Basic Remarks on the Orbital Parameters of Galactic Mergers and their State Vectors

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***Abstract*— The study of galactic mergers are important from reason such as that they determine on great part the structure universe to other as the advance in other scientific fields. In this work we present a way in which some set of parameters could determine a given merge and try to study relationships between the vector state of the pair interactive galaxies in moment in which the orbit was approximately by a two body problem. For such tasks the study of the papers [1]-[4] is done, some result are reproduced and then we make a contribution that could be make in the procedure in order to refine the results. The method proposed show to be consistent with a simple example and illustrate well some critical behaviors that a merge can have.**

I. Introduction

Galaxies are complex structures that are full of physics from every field you can imagine. The study of galactic mergers have shown to be important in galactic evolution, cosmology [1] as well as for other areas because the applications of the algorithms created to study galactic evolution can be used in several other fields. For example, as the N-body or non-collisionless problems arise in several other physical fields (as planetary or plasma physics), the algorithms that are develop on galactic evolution (specifically mergers) are sufficiently rich to permit the study of several phenomena; therefore it is important to develop efficient methods in order to study such systems.

The fact that simulations of this system are more of a necessity than only useful relays in the fact that the information about the space configuration system is very limited and the problems are highly non-linear. In fact it is only possible to observe (at most) the X,Y position in the sky and the radial velocity. Nevertheless, the physical information is still hidden, because the study of the merger in a consistent way depend upon the orientation of the merger respect the visual, and on other observations that could be made to bound the parameters; some of the are explained in [1][2][3].

The characteristics observed in them (mergers) have been explained extensively based only in the gravitational interaction, over time, of its components and tested in the observed movements of its light emitting parts. In order to study the parameters that could characterize a galactic merger, several methods have been explored, some examples of them are: classical N body simulations which represents each star or a set of them as a particle with some softening function that could correct for non-physical behaviours, hierarchical N-body codes that reduces the N^2 computations of the before type to orders of $Nlog(N)$ or N, fields methods of the order of N computations, between others.

Now, from the fact that some particles in the disk interact with both galactic halos more considerably and tend to change its behavior and acquire another configurations (tidal regions from now on), it is plausible to say, that such regions contain important physical information about the merger. Now, in the papers [2]-[3] a methodology to reconstruct a probability function of the initial merger orbital parameters from the observations of such regions is proposed. Being specific: the program uses techniques to define a mapping from the current morphology and kinematics of a tidal encounter back to the initial conditions (orbital parameters and time since pericenter). By requiring that various regions along a tidal feature all originate from a single disc with a unique orientation (this mapping can be used to derive the initial collision geometry). The galaxies components are assumed to follow a given model; that is, an exponential disk and NFW halo and bulb. It is remarkable to say, that the program do not take more than 20 minutes to compute a probability density function for the initial parameters based in some initial evolution system as input.

In this work we intend to do a first approximation to understand the identikit methodology as well as to try to contribute something to it by studying the relationship of the orbital parameters with the state vector of the galactic merger; when two velocity values are not known in the last one as well as one position parameter.

II. Objectives

Based in the state of the art, parameters for certain galactic mergers can be well bounded in little computational time, nevertheless an error of measurement in such parameters determined is not still clear.

For this reason and others the general idea of this project is to advance, in the solution of the system configuration space, by creating a program (from now on PP) which permit, with an error measurement, find the parameters that are enough to reproduce the morphology and kinematics of the interacting galaxies (specifically in a minor merger); trying to make the program to work with artificial intelligence (for which a rigorous statistic could be use), such that it associates automatically for the tidal regions the parameters determined for certain encounter between galaxies.

So, what is expected to do in this project is a first step for such goal. Defined by the following points:

1. Learn to manage some codes to create a given initial configuration of a merger between galaxies; without going in all the detail.
2. Learn how to manage at least two programs that evolve the system from a given configuration to another one after a given time.
3. Understand the method used to obtain from tidal features a probability density function of the possible parameters (in the way is proposed in 1.1 [2]).
4. Try to do basic programs that illustrate the principal ideas that underlie the method in 3 and propose something simple and new (contribution from this project).
5. Polish and advance further if it is possible; principally in the extraction from tidal features and the construction of the probability density function for the spins of the galaxies.

