homework2

October 14, 2018

1 Problem 1: Linear Regression

1.1 part 1

```
In [8]: import numpy as np
        import matplotlib.pyplot as plt
        import mltools as ml

        data = np.genfromtxt("data/curve80.txt",delimiter=None) # load the data
        X = data[:,0]
        X = np.atleast_2d(X).T # code expects shape (M,N) so make sure it's 2-dimensional
        Y = data[:,1] # doesn't matter for Y
        Xtr,Xte,Ytr,Yte = ml.splitData(X,Y,0.75) # split data set 75/25
        Ytr = Ytr[:, np.newaxis]
        Yte = Yte[:, np.newaxis]
        print(np.shape(Xtr))
        print(np.shape(Xte))
        print(np.shape(Ytr))
        print(np.shape(Yte))
```

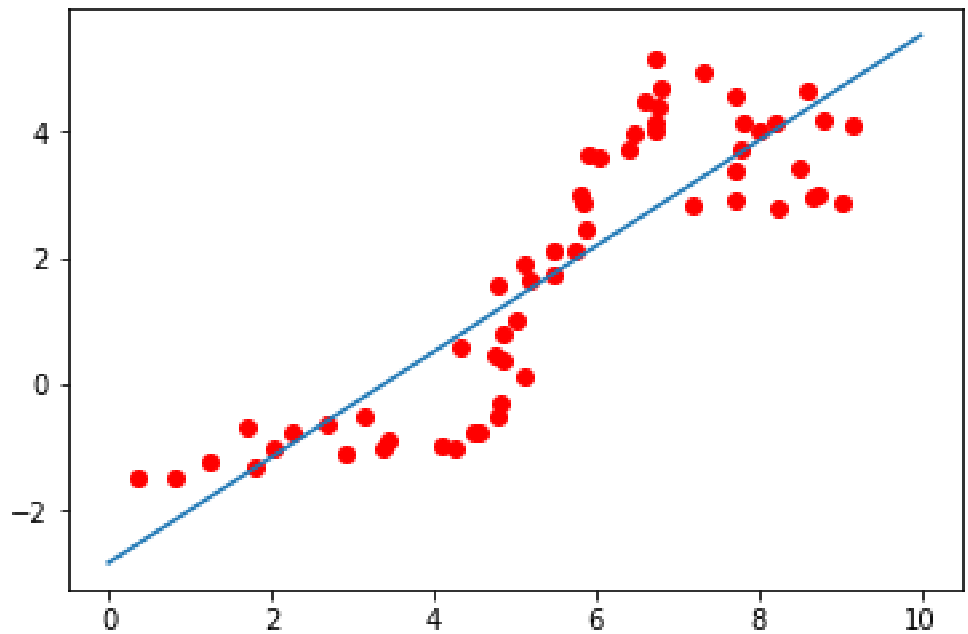
```
(60, 1)
(20, 1)
(60, 1)
(20, 1)
```

The Xtr and Ytr are lists with 60 elements. The Xte and Yte are lists with 20 elements.

1.2 part 2

```
In [9]: lr = ml.linear.linearRegress( Xtr, Ytr ) # create and train model
        xs = np.linspace(0,10,200)
        xs = xs[:,np.newaxis]
        ys = lr.predict( xs )
        plt.scatter(Xtr,Ytr,c='r')
        plt.plot(xs,ys)
        plt.show()
        print (lr.theta)
```

```
YTrainPred = lr.predict(Xtr)
YTestPred = lr.predict(Xte)
mseTrain = np.mean((YTrainPred - Ytr) ** 2)
mseTest = np.mean((YTestPred - Yte) ** 2)
print ( mseTrain)
print ( mseTest)
```



