Experimental Design and Data Analysis: Assignment 4

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Exercise 1

1

To randomise the 6 combinations of conditions for the slices of bread, we may implement the following R code.

$$sample_slices = sample(1:18, 18)$$

Which simply samples 18 numbers from the range 1-18 randomly. In this way, if we have the 18 random id's that correspond to the various conditions already decided upon, and their numerical results for time.

$\mathbf{2}$

From the data bread.txt we make boxplots of hours before bread becomes mouldy versus the two factors, temperature environment and humidity:

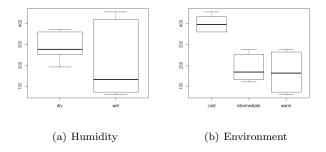


Figure 1: Boxplots of Hours with Humidity and Environment

There are a few notable features in these boxplots, mainly that the variances in the two humidity factors are significantly differe, and secondly that there is

zero overlap between the values obtained from the cold environment and the other two.

3

Next, by creating interaction plots, by letting one of our factors be the continuous x-axis, and plotting for individual values of the other factor, we obtain the following:

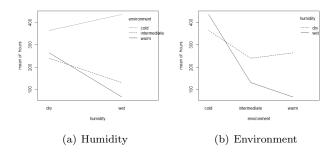


Figure 2: Interaction plots of Hours with Humidity and Environment

Notice in Fig 2:(a) the plot of cold environment is significantly different from those of the other two environments, and again in 2:(b) where the lines for wet and dry cross. Given the relatively extreme values seen in the box plots 1, it is plausible there is an interaction between environment and humidity taking place.

4

Performing an ANOVA test, to test the null hypothesis that the environment, humidity and the interaction between the two are insignificant we obtain the following:

Analysis of Variance Table

Response: hours

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
environment	2	201904	100952	233.685	$2.461\mathrm{e}{-10}$	***
humidity	1	26912	26912	62.296	$4.316\mathrm{e}\!-\!06$	***
$\mathbf{environment}: \mathbf{humidity}$	2	55984	27992	64.796	$3.705\mathrm{e}{-07}$	***
Residuals	12	5184	432			

The results are quite compelling, we reject all the null hypothesis with all p-values <<0.5, therefore there is an effect of environment, humidity as well as an interaction between the two.

5

This interaction effect, which can be more easily seen in Fig:2, appears to is surprisingly complex. For wet humidity, the cold temperature allowed the bread to resist mould the longest, but that drops off dramatically as the temperature moves to intermediate, and again to warm. The effect for the dry humidity was also the longest in the cold environment, and again this decreased when moving to an intermediate temperature, but the time to mould again increased as the environment went to warm. Clearly there are strong effects, that are non-obvious.

6

As we can see from the F-Value in the table of exercise 1.4, environment has the largest numerical influence. In our case this is not really a good question, especially considering that environment had a larger range of values than humidity, thus in this case it is more useful to instead consider the relative influence of our factors.

7

In Fig3 the QQ-plot of the residuals is shown. Here you can see that the data is roughly normally distributed.

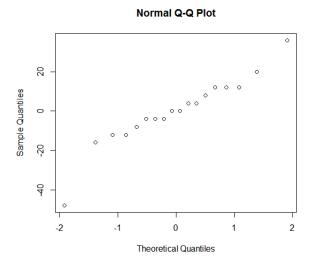


Figure 3: QQ-plot of the residuals

In Fig4 the fitted values versus residual values are shown. It seems that there is no heteroscedasticity present in this data.

