Assignment 3 Option 2
Quantity To Produce Quality

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***Abstract*—This paper investigates the influence of the size of the bags, the number of randomly selected features and the number of members in linear and non-linear logistic regression ensemble models. Both the ensemble models are applied to classify five binary classification datasets that vary in complexity. The non-linear logistic regression ensemble can capture non-linear relationships between descriptive features in a dataset, which generally allows it to perform better than the linear logistic regression ensemble model. The size of the bags, the number of randomly selected features, and the number of ensemble members are highly problem-dependent and should all be considered together to optimise model performance on a given dataset.**

I. Introduction

Three critical parameters that must be optimised in an ensemble model are the size of the bags, the number of randomly selected features and the number of members in linear and non-linear logistic regression ensembles. Each of these parameters can significantly influence the performance of the ensemble if inappropriate values are selected.

This paper investigates the influence of the size of the bags, the number of randomly selected features and the number of members in linear and non-linear logistic regression ensemble models across five binary classification datasets that varies in complexity.

The non-linear logistic regression ensemble model generally performed better than the linear logistic regression ensemble model, as the non-linear ensemble was able to capture more complex relationships between descriptive features in the datasets. To create an optimal ensemble of a dataset, the size of the bags, the number of randomly selected features, and the number of members should all be considered together. All of these elements significantly influence how well the ensemble performs.

The rest of this paper is structured as follows: Section II provides background information on gradient descent optimisation, the logistic regression model, basis functions, ensemble learning. Additionally, background on bootstrap aggregating and the performance metrics used in this paper. Section III provides the implementation of the linear and non-linear logistic regression ensemble models. Section IV provides the empirical procedure and Section V provides the research results.

II. Background

This section presents background information on the gradient descent optimisation algorithm, logistic regression model, and ensemble learning. Additionally, background information on bootstrap aggregating, basis functions, and performance metrics used in this report.

A. Gradient Descent

The gradient descent algorithm was first introduced by Augustin-Louis Cauchy in 1847 [4]. Cauchy introduced gradient descent to solve optimisation systems of simultaneous equations through iterative optimisation to find the minimum of a function. Cauchy also introduced the step size parameter, now commonly referred to as the learning rate, to control how large the steps are for each iteration as the algorithm updates model parameters to reach an optimal solution.

The generic learning algorithm of gradient descent is represented by Algorithm 1.

Algorithm 1 Gradient Descent Learning Algorithm

Preprocess the training set D_T as necessary
Initialise parameter vector, $\mathbf{w}(t)$, $t = 0$
Initialise the learning rate η
while stopping condition not satisfied **do**
    for each $i = 1, ..., n_T$ **do**
        Calculate error signal, $\delta(t)$
        Calculate a search direction, $\mathbf{q}(t) = f(\mathbf{w}(t), \delta(t))$
        Update parameter vector: $\mathbf{w}(t+1) = \mathbf{w}(t) + \eta\mathbf{q}(t)$
    end for
    $t = t + 1$
    Compute prediction error
end while
Return $\mathbf{w}(t-1)$ as solution

B. Logistic Regression

The logistic regression model was first introduced by David Cox in 1958 as a method to perform binary classification [5]. Cox specifically designed the logistic regression model to model the probability of a binary outcome as a function of descriptive features.

To construct a logistic regression model that makes use of gradient descent as an optimisation algorithm, a threshold function that is continuous, and therefore differentiable is needed. This function is known as the logistic function and is represented by the mathematical equation below.

$$Logistic(z) = \frac{1}{1+e^{-z}} \tag{1}$$

where z is a numeric value.

Before the logistic regression model is constructed the binary target features are mapped to 0 or 1. The logistic regression model is then constructed by use of the equation that follows.

$$\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i) = \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{d}_i}} \tag{2}$$

where $\mathbf{d}_i$ is a vector of the i-th descriptive features, with the bias term represented by $\mathbf{d}_0$ and equal to one, $\mathbf{w}$ is a vector of weights, where $\mathbf{w}_0$ represents the weight of the bias term, and the weights that remain corresponds to their respective descriptive features in $\mathbf{d}_i$. The term $\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i)$ represents the predicted output for the i-th instance of the logistic regression model. The output of the logistic regression model can be interpreted as probabilities of the occurrence of a target instance that belongs to a specific class. The probability the i-th target instance that belongs to class one is given by the equation below.

$$P(y_i = 1|\mathbf{d}_i) = \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i) \tag{3}$$

where y_i is the true label for the i-th observation. Similarly, the probability of the i-th target instance that belongs to class zero is given by the equation below.

$$P(y_i = 0|\mathbf{d}_i) = 1 - \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i) \tag{4}$$

To classify the i-th target instance, $\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i)$ is compared to a threshold of 0.5. The equation used to classify the i-th target feature that belongs to either class zero or class one is given as follows.

$$\hat{y}_i = \begin{cases} 0 & \text{if } \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i) < 0.5 \\ 1 & \text{if } \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i) \geq 0.5 \end{cases} \tag{5}$$

where $\hat{y}_i$ is the predicted class of the i-th binary target variable.

Gradient descent is used as the optimisation algorithm to find the optimal decision boundary for a logistic regression model. The optimal decision boundary is defined as the set of weights that minimise the sum of squared error (SSE) based on the training set. The mathematical represntation of the SSE is as follows.

$$L_2\left(\mathbb{M}_{\mathbf{w}}, \mathcal{D}\right) = \frac{1}{2}\sum_{i=1}^{n}\left(y_i - \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i)\right)^2 \tag{6}$$

where $\mathcal{D}$ is the training dataset and n is the number of instances in the training dataset and L_2 is the SSE of the training dataset.

The equation used to represent the error signal used in the gradient descent optimisation algorithm to update the weights of the logistic regression model is as follows.

$$\delta(\mathcal{D}, w_j) = \sum_{i=1}^{n}((y_i - \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i))\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i)(1-\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i))d_{i,j}) \tag{7}$$

where w_j is the j-th weight of the logistic regression model and $d_{i,j}$ is the value of the j-th feature for the i-th instance of the training dataset.

The equation used to update the weights of the logistic regression model by use of the gradient descent optimisation algorithm is as follows.

$$w_j = w_j + \eta\sum_{i=1}^{n}((y_i - \mathbb{M}_{\mathbf{w}}(\mathbf{d}_i))\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i)(1-\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i))d_{i,j}) \tag{8}$$

where η is the learning rate.

The logistic regression model is quite robust to noise and outliers in the dataset. However, the logistic regression can not handle missing values and sensitive to imbalanced classes. Additionally, the logistic regression model requires categorical features to be encoded into numerical representation by either the ordinal encoded or the one hot encoded technique and the data needs to be scaled or normalised. Feature subset selection also applies, as logistic regression tends to overfit on data with a large number of features. The logistic regression also assumes a linear relationship between the descriptive features, where the actual relationship between descriptive features might be non-linear.

C. *Basis Functions*

Basis functions are non-linear elements which transforms the linear inputs to the logistic regression into non-linear represntations, while the model itself remains linear in terms of the weights [6]. The addition of basis functions allows logistic regression model to capture relationships between descriptive features which are non-linear.

The data is transformed by use of a series of basis functions, which enables the logistic regression model to effectively manage non-linear relationships between descriptive features. A logistic regression model that makes use of basis functions is represented by the equation below.

$$\mathbb{M}_{\mathbf{w}}(\mathbf{d}_i) = \frac{1}{1+e^{-\sum_{j=0}^{b} w_j\phi_j(\mathbf{d}_i)}} \tag{9}$$

where ϕ_0 to ϕ_b are a series of b basis functions that each transform the i-th input vector $\mathbf{d}_i$ in a different way. Usually b is larger than n, which means that there are more basis functions than there are descriptive features.

There are two main disadvantages when basis functions are used in a logistic regression model to capture non-linear relationships between descriptive features. Firstly, some prior knowledge of these non-linear relationships is required to select appropriate basis functions. Secondly, an increased number of basis functions results in larger gradient descent search spaces, which can lead to longer convergence times and complicate the optimisation process.

D. *Ensemble Learning*

Ensemble learning combines several individual models to to obtain better generalisation performance and predict a new instance based on multiple models opposed to a single model [8].

Each model in an ensemble is trained on the same dataset and yields slightly different results due to variations in training data, model configurations or model architectures. Each model performs differently on certain data patterns. By aggregating these diverse models, the ensemble will balance individual errors and achieve better overall performance than any single model alone. This diversity helps the ensemble to cover the limitations of each model, which results in more accurate and robust predictions. An ensemble also helps to decrease the variance in the model predictions.

Approaches used to create these diverse models are

- Train the the same type of model on different subsets of the observation of the training data.
- Train the same type of model which uses different features of the training data.
- Use different types of models, that results in heterogeneous ensembles and cancels out the inductive bias of each model.
- Use different training or optimisation algorithms.
- Use different control parameters
- Use different model architectures.

An ensemble that contains only one type of model is called a homogeneous ensemble.

Ensemble methods usually use one of the voting schemes that follow to aggregate individual predictions [11].

- Majority voting: This method counts the predictions from each model in the ensemble and selects the class with the most votes as the final output.
- Weighted voting: This variant of majority voting assigns a weight to the vote of each model in the ensemble based on the performance or importance of the model. Models with a higher weight has more influence on the final prediction.
- Soft voting: This method is preferred when the output of a model produces probabilities over all classes. The ensemble averages these probabilities to make a final prediction.

