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The Rare Decays $K \to \pi\nu\bar\nu$ and $K_L \to \pi^0\ell^+\ell^-$

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Abstract. Standard-model and possible New-Physics contributions can compete in magnitude in the rare Kaon decay modes $K \to \pi\nu\bar\nu$ and $K_L \to \pi^0\ell^+\ell^-$. In particular the $K \to \pi\nu\bar\nu$ decays, the structure of short-distance physics is unclouded by non-perturbative, low-energy effects. This makes these decays excellent probes of New Physics. We review the status of the standard-model prediction of both $K \to \pi\nu\bar\nu$ and $K_L \to \pi^0\ell^+\ell^-$ decays and discuss the recent progress in the perturbative part of the $K \to \pi\nu\bar\nu$ decays.

1. Introduction

One approach to searching for deviations from the predictions of the Standard Model (SM) of particle physics is to examine low-energy observables, which are sensitive to New Physics (NP) effects and can also be predicted with a good accuracy within the SM. Four such observables are the branching ratios of the rare kaon decays $K_L \to \pi^0\nu\bar\nu$, $K^+ \to \pi^+\nu\bar\nu$, $K_L \to \pi^0\mu^+\mu^-$, and $K_L \to \pi^0 e^+ e^-$.

Within the SM short-distance (SD) physics accounts for approximately 90% and 40% of the branching fractions of the $K \to \pi\nu\bar\nu$ and $K_L \to \pi^0\ell^+\ell^-$ decays, respectively. The decays are all governed by a flavour-changing neutral current (FCNC) transition and are therefore loop induced within the SM. In loop induced processes, heavy new particles may contribute to the decay amplitudes on the same footing as SM particles. Thus, they represent ideal probes for searching and disentangling NP.

A precise SM prediction is therefore essential to disentangle NP effects from the SM background. Often, non-perturbative effects originating from low-energy dynamics spoil the SM precision. This is not the case for the two $K \to \pi\nu\bar\nu$ decays, which are almost completely free from non-perturbative uncertainties and thus rank among the cleanest NP probes in flavour physics. Long-distance (LD) effects are larger in the $K_L \to \pi^0\ell^+\ell^-$ decays, but useful information about the structure of NP can still be extracted from a combined analysis of both branching ratios.

In the following we briefly discuss the general structure of rare Kaon decays, we then review the SM prediction of the above-mentioned modes and also present the recent calculation of the electroweak (EW) top-quark contribution of $K \to \pi\nu\bar\nu$.
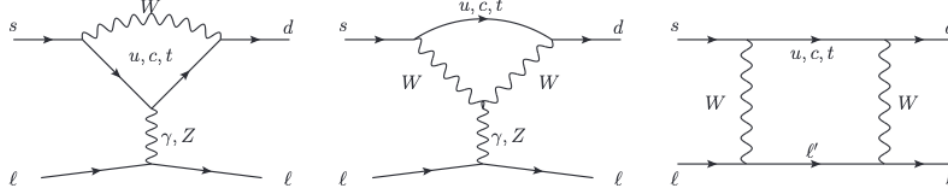
Figure 1. γ, Z penguins and W boxes contributing to the $s \to d$ transition at the one-loop level. In the case $l \equiv \nu$ the γ-penguin is absent.

2. Structure of Rare Kaon Decays

In the SM the FCNC transition $s \to d$ relevant for the rare Kaon decays is only possible at the one-loop level via the box and γ, Z-penguin diagrams of Fig. 1. There is a large energy gap between the scale in which the Feynman diagrams of Fig. 1 are to be evaluated and the scale of the outgoing physical states. The first scale is approximately the Z boson mass, while the second the mass of the Kaon, approximately 500 MeV. The gap is closed by using an appropriate effective field theory (EFT) to describe the low-energy physics, where heavy particles are no longer dynamical degrees of freedom. In this way the effect of SD physics is separated from LD effects. A master amplitude is then valid for all meson decays [1]:

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle \, V_{\text{CKM}}^i \, F_i. \tag{1}$$

$\langle Q_i \rangle$ are matrix elements of effective operators, that cannot be calculated perturbatively, V_{CKM}^i comprise products of CKM matrix elements and F_i are the so-called Wilson coefficients, the effective couplings incorporating the SD contributions. The sum in Eq. (1) extends over all possible operators Q_i generated by the SM or a given NP model and similarly over all possible internal particle contributions in the loop of Fig. 1. While the matrix elements of semileptonic decays can be extracted from experimental data, the Wilson coefficients depend on the high-energy model and can be calculated perturbatively within the SM.