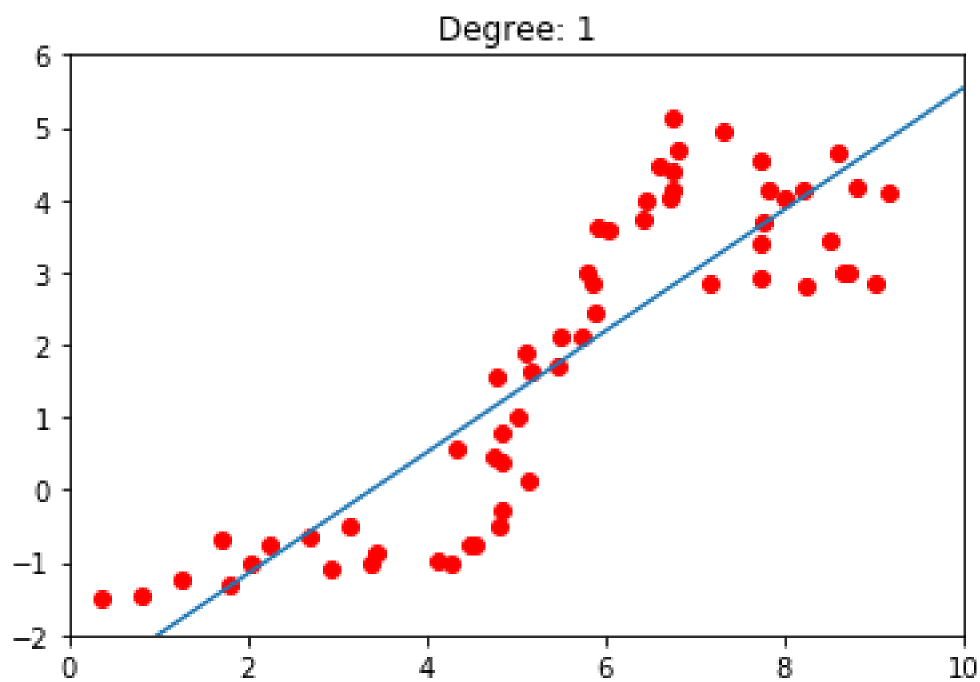
```
[[-2.82765049  0.83606916]]
1.1277119556093909
2.242349203010124
```

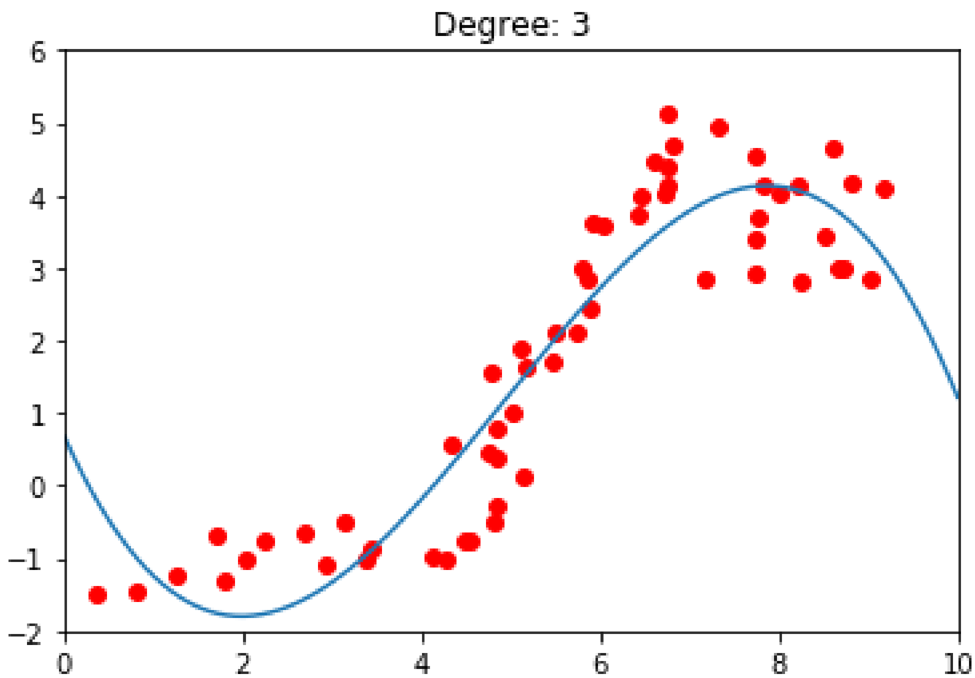
linear regression coefficients is -2.82765049 0.83606916
The mse for training data is 1.127, for test data is 2.24

