

$$\begin{aligned}
 g) \text{ dada } f(x) &\approx f(x_0) + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) \\
 &\quad \quad \quad - x_1 - x_0 \\
 &\cong f(x_0) + f[x_0, x_1]x - f[x_0, x_1]x_0 + f[x_0, x_1, x_2](x^2 - x(x_1+x_0) + x_0x_1) \\
 &\cong f(x_0) + f[x_0, x_1]x - f[x_0, x_1]x_0 + f[x_0, x_1, x_2]x^2 - x(x_1+x_0)f[x_0, x_1, x_2] \\
 &\quad + f[x_0, x_1, x_2]x_0x_1 \\
 &\cong \underbrace{f[x_0, x_1, x_2]x^2}_a + \underbrace{(f[x_0, x_1] - (x_1+x_0)f[x_0, x_1, x_2])x}_b + \underbrace{f(x_0) - f[x_0, x_1]x_0 + f[x_0, x_1, x_2]x_0x_1}_c
 \end{aligned}$$

h) $f(x) \cong a(x-x_2)^2 + b(x-x_2) + c$
 Vemos que si evaluamos $f(x)$ en el punto x_2 ,
 entonces: $\boxed{f(x_2) = c}$

Reemplazando: $f(x_1) = a(x_1-x_2)^2 + b(x_1-x_2) + f(x_2)$
 $f(x_1) - f(x_2) = f(x_2) - f(x_1) = -a(x_1-x_2)^2 - b(x_1-x_2)$

$$\begin{aligned}
 f(x_2) - f(x_1) + a(x_1-x_2)^2 &= -b(x_1-x_2) \\
 -b &= \frac{f(x_2) - f(x_1)}{(x_1-x_2)} + a(x_1-x_2) = -\frac{f(x_2) - f(x_1)}{(x_2-x_1)} - a(x_2-x_1)
 \end{aligned}$$

$$b = \frac{f(x_2) - f(x_1)}{(x_2-x_1)} + a(x_2-x_1) = \boxed{f[x_1, x_2] + ah_2 = b}$$

Ahora, para hallar a , evaluemos $f(x)$ en x_0

$$f(x_0) = a(x_0 - x_2)^2 + b(x_0 - x_2) + f(x_2)$$

$$f(x_0) - f(x_2) = -(f(x_2) - f(x_0)) \rightarrow \text{Lo que podemos escribir como}$$

$$- \left(\frac{f(x_1) - f(x_0)}{(x_1 - x_0)} (x_1 - x_0) + \frac{f(x_2) - f(x_1)}{(x_2 - x_1)} (x_2 - x_1) \right)$$

$$= -(f[x_0, x_1](x_1 - x_0) + f[x_1, x_2](x_2 - x_1)) = a(x_0 - x_2)^2 + b(x_0 - x_2)$$

$$\text{Donde } h_1 = x_1 - x_0 \text{ y } h_2 = x_2 - x_1$$

$$f[x_0, x_1]h_1 + f[x_1, x_2]h_2 = b(h_1 + h_2) - (h_1 + h_2)^2 a$$

$$f[x_0, x_1]h_1 + f[x_1, x_2]h_2 = bh_1 + bh_2 - (h_1 + h_2)^2 a$$

$$f[x_0, x_1]h_1 + f[x_1, x_2]h_2 = f[x_1, x_2]h_1 + ah_2h_1 + f[x_1, x_2]h_2 + ah_2^2 - (h_1 + h_2)^2 a$$

$$(f[x_0, x_1] - f[x_1, x_2])h_1 = ah_2h_1 + ah_2^2 - (h_1 + h_2)^2 a$$

$$= (h_2h_1 + h_2^2 - h_1^2 - 2h_1h_2 - h_2^2) a$$

$$= -(h_1h_2 + h_1^2) a$$

$$f[x_1, x_2] - f[x_0, x_1] = (h_2 + h_1) a$$

$$\boxed{\frac{f[x_1, x_2] - f[x_0, x_1]}{h_2 + h_1} = a}$$