Sea $A(x_1, x_2, \cdots, x_n)$, donde $A \sim N(\mu, \sigma)$

Mostrar

$$L(\vec{x}; (\mu,\sigma)) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\rightarrow \ln(L(\vec{x}; (\mu,\sigma))) = n \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\rightarrow \frac{\partial}{\partial \mu} \ln(L(\vec{x}; (\mu,\sigma))) = \frac{-1}{2\sigma^2} \sum_{i=1}^{n} (-2)(x_i-\mu)^2$$

$$= \frac{1}{\sigma^2}\left(-n\mu + \sum_{i=1}^{n} x_i\right) = 0$$

$$\Rightarrow n\mu = \sum_{i=1}^{n} x_i \Rightarrow \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{X}$$

$$\frac{\partial}{\partial \sigma^2} \ln(L(\vec{x}; (\mu,\sigma))) = \frac{-n}{2} \frac{2\pi}{2\pi\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^{n} (\cancel{x_i} - \mu)^2$$

$$\frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^{n} (x_i-\mu)^2 = 0 \longrightarrow \frac{n}{2\sigma^4} \cdot 2\sigma^2 = \sum_{i=1}^{n} (x_i-\mu)^2$$

$$\rightarrow n\sigma^2 = \sum_{i=1}^{n} (x_i-\mu)^2 \rightarrow \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\hat{\mu})^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{X})^2$$