Approximation of Dynamic Thrust

Alpha UAV

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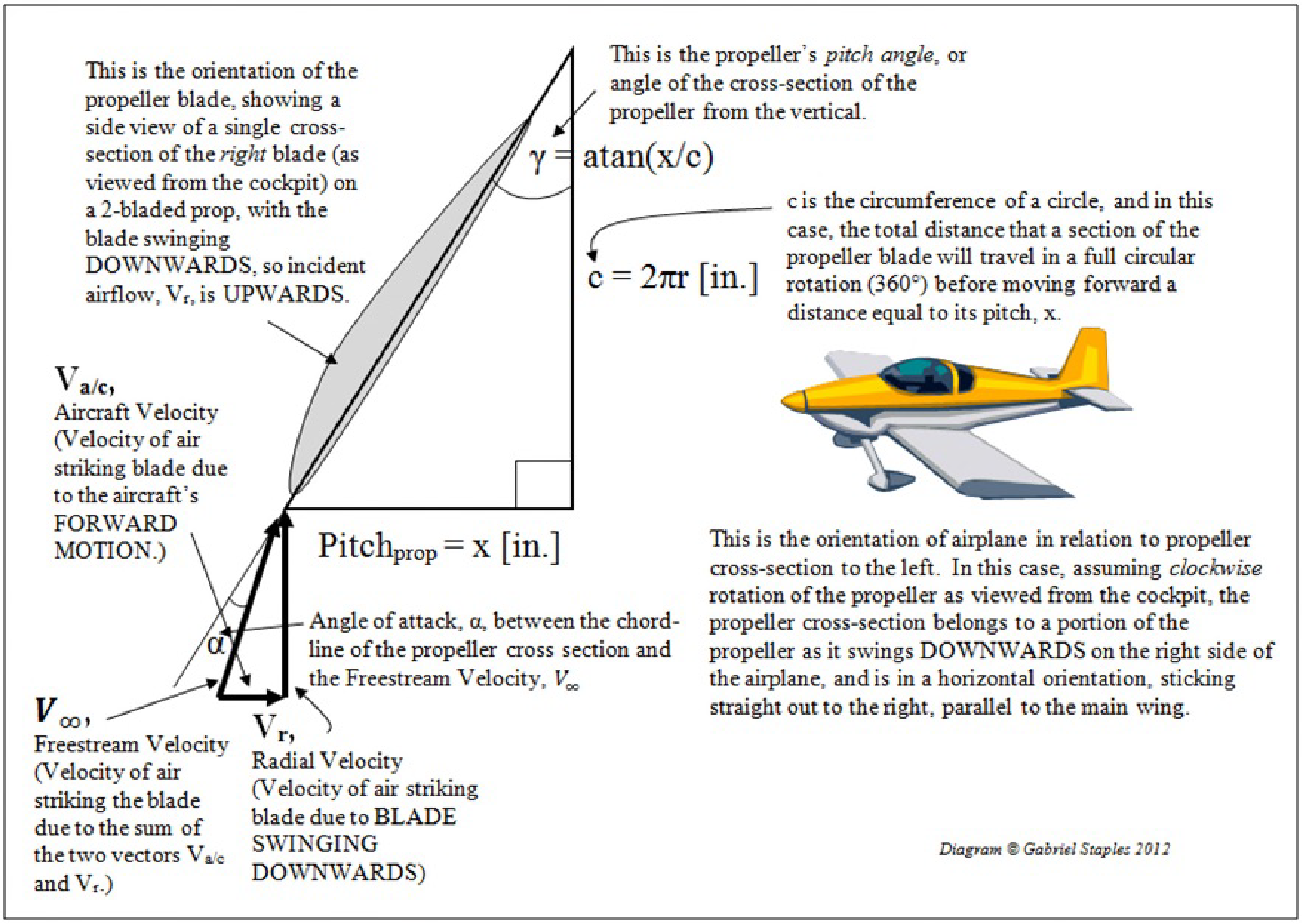
An attempt was made to approximate the dynamic thrust by analytical methods, which proved to be very inaccurate compared to experimental data in wind tunnels of similar propellers. Instead an experimental correction factor was used, as presented by Gabriel Staples' blog

Propeller basics

Some information about the propeller is needed to approximate its performance



For the current propeller (units in inches):

$$\underbrace{5''}_{radius} \times \underbrace{4.3''}_{pitch} \times \underbrace{3}_{n^o\ blades}$$

The pitch can be understood as: Advance through a soft solid (infinite friction) in one rotation.

