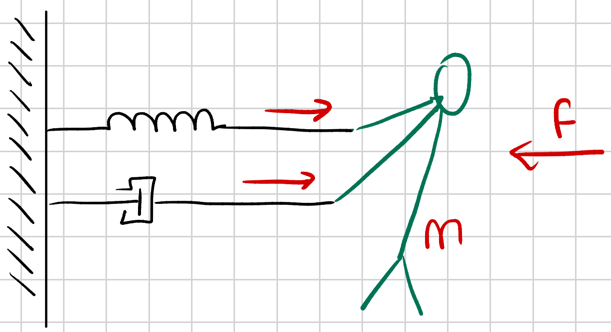
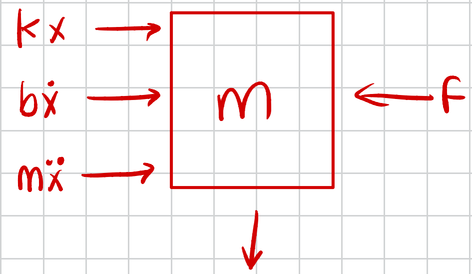
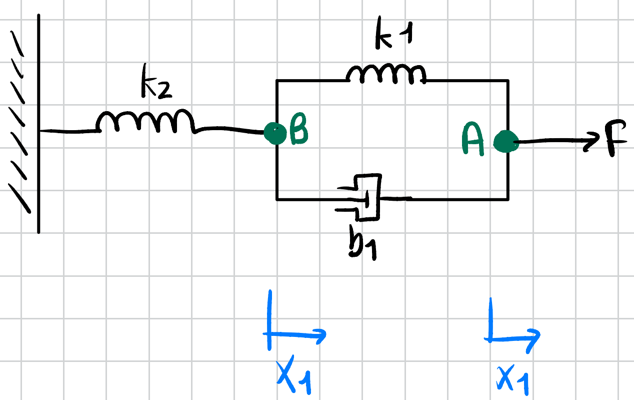
# Clase 11 abril

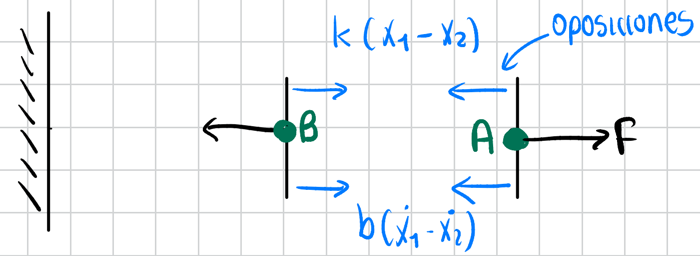


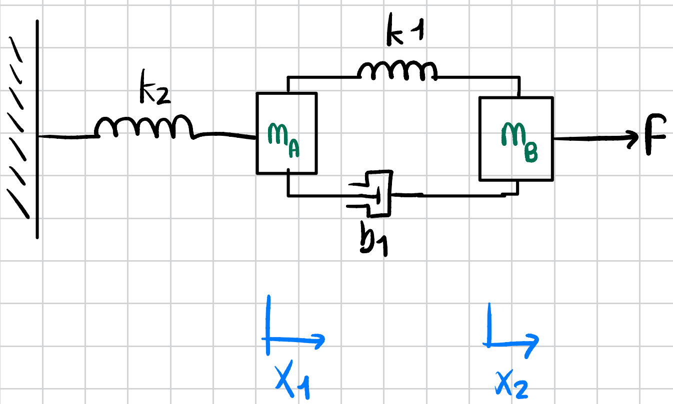



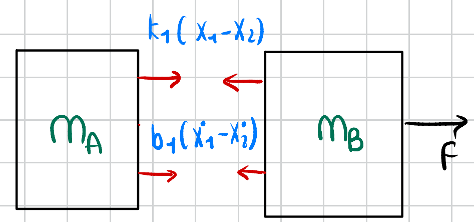

## Sistema mecanico rotacional



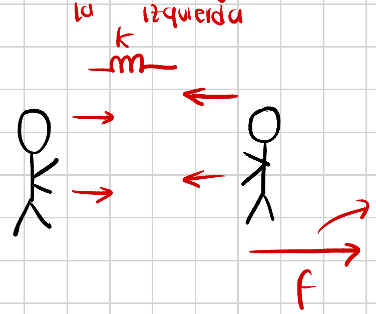

Masas puntuales:



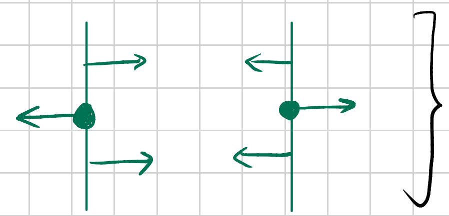




El resorte jala hacia la izquierda




Se ejerce una fuerza

masas puntuales



revisar sus oposiciones

$m\ddot{x} = 0$

masa: 0 aceleracion

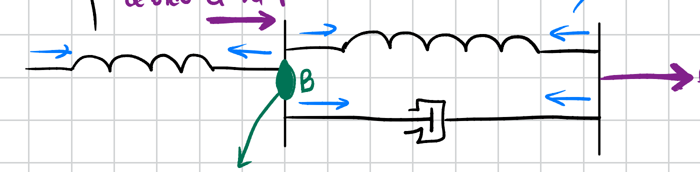
## Caso 1. Aplicando fuerza hacia la derecha → F

El resorte se estira:

El resorte se opone a la fuerza

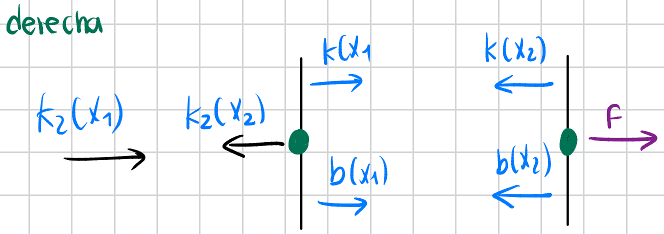
movimiento debido a la F

oposiciones: lucha de contrarios




Fuerza aplicada forzando el sistema a la derecha

la masa B siente que se mueve a la derecha

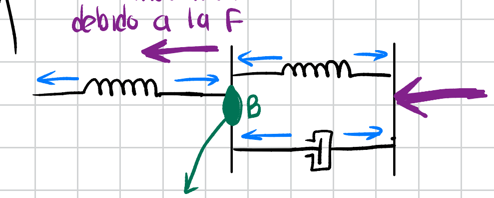



## Caso 2: Aplicando fuerza hacia la izquierda ← F

El resorte se estira:

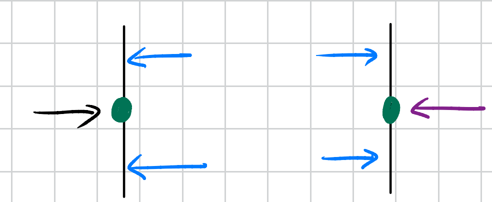
El resorte se opone a la fuerza

movimiento debido a la F

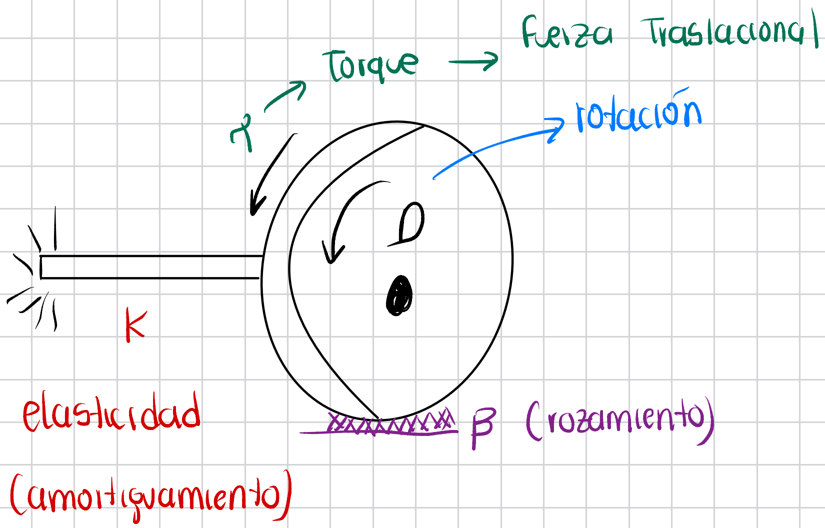


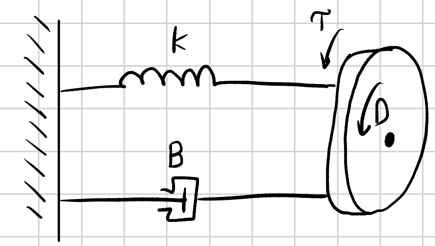

Fuerza aplicada forzando el sistema a la izquierda

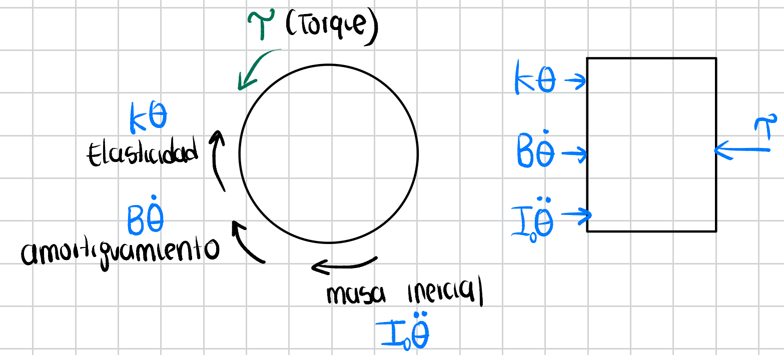
la masa B siente que se mueve hacia la izquierda



# Sistemas rotacionales



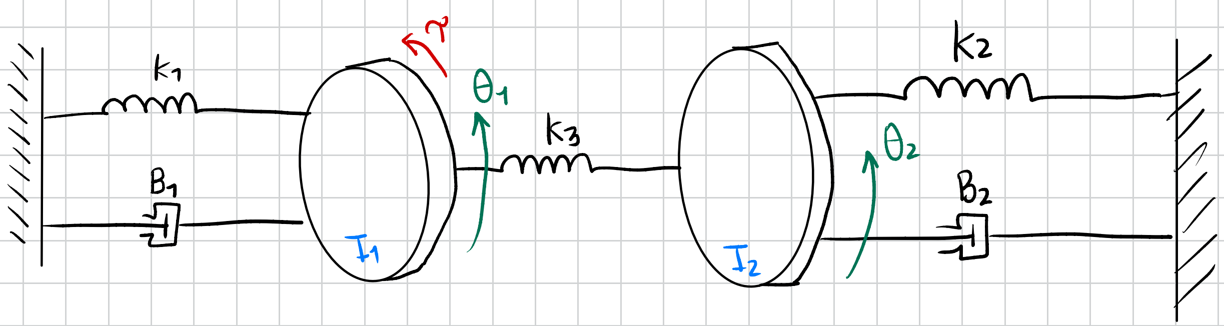

salida:

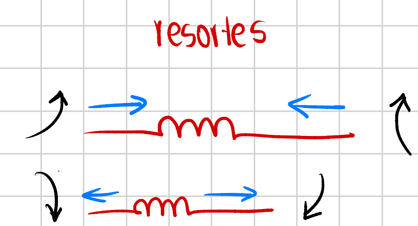
Aceleración velocidad posición

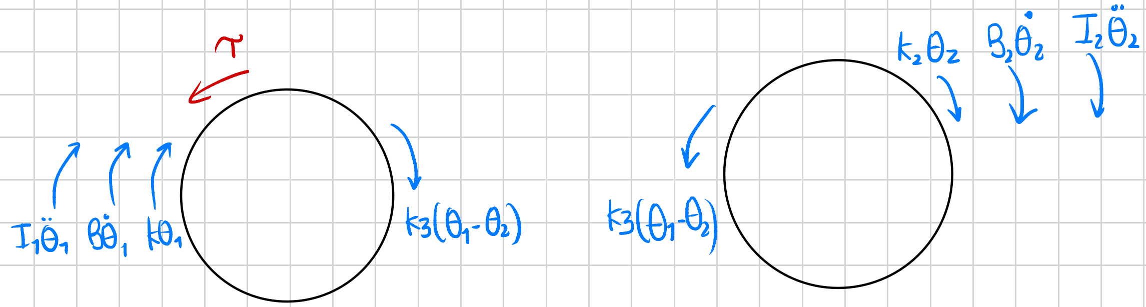
$$I\ddot{\theta} + B\dot{\theta} + k\dot{\theta} = \tau$$

$$q_1 = \theta$$
$$q_2 = \dot{q}_1 = \dot{\theta}$$
$$q_3 = \ddot{q}_1 = \ddot{\theta}$$

ejemplo:




$$\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + k_1\theta_1 + k_3(\theta_1 - \theta_2)$$

$$\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + k_1\theta_1 + k_3\theta_1 - k_3\theta_2$$

$$\boxed{\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + (k_1 + k_3)\theta_1 - k_3\theta_2} \quad (1)$$

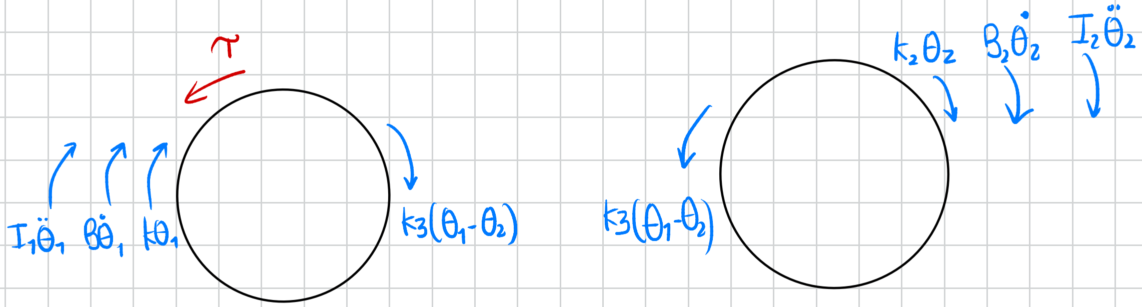
$$k_3(\theta_1 - \theta_2) - k_2\theta_2 - B_2\dot{\theta}_2 - I_2\ddot{\theta}_2 = 0$$

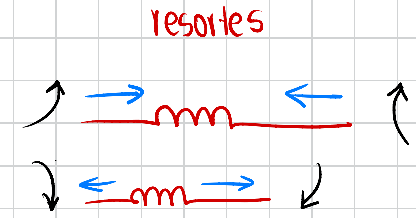
$$k_3\theta_1 - k_3\theta_2 - k_2\theta_2 - B_2\dot{\theta}_2 - I_2\ddot{\theta}_2 = 0$$

$$\boxed{I_2\ddot{\theta}_2 + B_2\dot{\theta}_2 - k_3\theta_1 + (k_2 + k_3)\theta_2 = 0} \quad (2)$$