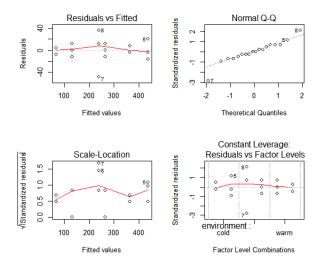


Figure 4: Fitted Values VS Residual Values

Exercise 2

1

The used code can be found in 1.2. It makes an array with the numbered students and an array with the skill level of the related student. The students are randomly distributed among the three different search engines by using this code:

```
block_design = matrix(c(samples[0:5], samples[6:10], samples[11:15]),
nrow = 3, ncol = 5)
```

$\mathbf{2}$

Making boxplots of our two factors versus time from the data in search.txt, we obtain the following:

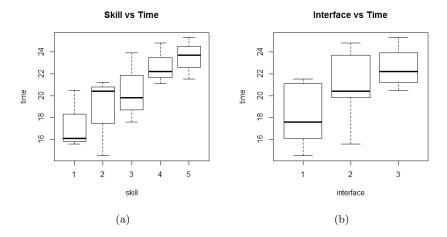


Figure 5: Box Plots of Skill and Interface vs Time

There are no surprises in Fig 5(a), it stands to reason that those with the lower score (higher computer experience) will have lower times, however in 5(b) it is worth noting that interface 3 seems to be related to higher time scores.

Continuing we created two interaction plots:

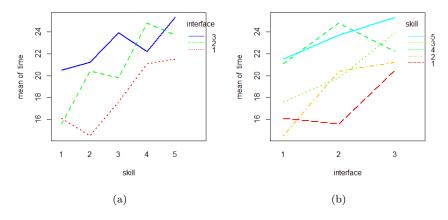


Figure 6: Interaction plots of Skill and Interface vs Time

It is difficult to conclude that there is an interaction between skill and interface as they are clearly not parallel, but they follow the same general trajectory, so this interaction may be due to noise.

3

Using the Kurskal-Wallis rank sum test, we test if the distributions of our populations in regards to the time measured for different interfaces are the same, and we obtain the following results:

Kruskal-Wallis rank sum test

data: time and interface
Kruskal-Wallis chi-squared = 4.22, df = 2, p-value =0.1212

Thus with a p-value of 0.1212 we reject the null hypothesis that our populations are the same, therefore the search time for all interfaces is not equal.

4

5

We next make diagnostic plots, a qq-plot of the risiduals, and a plot of fitted values versus residuals:

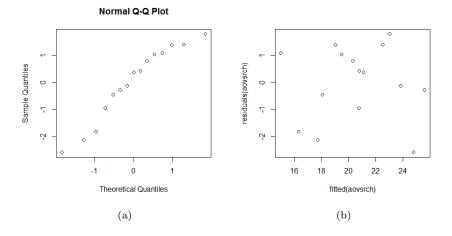


Figure 7: Diagnostic Plots

It is difficult to say for certain, there may be a slight curve in the qq-plot 7(a) but it looks approximately normal, and the fitted value plot 7(b) suggests there is no significant difference in the population variances.

6

Performing the non-parametric Friedman test:

Friedman rank sum test

```
data: time, interface and skill
Friedman chi-squared = 6.4, df = 2, p-value = 0.04076
```

With a p-value of 0.04076 we reject the null hypothesis, thus we conclude there is an effect of interfaces.

7

Testing the null hypothesis that the search time is the same for all interface by a ANOVA test, ignoring skill we obtain the following:

Analysis of Variance Table

With a p-value of 0.09642 we do not reject the null hypothesis, thus the time is not the same for all interfaces. This test is useful to use, if some conditions are met, as of right now, we do not have convincing evidence there is no interaction between skill and interface. It would be more useful to test along with the variable skill, and the interaction between the two as follows:

Analysis of Variance Table

Response: time

```
Df Sum Sq Mean Sq F value
                                                \Pr(>F)
interface
                  149.729
                             49.729 21.4145 0.0007313 ***
skill
                  178.732
                             78.732 33.9039 0.0001154 ***
interface: skill
                  1
                     2.312
                              2.312
                                     0.9956 \ 0.3398204
                 11 25.544
Residuals
                              2.322
```

This way we are able to measure the interaction between the data sets, and determine whether there is a factor which has a greater effect on our results. From the previous table we see that with a p-value of 0.3398 we accept the null hypothesis that there is no significant interaction between skill in and interface, which is what we require for the one-way ANOVA ignoring skill to be valid.