E. Bootstrap Aggregating

Bootstrap aggregating, also know as bagging, was first introduced by Leo Breiman, in 1996, as a method used to generate a diverse set of models to produce an aggregated model [3]. This diverse set of models is formed by training different models on a subset of the original data.

Datasets that differ from one another are created by use of the bootstrapping technique, where the original dataset is sampled with replacement. The datasets that results from this are the same size as the original dataset but contains different instances, where duplicates of instances may be present in the same dataset. This variation enables each dataset to introduce unique patterns, which helps create a diverse set of models in the ensemble.

A validation set can be created from the dataset with the observations that was not included in the bootstrap sample. The error obtained when this validation set is used to prevent the models from overfitting to the training data is known as the out-of-bag (OOB) error estimation.

The equation of the probability that a specific instance is included in a bootstrap sample is as follows

$$\begin{aligned} P(x \in \mathcal{D}_m) &= 1 - \left(1 - \frac{1}{n}\right)^2 \\ P(x \in \mathcal{D}_m) &\approx 1 - e^{-1} \\ P(x \in \mathcal{D}_m) &\approx 0.6322 \end{aligned} \tag{10}$$

where $\mathcal{D}_m$ is the training dataset used by the m-th model in the ensemble. On average less than $\frac{2}{3}$ of the instances will appear in a bootstrap sample. This phenomenon could lead to problems when the original dataset is small, as some instances may never be seen in the ensemble. To address the issue of instances that may never be seen in the ensemble, either the size of the bag or the number of models in the ensemble needs to be increased. However, both approaches lead to an increase in the computational complexity of the ensemble.

F. Performance Metrics

Performance metrics are essential tools when the effectiveness of classification models are evaluated. Performance metrics provide a quantitative measure of how reliable and accurate a prediction model performs classification on a dataset. Key metrics include accuracy, precision, recall, and F1-score, each offering unique insights into different aspects of model performance [2].

a) Accuracy: Accuracy is a common method used to evaluate the performance of classification models. The accuracy of a predictive classification model is determined by the proportion of correctly predicted labels against the total number of predictions. The calculation of the accuracy of a predictive model is as follows:

$$\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Predictions}} \tag{10}$$

Accuracy is a popular choice of performance measure mainly beacause it is fairly easy to understand and compute. Accuracy generally perform well on well balanced datasets. On imbalanced datasets, accuracy can produce values that are misleading.

b) Precision and Recall: Precision is the proportion of true positive (TP) predictions against all of the TP and false positive (FP). The equation to calculate the precision of a classification model is as follows:

$$\text{Precision} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Positives}} \tag{11}$$

Recall is the proportion of TP predictions against all of the TP and false negative (FN). The equation to calculate the recall of a classification model is as follows:

$$\text{Recall} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}} \tag{12}$$

c) F1-score: When a binary classification dataset has imbalanced classes, the accuracy of a model can present a high score that does not represent good performance, as the majority group could be overclassified. Therefore, when an imbalanced binary classification dataset is used, it is better to use multiple performance metrics.

The binary F1-score, also known as the Dice similarity coefficient, is the harmonic mean of precision and recall, that provides a balance between the precision and recall [7]. The equation used to calculate the binary F1-score is as follows.

$$F1 = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \tag{13}$$

The binary F1-score proves especially useful when model performance is assessed on imbalanced binary classification datasets.

III. Implementation

This section provides the approach taken to implement both the linear logistic regression ensemble model and the non-linear logistic regression ensemble model.

A. Linear Logistic Regression Ensemble

The linear logistic regression model is implemented as described in Section II-B. The logistic regression is then used to create a homogeneous ensemble of linear logistic regression models to enhance predictive accuracy and robustness and the ensemble uses a soft voting scheme to make predictions.

Each model in the ensemble is trained on different subsets of the dataset generated through bootstrapping. Additionally, a random set of features is selected from the original dataset for each model, which further enhances diversity within the ensemble and mitigates the possibility of the linear logistic regression models overfitting to the training dataset as the models are all constructed on diverse feature subsets.

This combination of bootstrapping and feature randomness allows the ensemble to capture a wider range of patterns, which improves the overall predictive performance of the ensemble on imbalanced datasets, as individual predictions aggregate through a soft voting scheme to produce a final output.

An OOB validation set is also constructed to ensure that the models don't overfit on the training data.

The implementation of the linear logistic regression model is provided in Algorithm 2.

Algorithm 2 Linear Logistic Regression Ensemble Learning Algorithm

Preprocess the training set D_T as necessary
Set parameters: num_models, η, $iterations$, bag_size, $num\_features$, $patience$
for each model $m \in [1, \ldots, num\_models]$ **do**
  Let $n_B = \lfloor bag\_size \times n \rfloor$
  Sample D_T with replacement to create a bagged dataset D_B with the size of n_B
  Create a validation set D'_B from the instances not included in D_B for out-of-bag error estimation with n'_B observations
  Randomly select a subset of features F_S from D_B
  Use only the randomly selected features to create the subsets $D_S = D_B[:, F_S]$ and $D'_S = D'_B[:, F_S]$
  Add a bias term of 1 to all observations in D_S and D'_S
  Initialise $patience\_counter$ = 0 and weights $\mathbf{w}_m$ for model m
  $best\_error = \infty$
  for iteration $t = 1$ to $iterations$ **do**
    Compute the error signal: $\boldsymbol{\delta}_m(D_S, \mathbf{w}_m) = \sum_{i=1}^{n_B} ((y_i - \mathbb{M}_{\mathbf{w}_m}(\mathbf{d}_i))\mathbb{M}_{\mathbf{w}_m}(\mathbf{d}_i)(1 - \mathbb{M}_{\mathbf{w}_m}(\mathbf{d}_i))\mathbf{d}_i)$ for $\mathbf{d}_i \in D_S$
    Update weights: $\mathbf{w}_m = \mathbf{w}_m + \eta \cdot \boldsymbol{\delta}_m(D_S, \mathbf{w}_m)$
    Compute out-of-bag error: $L_2(\mathbb{M}_{\mathbf{w}_m}, D'_S) = \sum_{i=1}^{n'_B} (y' - \mathbb{M}_{\mathbf{w}}(\mathbf{d}'_i))^2$ for $\mathbf{d}'_i \in D'_S$
    if $error < best\_error$ **then**
      $best\_error = error$
      $\mathbf{w}_m^{best} = \mathbf{w}_m$
      $patience\_counter = 0$
    else
      $patience\_counter = patience\_counter + 1$
    end if
    if $patience\_counter \geq patience$ **then**
      break
    end if
  end for
  Store model $(\mathbf{w}_m^{best}, F_S)$ in the ensemble
end for

The weights of all the linear logistic regression models in the ensemble is initialised by use of the equation below.

$$\mathbf{w}_m \sim Uniform\left(\frac{-1}{|F_S| + 1}, \frac{1}{|F_S| + 1}\right) \tag{14}$$

where $|F_S| + 1$ represents the number of the randomly slected features with the added bias feature.

B. Non-Linear Logistic Regression Ensemble

The non-linear logistic regression ensemble is implemented as described in Section III-A with the addition of basis functions as described in Section II-C. The transformation of the linear dataset to a non-linear dataset is given in Algorithm 3.

Algorithm 3 Transform The Linear Dataset To A Non-Linear Dataset

function GENERATEPOLYNOMIALTERMS($n\_features$, $polynomial\_degree$)
  Initialize empty list of terms T
  for $i \in [1, \ldots, n\_features]$ **do**
    Add single feature term $[i]$ to T
  end for
  for $degree = 2$ to $polynomial\_degree$ **do**
    for $i \in [1, \ldots, n\_features]$ **do**
      Add self-interaction term $[i, \ldots, i]$ of length $degree$ to T
    end for
    if $degree > 1$ **then**
      for $i \in [1, \ldots, n\_features]$ **do**
        for $j \in [i+1, \ldots, n\_features]$ **do**
          Add interaction term $[i, \ldots, i, j]$ of length $degree$ to T
          if $degree > 2$ **then**
            Add reverse interaction term $[j, \ldots, j, i]$ of length $degree$ to T
          end if
        end for
      end for
    end if
  end for
  return T
end function

A random subset of the generated basis functions is selected to mitigate the challenge described in Section II-C. This approach addresses the need for prior knowledge of the non-linear relationships in the dataset, which would otherwise be required to select appropriate basis functions. Each logistic regression model is then trained on different subset of features, which is first transformed to be non-linear by use of the $polynomial\_degree$ parameter. A random subset of this newly created non-linear feature set is then selected to ensure to ensure greater model diversity and to capture a broader range of patterns within the data, which enhances the overall predictive performance of the ensemble. The implementation of the non-linear logistic regression model is provided in Algorithm 4.

The weights of all the non-linear logistic regression models in the ensemble is initialised by use of the equation below.

$$\mathbf{w}_m \sim Uniform\left(\frac{-1}{|n\_nl|+1}, \frac{1}{|n\_nl|+1}\right) \tag{15}$$

where $|n\_nl| + 1$ represents the number of the randomly slected features from the transformed non-linear dataset with the added bias feature.

