**Strategic Produced Water Management Optimization Model**

November 2021

Mathematical Notation

**Sets**

Time periods (i.e. weeks)

Well pads

Production pads (subset of well pads )

Completions pads (subset of well pads )

Freshwater sources

Disposal sites

Storage sites

Treatment sites

Beneficial reuse options

Network nodes

Locations (superset of well pads, disposal sites, nodes, …)

Pipeline diameters

Storage capacities

Treatment capacities

Injection (i.e. disposal) capacities

Production-to-completions pipeline arcs

Production-to-node pipeline arcs

Production-to-production pipeline arcs

Completions-to-node pipeline arcs

Completions-to-completions pipeline arcs

Node-to-node pipeline arcs

Node-to-completions pipeline arcs

Node-to-disposal pipeline arcs

Node-to-storage pipeline arcs

Node-to-treatment pipeline arcs

Node-to-beneficial reuse pipeline arcs

Freshwater-to-completions pipeline arcs

Treatment-to-node pipeline arcs

Treatment-to-completions pipeline arcs

Treatment-to-disposal pipeline arcs

Treatment-to-storage pipeline arcs

Storage-to-node pipeline arcs

Storage-to-completions pipeline arcs

Storage-to-disposal pipeline arcs

Storage-to-treatment pipeline arcs

Storage-to-beneficial reuse pipeline arcs

Production-to-completions trucking arcs

Freshwater-to-completions trucking arcs

Production-to-disposal trucking arcs

Production-to-storage trucking arcs

Production-to-treatment trucking arcs

Production-to-beneficial reuse trucking arcs

Completions-to-disposal trucking arcs

Completions-to-storage trucking arcs

Completions-to-treatment trucking arcs

Completions-to-completions trucking arcs (flowback reuse)

Storage-to-completions trucking arcs

Storage-to-disposal trucking arcs

Treatment-to-disposal trucking arcs

**Continuous Variables**

Produced water piped from one location to another location

Produced water trucked from one location to another location

Fresh water sourced from source to completions pad

Water put into completions pad storage

Water removed from completions pad storage

Produced water delivered for completions reuse

Produced water delivered for disposal

Water level at storage site at the end of the time period

Water level in completions pad storage at the end of the time period

Total volume of water trucked

Total volume of freshwater sourced

Total volume of produced water disposed

Total volume of produced water reused

Cost of piping produced water from one location to another location

Cost of trucking produced water from one location to another location

Cost of sourcing fresh water from source to completions pad

Cost of injecting produced water at disposal site

Cost of treating produced water at treatment site

Cost of reusing produced water at completions site

Cost of storing produced water at storage site (incl. treatment)

Credit for retrieving stored produced water from storage site

Total cost of sourcing freshwater

Total cost of injecting produced water

Total cost of treating produced water

Total cost of reusing produced water

Total cost of piping produced water

Total cost of storing produced water

Total cost of trucking produced water

Total cost of slack variables

Total credit for withdrawing produced water

Disposal capacity in a given time period at disposal site

Storage capacity in a given time period at storage site

Treatment capacity in a given time period at treatment site

Flow capacity in a given time period between two locations

Capital cost of constructing or expanding disposal capacity

Capital cost of constructing or expanding piping capacity

Capital cost of constructing or expanding storage capacity

Capital cost of constructing or expanding treatment capacity

Slack variable to meet the completions water demand

Slack variable to process produced water production

Slack variable to process flowback water production

Slack variable to provide necessary pipeline capacity

Slack variable to provide necessary storage capacity

Slack variable to provide necessary disposal capacity

Slack variable to provide necessary treatment capacity

Slack variable to provide necessary beneficial reuse capacity

**Binary Variables**

New pipeline installed between one location and another location with specific diameter

New or additional storage facility installed at storage site with specific storage capacity

New or additional treatment facility installed at treatment site with specific treatment capacity