III. Chronogram

In order to do it, the following steps were proposed:

March 26-2
1. Study the program IDENTIKIT [1][2][3] that makes great part of the PP that is desired to do.
April 2-9
2. Begin with the study of the codes that reproduce a given initial configuration.
April 9-16
3. Begin with the study of at least two codes that permits to evolve the system and apply them first to simple systems.
April 16-23
4. Carry out objective 3.
April 23-30
5. Reproduce some of the result illustrated in [2] in order to justify the advancement in the project in the way proposed (try to do it with own data too).
May 1-7
6. Study Montecarlo methods and do basic codes. (Rejection and inversion examples).
May 7-14
7. Try to obtain a code and a process for the reconstruction of the probability density function of the parameters.
8 Two body problem study from the fact that galaxies have not forget its initial conditions; do it as analytical as possible.
May 14-21
9. Polish and advance further.

Each was designed to be carried out in seven days. Evidently, because the advancement in this program was complex, the initial schedule was changed in the way shown.

IV. Procedure results and analysis

About IDENTIKIT

The tool IDENTIKIT together with the library Zeno (necessary to use it) is useful to model the morphology of galaxies, the magnitude of tidal forces over test particles, the reconstruction and evolution of initial merger conditions, the efficient management of binary structures for N-body simulation data and efficient visualization. Additionally, the program uses Montecarlo methods to associate a probability density distribution to the stars in the configuration space, such that this one recreates in some degree the observational data. This is accomplish based in the behaviour that the system of particles have respect to the collisionless Boltzmann equation. Now, according to the article [2], the program is capable to bound the parameters that define an initial configuration of the merger, doing for that a search of the initial spins of each particle, that have a position respect to the center to the respective galaxy where it belong.

On the other hand, the program also permits to find the spins based in the observational information that could be obtained from the regions where tidal force are not negligible, which has a lot of sense, because such zones keep information about the dynamics of the pair interaction galaxies. It has been also understood that: for the obtained spins, the program works better when the studied zones are carefully selected, such that the regions were formed for the particles that belong to only one galaxy; under what models and physical suppositions creates the program the initial data for the galaxies, how the identification of the centers of each galaxy decrease the degrees of freedom that describe the merger; the reason for which worse results are obtained with galaxies with inclinations near to 90 degrees; how are mapped all the possible orientations (also parameters that are subject to be determined), useful criteria for chosen certain tidal regions, between others.

Generation of initial configurations

As a first step it was learned how to install locally the library Zeno; as it is in some degree specified at [4]. Then the installation of identikit was carried out as specified in [5].

The input for IDENTIKIT is the angular momentum of the particles (from now on called spins), each one respect to the galaxy where it belongs, and the evolution frames of the system.

To generate the initial spins of the particles how to manage Makefiles was learned, given the fact that there was a file name InitialData that had such format and that was designed to generate the initial conditions for the galaxies used in [2] for IDENTIKI. It was found that InitialData permitted to create galaxies characterized by a two body orbit. So, with this two class of parameters (orbital parameters, structure parameters stick to some given models) the configurations were created. The orbital parameters were specified by $r_{peri}, eccent\ \ and\ \ t_{peri}$, that are the minimum distance between the bodies in the orbit, its eccentricity and the time in which the observation is done happens, with zero time in the first pass through pericenter. Structure parameters were fixed to model the galaxies by exponential disk and NFW halos and bulbs. This parameters are the masses and scale parameters; the velocities on the other hand are constructed as specified in [1] ; already given all above parameters as input.

Evolution codes

Now, given such initial conditions the study of programs to evolve was done, from one side the code done in class was used and for a given system; results and code are found in [6], for the visualization, glnemo2 [7] was used; a short analysis for an example situation is present in [6]; where gadget units and format was used as a convection [8], to make easier the program and the analysis.

