Andrew Alford
11/15/2023

Homework 6

1. The data sets package (installed in R by default) contains a data set called InsectSprays that shows the results of an experiment with six different kinds of insecticides. For each kind of insecticide, n = 12 observations were conducted. Each observation represented the count of insects killed by the spray. In this experiment, what is the dependent variable (outcome) and what is the independent variable? What is the total number of observations?

The dependent variable in this experiment is the count of insects killed by the spray. This is the variable that is being measured and is expected to change in response to the different treatments.

2. After running the aov() procedure on the InsectSprays data set, the “Mean Sq” for spray is 533.8 and the “Mean Sq” for Residuals is 15.4. Which one of these is the between groups variance and which one is the within-groups variance? Explain your answers briefly in your own words.

The “Mean Sq” for Spray (533.8) is the between groups variance. It represents the variability in the insect counts that can be attributed to the differences between the different types of sprays. It’s a measure of how much the insecticide types differ from each other in terms of their effectiveness. The “Mean Sq” for Residuals (15.4) is the within-groups variance. It represents the variability within each type of insecticide spray. This is the variation in insect counts that cannot be explained by the type of spray used and is attributed to other, random factors.

3. Based on the information in question 2 and your response to that question, calculate an *F*-ratio by hand or using a calculator. Given everything you have learned about *F*-ratios, what do you think of this one? Hint: If you had all the information you needed for a Null Hypothesis Significance Test, would you reject the null? Why or why not?

To calculate the F-ratio, we can use the following equation:

$$F = \frac{Between\text{−}groups\ Variance}{Within\text{−}groups\ Variance} = \frac{533.8}{15.4} \approx 34.66$$

An F-ratio of 34.66 is quite high, suggesting that the observed differences in means across the groups are likely not due to random chance. This high F-ratio indicates that the variability between the groups is much larger than the variability within the groups. In the context of an NHST, a high F-ratio like this provides strong evidence against the null hypothesis. It implies that at least one insecticide type is significantly different in effectiveness compared to the others. Given the high value of the F-ratio here, it's likely that the null hypothesis would be rejected, indicating significant differences among the insecticide types.

4. Continuing with the InsectSprays example, there are six groups where each one has n = 12 observations. Calculate the degrees of freedom between groups and the degrees of freedom within groups. Explain why the sum of these two values adds up to one less than the total number of observations in the data set.

The degrees of freedom between groups is calculated as the number of groups minus one. Since there are 6 groups in the InsectSprays dataset, the between groups is calculated as 6 - 1 = 5. This represents the number of independent comparisons that can be made between the group means. On the other hand, the degree of freedom within groups is calculated as the total number of observations minus the number of groups. Given that each group has 12 observations, totaling to 6 X 12 = 72, the within groups can be calculated as 72 - 6 = 66. This figure represents the number of observations that are free to vary within each group after considering the group mean.

The sum of the between groups and within groups equates to one less than the total number of observations in the dataset. The total number of observations in the InsectSprays dataset is 72. In the variance between groups, one degree of freedom is used for each group, corresponding to the estimation of each group's mean, leaving 5 degrees of freedom. The remaining 66 degrees of freedom within groups correspond to each observation minus the number of groups. When these two values are combined (5 + 66), the result is 71, which is exactly one less than the total number of observations. This discrepancy is attributed to the estimation of the overall mean of the data, which consumes one degree of freedom, thereby aligning the total degrees of freedom with the total number of independent pieces of information in the dataset.

5. Use R or R-Studio to run the aov() command on the InsectSprays data set. You will have to specify the model correctly using the "~" character to separate the dependent variable from the independent variable. Place the results of the aov() command into a new object called insectResults. Run the summary() command on insectResults and interpret the results briefly in your own words. As a matter

of good practice, you should state the null hypothesis, the alternative hypothesis, and what the results of the null hypothesis test lead you to conclude.

```
> insectResults <- aov(count ~ spray, data = InsectSprays)
> summary(insectResults)
            Df Sum Sq Mean Sq F value Pr(>F)    
spray        5   2669   533.8    34.7 <2e-16 ***
Residuals   66   1015    15.4                   
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
```

In the ANOVA analysis conducted on the InsectSprays dataset, the results reveal some insightful aspects of the effectiveness of different insect sprays. This analysis, which is characterized by degrees of freedom for the spray variable at 5 and residuals at 66, aligns with the theoretical calculations, reflecting a comprehensive assessment of variance across the groups and within them. The Sum of Squares, partitioned into 2669 for the spray variable and 1015 for residuals, denotes the distribution of total variability. This partitioning indicates that a significant portion of the variance in the dataset is attributed to the differences in sprays, with the Mean Square values of 533.8 for sprays and 15.4 for residuals further elucidating this point. A particularly notable aspect of the analysis is the high F-value of 34.7, underscoring a substantial variance ratio between the groups compared to within them, suggesting that the model explains a significant proportion of the variance in the data.