Exercise 3

1

We begin with analysing the data presented in cream.txt in a three-way experiment without interactions.

Analysis of Variance Table

Response: acidity

Residuals 15 20.475 1.3650

$\mathbf{2}$

Call

 $lm(formula = acidity \ \tilde{\ } starter + batch + position, data = cream)$

Residuals:

Coefficients:

	Estimate	Std. Error	${f t}$ value	$\Pr(> \mathbf{t})$	
(Intercept)	7.8260	0.8586	9.115	$1.67{\rm e}\!-\!07$	***
starter2	-0.1500	0.7389	-0.203	0.84186	
starter3	-0.9800	0.7389	-1.326	0.20459	
starter4	2.8100	0.7389	3.803	0.00173	**
starter5	-0.4840	0.7389	-0.655	0.52238	
batch	0.3116	0.1652	1.886	0.07883	•
position 2	-0.6180	0.7389	-0.836	0.41608	
position3	-0.0380	0.7389	-0.051	0.95966	
position 4	-0.7640	0.7389	-1.034	0.31755	
position 5	-0.2640	0.7389	-0.357	0.72586	

Residual standard error: 1.168 on 15 degrees of freedom Multiple **R**-squared: 0.7149, Adjusted **R**-squared: 0.5438 F-statistic: 4.179 on 9 and 15 DF, p-value: 0.007304

3

Linear Hypotheses:

```
Estimate Std. Error \mathbf{t} value \Pr(>|\mathbf{t}|)
2 - 1 == 0
            -0.1500
                           0.4673
                                    -0.321
                                                0.997
3 - 1 == 0
             -0.9800
                           0.4673
                                    -2.097
                                                0.282
4 - 1 == 0
             2.8100
                           0.4673
                                     6.013
                                               <0.001 ***
5 - 1 == 0
             -0.4840
                           0.4673
                                    -1.036
                                                0.834
3 - 2 = 0
             -0.8300
                           0.4673
                                    -1.776
                                                0.429
```

```
4 - 2 == 0
               2.9600
                           0.4673
                                      6.334
                                               < 0.001 ***
5 - 2 = 0
              -0.3340
                           0.4673
                                     -0.715
                                                0.949
4 - 3 == 0
               3.7900
                           0.4673
                                      8.110
                                                < 0.001 ***
5 - 3 == 0
               0.4960
                           0.4673
                                      1.061
                                                0.822
5 - 4 == 0
              -3.2940
                           0.4673
                                     -7.048
                                               < 0.001 ***
```

Starter 1 and 4 produce significantly different acidity. This is apparent first from exercise 3.2 we obtain: $\mu_1 = 7.8260$ and $\mu_4 = 7.8260 + 2.8100 = 10.636$ which are the estimates for starter 1 and 4 respectively. Furthermore the p-value for starter 1 obtained from exercise 3.2, of 1.67e-07 suggests that we reject the null hypothesis that our estimate for starter 1 is equal to that of the rest of the population, and the p-values obtained from the simultaneous method, of 0.001 suggests that we reject the null hypothesis that the estimate for the treatment effect of 4 is equal to each of the other respective treatment effects.

4

The p-value for the test $H_0: \alpha_2 = \alpha_1$, where α_i is our estimate of the treatment effects is 0.997, whereas in exercise 3.2 our p-value for the null hypothesis $H_0: \alpha_2 = \alpha_1$ is 0.84186. It is no coincidence that the p-value obtained from exercise 3.2, is different from that of the simultaneous calculation, this is due to the fact that when calculating the simultaneous p-values for the null hypothesis $H_0: \alpha_i = \alpha_1$ we are doing so in such a way that the probability of rejecting any null hypothesis in error is less than 0.5. In contrast to the method used in exercise 3.2, where for every null hypothesis our chance of making such an error is N*0.05 where N is the number of possibilities to make such an error. In this way the Simultaneous p-values method gives us a higher confidence in our conclusions.

