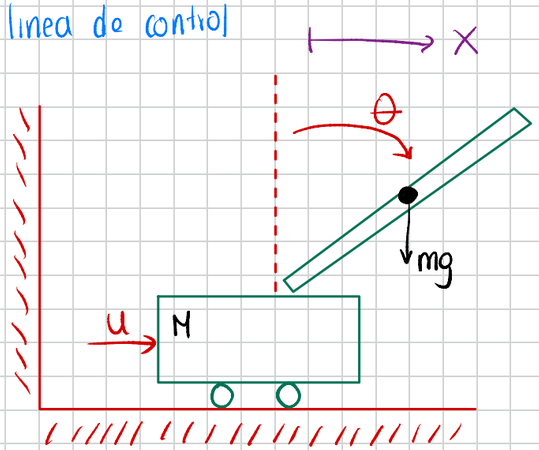
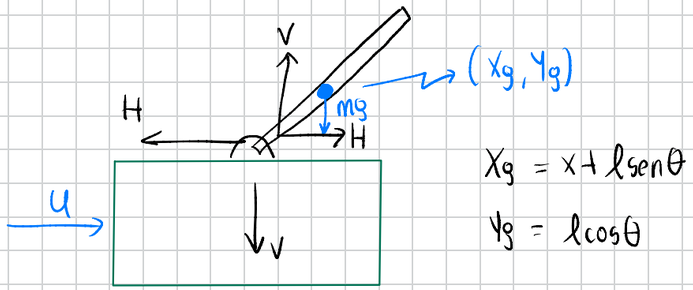
Clase 22 abril

libio ogata 5ta edición

linea de control





$X_g = x + l\,sen\theta$

$Y_g = l\cos\theta$

Movimiento Rotacional

$$I\ddot{\theta} = Vl\,sen\theta - Hl\,sen\theta \quad (3,9)$$

Movimiento Horizontal

$$H = \frac{md^2(x + l\,sen\theta)}{dt^2}$$

Derivando:

$$H = m\ddot{x} + m\frac{d^2(l\,sen\theta)}{dt^2}$$

$$H = m\ddot{x} + m\frac{d(l\cos(\theta)\dot{\theta})}{dt}$$

$$H = m\ddot{x} + ml\frac{d\cos(\theta)\dot{\theta}}{dt}$$

$$H = m\ddot{x} + ml[-sen(\theta)\dot{\theta}\dot{\theta} + \cos(\theta)\ddot{\theta}]$$

$$H = m\ddot{x} - ml\,sen(\theta)\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta} \quad (3,10)$$

Movimiento Vertical

$$V - mg = m\frac{d^2(l\cos(\theta))}{dt^2} \quad (3,11)$$

Movimiento Horizontal del carro:

$$M\ddot{x} = u - H \quad (3,12)$$

→ Problema: alinealidad

Tratamos con problemas LTI

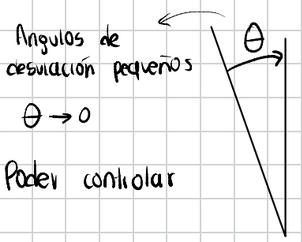
→ Cumple con la
- Homogeneidad
- Superposición

• Se debe linealizar

Angulos de desviación pequeños

$\theta \to 0$

Poder controlar



→ Teniendo en cuenta las ecuaciones

$$\theta \to 0 \begin{cases} sen\,\theta \approx \theta \\ \cos\theta \approx 1 \\ \theta\dot{\theta}^2 \approx 0 \end{cases}$$

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$$\theta \to 0 \begin{cases} \operatorname{sen}\theta \approx \theta \\ \cos\theta \approx 1 \\ \theta\dot{\theta}^2 \approx 0 \end{cases}$$

From (3-9): $I\ddot{\theta} = V l \operatorname{sen}\theta - H l \cos\theta$

$I\ddot{\theta} = V l \theta - H l$ (3-13)

From (3-10): $H = m\ddot{x} - ml\operatorname{sen}(\theta)\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta}$

$H = m\ddot{x} + ml\ddot{\theta}$

$H = m(\ddot{x} + l\ddot{\theta})$ (3,14)

From (3,11) $m\frac{d^2(l\cos\theta)}{dt^2} = V - mg$

$0 = V - mg$ (3,15)

From (3,12) y (3,14)

$M\ddot{x} = u - H \qquad H = m(\ddot{x} + l\ddot{\theta})$

$M\ddot{x} = u - m(\ddot{x} + l\ddot{\theta})H$

$M\ddot{x} = u - m\ddot{x} - ml\ddot{\theta}H$

$M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = u$

$(M+m)\ddot{x} + ml\ddot{\theta} = u$ (3,16)

From (3,13), (3,14) y (3,15)

$I\ddot{\theta} = Vl\theta - Hl \qquad H = m(\ddot{x} + l\ddot{\theta}) \qquad 0 = V - mg$ ($V = mg$)

$I\ddot{\theta} = mgl\theta - m(\ddot{x} + l\ddot{\theta})l$

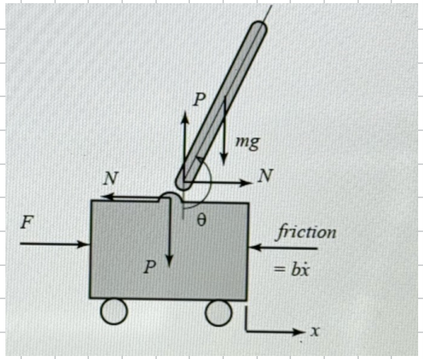
$I\ddot{\theta} = mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$

$I\ddot{\theta} = mgl\theta - l(m\ddot{x} + ml\ddot{\theta})$ → libro ogata 5ta edición

ejemplo 3-5, pg 68, cap 3

Tarea: Hacerlo en S.S → libro ogata 3ra edicion mismo ejemplo pero con valores

Solución Tarea 3:



$$M\ddot{x} + b\dot{x} + N = F$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Ml^2} & \frac{m^2gl^2}{I(M+m)+Ml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Ml^2} & \frac{mgl(M+m)}{I(M+m)+MmI^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Ml^2} \\ 0 \\ \frac{ml}{I(M+m)+Ml^2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$