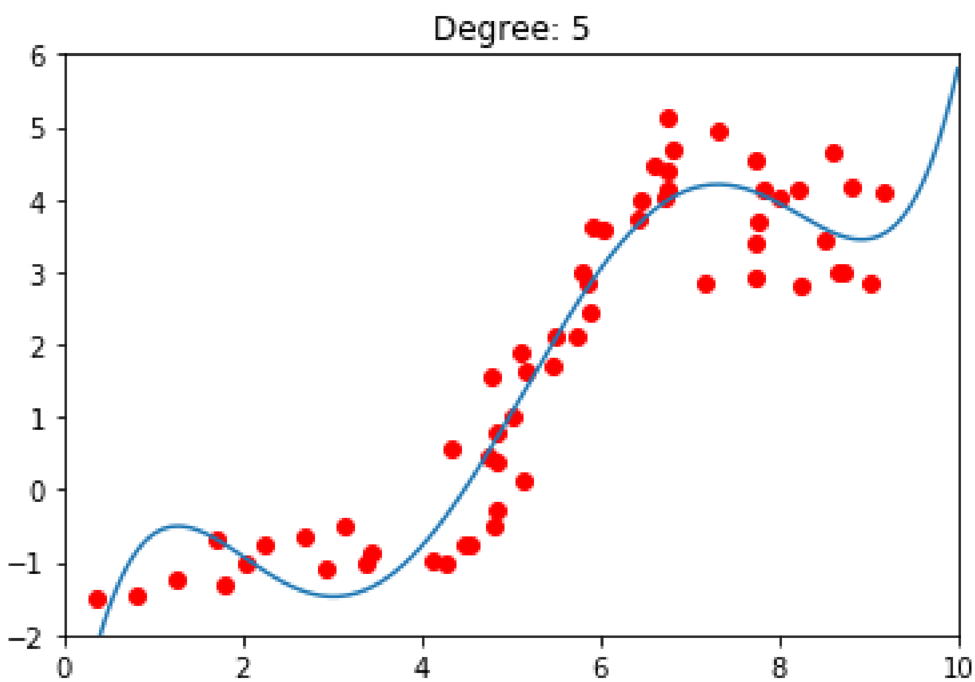
1.3 part 3

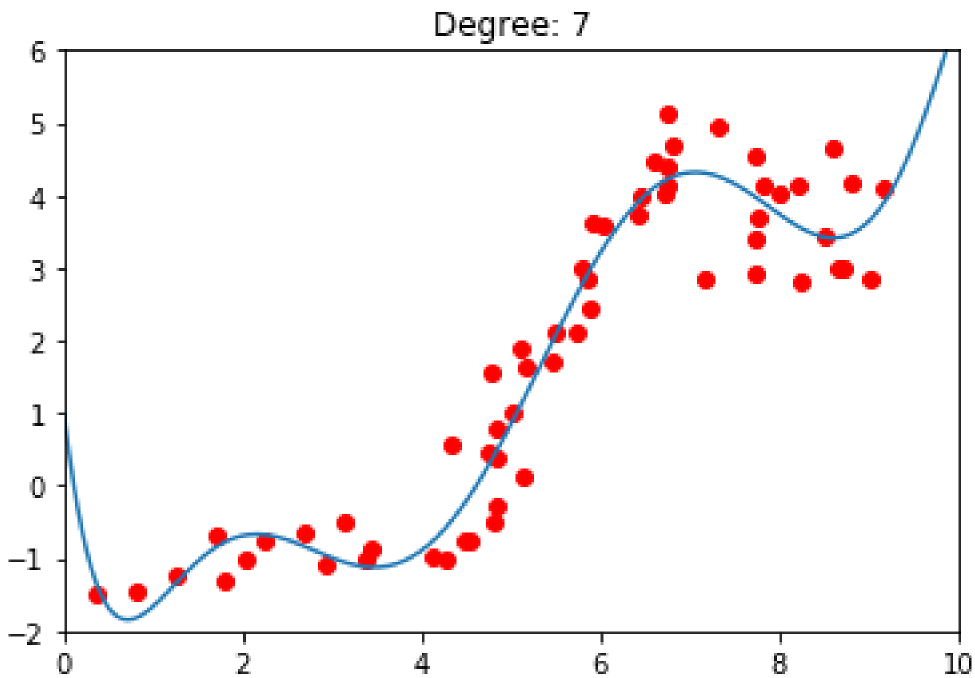
```
In [10]: degrees =[1, 3, 5, 7, 10, 18]
         mseTr=[]
         mseTe=[]
         for degree in degrees:
             XtrP = ml.transforms.fpoly(Xtr, degree, bias=False)
             # Rescale the data matrix so that the features have similar ranges / variance
             XtrP,params = ml.transforms.rescale(XtrP)
             # "params" returns the transformation parameters (shift & scale)
             # Then we can train the model on the scaled feature matrix:
```

```
    lr = ml.linear.linearRegress( XtrP, Ytr ) # create and train model
    # Now, apply the same polynomial expansion & scaling transformation to Xtest:
    XteP,_ = ml.transforms.rescale( ml.transforms.fpoly(Xte,degree,False), params)
    XsP,_= ml.transforms.rescale( ml.transforms.fpoly(xs,degree,False), params)
    ys = lr.predict( XsP )
    plt.scatter(Xtr,Ytr,c='r')
    ax = (0,10,-2,6) #set the axis
    plt.title("Degree: " + str(degree))
    plt.axis(ax)
    plt.plot(xs,ys)
    plt.show()
    Ytrp = lr.predict(XtrP)
    Ytep= lr.predict(XteP)
    mseTrain = np.mean((Ytrp - Ytr) ** 2)
    mseTest = np.mean((Ytep - Yte) ** 2)
    mseTr.append(mseTrain)
    mseTe.append(mseTest)
plt.semilogy(degrees, mseTr, c = 'red',label="training")
plt.legend()
plt.semilogy(degrees, mseTe, c = 'green',label="testing")
plt.legend()
plt.title("Error-Degree")
plt.show()
```