Tarea: hacerlo en SS

# Solución Tarea 1




Tarea: hacerlo en SS

$$\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + k_1\theta_1 + k_3(\theta_1-\theta_2)$$

$$\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + k_1\theta_1 + k_3\theta_1 - k_3\theta_2$$

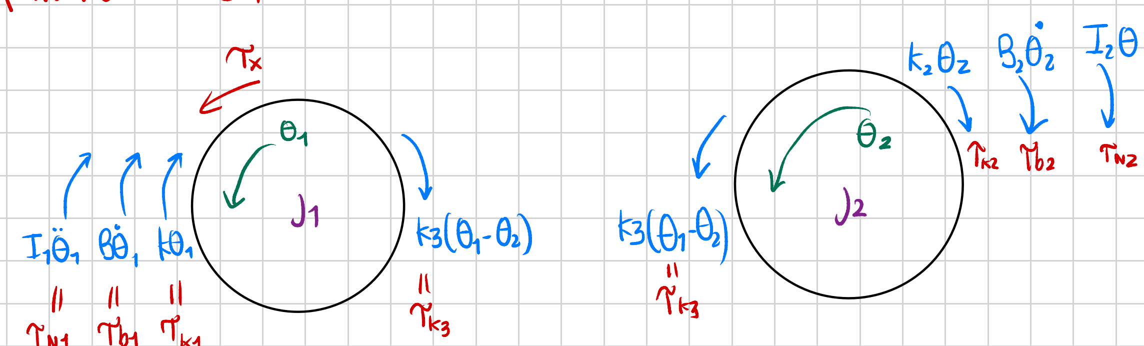
$$\boxed{\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + (k_1+k_3)\theta_1 - k_3\theta_2} \quad (1)$$

$$k_3(\theta_1-\theta_2) - k_2\theta_2 - B_2\dot{\theta}_2 - I_2\ddot{\theta}_2 = 0$$

$$k_3\theta_1 - k_3\theta_2 - k_2\theta_2 - B_2\dot{\theta}_2 - I_2\ddot{\theta}_2 = 0$$

$$\boxed{I_2\ddot{\theta}_2 + B_2\dot{\theta}_2 - k_3\theta_1 + (k_2+k_3)\theta_2 = 0} \quad (2)$$

Definiendo: $\theta_1 > \theta_2$




Definimos estados:

Definimos la salida $u = \tau_x$

$x_1 = \theta_1$
$x_2 = \dot{x}_1$
$x_3 = \theta_2$
$x_4 = \dot{x}_3$

$\tau_{k1} = k_1 x_1$
$\tau_{k2} = k_2 x_3$
$\tau_{k3} = k_3(x_1 - x_3)$

$\tau_{b1} = b_1 x_2$
$\tau_{b2} = b_2 x_4$

$\tau_{N1} = J_1 \dot{x}_2$
$\tau_{N2} = J_2 \dot{x}_4$

Salidas $x_1$ y $x_3$

$$u = J_1\dot{x}_2 + k_1x_1 + b_1x_2 + k_3(x_1 - x_3)$$

$$k_3(x_1 - x_3) = J_2\dot{x}_4 + k_2x_3 + b_2x_4$$

$$\dot{x}_2 = -\left(\frac{k_1}{J_1} + \frac{k_3}{J_1}\right)x_1 - \frac{b_1}{J_1}x_2 + \frac{k_3}{J_1}x_3 + \frac{u}{J_1}$$

$$\dot{x}_4 = \frac{k_3}{J_2}x_1 - \left(\frac{k_2}{J_2} + \frac{k_3}{J_2}\right)x_3 - \frac{b_2}{J_2}x_4$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1+k_3}{J_1} & \frac{-b_1}{J_1} & \frac{k_3}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_3}{J_2} & 0 & \frac{-k_2+k_3}{J_2} & \frac{-b_2}{J_2} \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 1/J_1 \\ 0 \\ 0 \end{bmatrix} u$$

$$\vec{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \vec{x} + \vec{0}u$$