NP contributions can enter Eq. (1) in two ways. Firstly, they can modify the Wilson coefficients. Since these are the same in rare B-, D-, and K-meson decays in the SM, the resulting correlation can be used to test the universality of the loop function in a model-independent way. Secondly, NP can generate extra operators that will contribute to the decay amplitude. This would signal a departure from Minimal Flavour Violation [2].

The master amplitude of Eq. (1) is also suited to compare the size of different contributions. The size of each contribution depends on both the CKM elements, conveniently parametrised in terms of $\lambda = |V_{us}| \approx 0.22$, and on the loop functions F_i. Within the SM, low-energy contributions are suppressed with respect to the top contribution due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [3]. When SD photon penguins contribute, this suppression is only logarithmic, but in their absence (e.g. in the $K \to \pi \nu \bar{\nu}$ decays) the loop functions are proportional to m_{top}^2, m_{charm}^2, or Λ_{QCD}^2 for the up-quark, and a quadratic GIM mechanism is at play.

In addition, for the Kaon decays the CKM factor of the top-quark contribution $\lambda_t = V_{ts}^* V_{td}$ is proportional to λ^5 and therefore highly suppressed. This strong suppression, absent in B- and D-decays, can imply a significant charm-contribution depending on the mode, but also renders rare Kaon decays sensitive to even very small deviations from the CKM picture of CP violation.
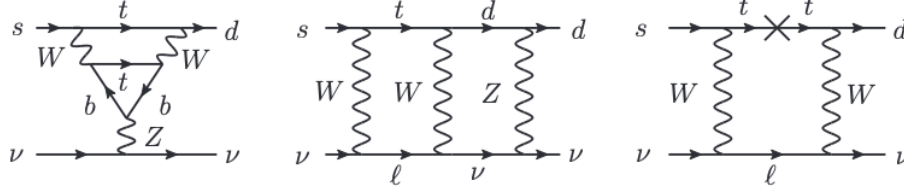
Figure 2. Sample penguin, box and counterterm diagrams contributing to X_t at NLO.

3. $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

The weak SM effective Hamiltonian for the two $K \to \pi\nu\bar{\nu}$ decays reads [4]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} \left(\lambda_c X^l + \lambda_t X_t \right) (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{lL} \gamma^\mu \nu_{lL}) + h.c. \qquad (2)$$

and involves to a good approximation only one effective operator below $\mu = \mu_c$.

The LD part of both decays is known with high precision. The matrix elements are extracted from $K_{\ell 3}$ decays using the approximate isospin symmetry. Corrections due to isospin-breaking ($\kappa_\nu^{+,L}$) and long-distance QED effects have also been considered in [5]. At the level of accuracy reached in the SM prediction, subleading effects from light quarks and higher dimensional operators can no longer be neglected; they have been estimated using chiral perturbation theory (ChPT). Their contribution to the branching ratios is parametrised by the phenomenological parameter $\delta P_{c,u}$ [6].

The small uncertainties due to LD effects renders higher order corrections on the SD parts essential for the improvement of the SM prediction. The SD charm contribution, denoted by $P_c = \frac{1}{\lambda^4} \left(\frac{2}{3} X^e + \frac{1}{3} X^\tau \right)$, is negligible in $K_L \to \pi^0 \nu\bar{\nu}$, but not in $K^+ \to \pi^+ \nu\bar{\nu}$, where it amounts to approximately 30% of its branching ratio. So far P_c is known up to next-to-next-to-leading order (NNLO) in QCD [7] and also the next-to-leading order (NLO) EW corrections have been calculated [8]. NLO QCD corrections [4, 9] are known for the top-quark contribution, but until recently the NLO EW corrections were only estimated in the large-m_t limit [10]. However, these poorly approximate the full EW corrections (see Fig. 3) [10]. Therefore, the renormalisation scheme of the EW input parameters $\alpha, \sin\theta_W, M_W, M_t$, appearing at LO in Eq. (2), was far from being clear and accounted for an uncertainty of approximately 2% in X_t.

