Video 1

State feedback

example 12.1

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$$

Overshoot = 9,5%

$t_s = 0{,}74\,s$

Realizar expresion es SS

$$U(s) \rightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \xrightarrow{Y(s)}$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s}$$

$$(s^3+5s^2+4s)\,X_1(s) = U(s)$$

$$\underbrace{\cancel{\dddot{x}_1}}_{\dot{x}_3} + 5\underbrace{\cancel{\ddot{x}_1}}_{x_3} + 4\underbrace{\cancel{\dot{x}_1}}_{x_2} = u$$

$x_1 = x_1$

$x_2 = \dot{x}_1$

$x_3 = \ddot{x}_1$

$\dot{x}_3 = \dddot{x}_1$

$$\dot{x}_3 = -5x_3 - 4x_2 + u \quad (1)$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)\,X_1(s)$$

$$Y(s) = (0s^2 + 20s + 100)\,X_1(s)$$

$$Y(s) = (20s + 100)\,X_1(s)$$

$\downarrow \mathcal{L}^{-1}$

$$y(s) = 20\underbrace{\cancel{\dot{x}_1}}_{x_2} + 100\underbrace{\cancel{x_1}}_{x_1}$$

$$y(s) = 20x_2 + 100x_1 \quad (2)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Para el overshoot:

$$e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \cdot 100 = 9{,}5\%$$

Aplicando Ln

$$\ln(0{,}095) = \ln\left(e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)}\right)$$

$$-2{,}3539 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\left(\cancel{-}2{,}3539\left(\cancel{\sqrt{}}\,1-\zeta^2\right)\right)^2 = \left((\cancel{-}\zeta\pi)\right)^2$$

$$5{,}5407\,(1-\zeta) = (\zeta^2\pi^2)$$

$$5{,}5407 - 5{,}5407\,\zeta^2 = \zeta^2\pi^2$$

$$5{,}5407 = \zeta^2\pi^2 + 5{,}5407\,\zeta^2$$

$$5{,}5407 = \zeta^2(\pi^2 + 5{,}5407)$$

$$\zeta^{\cancel{2}} = \sqrt{\frac{5{,}5407}{\pi^2 + 5{,}5407}}$$

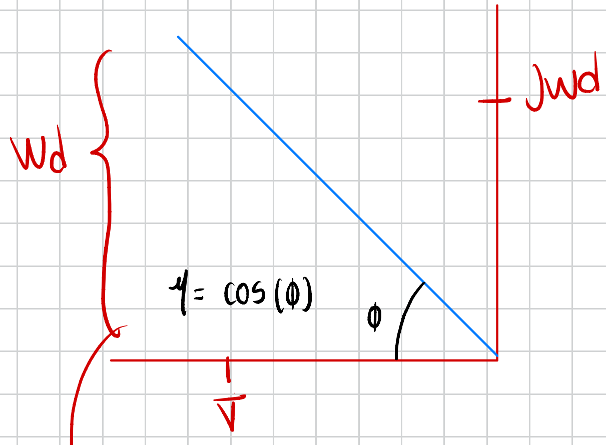
$$\zeta = 0{,}5996$$

$$s = \sigma + j\omega_d$$

$$s = \zeta\omega_n + j\omega_d$$

$\downarrow$

Conociendo la frecuencia natural, obtenemos la localización de un polo dominante de malla cerrada para un OS% = 9,5%



Conociendo $\zeta = 0{,}5996$

$\downarrow$ Hallar ϕ

Aplicar $\mathrm{acos}(\phi)$

$\downarrow$

$$\mathrm{acosd}(0{,}5996) \approx \underbrace{53{,}16^\circ}$$

Encontramos la posición del polo

(IR) (Img)