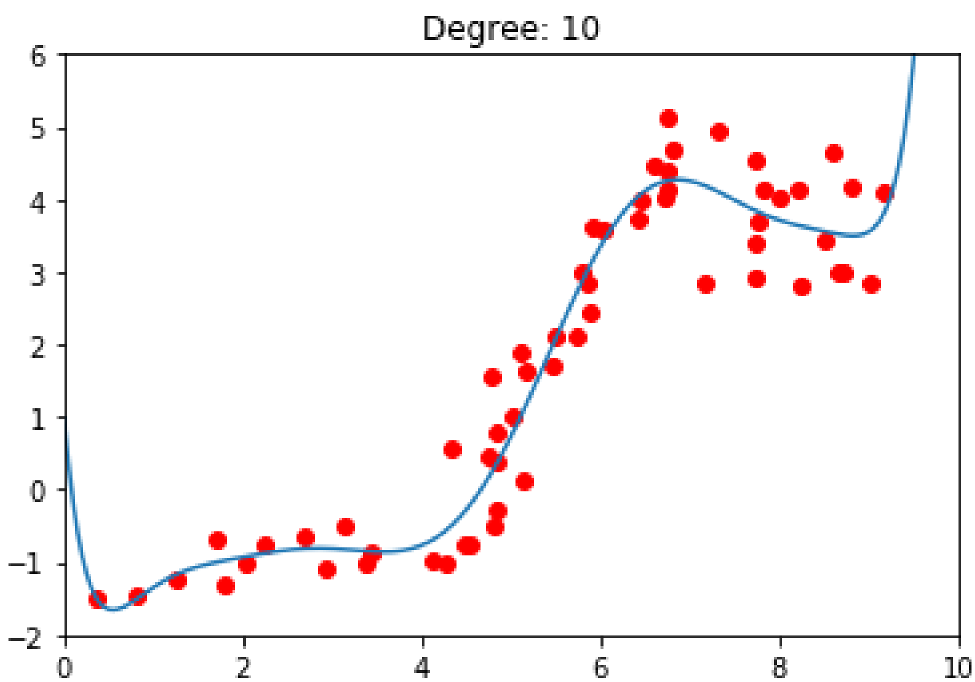



Degree: 3
6
5
4
3
2
1
0
−1
−2
0
2
4
6
8
10

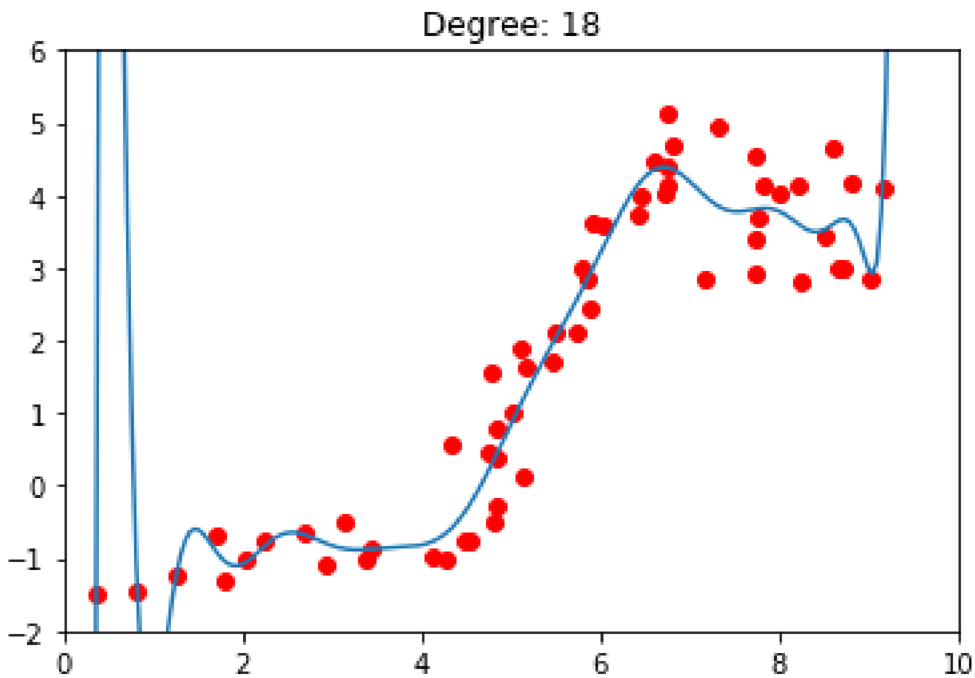

Degree: 5
6
5
4
3
2
1
0
−1
−2
0
2
4
6
8
10



Degree: 7
6
5
4
3
2
1
0
−1
−2
0
2
4
6
8
10



Degree: 10
6
5
4
3
2
1
0
−1
−2
0
2
4
6
8
10

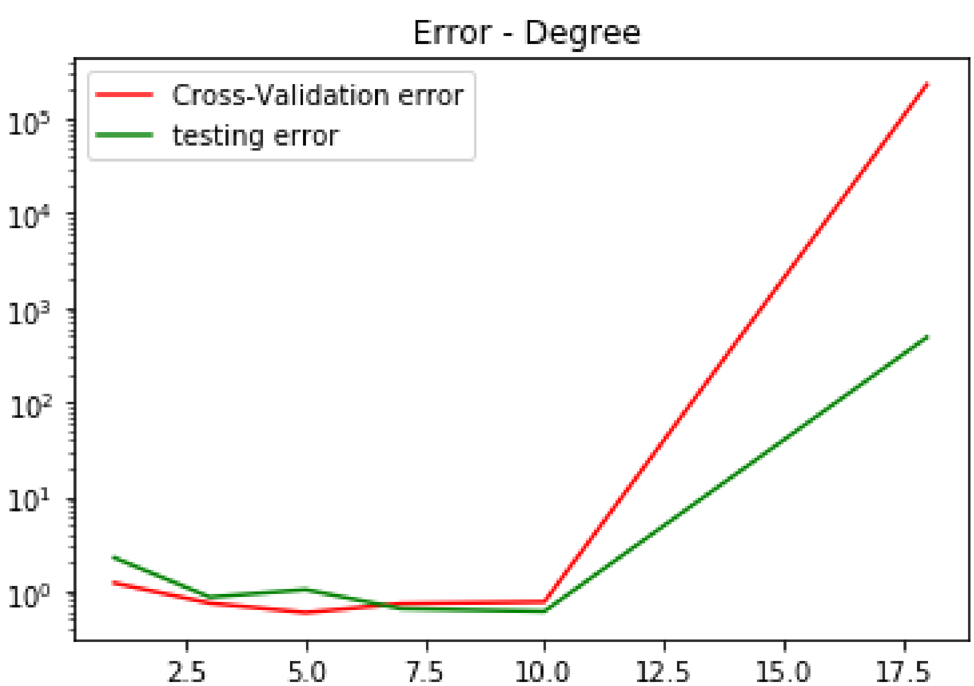


From the picture above, we can see that when degree is 10, the mse for Training reach there minimun while mse for Testing data is pretty small, so i will recommend degree to be 10.

2 Problem 2: Cross-validation

2.1 part 1

```
In [11]: d =[1, 3, 5, 7, 10, 18]
         res=[]
         for degree in d:
             nFolds = 5
             J=[]
             for iFold in range(nFolds):
                 Xti,Xvi,Yti,Yvi = ml.crossValidate(Xtr,Ytr,nFolds,iFold) # use ith block as va
                 Yti = Yti[:, np.newaxis]
                 Yvi = Yvi[:, np.newaxis]
                 XtiP = ml.transforms.fpoly(Xti, degree, bias=False)
                 # Rescale the data matrix so that the features have similar ranges / variance
                 XtiP,params = ml.transforms.rescale(XtiP)
                 learner = ml.linear.linearRegress(XtiP,Yti) #  train on Xti, Yti
                 XviP,_= ml.transforms.rescale( ml.transforms.fpoly(Xvi,degree,False), params)
                 yip = learner.predict(XviP)
                 J.append(np.mean((yip - Yvi) ** 2))#cross-validation error
             res.append(np.mean(J))
         plt.semilogy(d, res, c = 'r',label="Cross-Validation error")
         plt.legend()
         plt.semilogy(d, mseTe, c = 'g',label="testing error")
         plt.legend()
         plt.title(' Error - Degree')
         plt.show()
```