New or additional disposal facility installed at disposal site with specific injection capacity

Directional flow between two locations

Timing of pipeline installation between one location and another location with specific diameter

Timing of storage facility installation at storage site with specific storage capacity

Timing of disposal facility installation at disposal site with specific injection capacity

**Parameters**

Completions demand at a completions site in a time period

Total water demand over the planning horizon

Produced water supply forecast for a production pad

Flowback supply forecast for a completions pad

Total water production (production & flowback) over the planning horizon

Initial weekly pipeline capacity between two locations

Initial weekly disposal capacity at a disposal site

Initial storage capacity at a storage site

Storage capacity at completions site

Initial weekly treatment capacity at a treatment site

Initial weekly reuse capacity at a reuse site

Weekly freshwater sourcing capacity at freshwater source

Weekly truck offloading sourcing capacity per pad

Weekly truck offloading sourcing capacity per storage site

Weekly processing (e.g. clarification) capacity per pad

Weekly processing (e.g. clarification) capacity at storage site

Treatment efficiency at treatment site

Pipeline diameter installation or expansion increments [inch]

Pipeline capacity installation or expansion increments

Disposal capacity installation or expansion increments

Storage capacity installation or expansion increments

Treatment capacity installation or expansion increments

Truck capacity

Disposal construction or expansion lead time

Storage construction or expansion lead time

Pipeline construction or expansion lead time

Drive time between two pads

Drive time from a pad to a disposal site

Drive time from a pad to a storage site

Drive time from a pad to a treatment site

Drive time from a pad to a reuse site

Drive time from a storage site to a completions site

Drive time from a storage site to a disposal site

Drive time from a treatment site to a disposal site

Initial storage level at storage site

Initial storage level at completions site

Terminal storage level at storage site

Terminal storage level at completions site

Pipeline segment length

Disposal construction or expansion capital cost for selected capacity increment

Storage construction or expansion capital cost for selected capacity increment

Treatment construction or expansion capital cost for selected capacity increment

Pipeline construction or expansion capital cost for selected diameter [$/inch-mile]

Disposal operational cost

Treatment operational cost (may include “clean brine”)

Completions reuse operational cost

Storage deposit operational cost

Storage withdrawal operational credit

Pipeline operational cost

Trucking hourly cost (by source)

Fresh sourcing cost

Big-M flow parameter

Slack cost parameter

Slack cost parameter

Slack cost parameter

Slack cost parameter

Slack cost parameter

Slack cost parameter

Slack cost parameter

Slack cost parameter

Mathematical Program Formulation

**Objectives**

Two objective functions can be considered for the optimization of a produced water system: first, the **minimization of costs**, which includes operational costs associated with procurement of fresh water, the cost of disposal, trucking and piping produced water between well pads and treatment facilities, and the cost of storing, treating and reusing produced water. Capital costs are also considered due to infrastructure build out such as the installation of pipelines, treatment, and storage facilities. A credit for (re)using treated water is also considered, and additional slack variables are included to facilitate the identification of potential issues with input data. The second objective is the **maximization of water reused** which is defined as the ratio between the treated produced water that is used in completions operations and the total produced water coming to surface.

*(1) Minimize Costs*

*(2) Maximize Reuse*

**Completions Pad Demand Balance**

Completions pad demand can be met by trucked or piped water moved into the pad in addition to water in completions pad storage. For each completions pad and for each time period, completions demand at the given pad is equal to the sum of all piped and trucked water moved into the completions pad plus water removed from the pad storage minus water put into the pad storage plus a slack.

**Completions Pad Storage Balance**

Sets the storage level at the completions pad. For each completions pad and for each time period, completions pad storage is equal to storage in last time period plus water put in minus water removed. If it is the first time period, the pad storage is the initial pad storage.

**Completions Pad Storage Capacity**

The storage at each completions pad must always be at or below its capacity in every time period.

**Terminal Completions Pad Storage Level**

The storage in the last period must be at or below its terminal storage level.