$$S = \sigma + jWd$$

$$S = \zeta W_n \pm j\left(W_n \sqrt{1-\zeta^2}\right)$$

$t_s = 0{,}74\,s$ $\qquad t_s = \dfrac{4}{\sigma}$ $\rightarrow$ Hallar W_n

$$\sigma = \frac{4}{0{,}74}$$

$$\sigma = 5{,}405$$

$$\sigma = \zeta W_n$$

$$W_n = \frac{5{,}405}{0{,}5996}$$

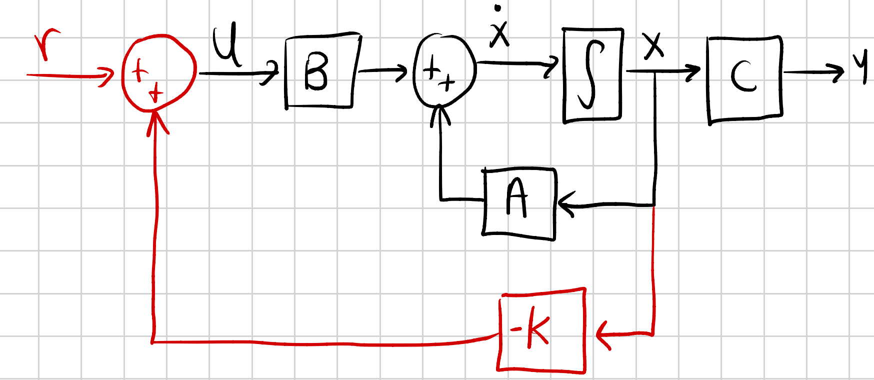
$$W_n = 9{,}02 \text{ rad/s}$$

Teniendo los siguientes datos:

	Posición en la parte Real =	Posición en la parte Img
$\zeta = 0{,}5996$		
$W_n = 9{,}02$ rad/s		
$\phi = 53{,}16^\circ$	$\sigma = \zeta W_n$	$Wd = W_n\sqrt{1-\zeta^2}$
$\sigma = 5{,}405$	$\sigma = (0{,}5996)(9{,}02)$	$Wd = (9{,}02)\sqrt{1-(0{,}5996)^2}$
$t_s = 0{,}74\,s$	$\sigma = 5{,}405$	$Wd = 7{,}21$ rad/s

$$\dot{x} = Ax + bx$$

$$y = Cx$$



$$\dot{x} = Ax + Bu$$

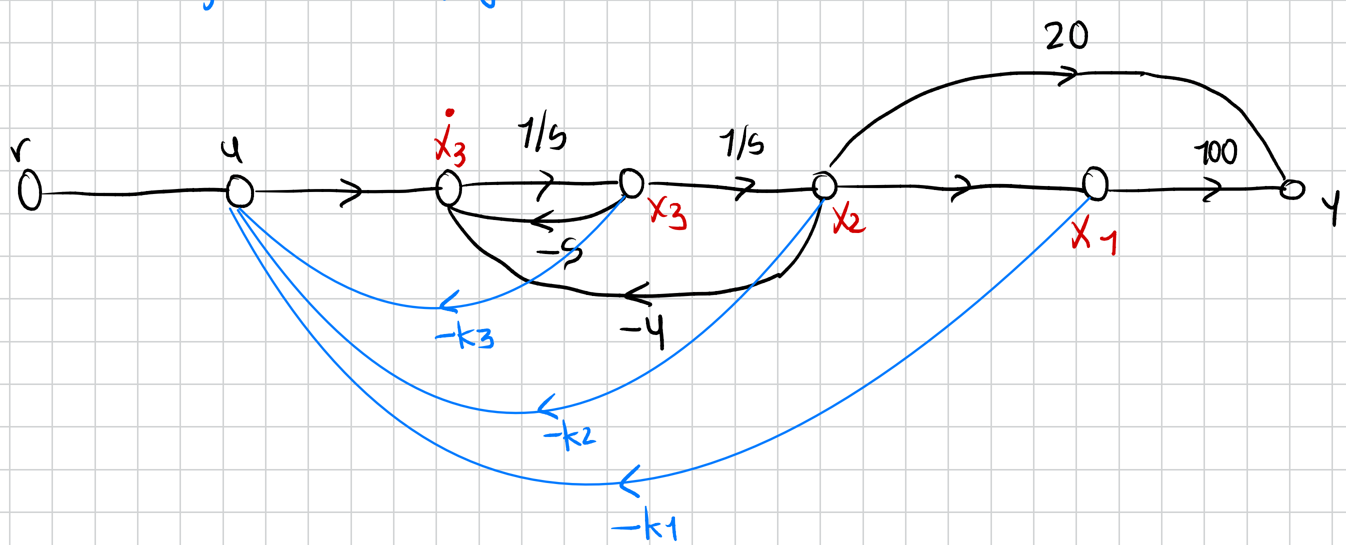
$$= Ax + B(-kx + r)$$

$$= Ax - Bkx + Br)$$

$$\dot{x} = (A - Bk)x + Br$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\0&-4&-5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad Y=\begin{bmatrix}100 & 20 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

Diagrama de flujo de señal



$$\dot{x}_3 = -4x_2 - 5x_3 + u + r$$
$$-4x_2 - 5x_3 + [-k_3x_3 - k_2x_2 - k_1x_1] + r$$
$$-4x_2 - 5x_3 - k_3x_3 - k_2x_2 - k_1x_1 + r$$
$$-k_1x_1 - (4+k_2)x_2 - (5+k_3) + r$$

Nueva representación en S.S

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-k_1&(-4+k_1)&(-5+k_3)\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}r \qquad Y=\begin{bmatrix}100 & 20 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$