Chapter 2 Frequency Distributions and Graphs

Formula for the percentage of values in each class:

$$\% = \frac{f}{n} \cdot 100$$

where

$$f = \text{frequency of class}$$
$$n = \text{total number of values}$$

Formula for the range:

$$R = \text{highest value} - \text{lowest value}$$

Formula for the class width:

$$\text{Class width} = \text{upper boundary} - \text{lower boundary}$$

Formula for the class midpoint:

$$X_m = \frac{\text{lower boundary} + \text{upper boundary}}{2}$$

or

$$X_m = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Formula for the degrees for each section of a pie graph:

$$\text{Degrees} = \frac{f}{n} \cdot 360°$$

Chapter 3 Data Description

Formula for the mean for individual data:

$$\text{Sample} \quad \overline{X} = \frac{\Sigma X}{n} \qquad \text{Population} \quad \mu = \frac{\Sigma X}{N}$$

Formula for the mean for grouped data:

$$\overline{X} = \frac{\Sigma f \cdot X_m}{n}$$

Formula for the weighted mean:

$$\overline{X} = \frac{\Sigma wX}{\Sigma w}$$

Formula for the midrange:

$$\mathbf{MR} = \frac{\textbf{lowest value} + \textbf{highest value}}{\mathbf{2}}$$

Formula for the range:

$$\boldsymbol{R} = \textbf{highest value} - \textbf{lowest value}$$

Formula for the variance for population data:

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

Formula for the variance for sample data (shortcut formula for the unbiased estimator):

$$s^2 = \frac{n\left(\Sigma X^2\right) - (\Sigma X)^2}{n(n-1)}$$

Formula for the variance for grouped data:

$$s^2 = \frac{n\left(\Sigma f \cdot X_m^2\right) - (\Sigma f \cdot X_m)^2}{n(n-1)}$$

Formula for the standard deviation for population data:

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$

Formula for the standard deviation for sample data (shortcut formula):

$$s = \sqrt{\frac{n\left(\Sigma X^2\right) - (\Sigma X)^2}{n\,(n-1)}}$$

Formula for the standard deviation for grouped data:

$$s = \sqrt{\frac{n\left(\Sigma f \cdot X_m^2\right) - (\Sigma f \cdot X_m)^2}{n\,(n-1)}}$$

Formula for the coefficient of variation:

$$\mathbf{CVar} = \frac{s}{\overline{X}} \cdot \mathbf{100} \qquad \text{or} \qquad \mathbf{CVar} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\mu}} \cdot \mathbf{100}$$

Range rule of thumb:

$$s \approx \frac{\textbf{range}}{\mathbf{4}}$$

Expression for Chebyshev's theorem: The proportion of values from a data set that will fall within k standard deviations of the mean will be at least

$$\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{k}^{\mathbf{2}}}$$

where k is a number greater than 1.

Formula for the z score (standard score):

$$\underset{\text{Sample}}{z = \frac{X - \overline{X}}{s}} \qquad \text{or} \qquad \underset{\text{Population}}{z = \frac{X - \boldsymbol{\mu}}{\boldsymbol{\sigma}}}$$

Formula for the cumulative percentage:

$$\textbf{Cumulative \%} = \frac{\begin{array}{c}\textbf{cumulative}\\ \textbf{frequency}\end{array}}{\boldsymbol{n}} \cdot \mathbf{100}$$

Formula for the percentile rank of a value X:

$$\textbf{Percentile} = \frac{\begin{array}{c}\textbf{number of values}\\ \textbf{below } \boldsymbol{X} + \mathbf{0.5}\end{array}}{\begin{array}{c}\textbf{total number}\\ \textbf{of values}\end{array}} \cdot \mathbf{100}$$

Formula for finding a value corresponding to a given percentile:

$$\boldsymbol{c = \frac{n \cdot p}{100}}$$

Formula for interquartile range:

$$\mathbf{IQR} = \boldsymbol{Q_3 - Q_1}$$

Chapter 4 Probability and Counting Rules

Formula for classical probability:

$$P(E) = \frac{\text{number of outcomes in } E}{\text{total number of outcomes in sample space}} = \frac{n(E)}{n(S)}$$

Formula for empirical probability:

$$P(E) = \frac{\text{frequency for class}}{\text{total frequencies in distribution}} = \frac{f}{n}$$

Addition rule 1, for two mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)$$

