# ECON 607 Assignment 10

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# Contents


**1 Model Fitting** **3**

1.1 Finding the median price . . . . . . . . . . . . . . . . . . . . . . . 3

1.2 Setting our parameters . . . . . . . . . . . . . . . . . . . . . . . . . 4

1.3 Finding the search cost . . . . . . . . . . . . . . . . . . . . . . . . . 5

1.4 Plot . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5

**2 References** **7**

**3 Code** **8**

# 1 Model Fitting

Consider a version of the Burdett-Judd (1983) model of posted price selling in which buyers may observe one or two prices with probabilities $q_1 = q > 0$ and $q2 = 1 - q > 0$. Buyers buy either one unit of the good or none. If they acquire the good, they receive utility v. Sellers can produce the good at constant marginal cost, $r < v$.

## 1.1 Finding the median price

Let's denote the median price as $\hat{p}$ in the distribution of posted prices in a seller equilibrium. To find the median price $\hat{p}$, we need to consider the equilibrium pricing strategy of sellers. The median price is the price at which half of the buyers are willing to buy, and half are not. Let's denote the cumulative distribution function (CDF) of the a price $p$ as $F(p)$. $F(p)$ indicates the proportion of firms that charge a price no greater than $p$, for any $p$. The median price is such that $F(\hat{p}) = 0.5$.

$$F(\hat{p}) = \frac{1}{2} = 1 - (\frac{1}{2})\frac{p^* - \hat{p}}{\hat{p} - r}\frac{q}{1-q} \tag{1}$$

$$\implies \frac{1}{2} = \frac{1}{2}(\frac{p^* - \hat{p}}{\hat{p} - r})(\frac{q}{1-q})$$

$$\implies \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{p^* - \hat{p}}{\hat{p} - r}\frac{q}{1-q}$$

$$\implies \frac{1}{\frac{q}{1-q}} = \frac{p^* - \hat{p}}{\hat{p} - r}$$

$$\implies \frac{1-q}{q} = \frac{p^* - \hat{p}}{\hat{p} - r}$$

$$\implies \frac{1-q}{q}(\hat{p} - r) = p^* - \hat{p}$$

$$\implies \frac{1-q}{q}(\hat{p} - r) = p^* - \hat{p}$$

Isolating for $\hat{p}$ we get:

$$\implies \hat{p} = \frac{(\frac{1-q}{q})r + p^*}{\frac{1-q}{q} + 1}$$

$$\implies \hat{p} = \frac{(\frac{1-q}{q})r + p^*}{\frac{1-q}{q} + \frac{q}{q}}$$

$$\implies \hat{p} = \frac{(\frac{1-q}{q})r + p^*}{\frac{1}{q}}$$

$$\implies \hat{p} = q((\frac{1-q}{q})r + p^*)$$

Finally we get

$$\implies \hat{p} = (1-q)r + qp^* \tag{2}$$

An increase in $q$ implies that buyers are more likely to observe two prices ($q_2$) rather than just one ($q_1$). Observing our equation 2, an increase in $q$ is likely to increase the median price $\hat{p}$. We assume that $p^* > r$ since no market will exist if $p* < r$ (Burdett & Judd, 1983). Using our assumtion we see that for any value increase in $q$ the value inside $(1-q)$ will decrease and be multiplied by $r$, then that term will be added to $qp^*$ which will have increased by a larger margin than the margin that $(1-q)r$ decreased. Therefore, an increase in $q$ is expected to increase the median price in the distribution of posted prices in the Burdett-Judd (1983) model.

## 1.2 Setting our parameters

We are given the following set of observed prices:

$$(23.75, 23.73, 24.35, 24.47, 26.14, 26.47, 27.95)$$

$p^* = 27.95$
$p_{min} = 23.72$
$\hat{p} = 24.47$

Using our CDF equation from equation 1 to get $r$:

$$\implies F(\hat{p}) = 1 - \frac{p^* - \hat{p}}{\hat{p} - r}(0.5)\frac{q}{1-q}$$

$$\implies 1 - F(\hat{p}) = \frac{p^* - \hat{p}}{\hat{p} - r}(0.5)\frac{q}{1-q}$$

$$\implies r = \frac{p^* - \hat{p}}{1 - F(\hat{p})}(0.5)\frac{q}{1-q} \tag{3}$$

To find our $q$:

$$p_{min} = \frac{2(1-q)}{2-q}r + \frac{q}{2-q}p^*$$

Subbing equation 3 for $r$ in:

$$\implies p_{min} = \frac{2(1-q)}{2-q}(\frac{p^* - \hat{p}}{1 - F(\hat{p})}(0.5)\frac{q}{1-q}) + \frac{q}{2-q}p^*$$

After simplifying:

$$\implies p_{min} = \frac{2\hat{p}(1-q) - 2(p^* - \hat{p}) + qp^*}{2-q} \tag{4}$$