Algorithm 4 Non-linear Logistic Regression Ensemble Learning Algorithm

Preprocess the training set D_T as necessary
Set parameters: num_models, η, $iterations$, bag_size, $num\_features$, $patience$, $num\_nl\_features$, $polynomial\_degree \geq 2$
for each model $m \in [1, \ldots, num\_models]$ **do**
  Let $n_B = \lfloor bag\_size \times n \rfloor$
  Sample D_T with replacement to create a bagged dataset D_B with size n_B
  Create validation set D'_B from instances not in D_B for out-of-bag error estimation
  Randomly select feature subset F_S from D_B
  Use only the randomly selected features to create the subsets $D_S = D_B[:, F_S]$ and $D'_S = D'_B[:, F_S]$
  $terms =$ GENERATEPOLYNOMIALTERMS($|F_S|$, $polynomial\_degree$)
  Transform D_S and D'_S using polynomial terms in $terms$
  $n\_nl = \lfloor num\_nl\_features \times |terms| \rfloor$
  Randomly select n_nl terms from transformed features
  Update $D_S = D_S[:, n\_nl]$ and $D'_S = D'_S[:, n\_nl]$
  Add bias term of 1 to all observations in D_S and D'_S
  Initialize $patience\_counter = 0$ and weights $\mathbf{w}_m$ for model m
  $best\_error = \infty$
  for iteration $t = 1$ to $iterations$ **do**
    Compute the error signal: $\boldsymbol{\delta}_m(D_S, \mathbf{w}_m) = \sum_{i=1}^{n_B} ((y_i - \mathbb{M}_{\mathbf{w}_m}(\mathbf{d}_i))\mathbb{M}_{\mathbf{w}_m}(\mathbf{d}_i)(1 - \mathbb{M}_{\mathbf{w}_m}(\mathbf{d}_i))\mathbf{d}_i)$ for $\mathbf{d}_i \in D_S$
    Update weights: $\mathbf{w}_m = \mathbf{w}_m + \eta \cdot \boldsymbol{\delta}_m(D_S, \mathbf{w}_m)$
    Compute out-of-bag error: $L_2(\mathbb{M}_{\mathbf{w}_m}, D'_S) = \sum_{i=1}^{n'_B} (y' - \mathbb{M}_{\mathbf{w}}(\mathbf{d}'_i))^2$ for $\mathbf{d}'_i \in D'_S$
    if $error < best\_error$ **then**
      $best\_error = error$
      $\mathbf{w}_m^{best} = \mathbf{w}_m$
      $patience\_counter = 0$
    else
      $patience\_counter = patience\_counter + 1$
    end if
    if $patience\_counter \geq patience$ **then**
      break
    end if
  end for
  Store model ($\mathbf{w}_m^{best}, F_S, terms$) and n_nl in ensemble
end for

IV. Empirical Procedure

This section of the report outlines the systematic approach used to evaluate the performance of the linear logistic regression ensemble model and non-linear logistic regression ensemble model across multiple binary classification tasks. Additionally, this section provides the approach used to process each dataset to ensure optimal performance of the ensemble models and the performance metrics used to investigate the influence of the number of members in the ensembles, the number of randomly selected features, and the size of the bags. The section provides the control parameters used to construct each linear and non-linear logistic regression ensemble model for each of the binary classification tasks, the experimental setup and the statistical significance tests used to ensure the reliability and statistical soundness of the results.

A. Performance Metrics

Accuracy is used to investigate the influence of the number of members in the ensembles, the number of randomly selected features, and the size of the bags if the original dataset contains balanced classes. If the original dataset contains imbalanced classes, the F1-score metric is used to investigate the influence of the number of members in the ensembles, the number of randomly selected features, and the size of the bags.

B. Data Preprocessing

The data has to be preprocessed to ensure optimal and reliable results on each dataset from both the linear logistic regression ensemble model and the non-linear logistic regression ensemble model. The procedures to preprocess each dataset differ. Therefore, each dataset must be explored separately and processed into an optimal form to use in the ensemble models.

1) Breast Cancer Dataset: The breast cancer dataset contains numerical features computed from a digitised image of a fine needle aspirate (FNA) of a breast mass, that describe characteristics of the cell nuclei present in the image [10]. The dataset includes a binary target feature labeled 'diagnosis,' which indicates whether the tumor is benign or malignant, denoted by 'B' or 'M', respectively. The dataset consists of 32 features and 569 observations.

The dataset contains a unique identification feature 'id' that is removed from the dataset, as it does not provide meaningful infomation. There are no missing values present in the dataset and the classes are imbalanced.

All descriptive features that remain are numerical and normalised to a range of $[-1, 1]$. The equation used to normalise the descriptive features is as follows

$$d'_{i,j} = \frac{2\,(d_{i,j} - min(\mathbf{d}_j))}{max(\mathbf{d}_j) - min(\mathbf{d}_j)} - 1 \tag{16}$$

where $\mathbf{d}_j$ is a vector of all values of the j-th feature, $min(\mathbf{d}_j)$ and $max(\mathbf{d}_j)$ is the minimum and maximum values of the j-th feature, respectively and $d'_{i,j}$ is the normalised value of $d_{i,j}$ within the range $[-1, 1]$.

The target feature 'diagnosis' is encoded so that each instance of 'M' is represented as a zero and each instance of 'B' is represented as one. Since the dataset has imbalanced classes, the F1-score serves as the performance metric used to evaluate the influence of the number of ensemble members, the number of randomly selected features, and the bag sizes.

2) Ionosphere Dataset: The ionosphere dataset contains radar data which aims to detect structural patterns in the ionosphere [1]. The dataset consists of 351 instances, each described by 32 continuous descriptive features, with 2 complex values per pulse number across 16 pulse numbers, and a binary target feature which indicates whether the radar return is 'g', that indicates a structured ionospheric signal, or 'b', which indicates an unstructured signal that passed through the ionosphere.

The descriptive features are all numerical and should be normalised within the range $[-1, 1]$ by use of Equation (16). The target feature is encoded so that each instance of 'b' is represented as a zero and each instance of 'g' is represented as one. Since the dataset has imbalanced classes, the F1-score serves as the performance metric used to evaluate the influence of the number of ensemble members, the number of randomly selected features, and the bag sizes.

3) Diabetes Dataset: The diabetes dataset contains data from 1304 patients who tested positive for diabetes, with ages ranging from 21 to 99 years [9]. The dataset includes a binary target feature labeled 'Diabetes', which indicates whether the patient tested positive or negative for type 2 diabetes. A positive result is indicated by a zero and a negative result is indicated by a one. The dataset consists of 1304 instances and 18 descriptive features.

The descriptive features in this dataset combine categorical and numerical features, with categorical features already encoded as numerical values. All of these descriptive features are then normalised within the range $[-1, 1]$ by use of Equation (16). Since the dataset has imbalanced classes, the F1-score serves as the performance metric used to evaluate the influence of the number of ensemble members, the number of randomly selected features, and the bag sizes.

4) Banana Quality Dataset: The banana quality dataset contains data on 8,000 bananas, with a binary target feature labeled 'Quality' and 7 numerical descriptive features [12]. A banana with good quality is labeled as 'Good' and a banana with bad quality is labeled as 'Bad'.

The numerical descriptive features are normalised within the range $[-1, 1]$ by use of Equation (16). The target feature is encoded so that each instance of 'Bad' is represented as a zero and each instance of 'Good' is represented as one. Since the dataset has balanced classes, the prediction accuracy serves as the performance metric used to evaluate the influence of the number of ensemble members, the number of randomly selected features, and the bag sizes.

5) Spiral Dataset: The spiral dataset is a randomly generated dataset designed for binary classification, that contains two interwoven spirals. 1000 random points are generated on the spiral, where 500 instances belongs to class zero and 500 instances belongs tro class one.

A generated spiral radii vector is constructed to define the distance of each point from the origin in both spirals. The equation used to generate the generated spiral radii vector is as follows.

$$\mathbf{v} = \sqrt{random\,(500)} \times 780 \times \frac{2\pi}{360} \tag{17}$$

where $\mathbf{v}$ is the generated spiral radii vector and $random(500)$ generates a vector of 500 random values between 0 and 1. The random points are multiplied by a scale factor of 780, and then multiplied by $\frac{2\pi}{360}$ to convert the values to radians.

Noise is also introduced when the spiral dataset is generated. The noise helps to mitigate overfitting as the dataset becomes less predictable, which forces the models to learn broader patterns which improves the generalisation of the models on unseen data.

The equation used to calculate the vector of x coordinates of the first spiral is given as follows.

$$\boldsymbol{d1x} = -\cos(\mathbf{v}) \cdot \mathbf{v} + random(500) \cdot \alpha \tag{18}$$

where $\boldsymbol{d1x}$ is the vector of x coordinates of the first spiral and α is the amount of noise introduced in the dataset.

The equation used to calculate the vector of y coordinates of the first spiral is given as follows.

$$\boldsymbol{d1y} = \sin(\mathbf{v}) \cdot \mathbf{v} + random(500) \cdot \alpha \tag{19}$$

where $\boldsymbol{d1y}$ is the vector of y coordinates of the first spiral. All of the 500 points in the first spiral is then assigned a class label of zero.

The equation used to calculate the vector of x coordinates of the second spiral is given as follows.

$$\boldsymbol{d2x} = \cos(\mathbf{v}) \cdot \mathbf{v} + random(500) \cdot \alpha \tag{20}$$

where $\boldsymbol{d2x}$ is the vector of x coordinates of the second spiral.