We substantially reduced the renormalisation scheme ambiguity by calculating the NLO term in the α expansion of the top-quark Wilson coefficient X_t [11]. A sample of the corresponding diagrams is given in Fig. 2. To estimate the remaining scheme- and matching-scale dependence we use both $\overline{\text{MS}}$ and on-shell scheme, as well as a mixed scheme with on-shell masses but $\overline{\text{MS}}$ couplings. With the full NLO EW corrections on X_t we reduce the renormalisation scheme uncertainty to 0.3%. The comparison of LO and NLO results in the $\overline{\text{MS}}$ and on-shell scheme is illustrated in Fig. 3 together with the remaining scale dependence.

The theoretical predictions and the current experimental values for the branching ratios then read:

$$\text{BR}^{\text{theo}}_{K^+ \to \pi^+ \nu\bar{\nu}} = (7.81^{+0.80}_{-0.71} \pm 0.29) \times 10^{-11} \text{ [11]} \qquad \text{BR}^{\text{exp}}_{K^+ \to \pi^+ \nu\bar{\nu}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10} \text{ [12]}$$

$$\text{BR}^{\text{theo}}_{K_L \to \pi^0 \nu\nu} = (2.43^{+0.40}_{-0.37} \pm 0.06) \times 10^{-11} \text{ [11]} \qquad \text{BR}^{\text{exp}}_{K_L \to \pi^0 \nu\bar{\nu}} < 6.7 \times 10^{-8} \text{ [13]}$$

The first error in the theory prediction is parametric, while the second summarises the remaining theory uncertainty. For the charged mode the detailed main parametric contributions are ($V_{cb} : 56\%, \bar{\rho} : 21\%, m_c : 8\%, m_t : 6\%, \bar{\eta} : 4\%, \alpha_s : 3\%, \sin^2\theta_W : 1\%$), while the main
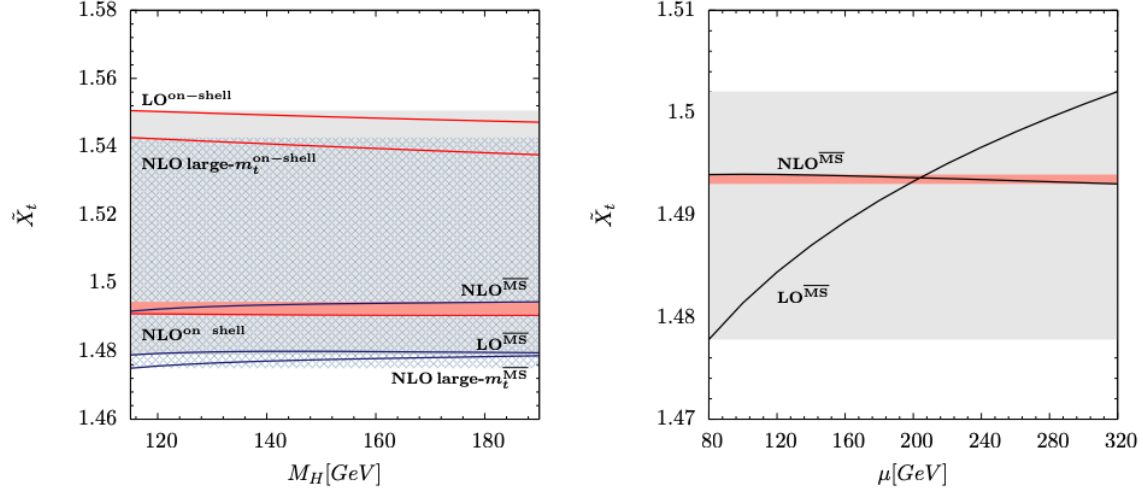
Figure 3. Left: comparison of the top-quark Wilson coefficient in the $\overline{\text{MS}}$ and on-shell schemes at LO, NLO large-m_t and NLO. The grey solid band, hashed band, and solid red band represent the estimated uncertainties of LO, NLO large-m_t, and NLO result as a function of the Higgs mass, respectively. Right: the uncertainty due to EW renormalisation scale at LO and NLO. Again the grey and red band represent the estimated uncertainty before and after the NLO calculation, respectively.

theoretical contributions are ($\delta P_{c,u} : 46\%$, $X_t(\text{QCD}) : 24\%$, $P_c : 20\%$, $\kappa_\nu^+ : 7\%$, $X_t(\text{EW}) : 3\%$), respectively. Similarly for the neutral mode, the parametric uncertainties are ($V_{cb} : 54\%$, $\bar{\eta} : 39\%$, $m_t : 6\%$) and the theoretical ($X_t(\text{QCD}) : 73\%$, $\kappa_\nu^L : 18\%$, $X_t(\text{EW}) : 8\%$, $\delta P_{c,u} : 1\%$).