Addition rule 2, for events that are not mutually exclusive:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Multiplication rule 1, for independent events:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Multiplication rule 2, for dependent events:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Formula for conditional probability:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Formula for complementary events:

$$P(\overline{E}) = 1 - P(E) \quad \text{or} \quad P(E) = 1 - P(\overline{E})$$

$$\text{or} \quad P(E) + P(\overline{E}) = 1$$

Fundamental counting rule: In a sequence of n events in which the first one has k_1 possibilities, the second event has k_2 possibilities, the third has k_3 possibilities, etc., the total number of possibilities of the sequence will be

$$k_1 \cdot k_2 \cdot k_3 \cdots k_n$$

Permutation rule 1: The number of permutations of n objects taking r objects at a time when order is important is

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutation rule 2: The number of permutations of n objects when r_1 objects are identical, r_2 objects are identical, . . . , r_p objects are identical is

$$\frac{n!}{r_1!r_2!\cdots r_p!}$$

Combination rule: The number of combinations of r objects selected from n objects when order is not important is

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

Chapter 5 Discrete Probability Distributions

Formula for the mean of a probability distribution:

$$\mu = \Sigma X \cdot P(X)$$

Formulas for the variance and standard deviation of a probability distribution:

$$\sigma^2 = \Sigma\left[X^2 \cdot P(X)\right] - \mu^2$$
$$\sigma = \sqrt{\Sigma\left[X^2 \cdot P(X)\right] - \mu^2}$$

Formula for expected value:

$$E(X) = \Sigma X \cdot P(X)$$

Binomial probability formula:

$$P(X) = \frac{n!}{(n-X)!X!} \cdot p^X \cdot q^{n-X} \text{ where } X = 0, 1, 2, 3, \ldots, n$$

Formula for the mean of the binomial distribution:

$$\mu = n \cdot p$$

Formulas for the variance and standard deviation of the binomial distribution:

$$\sigma^2 = n \cdot p \cdot q \qquad \sigma = \sqrt{n \cdot p \cdot q}$$

Formula for the multinomial distribution:

$$P(X) = \frac{n!}{X_1! \cdot X_2! \cdot X_3! \cdots X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdots p_k^{X_k}$$

(The X's sum to n and the p's sum to 1.)

Formula for the Poisson distribution:

$$P(X; \lambda) = \frac{e^{-\lambda}\lambda^X}{X!} \qquad \text{where } X = 0, 1, 2, \ldots$$

Formula for the hypergeometric distribution:

$$P(X) = \frac{{}_aC_X \cdot {}_bC_{n-X}}{{}_{a+b}C_n}$$

Formula for the geometric distribution:

$$P(n) = p(1-p)^{n-1} \quad \text{where } n = 1, 2, 3, \ldots$$

Chapter 6 The Normal Distribution

Formula for the z score (or standard score):

$$z = \frac{X - \mu}{\sigma}$$

Formula for finding a specific data value:

$$X = z \cdot \sigma + \mu$$

Formula for the mean of the sample means:

$$\mu_{\overline{X}} = \mu$$

Formula for the standard error of the mean:

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

Formula for the z value for the central limit theorem:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

Formulas for the mean and standard deviation for the binomial distribution:

$$\mu = n \cdot p \qquad \sigma = \sqrt{n \cdot p \cdot q}$$

Chapter 7 Confidence Intervals and Sample Size

Formula for the confidence interval of the mean when σ is known (when $n \geq 30$, s can be used if σ is unknown):

$$\overline{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \overline{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Formula for the sample size for means:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

where E is the margin of error.

Formula for the confidence interval of the mean when σ is unknown:

$$\overline{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \overline{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

Formula for the confidence interval for a proportion:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

where $\widehat{p} = X/n$ and $\widehat{q} = 1 - \widehat{p}$.

Formula for the sample size for proportions:

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Formula for the confidence interval for a variance:

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

Formula for the confidence interval for a standard deviation:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\text{right}}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{\text{left}}}}$$

Chapter 8 Hypothesis Testing

Formula for the z test for means:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \qquad \textbf{if } n < 30\textbf{, variable must be normally distributed}$$

Formula for the t test for means:

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} \qquad \textbf{if } n < 30\textbf{, variable must be normally distributed}$$

Formula for the z test for proportions:

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} \qquad \textbf{if } np \geq 5 \textbf{ and } nq \geq 5$$

Formula for the chi-square test for variance or standard deviation:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad \textbf{if } n < 30\textbf{, variable must be normally distributed}$$