On the other hand, based in the paper of Joshua Barnes [2], auto-consistent data has been generated through the program treecode.c [2]. Therefore, the same tool was used to evolve the configurations generated with Zeno. The tool was learned to be manage under the instructions in [9]; a Makefile given with IDENTIKIT was used to evolve the systems.

How IDENTIKIT works?

Now, it is time what is so special about IDENTIKIT and how does it work. The following explanation is a rough summary:

From the the evolution frames it construct in a Montecarlo fashion (followed by and adjustment) the probability density function g_d, where $g_d d^3\boldsymbol{v} d^3\boldsymbol{r} d^3\boldsymbol{s_i}$ states what is the probability of having a particle with values between $\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{s_i}$ $(\boldsymbol{r} + d\boldsymbol{r}, \boldsymbol{v} + d\boldsymbol{v}, \boldsymbol{s_i} + d\boldsymbol{s_i})$. In mathematical terms g_d is constructed by doing the following counting:

$$g_d(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{s}) := \sum_{all\ i} \delta^3(\boldsymbol{r} - \boldsymbol{r_i})\delta^3(\boldsymbol{v} - \boldsymbol{v_i})\delta^2(\boldsymbol{s} - \boldsymbol{s_i})$$

and then doing an adjustment. Where the $\boldsymbol{s_i}$ are used to denote the initial angular momentum of a particle respect to its respective galaxy.

From this adjustment it is proposed to integrate over the tidal regions that contains the same feature, because only certain spins could have been capable to make this features to arise. Then, if we let $\boldsymbol{q'}\ \boldsymbol{R}^6$ be a region of phase space which is populated by tidal material from disc d, we can construct:

$$\Omega_d(\boldsymbol{s}, \boldsymbol{q}, t) := \sum_{all\ i} \delta^3(\boldsymbol{r} - \boldsymbol{r_i})\delta^3(\boldsymbol{v} - \boldsymbol{v_i}) g_d(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{s})$$

However, such feature as a whole is presumably populated by one disc with a unique set of spins $\boldsymbol{s_0}$, and these must lie within the intersection of the images of the individual regions; so assuming the probability density functions $\Omega(\boldsymbol{s}, \boldsymbol{q'_j})$ are independent for each region we can multiply them to find the probability of the disk of having a set of spins. As this is function of the orbital parameters from which the run was done, it depend on the orbital parameters, so , this function finishes being a density probability function of the orbital parameters and once constructed applying symmetries and marginalizing for the orbital parameters we obtain its probability density function.

Running IDENTIKIT

Finally, with the files that represented the evolution of the some of the systems presented in [2] the program IDENTIKIT was ran as specified in [5]. Now, given the lack of a manual to manage the program, this one was learned to be manage in an empirical way. Using in it, it was possible to reconstruct the density probability function of some fixed set of given orbital parameters (the more probable ones) as a function of two Euler angles that were let as free; figure 1 illustrates the result for a given configuration.

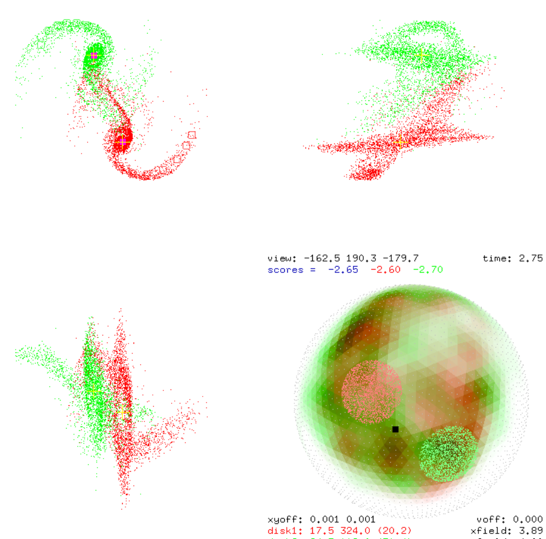


Fig. 1. The first three figures show the different observational projections: 1 above left the sky plane (X,Y), 2 above right (Vr,Y), 3 below left (X, Vr); where Vr represents the radial velocity. The last figure illustrates all the possible probability distributions that in terms of the independent Euler angles (their prouct is found in figure 2). It is proposed as a first approximation, the statistical independence of the distribution that arise from each region, such that the resultant probability is the product of all the density probabilities founded from each region.