4. $K_L \to \pi^0 \mu^+ \mu^-$ and $K_L \to \pi^0 e^+ e^-$

In the two $K_L \to \pi^0 \ell^+ \ell^-$ modes three different contributions compete in magnitude: direct-CP-violating (DCPV), indirect-CP-violating (ICPV) and CP-conserving (CPC) contribution. Long distance effects are much larger here. Still, the importance of these decay channels lies in their sensitivity to SD contributions from more than one operator. Let us now look at the three distinct contributions.

DCPV: high-energy top and charm contributions generate both vector, $Q_{7V} = (\bar{s}d)_V (\bar{\ell}\ell)_V$, and axial-vector, $Q_{7A} = (\bar{s}d)_V (\bar{\ell}\ell)_A$, dimension-six operators. Their Wilson coefficients are known to NLO QCD [2] and their matrix elements are also extracted with a few per mil precision from $K_{\ell 3}$ decays [5]. One difference between the electronic and the muonic channels is the suppression of the electronic axial-vector matrix element due to the smallness of the electron mass.

ICPV: in this case the K_L decays through its small CP even component via the γ-penguin in Fig. 4. The size of this smaller component is described by the parameter ϵ_K. We can therefore relate this contribution to the decay $K_S(\approx K_1) \to \pi^0 \ell^+ \ell^-$ using ChPT. The light meson loops are subleading and the amplitude is dominated by the chiral counterterm a_S [14], whose 20% uncertainty completely dominates the error for the $K_L \to \pi^0 \ell^+ \ell^-$ rates [15]. Although the absolute value of a_S has been extracted from NA48 measurements, its sign remains unknown. Therefore, it is still an open question whether the ICPV contribution interferes constructively or destructively with the DCPV contribution. Apart from a more precise $K_S \to \pi^0 \ell^+ \ell^-$ measurement, the sign of a_S can also be determined from the measurement of the integrated forward-backward asymmetry A_{FB}^ℓ in $K_L \to \pi^0 \mu^+ \mu^-$ [15].
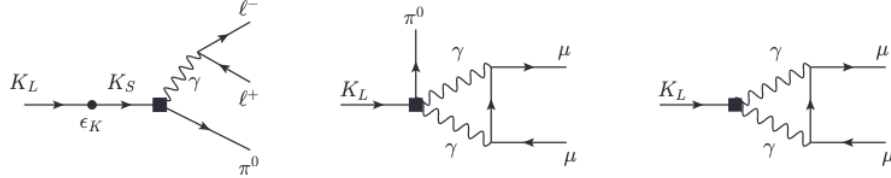
Figure 4. γ- and $\gamma\gamma$- penguins contributing to the LD part of the $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \mu^+\mu^-$ branching ratios.

CPC: is a purely LD effect described by the $\gamma\gamma$-penguin ChPT diagram in Fig. 4. The contribution is helicity suppressed for the electronic mode, a further difference between the two modes. For the muonic mode the meson loops are finite at $\mathcal{O}(p^4)$, while higher order effects of $\mathcal{O}(p^6)$ have been partially estimated from the $K_L \to \pi^0\gamma\gamma$ ratio, reducing the uncertainty to approximately 30% [16].

The current SM predictions and experimental values for the branching ratios for the two decays then read:

$$\mathrm{BR}^{\text{theo}}_{K_L\to\pi^0\mu^+\mu^-} = 1.41^{+0.28}_{-0.26}\,(0.95^{+0.22}_{-0.21}) \times 10^{-11}\,[15] \qquad \mathrm{BR}^{\text{exp}}_{K_L\to\pi^0\mu^+\mu^-} < 3.8 \times 10^{-10}\,[17]$$

$$\mathrm{BR}^{\text{theo}}_{K_L\to\pi^0 e^+e^-} = 3.54^{+0.98}_{-0.85}\,(1.56^{+0.62}_{-0.49}) \times 10^{-11}\,[15] \qquad \mathrm{BR}^{\text{exp}}_{K_L\to\pi^0 e^+e^-} < 2.8 \times 10^{-10}\,[18]$$

Theory predictions correspond to constructive interference of DCPV with ICPV contribution, while predictions in brackets to destructive interference.