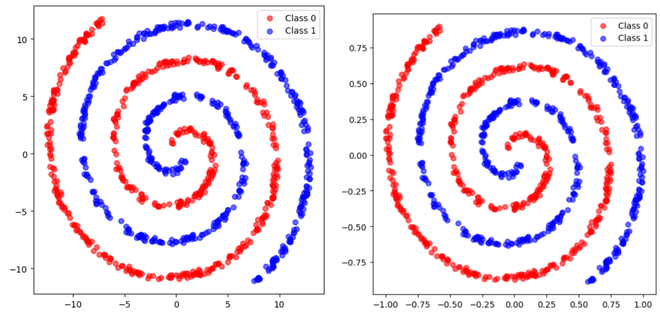
The equation used to calculate the vector of y coordinates of the second spiral is given as follows.

$$\boldsymbol{d2y} = -\sin(\mathbf{v}) \cdot \mathbf{v} + random(500) \cdot \alpha \tag{21}$$

where $\boldsymbol{d2y}$ is the vector of y coordinates of the second spiral. All of the 500 points in the second spiral is then assigned a class label of one.

For the purpose of this report, α has been assigned a value of 0.5 and 500 descriptive features are generated for both spirals. The descriptive and target features are then combined into one dataset and normalised within the range $[-1, 1]$ by use of Equation (16).

The original spiral, which contains noise, can be seen in Figure 1a, while the normalised spiral can be seen in Figure 1b.



(a) Original Spiral With Added Noise

(b) Normalised Spiral With Added Noise

Fig. 1: Randomly Generated Spiral Dataset

Since the dataset has balanced classes, the prediction accuracy serves as the performance metric used to evaluate the influence of the number of ensemble members and the bag sizes. A random seed is initialised to ensure reproducibility of the generated dataset.

C. Experimental Setup

1) Obtaining the results: The grids $[0.4, 0.5, 0.6, 0.7, 0.8]$, $[0.4, 0.5, 0.6, 0.7, 0.8]$ and $[5, 10, 20, 50, 100]$ were used to investigate the influence of the size of the bag, the number of randomly selected features and the number of ensemble members, respectively, for both the linear and non-linear logistic regression ensembles. However, the spiral dataset contains only 2 descriptive features and therefore, only the influence of the bag size and the number of ensemble methods were investigated with their respective grids of $[0.4, 0.5, 0.6, 0.7, 0.8]$ and $[5, 10, 20, 50, 100]$.

A 5-fold cross-validation was also conducted for each possible combination of the grid values, which is also known as the grid search technique. For each dataset, the cross validation splits the data up into five datasets, where four sets were used to construct the model with the associated values of the grids and the set that remains was used as the test set to evaluate the model performance. This process was repeated 5 times to ensure each subset is used as the test set once. The utilisation of the 5-fold cross validation technique ensured that the evaluation was fair and consistent across the entire dataset and the bias that might arise from relying on a single training-test split was reduced.

2) Finding the optimal control parameters: To find the optimal control parameters of each optimisation algorithm for each task, a 5-fold cross-validation was used, as described above, for each dataset. A Bayesian optimisation technique efficiently searched the hyperparameter space to identify the parameters that minimised the average test accuracy error for the banana quality and spiral datasets and the parameters that minimised the average test F1-score for the breast cancer, ionosphere, and diabetes datasets across the 5-fold cross validation.

D. Control Parameters

Bayesian optimisation is used to find the optimal control parameter for each dataset, as described in Section IV-C.

The optimal control parameters used for each linear logistic regression ensemble are represented in Table I.

TABLE I: Linear Logistic Regression Control Parameters

Dataset	Control Parameters		
	eta	*epochs*	*patience*
Breast Cancer	0.00239	10678	6
Ionosphere	0.00012	9809	6
Diabetes	0.01121	18690	9
Banana Quality	0.00023	4338	5
Spiral	0.06708	17335	7

The optimal control parameters used for each non-linear logistic regression ensemble are represented in Table II.

TABLE II: Non-Linear Logistic Regression Control Parameters

Dataset	Control Parameters				
	eta	*epochs*	*patience*	*poly*	*% poly*
Breast Cancer	0.00013	3812	5	3	50
Ionosphere	0.0001	2797	10	3	70
Diabetes	0.00783	1000	6	3	80
Banana Quality	0.01099	5054	5	3	80
Spiral	0.00011	8755	5	4	60

E. Statistical Significance and Analysis

The statistical significance of the results from the the grid search of each dataset for the linear and non-linear logistic regression ensemble is determined by the evaluation of the accuracy for banana quality and spiral dataset and the F1-score for the breast cancer, ionosphere and diabetes dataset, by the utilisation of a 5-fold cross validation. For each dataset, the Kruskal-Wallis test was used to evaluate the statistical significance of differences among the grid search results across various parameter combinations, with a significance level of 0.05. Additionally, for each dataset, the paired t-test was performed to compare the results of the linear and non-linear logistic regression ensemble models, which assessed the statistical significance between their performance distributions, with a significance level of 0.05.

V. Research Results

This section presents the results of the evaluation of the linear and non-linear logistic regression ensembles across the five binary classification datasets. Key factors examined include the influence of ensemble size, bag size, and feature subset size on classification performance.

A. Breast Cancer Dataset

By use of Bayesian optimisation, the optimal values of the control parameters in the linear and non-linear logistic regression ensembles are determined as seen in Section IV-D.

A 5-fold cross validation technique is then applied to the grid search of the linear and non-linear logistic regression ensemble models to investigate the influence of the size of the bag, the number of randomly selected features and the number of ensemble members as seen in Section IV-C.

1) Bag size: Table III presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE III: Summary Statistics of F1-Scores for Each Bag Size of the Linear Logistic Regression Ensemble on the Breast Cancer Dataset

Bag Size	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	97.66	0.0103	94.89	99.35
0.5	125	97.47	0.0099	94.96	99.35
0.6	125	97.88	0.0097	94.03	99.35
0.7	125	97.85	0.0099	95.65	99.35
0.8	125	97.91	0.0096	95.83	99.35

The Kruskal-Wallis test returned a p-value of 0.3895 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the bag size has a minimal effect on the performance of the linear logistic regression ensemble on the breast cancer dataset.

Table IV presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE IV: Summary Statistics of F1-Scores for Each Bag Size of the Non-Linear Logistic Regression Ensemble on the Breast Cancer Dataset

Bag Size	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	97.37	0.0100	94.52	99.35
0.5	125	97.69	0.0097	95.59	99.35
0.6	125	97.83	0.0088	95.83	99.35
0.7	125	97.81	0.0100	94.96	1
0.8	125	97.92	0.0105	95.59	1

The Kruskal-Wallis test returned a p-value of 0.004 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the bag size has a notable effect on the performance of the non-linear logistic regression ensemble on the breast cancer dataset.

The mean F1-scores obtained from each value of bag size in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 2.

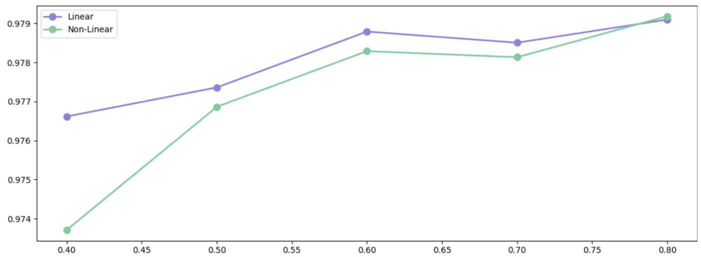


Fig. 2: Comparison of mean F1-score across the bag size grid when the breast cancer dataset is classified by use of 5-fold cross-validation.

From figure 2 it is seen that the linear logistic regression ensemble model performs slightly better at smaller bag sizes and maintains a more stable performance across the range. The non-linear logistic regression ensemble model starts with lower performance at small bag sizes but catches up to the linear model at larger bag sizes, at a bag size of 0.80 with an average F1-score of approximately 0.979 the models converge. The differences between the two models are quite small, with average F1-scores which only varys between about 0.974 and 0.979.

The paired t-test between the two models returned a p-value of 0.1880, which indicates there is no significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the breast cancer dataset.

The linear logistic regression model performs better for smaller bag sizes and the models performs similarly on larger bag sizes. Therefore the linear logistic ensemble model performs best on the breast cancer dataset when the bag size is considered.

2) Number of features: Table V presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE V: Summary Statistics of F1-Scores for Each Number of Features of the Linear Logistic Regression Ensemble on the Breast Cancer Dataset

Number of Featurs	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
0.4	125	97.62	0.0099	94.03	99.35
0.5	125	97.70	0.0111	94.89	99.35
0.6	125	97.90	0.0095	95.71	99.35
0.7	125	97.89	0.0096	95.83	99.35
0.8	125	97.92	0.0090	95.83	99.35

The Kruskal-Wallis test returned a p-value of 0.1813 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of randomly selected features has a minimal effect on the performance of the linear logistic regression ensemble on the breast cancer dataset.

Table VI presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE VI: Summary Statistics of F1-Scores for Each Number of Features of the Non-Linear Logistic Regression Ensemble on the Breast Cancer Dataset

Number of Features	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
0.4	125	97.28	0.0099	94.52	99.35
0.5	125	97.59	0.0093	94.89	99.35
0.6	125	97.77	0.0092	94.96	1
0.7	125	97.87	0.0097	94.89	1
0.8	125	98.11	0.0099	95.77	1

The Kruskal-Wallis test returned a p-value of 0 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the non-linear logistic regression ensemble on the breast cancer dataset.