Now, the process was not done for own data given the fact that although the format input for identikit was known after a great effort, the way in which the spins of the particles were saved (how are they measured in the configuration) is not still clear.

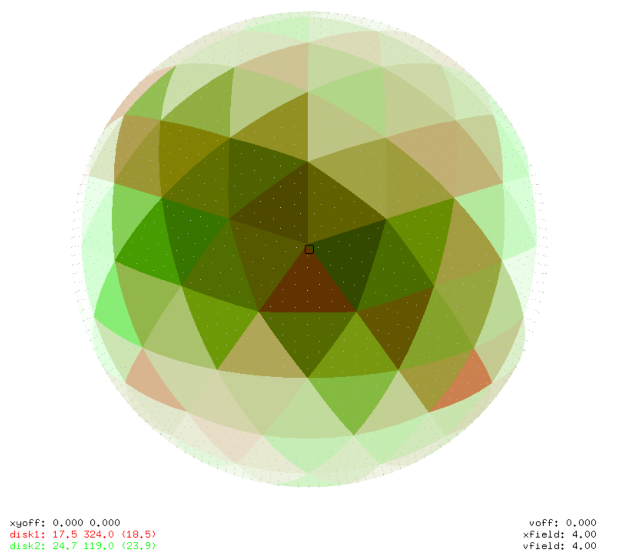


Fig. 2. Product of all the probability functions of a configuration for the more probable set of orbital parameter as a function of two Euler angles; the result is visually consistent with the one shown in [2].

Studying Montecarlo methods: inversion and rejection

Now, given the fact, as was explained above, that all this is based in the Montecarlo methods for the reconstruction of the density probabilities, the rejection method and inverse methods were programmed to construct a particular configuration (points distributed uniformly over a sphere of a given radius); another ones as the particles in the disk of a galaxy were programmed in python.

The principal idea from the inversion method is well summarized by the following procedure; in which U follows a uniform distribution and X a given cumulative function F_X:

$$u = F_U = P(u < U) = P(F^{-1}(u) = F^{-1}(U))$$
$$= P(x < X) = F_X(x)$$

this implies that values of x that underlies F_X can be simply generated as:

$$x = F^{-1}(u)$$

The synthetic sky follows: $F_{\Theta,\Phi} = \frac{1}{4\pi R^2}\sin\theta$ and for the inversion marginalization is first done. The result using this method is illustrated by 3 A.

The rejection method on the other hand is based in the fact that the probability of obtaining the value x can be approximated as $P(x)dx = P(x)\Delta x$, so the portion of the x are given by $N_x \approx P(x)\Delta x N_{sample}$. From this fact is observed that N_x is proportional to $P(x)$, that is the same to say that for a (x',P') value generated uniformly in some region that contain all relevant valus of the possible P's and x's, x will be accepted if $P' < P(x')$; the result from this method for the synthetic sky is observed in 3 B.

The consistency of both methods is evident from the fact that there is not visual difference between them and uniformity is clear.

Analytical solution of two body problem

The known two body problem is a good place to depart to try to make some contribution or to at least make important remarks.

The following method was proposed to gain insight about the relationship of the orbital parameters, which basically comes from the observation and justified priors for the distribution of the not observed (or at least not directly) parameters over time.

Our methodology consist into making the normal procedure that can be done to evolve the system from the observation of the vector state by supposing some distribution of probability for the not observe parameters of it, in order to evaluate later the distribution of some orbital parameter or any other characteristic; using Montecarlo for such reconstruction.