Equation model

Nomenclature:

1. V_e Ideal speed added to the flow due to the propeller rotation
2. V_{ac} Speed of the free stream i.e aircraft
3. ρ Density. ρ_0 Density of static thrust test conditions.
4. T Thrust. T_0 Static thrust
5. A. Propeller area, $A = \pi r^2$
6. RPM revolutions per minute, RPS revolutions per second.

Starting form simple conservation of momentum:

$$T = \dot{m}(V_e - V_{ac}), \quad \dot{m} = \rho \cdot V_e \cdot A$$

Therefore the thrust for a given aircraft speed is given by:

$$T = \rho A(V_e^2 - V_e V_{ac})$$

The issue here is how to approximate the induced speed V_e, which is quite complicated (could be done with blade element theory). A simplification is used that assumes V_e to be proportional to the revolutions per second and the pitch of the propeller.

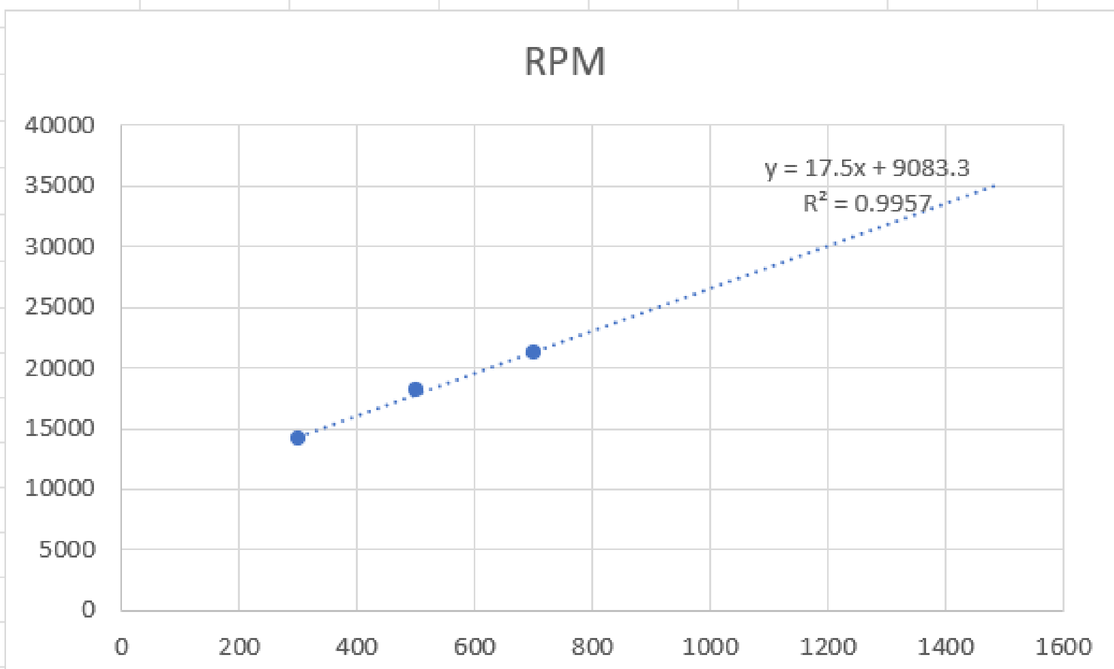
$$V_e = RPS \cdot pitch$$

In our case, as the thrust bench seemed to display wrong values of RPM, the throttle (originally -100 to +100, normalized after) was recorded instead. It is assumed that RPM are proportional to the throttle level.

$$V_e = V_e(throttle) = RPS_{max} \cdot throttle \cdot pitch$$

The maximum revolutions were extrapolated from the manufacturer data of the motor for the maximum achieved thrust at throttle = 1. Maximum RPM is approximated to be 28 000 for this motor.