The mean F1-scores obtained from each value of number of randomly selected features in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 3.

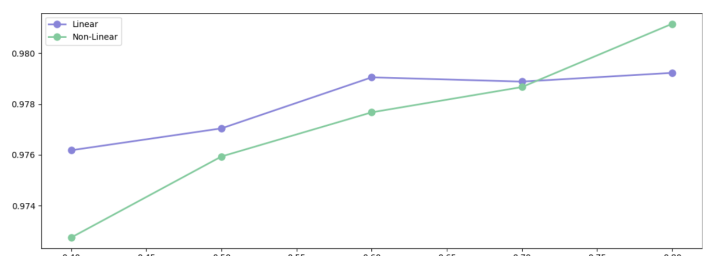


Fig. 3: Comparison of mean F1-score across the number of randomly selected features grid when the breast cancer dataset is classified by use of 5-fold cross-validation.

From figure 3 it is seen that the linear logistic regression ensemble model performs slightly better at less number of features and maintains a more stable performance across the range. The non-linear logistic regression ensemble model starts with lower performance at less number of features but catches up to the linear model at more number of features, at a number of features of 0.70 with an average F1-score of approximately 0.978 the models converge. The non-linear logistic regression ensemble model then performs much better than the linear logistic regression ensemble model at a number of features of 0.8. The differences between the two models are quite small, with average F1-scores which only varys between about 0.972 and 0.982.

The paired t-test between the two models returned a p-value of 0.3895, which indicates there is no significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the breast cancer dataset.

For less number of features, the linear logistic regression performs best and for more number of features, the non-linear logistic regression performs best. Therefore, there is no clear ensemble model that performs best on the breast cancer dataset when the number of features are considered.

3) Number of ensemble members: Table VII presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE VII: Summary Statistics of F1-Scores for Each Number of Ensemble Members of the Linear Logistic Regression Ensemble on the Breast Cancer Dataset

Number of Featurs	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
5	125	97.66	0.0106	94.03	99.35
10	125	97.74	0.0100	95.65	99.35
20	125	97.88	0.0094	95.83	99.35
50	125	97.86	0.0101	95.65	99.35
100	125	97.90	0.0093	95.83	99.35

The Kruskal-Wallis test returned a p-value of 0.4252 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of ensemble members has a minimal effect on the performance of the linear logistic regression ensemble on the breast cancer dataset.

Table VIII presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE VIII: Summary Statistics of F1-Scores for Each Number of Ensemble Members of the Non-Linear Logistic Regression Ensemble on the Breast Cancer Dataset

Number of Features	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
5	125	97.35	0.0125	94.52	1
10	125	97.62	0.0098	95.59	1
20	125	97.80	0.0091	95.65	99.35
50	125	97.90	0.0085	96.35	99.35
100	125	97.94	0.0084	96.35	99.35

The Kruskal-Wallis test returned a p-value of 0.0003 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of ensemble members has a notable effect on the performance of the non-linear logistic regression ensemble on the breast cancer dataset.

The mean F1-scores obtained from each value of number of ensemble members in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 4.

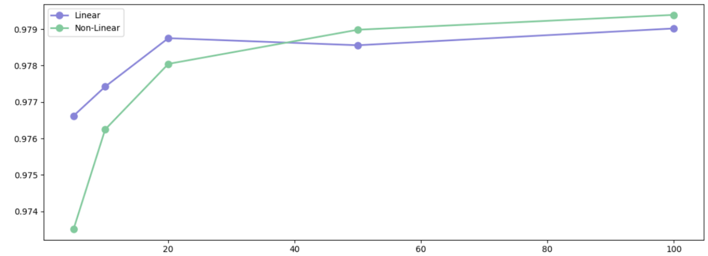


Fig. 4: Comparison of mean F1-score across the number of ensemble members grid when the breast cancer dataset is classified by use of 5-fold cross-validation.

From figure 4 it is seen that the linear logistic regression ensemble model performs slightly better at less ensemble members and maintains a more stable performance across the range. The non-linear logistic regression ensemble model starts with lower performance at less ensemble members but catches up to the linear model at more ensemble members, at a number of ensemble members of 40 with an average F1-score of approximately 0.9785 the models converge. The non-linear logistic regression ensemble model then performs much better than the linear logistic regression ensemble model at a number of ensemble members of 50 and higher. The differences between the two models are quite small, with average F1-scores which only varys between about 0.973 and 0.98.

The paired t-test between the two models returned a p-value of 0.2643, which indicates there is no significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the breast cancer dataset.

For a lower number of ensemble members, the linear logistic regression performs best and for a higher number of ensemble members, the non-linear logistic regression performs best. Therefore, there is no clear ensemble model that performs best on the breast cancer dataset when the number of ensemble members are considered.

B. Ionosphere Dataset

By use of Bayesian optimisation, the optimal values of the control parameters in the linear and non-linear logistic regression ensembles are determined as seen in Section IV-D.

A 5-fold cross validation technique is then applied to the grid search of the linear and non-linear logistic regression ensemble models to investigate the influence of the size of the bag, the number of randomly selected features and the number of ensemble members as seen in Section IV-C.

1) Bag size: Table IX presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE IX: Summary Statistics of F1-Scores for Each Bag Size of the Linear Logistic Regression Ensemble on the Ionosphere Dataset

Bag Size	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	88.22	0.0214	84.21	93.20
0.5	125	88.58	0.0180	85.11	92.31
0.6	125	88.39	0.0191	84.11	92.16
0.7	125	88.80	0.0164	84.21	92.16
0.8	125	88.88	0.0172	85.11	92.16

The Kruskal-Wallis test returned a p-value of 0.0078 between the scores of the linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the bag size has a notable effect on the performance of the linear logistic regression ensemble on the ionosphere dataset.

Table X presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE X: Summary Statistics of F1-Scores for Each Bag Size of the Non-Linear Logistic Regression Ensemble on the Ionosphere Dataset

Bag Size	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	91.72	0.0221	86.87	95.65
0.5	125	92.40	0.0220	83.33	96.00
0.6	125	92.88	0.0209	86.00	96.00
0.7	125	93.43	0.0213	88.66	96.70
0.8	125	93.63	0.0184	88.89	96.70

The Kruskal-Wallis test returned a p-value of 0 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the bag size has a notable effect on the performance of the non-linear logistic regression ensemble on the ionosphere dataset.

The mean F1-scores obtained from each value of bag size in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 5.

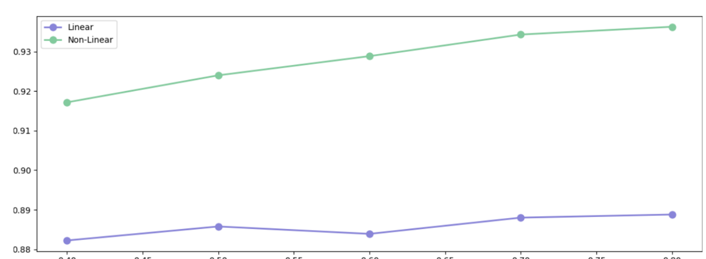


Fig. 5: Comparison of mean F1-score across the bag size grid when the ionosphere dataset is classified by use of 5-fold cross-validation.

The optimal bag size for both the linear and non-linear logistic regression ensemble models for the ionosphere dataset is 0.8, which indicates that the models performs better when constructed on more observations.

The paired t-test between the two models returned a p-value of 0.0001, which indicates there is a significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the ionosphere dataset.

From figure 5 it is seen that the non-linear logistic regression ensemble performs best across all different bag sizes. Therefore the non-linear logistic ensemble model performs best on the ionosphere dataset when the bag size is considered.

2) Number of features: Table XI presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE XI: Summary Statistics of F1-Scores for Each Number of Features of the Linear Logistic Regression Ensemble on the Ionosphere Dataset

Number of Featurs	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	88.31	0.0194	84.21	92.31
0.5	125	88.29	0.0195	84.11	92.31
0.6	125	88.62	0.0179	85.11	92.31
0.7	125	88.76	0.0181	85.11	93.20
0.8	125	88.89	0.0176	85.11	93.20

The Kruskal-Wallis test returned a p-value of 0.0118 between the scores of the linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the linear logistic regression ensemble on the ionosphere dataset.

Table XII presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE XII: Summary Statistics of F1-Scores for Each Number of Features of the Non-Linear Logistic Regression Ensemble on the Ionosphere Dataset

Number of Features	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	92.15	0.0261	83.33	96
0.5	125	92.94	0.0197	87.91	96.7
0.6	125	93.09	0.0202	88.89	96.7
0.7	125	93.03	0.0208	88.89	96.7
0.8	125	92.84	0.0217	87.91	96.7

The Kruskal-Wallis test returned a p-value of 0.0166 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the non-linear logistic regression ensemble on the ionosphere dataset.

The mean F1-scores obtained from each value of number of randomly selected features in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 6.