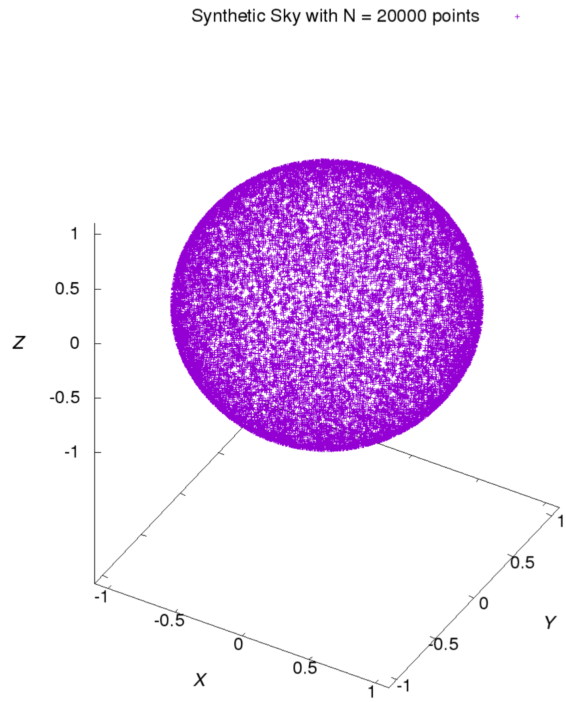
First, a transformation between the vector state and orbital parameters was carefully programmed.

To evaluate the consistency of it, the state vector of a known system was studied, that is, we supposes the Earth-Sun system in the following canonical units:

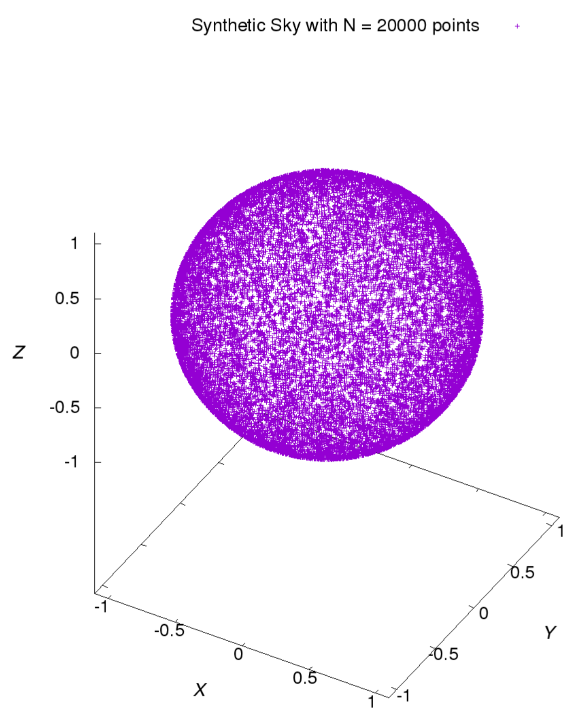
- 1UA = UL := Unit of length
- M_{sun} = UM := Unit of mass
- G_n=1, constraining in this way the system to have certain UT

Then with the initial conditions at the perigee, that is, $\boldsymbol{v} = (0, 0, 1)$ and $\boldsymbol{r} = (0, 1, 0)$ the system was rotated; task for which the rotation along any axis was programmed. After that the transformation was applied, being consistent with the parameters of (w,i,W,a,e,f) returned.

Now, to solve the the evolution of the system the transformation from the state vector to orbital parameters is applied, then with the values of a,e obtained, the orbital period obtained by:



(a) A



(b) B

Fig. 3. Synthetic uniform skies produced by the inverse (A) and rejection method (B).

$$w = \sqrt{\frac{G(m_1 + m_2)}{a^3}}$$

Then, with the time since pericenter specified, the normal anomaly is calculated, and from it we solve for the eccentric anomaly in the Kepler equation using bisection method; using consistent initial conditions for $M_{left} = -10$ and $M_{right} = 10$:

$$M = E - e\sin(E) \quad e <= 1$$

$$M = e\sinh E - E \quad > 1$$

It is valuable to remark that when the orbit is circular we defined W=0 and w=0, so by definition the nodal vector can be taken along the x-axis.