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**Freshwater Sourcing Capacity**

For each freshwater source and each time period, the outgoing water from the freshwater source is below the freshwater capacity.

**Completions Pad Truck Offloading Capacity**

For each completions pad and time period, the volume of water being trucked into the completions pad must be below the trucking offloading capacity.

**Completions Pad Processing Capacity**

For each completions pad and time period, the volume of water (excluding freshwater) coming in must be below the processing limit.

Note: this constraint has not actually been implemented yet.

**Storage Site Truck Offloading Capacity**

For each storage site and each time period, the volume of water being trucked into the storage site must be below the trucking offloading capacity for that storage site.

**Storage Site Processing Capacity**

For each storage site and each time period, the volume of water being trucked into the storage site must be less than the processing capacity for that storage site.

**Production Pad Supply Balance**

All produced water must be accounted for. For each production pad and for each time period, the volume of outgoing water must be equal to the forecasted produced water for the production pad.

**Completions Pad Supply Balance (i.e. Flowback Balance)**

All flowback water must be accounted for. For each completions pad and for each time period, the volume of outgoing water must be equal to the forecasted flowback produced water for the completions pad.

**Network Node Balance**

Flow balance constraint (i.e., inputs are equal to outputs). For each pipeline node and for each time period, the volume water into the node is equal to the volume of water out of the node.

**Bi-Directional Flow**

There can only be flow in one direction for a given pipeline arc in a given time period. Flow is only allowed in a given direction if the binary indicator for that direction is “on”.

Note: Technically this constraint should only be enforced for truly reversible arcs (e.g. *NCA* and *CNA*; and even then it only needs to be defined per one reversible arc (e.g. *NCA* only and not *NCA* and *CNA*).

**Storage Site Balance**

For each storage site and for each time period, if it is the first time period, the storage level is the initial storage. Otherwise, the storage level is equal to the storage level in the previous time period plus water inputs minus water outputs.

**Terminal Storage Level**

For each storage site, the storage in the last time period must be less than or equal to the predicted/set terminal storage level.

**Pipeline Capacity Construction/Expansion**

Sets the flow capacity in a given pipeline during a given time period. Different constraints apply depending on if the pipeline is realistically reversible or not.

Note: Parameter will be calculated as follows:

where is Hazen-Williams constant and is Hazen-Williams exponent as per Cafaro & Grossmann (2020) and represents the pipeline diameter as per the set .

**Storage Capacity Construction/Expansion**

Update the storage capacity variable. If expansion/construction is selected, expand the capacity by the set expansion amount. The water level at the storage site must be less than this capacity. As of now, the model considers that a storage facility is expanded or built at the beginning of the planning horizon.. The C0 notation indicates that we also include the 0th case, meaning that there is no selection in the set C where no capacity is added.

**Disposal Capacity Construction/Expansion**

Update the disposal capacity variable. If expansion/construction is selected, expand the capacity by the set expansion amount. The total disposed water in a given time period must be less than this new capacity.

**Treatment Capacity Construction/Expansion**

Similarly to Disposal and Storage Capacity Construction/Expansion constraints, the current treatment capacity can be expanded as required or new facilities may be installed.

**Treatment Balance**

Water input into treatment facility is treated with a level of efficiency, meaning only a given percentage of the water input is outputted to be reused at the completions pads.

where

**Beneficial Reuse Capacity**

For each beneficial reuse site and for each time period, water sent to a site must be less than or equal to the capacity.

**Fresh Sourcing Cost**

For each freshwater source, for each completions pad, and for each time period, the freshwater sourcing cost is equal to all output from the freshwater source times the freshwater sourcing cost.

**Total Fresh Sourced Volume**

The total fresh sourced volume is the sum of freshwater movements by truck and pipeline over all time periods, completions pads, and freshwater sources.