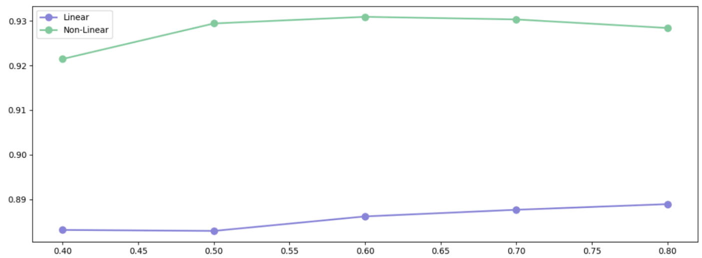


Fig. 6: Comparison of mean F1-score across the number of randomly selected features grid when the ionosphere dataset is classified by use of 5-fold cross-validation.

The linear logistic regression ensemble model has a slight increase in F1-score on the ionosphere dataset when the number of features are increased and the F1-score of the non-linear logistic regression ensemble model remains stable with values of 0.5 to 0.7 and decreases when 80% of the features are included when the model is constructed. Therefore the optimal value of features to use is 0.5.

The paired t-test between the two models returned a p-value of 0, which indicates there is a significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the ionosphere dataset.

From figure 6 it is seen that the non-linear logistic regression ensemble performs best across all different number of features. Therefore the non-linear logistic ensemble model performs best on the ionosphere dataset when the number of features is considered.

3) Number of ensemble members: Table XIII presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XIII: Summary Statistics of F1-Scores for Each Number of Ensemble Members of the Linear Logistic Regression Ensemble on the Ionosphere Dataset

Number of Featurs	Summary Statistics				
	count	***mean***	***std***	***min***	***max***
5	125	88.34	0.0205	84.21	92.31
10	125	88.57	0.0179	84.11	92.31
20	125	88.50	0.0182	84.21	92.16
50	125	88.70	0.0181	86.00	93.20
100	125	88.75	0.0181	86.00	92.31

The Kruskal-Wallis test returned a p-value of 0.4025 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of ensemble members has a minimal effect on the performance of the linear logistic regression ensemble on the ionosphere dataset.

Table XIV presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XIV: Summary Statistics of F1-Scores for Each Number of Ensemble Members of the Non-Linear Logistic Regression Ensemble on the Ionosphere Dataset

Number of Features	Summary Statistics				
	count	***mean***	***std***	***min***	***max***
5	125	91.92	0.0247	83.33	96.7
10	125	92.75	0.0212	86.87	96.7
20	125	92.95	0.0210	86.87	96.7
50	125	93.21	0.0201	87.76	96.7
100	125	93.23	0.0206	86.87	96.7

The Kruskal-Wallis test returned a p-value of 0.0001 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of ensemble members has a notable effect on the performance of the non-linear logistic regression ensemble on the ionosphere dataset.

The mean F1-scores obtained from each value of number of ensemble members in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 7.

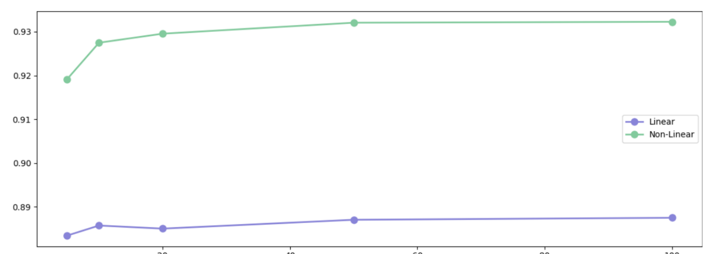


Fig. 7: Comparison of mean F1-score across the number of ensemble members grid when the ionosphere dataset is classified by use of 5-fold cross-validation.

The F1-score for both of the linear and logistic regression ensemble models converges after 20 members in the ensemble. Therefore, the optimal number of members to use in the linear and logistic regression ensemble models are 10 and 20 respectively.

The paired t-test between the two models returned a p-value of 0, which indicates there is a significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the ionosphere dataset.

From figure 7 it is seen that the non-linear logistic regression ensemble performs best across all different number of ensemble members. Therefore the non-linear logistic ensemble model performs best on the ionosphere dataset when the number of ensemble members is considered.

C. Diabetes Dataset

By use of Bayesian optimisation, the optimal values of the control parameters in the linear and non-linear logistic regression ensembles are determined as seen in Section IV-D.

A 5-fold cross validation technique is then applied to the grid search of the linear and non-linear logistic regression ensemble models to investigate the influence of the size of the bag, the number of randomly selected features and the number of ensemble members as seen in Section IV-C.

1) Bag size: Table XV presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE XV: Summary Statistics of F1-Scores for Each Bag Size of the Linear Logistic Regression Ensemble on the Diabetes Dataset

Bag Size	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	90.57	0.0225	85.78	94.05
0.5	125	90.73	0.0207	86.05	93.47
0.6	125	90.60	0.0237	84.21	94.05
0.7	125	90.97	0.0205	86.51	94.05
0.8	125	89.90	0.0246	82.96	93.25

The Kruskal-Wallis test returned a p-value of 0.0032 between the scores of the linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the bag size has a notable effect on the performance of the linear logistic regression ensemble on the diabetes dataset.

Table XVI presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE XVI: Summary Statistics of F1-Scores for Each Bag Size of the Non-Linear Logistic Regression Ensemble on the Diabetes Dataset

Bag Size	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	90.94	0.0211	85.25	93.48
0.5	125	91.39	0.0202	86.45	93.70
0.6	125	91.10	0.0213	84.63	93.48
0.7	125	91.27	0.0202	86.12	93.90
0.8	125	90.78	0.0210	86.33	94.00

The Kruskal-Wallis test returned a p-value of 0.0154 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the bag size has a notable effect on the performance of the non-linear logistic regression ensemble on the diabetes dataset.

The mean F1-scores obtained from each value of bag size in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 8.

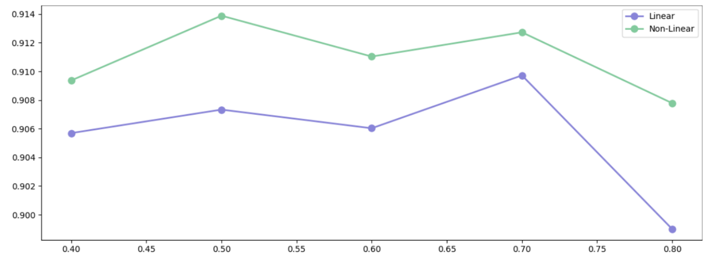


Fig. 8: Comparison of mean F1-score across the bag size grid when the diabetes dataset is classified by use of 5-fold cross-validation.

The optimal bag size for the linear and non-linear logistic regression ensemble models is a value of 0.5 and 0.7 respectively, therefore a larger bag size, such as 0.8 used for the classification of the diabetes dataset decreases the performance of the linear and non-linear logistic regression ensemble model.

The paired t-test between the two models returned a p-value of 0.0065, which indicates there is a significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the diabetes dataset.

From figure 8 it is seen that the non-linear logistic regression ensemble performs best across all different bag sizes. Therefore the non-linear logistic ensemble model performs best on the diabetes dataset when the bag size is considered.

2) Number of features: Table XVII presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE XVII: Summary Statistics of F1-Scores for Each Number of Features of the Linear Logistic Regression Ensemble on the Diabetes Dataset

Number of Featurs	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	88.94	0.0218	82.96	92.87
0.5	125	90.24	0.0189	85.78	93.23
0.6	125	90.96	0.0207	84.83	93.41
0.7	125	91.07	0.0212	84.83	93.39
0.8	125	91.57	0.0214	86.52	94.05

The Kruskal-Wallis test returned a p-value of 0 between the scores of the linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the linear logistic regression ensemble on the diabetes dataset.

Table XVIII presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE XVIII: Summary Statistics of F1-Scores for Each Number of Features of the Non-Linear Logistic Regression Ensemble on the Diabetes Dataset

Number of Features	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
0.4	125	90.02	0.0210	84.63	93.49
0.5	125	91.21	0.0205	86.12	94.00
0.6	125	91.29	0.0205	86.77	93.73
0.7	125	91.34	0.0195	86.77	93.73
0.8	125	91.63	0.0192	87.24	93.90

The Kruskal-Wallis test returned a p-value of 0 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the non-linear logistic regression ensemble on the diabetes dataset.

The mean F1-scores obtained from each value of number of randomly selected features in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 9.

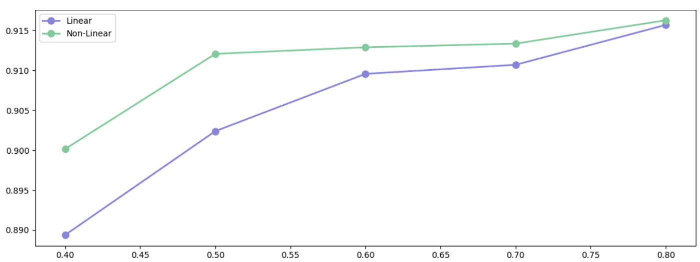


Fig. 9: Comparison of mean F1-score across the number of randomly selected features grid when the diabetes dataset is classified by use of 5-fold cross-validation.

The optimal bag size to use in both the linear and logistic regression ensemble models is 0.8.

The paired t-test between the two models returned a p-value of 0.0462, which indicates there is a significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the diabetes dataset.

From figure 9 it is seen that the non-linear logistic regression ensemble performs best across all different number of features. Therefore the non-linear logistic ensemble model performs best on the diabetes dataset when the number of features is considered.