The pivotal component of the analysis is the P-value, reported as less than 2e-16, which is practically zero and significantly lower than the standard significance level of 0.05. This exceptionally low P-value leads to the rejection of the null hypothesis. The rejection of the null hypothesis indicates a statistically significant difference in spray effectiveness, implying that not all sprays are equally efficient. This finding is crucial as it directs attention towards identifying which specific insect spray is most effective.

6. Load the BayesFactor package and run the anovaBF() command on the InsectSprays data set. You will have to specify the model correctly using the “~” character to separate the dependent variable from the independent variable. Produce posterior distributions with the posterior() command and display the resulting HDIs. Interpret the results briefly in your own words, including an interpretation of the BayesFactor produced by the grouping variable. As a matter of good practice, you should state the two hypotheses that are being compared. Using the rules of thumb offered by Kass and Raftery (1995), what is the strength of this result?

In total, here is the code I used to produce results:

```
library(BayesFactor)
data("InsectSprays")
bfResults <- anovaBF(formula = count ~ spray, data = InsectSprays)
posterioDist <- posterior(bfResults, iterations = 10000)
HDIs <- HPDinterval(posteriorDist)
posteriorDist
HDIs
```

The Bayes Factor (BF) for the effect of the spray is an astonishingly large value of approximately 1.51×10^{14}.

```
Bayes factor analysis
--------------
[1] spray : 1.506706e+14 ±0%

Against denominator:
  Intercept only
---
Bayes factor type: BFlinearModel, JZS

>
```

This figure indicates extremely strong evidence against the null hypothesis. According to the scale provided by Kass and Raftery, the BF obtained here far surpasses this threshold, placing it firmly in the category of very strong evidence. This result strongly supports the alternative hypothesis.

Complementing this conclusion are the HDIs derived from the Bayesian analysis.

```
> HDIs
                lower     upper
mu          8.5509257 10.422179
spray-A     2.6640195  6.813572
spray-B     3.5112487  7.638754
spray-C    -9.1072894 -4.973382
spray-D    -6.4444248 -2.350782
spray-E    -7.7412026 -3.630771
spray-F     4.8396258  8.901202
sig2       11.1044376 22.207758
g_spray     0.3982281  8.969179
attr(,"Probability")
[1] 0.95
```

These HDIs provide a 95% probability range for the effects of each individual spray, as well as for the overall mean. For instance, negative intervals for some sprays, such as spray-C, suggest their lower effectiveness in comparison to the baseline, indicating a reduction in count. In contrast, positive intervals for other sprays like Spray-A and spray-F indicate a higher level of effectiveness.

7. In situations where the alternative hypothesis for an ANOVA is supported and there are more than two groups, it is possible to do post-hoc testing to uncover which pairs of groups are substantially different from one another. Using the InsectSprays data, conduct a *t*-test to compare groups C and F (preferably a Bayesian *t*-test). Interpret the results of this *t*-test.

First things first, here is the code that I used to conduct the t-test:

```
> ttestBF(InsectSprays$count[InsectSprays$spray == "C"], InsectSprays$count[InsectSprays$spray == "F"])
Bayes factor analysis
--------------
[1] Alt., r=0.707 : 90005.78 ±0%

Against denominator:
  Null, mu1-mu2 = 0
---
Bayes factor type: BFindepSample, JZS
```

The Bayesian t-test conducted to compare the effectiveness of insect sprays C and F in the InsectSprays dataset yielded a Bayes Factor (BF) of approximately 90,005.78. This value, significantly greater than 1, indicates evidence in favor of the alternative hypothesis. Such a high BF value is a robust indicator that the effectiveness of these two sprays is not the same, favoring rejection of the null hypothesis. In statistical terms, a BF exceeding 100 is generally interpreted as decisive evidence, according to the scale proposed by Kass and Raftery. Therefore, the BF of 90005.78 provides decisive evidence against the null hypothesis.