5

Linear Hypotheses:

```
Estimate
                     lwr
                               upr
2 - 1 = 0 -0.1500
                      -1.6391
                                1.3391
3 - 1 = 0 -0.9800
                      -2.4691
                                0.5091
4 - 1 == 0
             2.8100
                       1.3209
                                4.2991
5 - 1 = 0 -0.4840
                      -1.9731
                                1.0051
3 - 2 = 0 -0.8300
                      -2.3191
                                0.6591
4 - 2 = 0
             2.9600
                       1.4709
                                4.4491
5 - 2 = 0 -0.3340
                      -1.8231
                                1.1551
4 - 3 == 0
             3.7900
                       2.3009
                                5.2791
5 - 3 = 0 \quad 0.4960
                      -0.9931
                                1.9851
5 - 4 = 0 -3.2940
                      -4.7831
                               -1.8049
```

The confidence intervals for testing all differences $\alpha_j - \alpha_{j'}$ for $(i, i' \in 1, 2..., 5)$ of the main effects for starter with simultaneous confidence level 95% are (4-1),(4-2),(4-3),(4-5),(5-4) so all those containing starter 4. This is likely due to the fact that the estimated value for starter 4, which was earlier calculated as 10.636, is

so much larger than every other estimate, for example all the confidence intervals for (4-i) are greater than 0, and the one interval (5-4) is less than 0.

Exercise 4

1

Performing ANOVA test using an ordinary fixed effects model fitted with lm, the follow results were obtained:

Analysis of Variance Table

```
Response: milk
           Df
               Sum Sq Mean Sq F value Pr(>F)
                  0.27
                                 0.0016 \ 0.9682
treatment
            1
                         0.269
                                 0.3268 \ 0.5760
order
            1
                53.52
                        53.515
Residuals 15 2456.69 163.779
```

Residuals:

```
Min
                  1\mathbf{Q}
                       Median
                                          3\mathbf{Q}
                                                    Max
-16.602 -10.103
                          0.170
                                                22.042
                                      9.431
```

Coefficients:

```
Estimate Std. Error \mathbf{t} value \Pr(>|\mathbf{t}|)
(Intercept)
                37.1722
                               5.4379
                                          6.836
                                                 5.64e - 06 ***
                -0.2444
                               6.0329
treatmentB
                                         -0.041
                                                     0.968
orderBA
                -3.4700
                               6.0704
                                         -0.572
                                                      0.576
```

```
Signif. codes:
             0.001
```

```
Residual standard error: 12.8 on 15 degrees of freedom
```

0.05

0.1

1

0.01

Multiple R-squared: 0.02142, Adjusted R-squared: F-statistic: 0.1642 on 2 and 15 DF, p-value: 0.8501

We can conclude from the F-value and p-value of 0.0016 and 0.9682 respectively for the treatment effect on milk production, we do not reject the null hypothesis, therefore there is no treatment effect on milk production. Further we observe that the p-value for the null hypothesis that the individual treatments effects are all greater than 0.05, thus we accept the null hypothesis that the treatment effects are equal. This stands to reason as if we do not conclude there is any effect for all treatments, there would be no effect for individual treatments when sample sizes are equal.

2

For the treatment order AB, the summary of milk is:

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 26.50 28.32 34.15 37.05 43.62 54.00
```

And for the treatment order BA, the summary of milk is:

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 21.40 30.45 37.80 37.56 44.90 55.50
```

We may observe that the cows with the BA order have a small increase milk production than the ones with the AB order, but this may be misleading as the effect is quite small.

