Ejercicios de integración:

1) Sea $I = \int_a^b f(x) \cong \int_a^b P_1(x)\,dx$, donde $P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

$$\cong \frac{f(a)}{a-b}\int_a^b x-b\,dx + \frac{f(b)}{b-a}\int_a^b x-a\,dx$$

donde, resolviendo cada integral:

$$\int_a^b x-b\,dx = \left(\frac{x^2}{2} - bx\right)\Big|_a^b = \frac{b^2}{2} - b^2 - \frac{a^2}{2} + ab = -\frac{b^2}{2} - \frac{a^2}{2} + ab$$

$$\int_a^b x-a\,dx = \left(\frac{x^2}{2} - ax\right)\Big|_a^b = \frac{b^2}{2} - ab - \frac{a^2}{2} + a^2 = \frac{b^2}{2} + \frac{a^2}{2} - ab$$

$$\frac{f(a)}{a-b}\left(-\frac{b^2}{2} - \frac{a^2}{2} + ab\right) + \frac{f(b)}{-b+a}\left(\frac{b^2}{2} - \frac{a^2}{2} + ab\right)$$

$$(f(a)+f(b))\left(\frac{1}{a-b}\left(-\frac{b^2}{2} - \frac{a^2}{2} + ab\right)\right)$$

$$= (f(a)+f(b))\left(\frac{-1}{2(a-b)}\left(a^2 - 2ab + b^2\right)\right) = (f(a)+f(b))\left(-\frac{(a-b)^2}{2(a-b)}\right)$$

$$= (f(a)+f(b))\left(\frac{b-a}{2}\right) \cong \int_a^b f(x)\,dx$$