

## Lecture 8

Tuesday, April 18, 2023 11:10 AM

- Topics:
  - Recursively Enumerable and Recursive
  - Enumerating a Lang
  - More general Grammars
  - Context-Sensitive languages and the Chomsky hierarchy
  - Not every lang is recursively enumerable
- Recursively Enumerable Languages
  - Lang that can be accepted by a TM
  - Recursive languages are – can be decided by a TM
  - Only in second case (^) we guaranteed answer to question:
    - String  $x$ , is  $x$  and element of the lang
  - First case (^^) doesn't guarantee that the TM will halt w/ an answer
- Recursively Enumerable and Recursive
  - A TM  $T$  w/ input alphabet  $\Sigma$  accepts a lang  $L$  is a subset of  $\Sigma^*$  if it accepts the string in  $L$  and no others
  - $T$  decides  $L$  if  $T$  computes the char fn  $x \mapsto \Sigma^* \rightarrow$
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  - Every recursive lang is recursively enumerable
  - If  $L$  is a subset of  $\Sigma^*$  is accepted by a TM  $T$  that halts on every input string, then  $L$  is recursive
  - If  $L_1$  and  $L_2$  are both recursively enumerable langs over  $\Sigma$ , the  $L_1 \cup L_2$  and  $L_1 \cap L_2$  are also recursively enumerable
  - If  $L_1$  and  $L_2$  are both recursive langs over  $\Sigma$  then  $L_1 \cup L_2$  and  $L_1 \cap L_2$  are also recursive
  - If  $L$  is a recursive language over  $\Sigma$ , then its complement  $L'$  is also recursive
  - If  $L$  is a recursively enumerable lang, and its complement  $L'$  is also recursively enumerable, then  $L$  is recursive
- Enumerating a Language
  - Let  $T$  be a  $k$ -tape Turing Machine for some  $k \geq 1$ , and let  $L$  be a subset of  $\Sigma^*$ 
    - We say  $T$  enumerates  $L$  if it operates such that these conditions satisfied:
      - Tape head on first tape never moves left, and nonblank symbol printed on tape 1 is subsequently moved/erased
      - For every  $x$  an element of  $L$ , there is some point during the operation of  $T$  when tape 1 has stuff for some  $n \geq 0$ , where the  $x$ 's are also elements of  $L$  and  $x_1, x_2, \dots, x_n, x$  are distinct
  - For every lang  $L$  is a subset of  $\Sigma^*$ ,
    - $L$  is recursively enumerable if and only if there is a TM enumerating  $L$ , and
    - $L$  is recursive if and only if there is a TM that enumerates the strings in  $L$  in canonical order
  - Canonical is so it goes  $a, b, \dots$ , then  $aa, ab, ac, \dots$ , then  $aaa, \dots$ 
    - Lexicographic order enum is  $a, aa, aaa, \dots$
- More General Grammars
  - At beginning of each pass,  $D$  is intro at the left end of the string, think of each application of the production as allowing  $D$  to move past an  $a$ , doubling it in the process
  - Complete grammar has the productions:
    - $P \rightarrow aP$
  - Ex:  $anbn$
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