Thrust	RPM
300	14200
500	18100
700	21200
1050	27458.3



Experimental correction

This V_e approximation is still really poor as assuming that each revolution moves the flow perfectly is far from realistic, so an experimental correction coefficient is additionally used on T, based on the static thrust data and other experimental data for similar propellers.

$$T = \rho A(V_e^2 - V_e V_{ac}) \cdot \left(k_1 \frac{2r}{pitch}\right)^{k_2}$$

$$\text{with} \quad V_e = V_e(throttle)$$

The correction factor depends on the diameter to pitch ratio which try to signify the efficiently of the propeller. This means that changing these parameters would allow us to see the proper trend in increase/decrease of thrust if desired. Additionally, the factor k_1 allows to tune the correction to our obtained static data, plus k_2 takes into account the non-linear relation with the diameter-pitch ratio.

Gabriel Staples proposes the value $k_2 = 1.5, k_1 = 0.035$ after fitting experimental data of several propellers with this model. In our case, the same value of k_2 is used as better approximation would only be useful if the changing of a parameter of the propeller was to be studied (for now, not important as we use a defined propeller). Instead, we approximate the k_1 value for each data point obtained in the static test (Thrust for a given throttle level, with $V_{ac} = 0$):

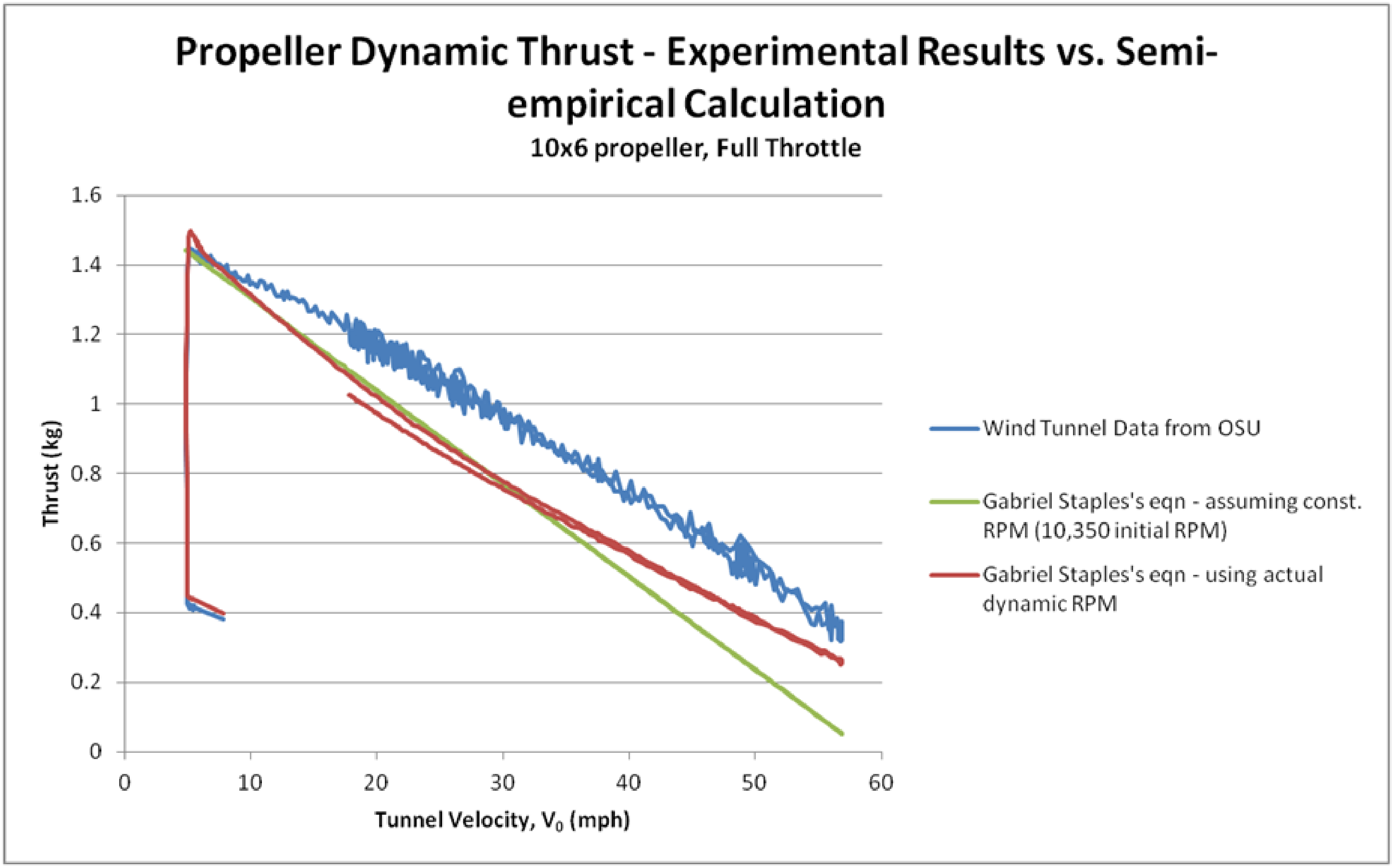
$$k_1 = \left(\frac{T_0}{\rho_0 A V_e^2}\right)^{1/k_2} \cdot \left(\frac{pitch}{2r}\right)$$