3

Analysis of Variance Table

Df Sum Sq Mean Sq F value
treatment 1 0.26889 0.26889 0.0503

order 1 0.82889 0.82889 0.1552

AIC BIC logLik deviance df.resid 125.4 129.9 -57.7 115.4 13

Scaled residuals:

Random effects:

 $\begin{array}{cccc} Groups & Name & Variance & Std.Dev.\\ id & (Intercept) & 169.755 & 13.029\\ Residual & & 5.341 & 2.311\\ Number of obs: 18, groups: id, 9 \end{array}$

Fixed effects:

Estimate Std. Error **t** value (Intercept) 37.1722 6.5881 5.642 treatmentB -0.2444 1.0895 -0.224 orderBA -3.4700 8.8086 -0.394

Correlation of Fixed Effects:

 $\begin{array}{c} (Intr) \ trtmnB \\ treatmentB \ -0.083 \\ orderBA \ -0.743 \ 0.000 \end{array}$

4

Paired t-test

```
data: milk[treatment = "A"] and milk[treatment = "B"]
t = 0.2244, df = 8, p-value = 0.8281
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.267910 2.756799
sample estimates:
mean of the differences 0.2444444
    R-Code
1
```

1.1 Exercise 1

```
bread = read.table("bread.txt")
hours = bread$hours
humidity = bread$humidity
environment = bread$environment
#1.1
sample_slices = sample(1:18, 18)
#1.2
boxplot(hours humidity)
boxplot(hours~environment)
#1.3
interaction.plot(humidity, environment, hours)
interaction.plot(environment, humidity, hours)
aovenvhum = lm(hours ~ environment*humidity, data=bread)
anova(aovenvhum)
qqnorm(residuals(aovenvhum))
\mathbf{par}(\mathbf{mfrow} = \mathbf{c}(2,2))
plot (aovenvhum)
1.2
    Exercise 2
#Note: The interface is of greatest importance
search = read.table("search.txt")
```

```
time =search$time
skill = search skill
interface = search$interface
#2.1
students = 1:15
# set the lvls of the students
skill_lvls = rep(c(1,2,3,4,5), each=3)
# sample the students
samples = sample(students, 15)
\# make the block_design
block_design = matrix(c(samples[0:5],
                         samples [6:10],
                         samples [11:15]),
                      nrow = 3, ncol = 5)
skill\_block\_design = matrix(c(skill\_lvls[samples[0:5]],
                               skill_lvls[samples[6:10]],
                               skill_lvls[samples[11:15]]),
                             nrow = 3, ncol = 5)
#2.2
boxplot(time skill, xlab="skill", ylab="time", main="Skill_vs_Time")
boxplot(time~interface, xlab="interface", ylab="time", main="Interface_vs_Time")
interaction.plot(skill,interface,time,lwd=2,col=rainbow(3))
interaction.plot(interface, skill, time, lwd=2, col=rainbow(8))
#2.3
interface = factor(search$interface)
skill = factor(search$skill)
kruskal.test (time, interface)
#2.4
aovsrch=lm(time~skill+interface)
summary(aovsrch)
#2.5
qqnorm(residuals(aovsrch))
plot(fitted(aovsrch), residuals(aovsrch))
#2.6
```

```
friedman.test(time, interface, skill)
#2.7
anovinter = lm(time interface)
anova (anovinter)
anv = lm(time~interface*skill,data=search)
anova (anv)
     Exercise 3
1.3
library(multcomp)
cream = read.table("cream.txt", header=T)
cream$starter = factor(cream$starter)
cream$position = factor(cream$position)
#3.1
creamlm = lm(acidity~starter+batch+position, data=cream)
anova (creamlm)
#3.2
summary(creamlm)
#3.3
tmp = glht(creamlm, linfct=mcp(starter="Tukey"))
summary(tmp)
#3.4
confint (tmp)
1.4 Exercise 4
library (lme4)
cow = read.table("cow.txt", header=T)
cow$order = factor(cow$order)
order = cow\$order
milk = cow milk
cow$treatment = factor(cow$treatment)
treatment = cow$treatment
#4.1
cowlm = lm(milk ~ treatment + order, data=cow)
anova (cowlm)
```

```
summary(cowlm)
#4.2
summary(cow[0:8,]$milk)
summary(cow[9:16,]$milk)

#4.3
cowlmer = lmer(milk ~ treatment + order+(1|id), data=cow)
anova(cowlmer)
summary(cowlmer)

#4.4
attach(cow)
t.test(milk[treatment="A"], milk[treatment="B"], paired=TRUE)
```