3) Number of ensemble members: Table XIX presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XIX: Summary Statistics of F1-Scores for Each Number of Ensemble Members of the Linear Logistic Regression Ensemble on the Diabetes Dataset

Number of Featurs	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
5	125	90.05	0.0242	82.96	93.68
10	125	90.60	0.0234	84.21	94.05
20	125	90.64	0.0216	84.49	94.05
50	125	90.67	0.0222	84.41	93.89
100	125	90.81	0.0215	84.41	93.86

The Kruskal-Wallis test returned a p-value of 0.0764 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of ensemble members has a minimal effect on the performance of the linear logistic regression ensemble on the diabetes dataset.

Table XX presents the mean, standard deviation, minimum and maximum of the F1-scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XX: Summary Statistics of F1-Scores for Each Number of Ensemble Members of the Non-Linear Logistic Regression Ensemble on the Diabetes Dataset

Number of Features	Summary Statistics				
	count	*mean*	*std*	*min*	*max*
5	125	91.17	0.0223	85.25	94.
10	125	91.14	0.0212	84.63	93.88
20	125	91.07	0.0203	85.92	93.63
50	125	90.96	0.0207	85.04	93.44
100	125	91.13	0.0199	85.38	93.63

The Kruskal-Wallis test returned a p-value of 0.3451 between the scores of the non-linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of ensemble members has a minimal effect on the performance of the non-linear logistic regression ensemble on the diabetes dataset.

The mean F1-scores obtained from each value of number of ensemble members in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 10.

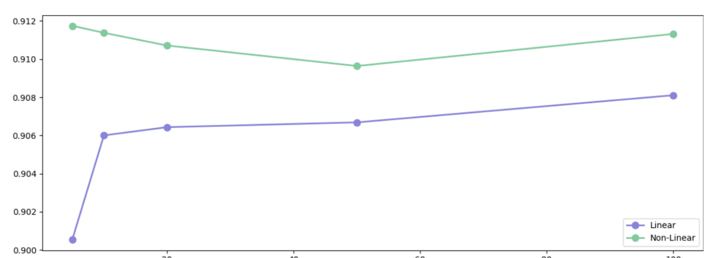


Fig. 10: Comparison of mean F1-score across the number of ensemble members grid when the diabetes dataset is classified by use of 5-fold cross-validation.

The average F1-score of the number of members for the non-linear logistic regression ensemble model starts to decrease from 10 members and is then more or less the same at 100 members. An optimal value of members for the non-linear logistic regression ensemble is 5 as the model would train faster and obtain more or less the same results.

The paired t-test between the two models returned a p-value of 0.0233, which indicates there is a significant difference between the average F1-scores of the linear and non-linear logistic regression ensembles based on the diabetes dataset.

From figure 10 it is seen that the non-linear logistic regression ensemble performs best across all different number of ensemble members. Therefore the non-linear logistic ensemble model performs best on the diabetes dataset when the number of ensemble members is considered.

D. Banana Quality Dataset

By use of Bayesian optimisation, the optimal values of the control parameters in the linear and non-linear logistic regression ensembles are determined as seen in Section IV-D.

A 5-fold cross validation technique is then applied to the grid search of the linear and non-linear logistic regression ensemble models to investigate the influence of the size of the bag, the number of randomly selected features and the number of ensemble members as seen in Section IV-C.

1) Bag size: Table XXI presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE XXI: Summary Statistics of Accuracy Scores for Each Bag Size of the Linear Logistic Regression Ensemble on the Banana Quality Dataset

Bag Size	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
0.4	125	85.88	0.0196	80.44	88.81
0.5	125	85.81	0.0219	79.06	88.75
0.6	125	86.58	0.0124	82.12	88.44
0.7	125	84.99	0.0449	68.81	88.69
0.8	125	86.38	0.0129	82.12	88.25

The Kruskal-Wallis test returned a p-value of 0.0708 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the bag size has a minimal effect on the performance of the linear logistic regression ensemble on the banana quality dataset.

Table XXII presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE XXII: Summary Statistics of Accuracy Scores for Each Bag Size of the Non-Linear Logistic Regression Ensemble on the Banana Quality Dataset

Bag Size	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
0.4	125	92.88	0.0367	81.75	97.00
0.5	125	93.09	0.0319	82.38	96.94
0.6	125	94.08	0.0269	87.00	97.06
0.7	125	92.00	0.0613	69.50	97.12
0.8	125	92.46	0.0435	71.00	96.88

The Kruskal-Wallis test returned a p-value of 0.0056 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the bag size has a notable effect on the performance of the non-linear logistic regression ensemble on the banana quality dataset.

The mean accuracy scores obtained from each value of bag size in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 11.

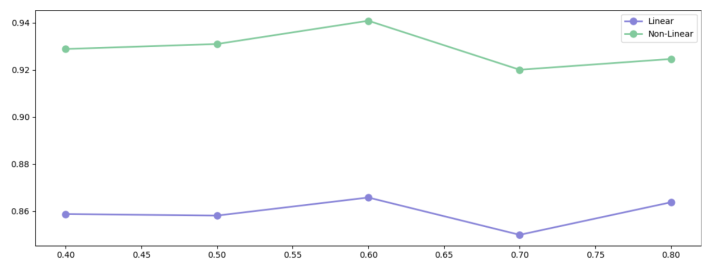


Fig. 11: Comparison of mean accuracy score across the bag size grid when the banana quality dataset is classified by use of 5-fold cross-validation.

The optimal bag size for the linear and non-linear logistic regression ensemble models is a value of 0.6. Both models predictions also become inconsistent for a bag size larger than 0.6, as the minimum average accuracy score for a bag size of 0.7 is 68.81% and 69.5% for the linear and non-linear logistic regression ensemble, respectively.

The paired t-test between the two models returned a p-value of 0, which indicates there is a significant difference between the average accuracy scores of the linear and non-linear logistic regression ensembles based on the banana quality dataset.

From figure 11 it is seen that the non-linear logistic regression ensemble performs best across all different bag sizes. Therefore the non-linear logistic ensemble model performs best on the banana quality dataset when the bag size is considered.

2) Number of features: Table XXIII presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE XXIII: Summary Statistics of Accuracy Scores for Each Number of Features of the Linear Logistic Regression Ensemble on the Banana Quality Dataset

Number of Featurs	Summary Statistics				
	count	***mean***	***std***	***min***	***max***
0.4	125	84.69	0.0367	68.81	88.19
0.5	125	85.45	0.0282	73.44	88.75
0.6	125	86.11	0.0197	78.56	88.81
0.7	125	86.11	0.0197	78.56	88.81
0.8	125	87.27	0.0078	84.12	88.75

The Kruskal-Wallis test returned a p-value of 0 between the scores of the linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the linear logistic regression ensemble on the banana quality dataset.

Table XXIV presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of randomly selected features value in the grid.

TABLE XXIV: Summary Statistics of Accuracy Scores for Each Number of Features of the Non-Linear Logistic Regression Ensemble on the Banana Quality Dataset

Number of Features	Summary Statistics				
	count	***mean***	***std***	***min***	***max***
0.4	125	87.22	0.0493	69.50	93.69
0.5	125	91.89	0.0328	72.88	95.31
0.6	125	94.58	0.0142	89.62	96.75
0.7	125	94.58	0.0142	89.62	96.75
0.8	125	96.24	0.0049	94.56	97.12

The Kruskal-Wallis test returned a p-value of 0 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of randomly selected features has a notable effect on the performance of the non-linear logistic regression ensemble on the banana quality dataset.

The mean accuracy scores obtained from each value of number of randomly selected features in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 12.

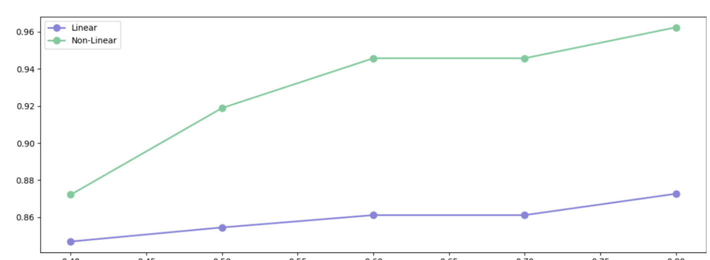


Fig. 12: Comparison of mean accuracy score across the number of randomly selected features grid when the banana quality dataset is classified by use of 5-fold cross-validation.

The optimal value for the number of features in the linear and non-linear logistic regression ensemble is 0.8, which indicates that both models performs better when constructed on more features.

The paired t-test between the two models returned a p-value of 0.0043, which indicates there is a significant difference between the average accuracy scores of the linear and non-linear logistic regression ensembles based on the banana quality dataset.

From figure 12 it is seen that the non-linear logistic regression ensemble performs best across all different number of features. Therefore the non-linear logistic ensemble model performs best on the banana quality dataset when the number of features is considered.

3) Number of ensemble members: Table XXV presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XXV: Summary Statistics of Accuracy Scores for Each Number of Ensemble Members of the Linear Logistic Regression Ensemble on the Banana Quality Dataset

Number of Featurs	Summary Statistics				
	count	***mean***	***std***	***min***	***max***
5	125	82.90	0.0393	68.81	88.31
10	125	85.74	0.0169	79.94	88.75
20	125	86.66	0.0111	82.12	88.69
50	125	87.08	0.0074	85.38	88.81
100	125	87.26	0.0065	85.62	88.75

The Kruskal-Wallis test returned a p-value of 0 between the scores of the linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of ensemble members has a notable effect on the performance of the linear logistic regression ensemble on the banana quality dataset.