5. New Physics and Outlook

The theoretical cleanness of the $K \to \pi\nu\bar{\nu}$ modes promotes both to excellent probes of NP, especially in view of the dedicated experiments, NA62 at CERN (see talk by G. Lamanna [19]) and KOTO at JPARC, which aim at measuring the charged and neutral mode, respectively, with an expected accuracy of 15%. These decays have therefore been studied extensively in models beyond the SM. A summary of predictions in different models is presented in [20]. This analysis points out the possibility of large effects in both decay modes and also illustrates the correlation between the predictions of the two decays in a large class of NP models.

NP effects on the branching ratios of the $K_L \to \pi^0\ell^+\ell^-$ modes have also been considered, in a model-independent way. A measurement of the decays could test contributions from operators not generated by the SM. Effects from scalar, pseudoscalar, tensor, and pseudotensor operators, generated by NP with or without helicity suppression, have been studied and can be disentangled by a more precise measurement of both modes [15].

To conclude, rare Kaon decays are very clean and sensitive probes of NP. The observation of deviations from the SM by the dedicated experiments would be a clear NP signal. Also, the use of both predictions and measurements as constraints of NP can shed light on NP patterns and the structure of flavour violation.

References

[1] Buchalla G, Buras A J and Lautenbacher M E 1996 *Rev. Mod. Phys.* **68** 1125–1144 (*Preprint* `hep-ph/9512380`)

[2] Buras A J, Gambino P, Gorbahn M, Jager S and Silvestrini L 2001 *Nucl. Phys.* **B592** 55–91 (*Preprint* `hep-ph/0007313`)

[3] Glashow S L, Iliopoulos J and Maiani L 1970 *Phys. Rev.* **D2** 1285–1292

[4] Buchalla G and Buras A J 1999 *Nucl. Phys.* **B548** 309–327 (*Preprint* `hep-ph/9901288`)

[5] Mescia F and Smith C 2007 *Phys. Rev.* **D76** 034017 (*Preprint* `0705.2025`)

[6] Isidori G, Mescia F and Smith C 2005 *Nucl. Phys.* **B718** 319–338 (*Preprint* `hep-ph/0503107`)

[7] Buras A J, Gorbahn M, Haisch U and Nierste U 2006 *JHEP* **11** 002 (*Preprint* `hep-ph/0603079`)

[8] Brod J and Gorbahn M 2008 *Phys. Rev.* **D78** 034006 (*Preprint* `0805.4119`)

[9] Misiak M and Urban J 1999 *Phys. Lett.* **B451** 161–169 (*Preprint* `hep-ph/9901278`)

[10] Buchalla G and Buras A J 1998 *Phys. Rev.* **D57** 216–223 (*Preprint* `hep-ph/9707243`)

[11] Brod J, Gorbahn M and Stamou E 2010 (*Preprint* `1009.0947`)

[12] Artamonov A V *et al.* (E949) 2008 *Phys. Rev. Lett.* **101** 191802 (*Preprint* `0808.2459`)

[13] Ahn J K *et al.* (E391a) 2010 *Phys. Rev.* **D81** 072004 (*Preprint* `0911.4789`)

[14] D'Ambrosio G, Ecker G, Isidori G and Portoles J 1998 *JHEP* **08** 004 (*Preprint* `hep-ph/9808289`)

[15] Mescia F, Smith C and Trine S 2006 *JHEP* **08** 088 (*Preprint* `hep-ph/0606081`)

[16] Isidori G, Smith C and Unterdorfer R 2004 *Eur. Phys. J.* **C36** 57–66 (*Preprint* `hep-ph/0404127`)

[17] Alavi-Harati A *et al.* (KTeV) 2000 *Phys. Rev. Lett.* **84** 5279–5282 (*Preprint* `hep-ex/0001006`)

[18] Alavi-Harati A *et al.* (KTeV) 2004 *Phys. Rev. Lett.* **93** 021805 (*Preprint* `hep-ex/0309072`)

[19] Lamanna G 2010 *these proceedings*

[20] Mescia F and Smith C URL `http://www.lnf.infn.it/wg/vus/content/Krare.html`