Subbing our values for $\hat{p}$, $p^*$ and $p_{min}$ into equation 4 to solve for $q$:

$$\implies 23.72 = \frac{2(24.47)(1-q) - 2(27.95 - 24.47) + q(27.95)}{2-q}$$

After rearraging and computing:

$$\implies q = 0.35$$

subbing $\hat{p}$, $p^*$, and $q$ into equation 3 to find $r$:

$$\implies r = \frac{p^* - \hat{p}}{1 - F(\hat{p})}(0.5)\frac{q}{1-q}$$

$$\implies r = \frac{27.95 - 24.47}{1 - 0.5}(0.5)\frac{0.35}{1 - 0.35}$$

$$\implies r = 22.56$$

## 1.3 Finding the search cost

From (Burdett & Judd, 1983) p 960:

$$\Pi = (p - r)\mu\Sigma_{k=1}^{\infty} nq_n[1 - F(p)]^{k-1} \tag{5}$$

$$\implies \Pi = (p^* - r)\mu q$$

From equation 1 we have our CDF:

$$F(\hat{p}) = \frac{1}{2} = 1 - \frac{p^* - \hat{p}}{\hat{p} - r}(\frac{1}{2})\frac{q}{1-q}$$

$$c = \Delta_1(q)$$

From matlab code, see section 3, we get:

$$c = 0.56$$

## 1.4 Plot

Plotting the empirical distribution of prices along with the equilibrium distribution of prices implied by the parameterized model.

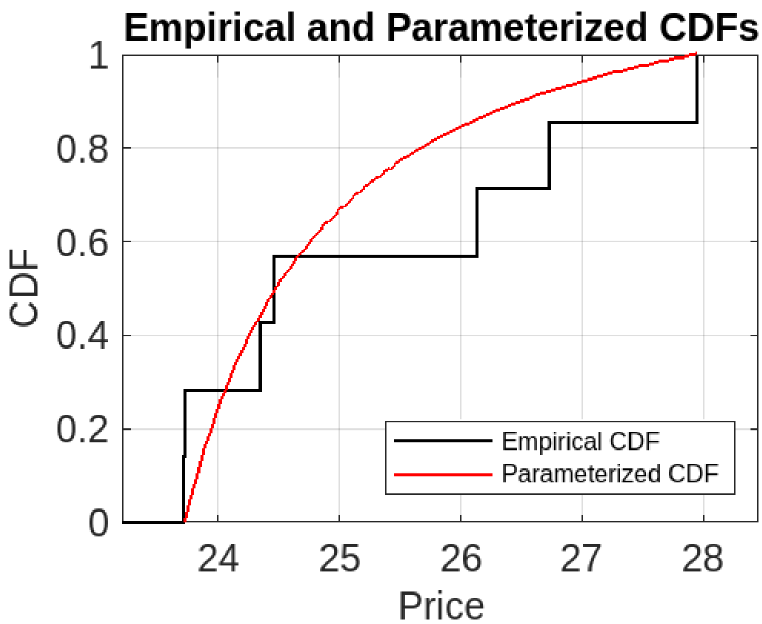


Figure 1: Plot of the empirical CDF of prices and the equilibrium distribution of prices implied by the parameterized model

## 2 References

## References

Burdett, K., & Judd, K. (1983). Equilibrium price dispersion. *Econometrica*, *51*(4), 955–69. https://EconPapers.repec.org/RePEc:ecm:emetrp:v:51:y:1983:i:4:p:955-69

# 3 Code

```
% Given prices
prices = [23.72; 23.73; 24.35; 24.47; 26.14; 26.74; 27.95];

% Calculating statistics
p_star = max(prices);
p_median = median(prices);
p_min = min(prices);

fprintf('Median price: %.4f\n', p_median)
fprintf('Max price: %.4f\n', p_star)
fprintf('Min price: %.4f\n\n', p_min)

% Given parameters
q = 1.5 / 4.23;
r = p_median - (p_star - p_median) * (q / (1 - q));

fprintf('q: %.4f\n', q)
fprintf('r (marginal cost of sellers): %.4f\n\n', r)

% Initialize arrays for price grid and cumulative distribution function
price_step = (p_star - p_min) / 100;
price_grid = linspace(p_min, p_star, 101)';
cdf_grid = 1 - (1/2) * (q / (1 - q)) * ((p_star - price_grid) ./ (price_grid - r));

% Store cdf
modified_cdf1 = cdf_grid;
modified_cdf2 = 1 - (1 - cdf_grid).^2;

% Calculate expected prices
expected_price1 = trapz(price_grid, modified_cdf1);
expected_price2 = trapz(price_grid, modified_cdf2);

% Calculate the difference between expected prices
cost = expected_price1 - expected_price2;

fprintf('Search cost: %.4f\n\n', cost)

%% Plotting the graph

% Calculate empirical cumulative distribution function
p_ecdf = ecdf(prices);

F1 = figure(1);
stairs([0; unique(prices)], p_ecdf, 'LineWidth', 1, Color="black");
```

```
hold on;

% Plotting the parameterized CDFs
plot(price_grid, modified_cdf1, 'LineWidth', 1, Color="r");


set(gca, 'FontSize', 12);
xlabel('Price', 'FontSize', 12);
ylabel('CDF', 'FontSize', 12);
xlim([(p_min - 0.5), (p_star + 0.5)]);
ylim([0, 1]);

% Adding legend
legend('Empirical CDF', 'Parameterized CDF ', 'Location', 'southeast', 'FontSize', 8);

% Adding title
title('Empirical and Parameterized CDFs', 'FontSize', 12);
```