Table XXVI presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XXVI: Summary Statistics of Accuracy Scores for Each Number of Ensemble Members of the Non-Linear Logistic Regression Ensemble on the Banana Quality Dataset

Number of Features	Summary Statistics				
	count	***mean***	***std***	***min***	***max***
5	125	90.15	0.0627	69.50	96.75
10	125	92.18	0.0423	71.00	96.69
20	125	93.36	0.0305	82.88	96.88
50	125	94.16	0.0235	87.75	97.00
100	125	94.66	0.0219	88.31	97.12

The Kruskal-Wallis test returned a p-value of 0 between the scores of the non-linear logistic regression ensemble model, which means there is a significant difference between the scores. Therefore, the number of ensemble members has a notable effect on the performance of the non-linear logistic regression ensemble on the banana quality dataset.

The mean accuracy scores obtained from each value of number of ensemble members in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 13.

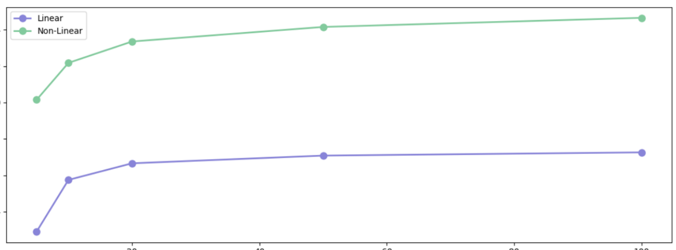


Fig. 13: Comparison of mean accuracy score across the number of ensemble members grid when the banana quality dataset is classified by use of 5-fold cross-validation.

From 20 members in both the linear and non-linear logistic regression ensemble models, the average accuracy score increases by a very small amount. To allow the model to train faster, an optimal value of 20 members is chosen for both ensemble models.

The paired t-test between the two models returned a p-value of 0, which indicates there is a significant difference between the average accuracy scores of the linear and non-linear logistic regression ensembles based on the banana quality dataset.

From figure 13 it is seen that the non-linear logistic regression ensemble performs best across all different number of ensemble members. Therefore the non-linear logistic ensemble model performs best on the banana quality dataset when the number of ensemble members is considered.

E. Spiral Dataset

By use of Bayesian optimisation, the optimal values of the control parameters in the linear and non-linear logistic regression ensembles are determined as seen in Section IV-D.

A 5-fold cross validation technique is then applied to the grid search of the linear and non-linear logistic regression ensemble models to investigate the influence of the size of the bag and the number of ensemble members as seen in Section IV-C.

1) Bag size: Table XXVII presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE XXVII: Summary Statistics of Accuracy Scores for Each Bag Size of the Linear Logistic Regression Ensemble on the Spiral Dataset

Bag Size	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
0.4	25	59.31	0.0259	54.4	62
0.5	25	59.65	0.0278	54.4	63.2
0.6	25	59.25	0.0274	54	62.8
0.7	25	59.46	0.0319	54	63.6
0.8	25	59.58	0.0294	54	63.6

The Kruskal-Wallis test returned a p-value of 0.8946 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the bag size has a minimal effect on the performance of the linear logistic regression ensemble on the spiral dataset.

Table XXVIII presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each bag size value in the grid.

TABLE XXVIII: Summary Statistics of Accuracy Scores for Each Bag Size of the Non-Linear Logistic Regression Ensemble on the Spiral Dataset

Bag Size	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
0.4	25	60.66	0.0347	54.4	64.4
0.5	25	60.69	0.0358	53.2	64.8
0.6	25	61.20	0.0368	54	65.6
0.7	25	61.26	0.0368	54	66.4
0.8	25	62.02	0.0383	54	66.8

The Kruskal-Wallis test returned a p-value of 0.3408 between the scores of the non-linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the bag size has a minimal effect on the performance of the non-linear logistic regression ensemble on the spiral dataset.

The mean accuracy scores obtained from each value of bag size in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 14.

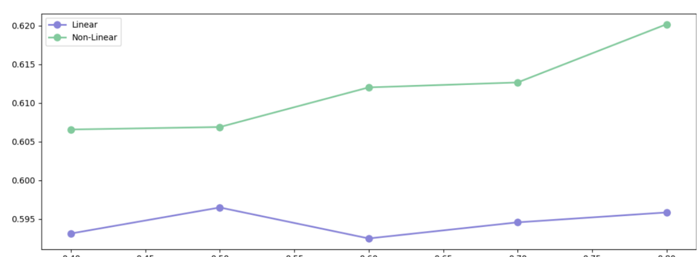


Fig. 14: Comparison of mean accuracy score across the bag size grid when the spiral dataset is classified by use of 5-fold cross-validation.

The optimal bag size for the linear and non-linear logistic regression ensemble models is a value of 0.8.

The paired t-test between the two models returned a p-value of 0.0021, which indicates there is a significant difference between the average accuracy scores of the linear and non-linear logistic regression ensembles based on the spiral dataset.

From figure 14 it is seen that the non-linear logistic regression ensemble performs best across all different bag sizes. Therefore the non-linear logistic ensemble model performs best on the spiral dataset when the bag size is considered.

2) Number of ensemble members: Table XXIX presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XXIX: Summary Statistics of Accuracy Scores for Each Number of Ensemble Members of the Linear Logistic Regression Ensemble on the Spiral Dataset

Number of Featurs	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
5	25	59.46	0.0296	54	63.6
10	25	59.34	0.0274	54	63.2
20	25	59.68	0.0298	54	63.6
50	25	59.49	0.0281	54	62
100	25	59.28	0.0278	54	62

The Kruskal-Wallis test returned a p-value of 0.9572 between the scores of the linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of ensemble members has a minimal effect on the performance of the linear logistic regression ensemble on the spiral dataset.

Table XXX presents the mean, standard deviation, minimum and maximum of the accuracy scores from the 5-fold cross-validation of the non-linear logistic regression ensemble, grouped by each number of ensemble members value in the grid.

TABLE XXX: Summary Statistics of Accuracy Scores for Each Number of Ensemble Members of the Non-Linear Logistic Regression Ensemble on the Spiral Dataset

Number of Features	**Summary Statistics**				
	count	***mean***	***std***	***min***	***max***
5	25	60.69	0.0320	54	66.4
10	25	61.14	0.0334	54.8	64.8
20	25	61.09	0.0363	54	64.4
50	25	61.65	0.0411	53.2	66.8
100	25	61.26	0.0399	54.4	65.6

The Kruskal-Wallis test returned a p-value of 0.4160 between the scores of the non-linear logistic regression ensemble model, which means there is no significant difference between the scores. Therefore, the number of ensemble members has a minimal effect on the performance of the non-linear logistic regression ensemble on the spiral dataset.

The mean accuracy scores obtained from each value of number of ensemble members in the grid from both the linear and non-linear logistic regression ensemble models is presented in Figure 15.

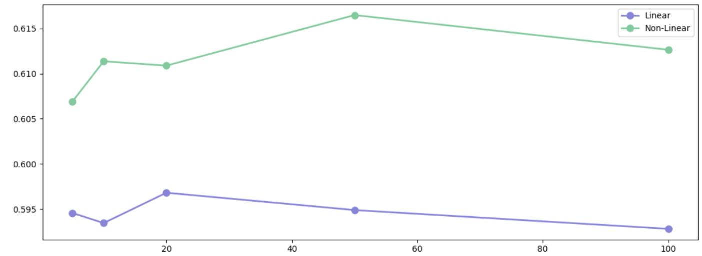


Fig. 15: Comparison of mean accuracy score across the number of ensemble members grid when the spiral dataset is classified by use of 5-fold cross-validation.

The optimal number of members for both of the linear and non-linear logistic regression ensembles is 20 and 50 respectively.

The paired t-test between the two models returned a p-value of 0.0006, which indicates there is a significant difference between the average accuracy scores of the linear and non-linear logistic regression ensembles based on the spiral dataset.

From figure 15 it is seen that the non-linear logistic regression ensemble performs best across all different number of ensemble members. Therefore the non-linear logistic ensemble model performs best on the spiral dataset when the number of ensemble members is considered.

VI. Conclusion

This paper investigated the influence of the size of the bags, the number of randomly selected features and the number of members in linear and non-linear logistic regression ensemble models across five binary classification datasets that varies in complexity. The results show that each factor plays a critical role in model performance, however, the degree of impact varies based on the complexity of the dataset and the type of model used.

The results indicated the number of statistically significant tests associated with each model configuration. Specifically, the linear model configuration with a bag size of 2, 3 features, and 1 model showed a lower count of significant tests, while the non-linear configuration with a bag size of 4, 4 features, and 3 models resulted in a higher count of significant tests across the datasets. To create an optimal ensemble on each dataset, the size of the bags, the number of randomly selected features, and the number of members should all be considered together. All of these elements significantly influence how well the ensemble performs.

The non-linear logistic regression ensemble performed the best on all but one of the datasets across all bag sizes, numbers of randomly selected features, and ensemble members. This consistency in performance suggests that non-linear ensembles are more adept at capturing complex patterns and interactions within the data.

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