Study of the probability density given some priors

To begin, it is important to clarify what do we observe from the state vector in a pair of interacting galaxies. If they are separated enough we can measure the their positions (x,y) in some arbitrary units and their velocities along the radial, that are well approximated as perpendicular to the x and y unitary vectors. Therefore, in some degree we can assume that we have the values of x,y and v_z, but measured in a system that is not aligned with the plane of the orbit; because the orientation of the orbit is not known.

So, supposing that x, y and v_z can be in fact measured, we begin to study with the methodology proposed the Earth-Sun system already mentioned. But instead we suppose $v = (Vx, Vy, 1)$ and $r = (0, 1, Z)$ where the variables Vx, Vy and Vz were chosen to lie an uniform distribution in the intervals (-0.1,0.1).

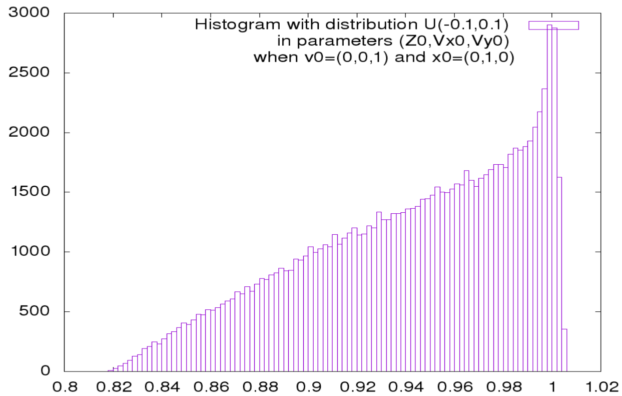
First, we studied the system distance after a period distribution for this case, using after the calculation of the orbital parameters the equation, evolution and calculation of E the following equation to calculate the distance:

$$r = a(1 - e\cos E) \quad e <= 1$$

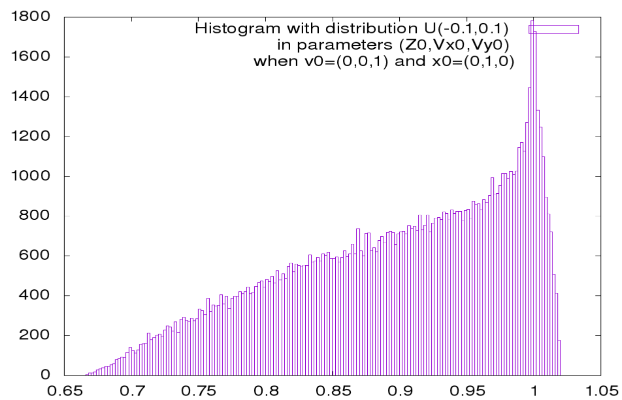
$$r = a(1 - e\cosh E) \quad e > 1$$

This in order to observe how it behaves and if the more probable value remained unchanged under such little "perturbation"; the result obtained is illustrates in figure 4 A; and in fact the more probable value is reproduced.

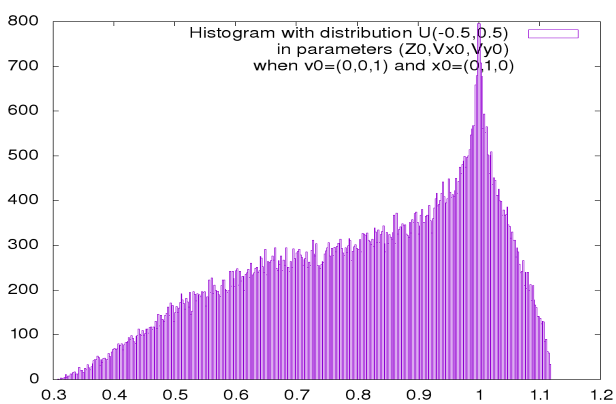
Also, a good exercise could be to increase the values of the uniform distributions, to see what happens with the set of possible orbits, we do it for U(-a,a) for Vx,Vy and Z. In figures 4 B, C and D we do it for values of a of 0.2, 0.5 and 0.7 respectively.



