel{\cos\theta}^{\,1}$

$I\ddot{\theta} = Vl\theta - Hl$ (3-13)

From (3-10): $H = m\ddot{x} - \underbrace{ml\cancel{\operatorname{sen}(\theta)}^{\,\theta}\cancel{\dot{\theta}^2}^{\,0}}_{0} + ml\cancel{\cos(\theta)}^{\,1}\ddot{\theta}$

$H = m\ddot{x} + ml\ddot{\theta}$

$H = m(\ddot{x} + l\ddot{\theta})$ (3,14)

From (3,11) $m\dfrac{d^2(l\cos\theta)}{dt^2} = V - mg$

$0 = V - mg$ (3,15)

From (3,12) y (3,14)

$M\ddot{x} = u - H$ $\quad H = m(\ddot{x} + l\ddot{\theta})$

$M\ddot{x} = u - m(\ddot{x} + l\ddot{\theta})H$

$M\ddot{x} = u - m\ddot{x} - ml\ddot{\theta}H$

$M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = u$

$(M+m)\ddot{x} + ml\ddot{\theta} = u$ (3,16)

From (3,13), (3,14) y (3,15)

$V = mg$

$I\ddot{\theta} = Vl\theta - Hl$ $\quad H = m(\ddot{x} + l\ddot{\theta})$ $\quad 0 = V - mg$

$I\ddot{\theta} = mgl\theta - m(\ddot{x} + l\ddot{\theta})l$

$I\ddot{\theta} = mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$

$I\ddot{\theta} = mgl\theta - l(m\ddot{x} + ml\ddot{\theta})$ ⟶ libro ogata 5ta edición ejemplo 3-5, pg 68, cap 3

Tarea: Hacerlo en S.S

⟶ libro ogata 3ra edicion mismo ejemplo pero con valores

Solución Parcial:

Primer punto

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4,5

1) Dada la siguiente ecuación diferencial represéntela en el espacio de estados y encuentre la función de transferencia

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t)$$

edo	1.0
eto	0.0
mech	0.0
exercis	~~0.0~~
video	~~1.0~~
paper	1.0

0,5 1,0

$$\cancel{f(t)} = \dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t)$$

$x = x_1$

$\dot{x} = \dot{x}_1 = x_2$

$\ddot{x} = \ddot{x}_1 = \dot{x}_2 = x_3$

$\dddot{x} = \dddot{x}_1 = \ddot{x}_2 = \dot{x}_3 = x_4$

$\dot{x}_1 = 0x_1 + 1x_2 + 0x_3 + 0x_4 + 0u$

$\dot{x}_2 = 0x_1 + 0x_2 + 1x_3 + 0x_4 + 0u$

$\dot{x}_3 = -x_1 - 2x_2 - x_3 + 2u$

espacio de estados

$\dot{y} = Ax + Bu$

$y = Cx + Du$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u$$

salida $x = x_1$

$$y \; [1\,0\,0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0u$$

Segundo punto:

Hallando la función de Transferencia:

$G(s) = C(sI - A)^{-1}B + \cancel{D}^{\,0}$ $\qquad \dot{x} = Ax(t) + Bu(t)$

calcular $(sI - A)$ $\qquad y(t) = Cx(t) + Du(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = 0$$

$$sI - A = s \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+1 \end{bmatrix} \Leftarrow \text{Invertirla}$$

$$(sI - A)^{-1} = \frac{\operatorname{adj}(sI - A)}{\det(sI - A)}$$

$$\operatorname{adj}(sI - A) = \begin{bmatrix} +\begin{bmatrix} s & -1 \\ 2 & s+1 \end{bmatrix} & -\begin{bmatrix} 0 & -1 \\ 1 & s+1 \end{bmatrix} & +\begin{bmatrix} 0 & s \\ 1 & 2 \end{bmatrix} \\ -\begin{bmatrix} -1 & 0 \\ 2 & s+1 \end{bmatrix} & +\begin{bmatrix} s & 0 \\ 1 & s+1 \end{bmatrix} & -\begin{bmatrix} s & -1 \\ 1 & 2 \end{bmatrix} \\ +\begin{bmatrix} -1 & 0 \\ s & -1 \end{bmatrix} & -\begin{bmatrix} s & 0 \\ 0 & -1 \end{bmatrix} & \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix} \end{bmatrix}^T$$

$$\operatorname{adj}(sI - A) = \begin{bmatrix} s(s+1) - (-1)(2) & -[(0)(s+1) - (-1)(1)] & (0)(2) - (s)(1) \\ -[(-1)(s+1) - (0)(2)] & (s)(s+1) - (0)(2) & -[(s)(2) - (-1)(1)] \\ (-1)(-1) - (0)(s) & -[(s)(-1) - (0)(0)] & (s)(s) - (-1)(0) \end{bmatrix}^T$$

$$\operatorname{adj}(sI - A) = \begin{bmatrix} s^2 + s + 2 & -1 & -2s \\ s+1 & s^2 + s & -(2s+1) \\ 1 & s & s^2 \end{bmatrix}^T$$

$$\operatorname{adj}(sI - A) = \begin{bmatrix} s^2 + s + 2 & s+1 & 1 \\ -1 & s^2 + s & s \\ -2s & -(2s+1) & s^2 \end{bmatrix}^T$$

$$(sI - A)^{-1} = \frac{1}{s^3 + s^2 + s + 2} \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+1 \end{bmatrix}$$

$$\det(sI - A) = (s)(s)(s+1) + (-1)(-1)(1) + (0)(2)(0) - [(0)(s)(1) + (-1)(0)(s+1) + (s)(2)(-1)]$$

$$(sI - A)^{-1} = \frac{\operatorname{adj}(sI - A)}{\det(sI - A)}$$

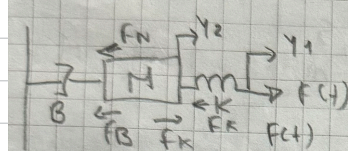
$$s^3 + 2s^2 + 1 - [-2s] = s^3 + 2s^2 + 2s = s^3 + 2s^2 + 2s + 1$$

función de Transferencia:

$$G(s) = \frac{2}{s^3 + s^2 + 2s + 1}$$

Tercer punto:

3) $\qquad y_1 > y_2$



$F = F_K \qquad 1$

$F_K = F_N + F_B \qquad 2$

equivalentes

y_2

$x_1 = y_2 \qquad x_2 = \dot{x}_1 = \dot{y}_2 \qquad f_K = u = k(y_1 - x_1)$

$u = F = F_K \qquad y_1 = x_1 + \dfrac{u}{K}$?

$u = \boxed{M\dot{x}_2} + \boxed{Bx_2} \qquad \boxed{\dot{x}_2 = -\dfrac{B}{M}x_2 + \dfrac{u}{M}}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \boxed{\;} + \begin{bmatrix} \frac{1}{K} \\ 0 \end{bmatrix} u$$

?

Reclame