2.2 part 2

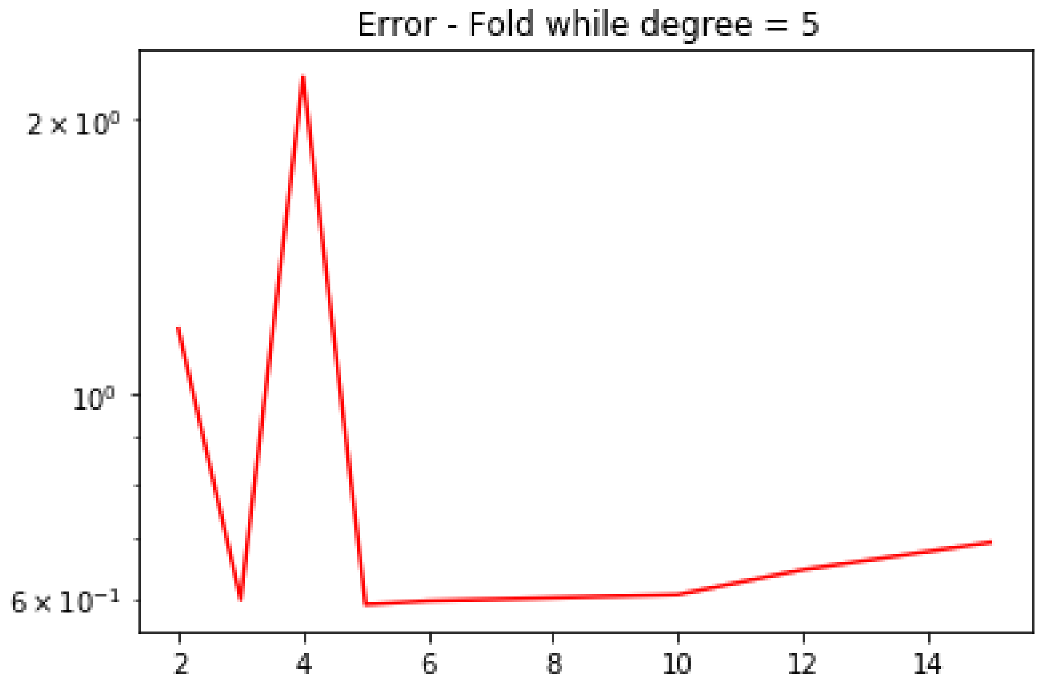
The MSE estimates from five-fold cross-validation has different lowest error point with degree = 5 compared to the MSEs evaluated on the actual test data while the lowest error point of it occurs when degree = 10, but the trend and pattern of them is pretty similar.

2.3 part 3

The mse for five-fold cross-validation reach the minimun when degree is 5, so i will recommend degree to be 5.

2.4 part 4

```
In [15]: Folds = [2, 3, 4, 5, 6, 10, 12, 15]
         tem=[]
         for Fold in Folds:
             J2=[]
             for iFold in range(Fold):
                 Xti,Xvi,Yti,Yvi = ml.crossValidate(Xtr,Ytr,Fold,iFold) # use ith block as val
                 Yti = Yti[:, np.newaxis]
                 Yvi = Yvi[:, np.newaxis]
                 XtiP = ml.transforms.fpoly(Xti, 5, bias=False)
                 # Rescale the data matrix so that the features have similar ranges / variance
                 XtiP,params = ml.transforms.rescale(XtiP)
                 learner = ml.linear.linearRegress(XtiP,Yti) #  train on Xti, Yti
                 XviP,_= ml.transforms.rescale( ml.transforms.fpoly(Xvi,5,False), params)
                 yip = learner.predict(XviP)
                 J2.append(np.mean((yip - Yvi) ** 2))#cross-validation error
             tem.append(np.mean(J2))
         plt.semilogy(Folds, tem, c = 'r')
         plt.title(' Error - Fold while degree = 5')
         plt.show()
```



We can see in the picture above, the error reaches its minimum when there are 5 folds. The cross-validation error changes sharpely when the number of folds is lower than 5, and is unusually high with 4-folds. Then the error increases as the number of folds increases from 5 to 15.

The reason is that small fold number may result in under-fitting while too much fold number results in over-fitting. Some data points in the data set that are clearly different from other data points cause a large error when divided by 4-folds .

3 Statement of Collaboration

I obey all the rules of UCI academic integrity and finish the project only by my own. Ziyang Zhang 14/10/2018