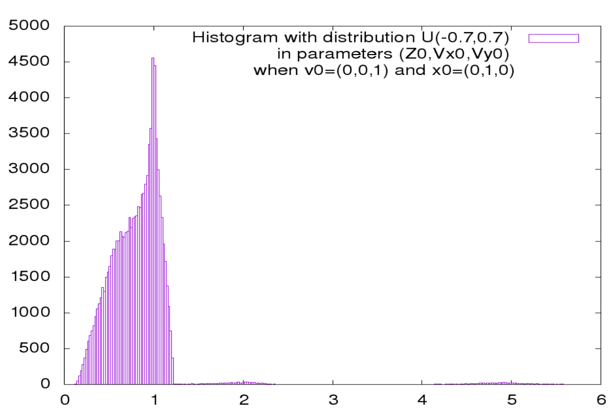
(a) A



(b) B



(c) C



(d) D

Fig. 4. A,B,C and D represent the probability density function of the distance between both bodies after a period of time for v0=(vx,vy,1) and r0=(0,1,z) where vx,vy and z follow U(-0.1,0.1), U(-0.2,0.2), U(-0.5,0.5) and U(-0.7,0.7) respectively.

It is observed that when the values of velocities are increase a greater number of mechanical states characterize the system, it is more clear in 4 where the values of velocity increase to such point in which it was capable to find a parabolic or hyperbolic orbit; note that the distance increase considerable with time.

Also, when the value of z increase and v_x and v_y are around 0, the orbit is less bound and, as energy is indirectly proportional to distance (rigorously $a \approx |E|^{-1}$) then some greater values for r are accepted after a "period". On the other hand when the values of z, vx and vy such that eccentricity does not change the angular momentum becomes greater and that makes the orbit to have more little distance after a "period".

V. Conclusions

- A good change and precise change between vector state and orbital parameters is of critical importance, numerical errors were presented in such transform and modifications were needed to be done.
- The analysis made, permit to observe that under conditions of little variation around the expected values the method is capable to reproduce a distribution of the possible distance after a period, that characterize a set of given orbits, such distribution can be a good place to star the possible orbits that the galaxies can have around each other; and then by using Bayesian inference repeatedly the configuration of the system could be actualized and more refined using the identikit procedure; this process could make possible to study the convergence of IDENTIKIT in some set of spins that reproduce the galaxies.
- This analysis also makes clear that the distribution does not vary smoothly when change from $e <= 1$ and $e > 1$, so care must be provided in such conditions.
- The method also shows that under the symmetry consideration of the uniform distribution the maximum expected distance was always the same; in some degree this permit us to see the importance of the possible obtainment of such condition in the determination of a simpler probability density function.
- Also, care when characterizing a given merger taking the maximum probability need to be done, because as observed the distribution of the parameter parameters does not need to be symmetric.
- Montecarlo methods have shown to be important because they permit a rapid study of the phenomena, are simple, not computational expensive and help us to construct theoretical model on solid basis.
- Some improvement that could be make to the program is to study the orbital parameters that result form making such assumptions in the state vector, the election of more centered distributions for the lack parameters as Gaussian ones, the implementation of dynamical friction in the evolution of the two body system under the assumption of a minor merge in order to make

simpler the problem, the study of the six dimensional distribution of the orbital parameters and the behaviour in its first statistical moments, the adjustment of the resultant probabilities, the implementation of a chi square adjustment test for the extended probability function g_d that could be used in some way to actualize later the priors done, between others.

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