This yielded values between 0.34 (low throttle) and 0.068 (max throttle) with average 0.123, which seem reasonable.

Accuracy

According to the data used by Gabriel Staples, the equation underestimates the thrust by approximately 15%~30%. This means that the real zero-thrust velocity may be around 30% greater than the predicted one.

A comparison with wind tunnel data and equation is provided here:



Possible improvements

The model could be easily improved if the RPM where used instead of the throttle approximation, which would require the proper usage of the thrust bench.

As the dynamic thrust trend is linear with V_{ac}, obtaining a zero-thrust speed value experimentally (could be done by observing the maximum speed of the aircraft once is operational) or other values with a wind tunnel could significantly improve the accuracy ot the dynamic thrust prediction.

Matlab Script

A Matlab script was made to calculate dynamic thrust given propeller information and a static thrust test. The data is stored in the `data_base.m` script.

The calculation of dynamic thrust and plots is handled by `dynamic_thrust_1.m`.

Data base script

The data is stored in a `struct` object, each index (entry) corresponds to a propeller. The required data is:

1. Name or label
2. Propeller radius (metres)
3. Propeller pitch (metres)
4. Maximum RPS (revolutions per second)
5. Throttle vector (-100 to +100 format)
6. Power vector (Watts)
7. Thrust vector (grams)

The easiest is to copy the previous entry and change with the new values accordingly. The calculations will be applied to each entry of the data base.

Dynamic thrust calculations script

The input parameters are:

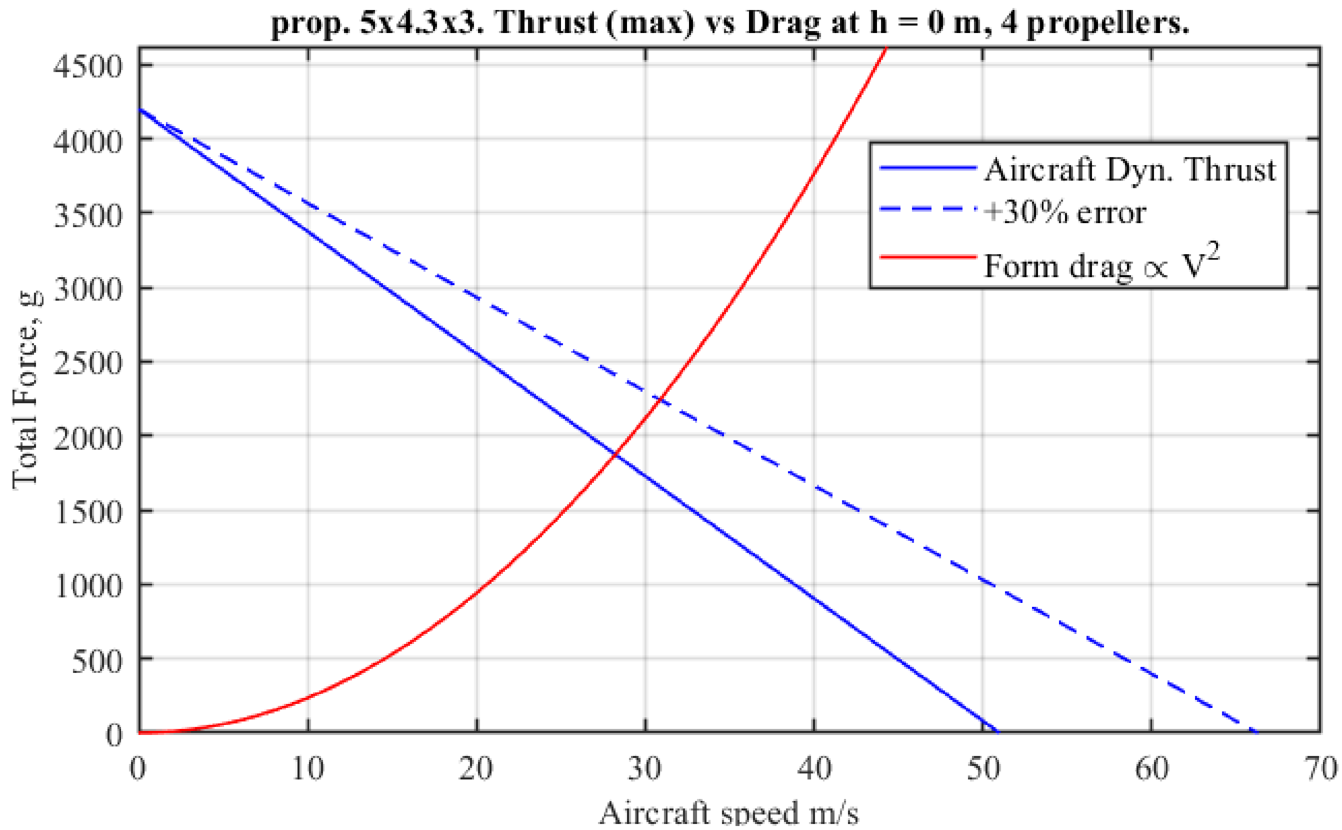
1. Altitude h, (geometric). This is used with an ISA atmosphere model to calculate ρ
2. Number of motors `n_motors`
3. Drag coefficient `CD = 0.0314` as of the current half scale design
4. Drag reference area `S_wing`
5. Propeller index to plot `prop_i`

The script calculates dynamic thrust for each throttle level added in the data base entry for each propeller in the data base. The plots are only performed on the propeller chosen but this could be easily changed if comparison of propellers required.

Results, half scale, 5x4.3x3 propeller

For the current half scale design and motors at full throttle, the predicted dynamic thrust and form drag is plotted below. the intersection of thrust and drag represents the maximal theoretical speed.

$$V_{max} \approx 30\ m/s = 108\ km/h$$



prop. 5x4.3x3. Thrust (max) vs Drag at h = 0 m, 4 propellers.
Aircraft Dyn. Thrust
+30% error
Form drag ∝ V2
Total Force, g
Aircraft speed m/s
0
500
1000
1500
2000
2500
3000
3500
4000
4500
0
10
20
30
40
50
60
70