CSc 59866

Assignment due March 25, 2021

Adapted slightly from Exercise 8.3 in the Textbook

The FitzHugh-Nagumo model is a simplified model of neuronal dynamics that can demonstrate both excited and resting behaviors of neurons. In a normal setting, this system's state converges and stays at a stable equilibrium point (resting), but when perturbed, the system's state moves through a large cyclic trajectory in the phase space before coming back to the resting state, which is observed as a big pulse when plotted over time (excitation). Moreover, under certain conditions, this system can show a nonlinear oscillatory behavior that continuously produces a sequence of pulses. The behavioral shift between convergence to the resting state and generation of a sequence of pulses occurs as a Hopf bifurcation, where the external current is used as a control parameter. Here are the model equations:

$$\frac{dx}{dt} = c\left(x - \frac{x^3}{3} + y + z\right)$$

$$\frac{dy}{dt} = -\left(\frac{x - a + by}{c}\right)$$

z is the key parameter that represents the external current applied to the neuron. Other parameters are typically constrained as follows:

$$\left(1-\frac{2}{3}b\right)<a<1$$

$$0<b<1$$

$$b<c^2$$

With a = 0.7, b = 0.8, and c = 3, do the following:

- Numerically obtain the equilibrium point of this model for the following values of z, z ϵ {-2.0,-1.0, -0.5, 0.}. There is only one real equilibrium point in this system.
- Apply the result obtained above to the Jacobian matrix of the model, and numerically evaluate the stability of that equilibrium point for each value of z.
- Estimate the critical thresholds of z at which a Hopf bifurcation occurs. There are two such critical thresholds.
- Draw a series of its phase spaces with values of z from above to confirm your analytical prediction.
- Discuss your results.

Include all of your analysis and discussion in your .ipynb file and submit the file through Blackboard. The name of the file you submit should be

lastname_firstname_AS03.ipynb.

Do not clear your results after your last run so that I will be able to see your results without rerunning your file.

If you collaborate with anyone on this assignment, be sure to follow the collaboration guidelines in the syllabus including listing with whom you collaborated in your ipynb file.

While collaboration is fine, DO NOT submit exactly the same file as your collaborations. Your code and your discussion must be your own.