

## Lecture 6

Thursday, March 23, 2023 12:26 PM

- Pumping Lemma for Context Free Lang
  - Easy to find lang that cannot be accepted by finite automation, even if proving is little harder
    - Ex:  $AnBn$  can't be accepted by a FA, bc w/ only a finite
  - Theorem
    - 5 substrings
    - Wy has least 1 letter, wxy has at most n letters
    - M should be the same on w and y
  - Proof
    - Input grammar G should be in Chomsky normal form
    - Makes binary search tree
    - G is generic
  - Every derivation tree in this grammar is a binary tree, of height h has no more than  $2^h$  leaf nodes
    - If u is in  $L(G)$  and h is the height of the derivation tree for u, the the length of u is less than/equal  $2^h$
  - Let n be  $2^{p+1}$  where p is the numb of distinct vars in G, and let u is a string in  $L(G)$  of length at least n
  - So in a derivation tree for u, there must be a path from the root to a leaf node w/ at least p+1 interior nodes
  - Ex:
    - Applying the pumping lemma to  $AnBnCn$ 
      - If  $AnBnCn$  is a context-free lang, and let n be the int in the pumping lemma
    - Let u be the string  $a^n b^n c^n$
    - The second,  $|wxy| \leq n$ , implies that wxy have no more than 2 distinct symbols
    - If sigma1 is one of the 3 symbols that happens in wy
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    - Keep making contradictions
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  - Theorem (Ogden's Lemma)(stronger version of the pumping lemma):
- Intersections and Complements of CFLs
  - The set of CFLs like the set of reg languages, is closed under the operations of union, concatenation, and Kleene \*
  - But, unlike the set of reular langs, its not closed under intersections/diff
  - Consider  $AnBnCn$ 
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  - Cousin of intersection is closed under union
  - Xx is not context free, but compliment is context free