**Disposal Cost**

For each disposal site, for each time period, the disposal cost is equal to all water moved into the disposal site multiplied by the operational disposal cost. Total disposal cost is the sum of disposal costs over all time periods and all disposal sites.

**Total Disposed Volume**

Total disposed volume over all time is the sum of all piped and trucked water to disposal summed over all time periods.

**Treatment Cost**

For each treatment site, for each time period, the treatment cost is equal to all water moved to the treatment site multiplied by the operational treatment cost. The total treatments cost is the sum of treatment costs over all time periods and all treatment sites.

**Completions Reuse Cost**

Completions reuse water is all water that meets completions pad demand, excluding freshwater. Completions reuse cost is the volume of completions reused water multiplied by the cost for reuse.

Note: Freshwater sourcing excluded from completions reuse costs.

**Total Completions Reuse Volume**

The total reuse volume is the total volume of produced water reused, or the total water meeting completions pad demand over all time periods, excluding freshwater.

**Piping Cost**

Piping cost is the total volume of piped water multiplied by the cost for piping.

Note: the constraints above explicitly consider freshwater piping via FCA arcs.

**Storage Deposit Cost**

Cost of depositing into storage is equal to the total volume of water moved into storage multiplied by the storage operation cost rate.

**Storage Withdrawal Credit**

Credits from withdrawing from storage is equal to the total volume of water moved out from storage multiplied by the storage operation credit rate.

**Trucking Cost (Simplified)**

Trucking cost between two locations for time period is equal to the trucking volume between locations in time t divided by the truck capacity [this gets # of truckloads] multiplied by the lead time between two locations and hourly trucking cost.

Note: the constraints above explicitly consider freshwater trucking via FCT arcs.

**Total Trucking Volume**

Get total trucking volume by summing trucking movements over all time periods and locations.

**Disposal Construction or Capacity Expansion Cost**

Cost related to expanding or constructing new disposal capacity. Takes into consideration capacity increment, cost for selected capacity increment, and if the construction/expansion is selected to occur.

**Storage Construction or Capacity Expansion Cost**

Cost related to expanding or constructing new storage capacity. Takes into consideration capacity increment, cost for selected capacity increment, and if the construction/expansion is selected to occur.

**Treatment Construction or Capacity Expansion Cost**

Cost related to expanding or constructing new treatment capacity. Takes into consideration capacity increment, cost for selected capacity increment, and if the construction/expansion is selected to occur.

**Pipeline Construction or Capacity Expansion Cost**

Cost related to expanding or constructing new pipeline capacity. Takes into consideration capacity increment, cost for selected capacity increment, and if the construction/expansion is selected to occur.

**Slack Costs**

Weighted sum of the slack variables. In the case that the model is infeasible, these slack variables are used to determine where the infeasibility occurs (e.g. pipeline capacity is not sufficient).

**Logic Constraints**

New pipeline or facility capacity constraints: e.g., only one injection capacity can be used for a given site.

**Deliveries Constraints**

Completions reuse deliveries at a completions pad in time period t is equal to all piped and trucked water moved into the completions pad, excluding freshwater. Disposal deliveries for disposal site k at time t is equal to all piped and trucked water moved to the disposal site k.

Terminology

**Beneficial Reuse Options:** This term refers to the reuse of water at mining facilities, farms, etc.

**Completions Demand:** Demand set by completions pads. This demand can be met by produced water, treated water, or freshwater.

**Completions Reuse Water:** Water that meets demand at a completions site. This does not include freshwater or water for beneficial reuse.

**Network Nodes:** These are branch points for pipelines only. Note: well pads are not a subset of network nodes.

**[t]:** This notation indicates that timing of capacity expansion has not yet been implemented.

**Terminal Storage Level:** These are goal storage levels for the final time period. Without this, the storage levels would likely be depleted in the last time period.

**Water Boosting:** Moving large volumes of water requires water pumps. Water boosting refers to the infrastructure required to maintain water pressure.