Task 1.

Consider 2D Ising model in the external magnetic field in mean-field approximation (MFA). In this approximation for each spin one can write

$$s_i = \langle s_i \rangle + \delta s_i,$$

where $\langle s_i \rangle$ is an average magnetization, $m = \langle s_i \rangle$, and δs_i is small.

1. Write down the model Hamiltonian in MFA
2. Calculate the partition function in MFA, find the critical temperature and critical exponents β, γ
3. Show that critical exponents of Van der Waals gas,

$$v_{\text{gas}} - v_{\text{liquid}} \sim (T_c - T)^{\beta}, \quad \kappa = -\frac{1}{v}\left.\frac{\partial v}{\partial p}\right|_T \sim (T - T_c)^{\gamma}$$

   coincide with critical exponents of 2D Ising model in MFA. Why?

Task 2.

Consider 2D Ising model on triangular lattice. Write down the expression for the partition function in terms of closed graphs. Then find the dual lattice and show that the following relation holds

$$C \exp(K'(s_1 s_2 + s_2 s_3 + s_3 s_1)) = 2 \cosh(k(s_1 + s_2 + s_3)), \tag{1}$$

where K' and k are the couplings on dual and original lattices, respectively. Using this relation, write down the equation for critical temperature and find the critical temperatures $T_c^{\triangle}$ and $T_c^{\hexagon}$.