

# OEMP 3 Part 2 Assignment

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$$1) M_L(x) = 0.5 w_0 x^2 - \frac{1}{3} w_0 \frac{x^3}{L}$$

$$M_w(x) = -\rho A \frac{(L-x)^2}{2}$$

$$M_{LW}(x) = M_L(x) + M_w(x) = 0.5 w_0 x^2 - \frac{1}{3} w_0 \frac{x^3}{L} - \rho A \frac{(L-x)^2}{2}$$

| Shape Type    | Cross Sectional Area | Factor of Safety |
|---------------|----------------------|------------------|
| T-beam        | 11.5871              | 1.5116           |
| Hollow square | 88.2178              | 1.5208           |
| Hollow square | 87.75                | 1.5113           |
| Hollow square | 87.2803              | 1.5018           |
| Circle        | 84.0322              | 1.5022           |
| Circle        | 84.5407              | 1.5175           |
| Rectangle     | 114.2227             | 1.5003           |
| Rectangle     | 114.8916             | 1.5157           |
| I-beam        | 10.2155              | 1.5036           |
| I-beam        | 10.2698              | 1.5157           |
| I-beam        | 10.3242              | 1.528            |

3) The above table is the shapes with the required safety factor using the aluminum 6061 as the chosen material. The cross-sectional area is in inches squares.




These data were all obtained with required safety factor of  $1.50 \leq FS \leq 1.53$ . What is noticed is that the lowest efficiency beam is the rectangle with highest cross-sectional area. In fact, the top three inefficient geometries were all rectangles. The second most inefficient geometry is the circle. The highest efficiency beam in terms of area is the I-beam with a value of ---. This list is reasonable given maximizing the material far away from the centroid or neutral axis is desired for high moments of inertia. And as a result can achieve the lowest stresses with little material.

From the above table we discovered that the lowest cost was the three I-beams and the next is the T-beam. This makes sense because it has the lowest area.

4)

- a) The dimensions of the top and bottom rectangles are 0.474 inches by 5.448 inches and the vertical rectangle measures 0.474 inches by 10.896 inches. The cost of the beam would be \$2642.55, excluding the cost to make a custom beam to fit these exact dimensions.
- b) The material we chose for our beam is aluminum 6061-T6. This material has a medium to high strength with a tensile strength of 45000 psi, a yield strength of 40000 psi and modulus of elasticity of 10000 ksi which makes it very resistant to the forces the wing experiences in flight. Its density is relatively low at 0.0975 lb/in<sup>3</sup> compared to most metals which makes it a great candidate to use when building an airplane wing. The Coefficient of Thermal Expansion at 20.0°C - 100 °C is 13.1µin/in-°F and the Thermal Conductivity is 1160 BTU - in/hr-ft<sup>2</sup>-°F. Aluminum 6061-T6 also has good corrosion resistance, weldability, workability, and Machinability. Which is perfect for a building material on a plane as it is resistant to the effects of weather but when repairs are needed it is easy to do so.

**Table 1. Possible Spar Beam Material Properties.**

| <b>Metal</b>                | <b>E<br/>(× 10<sup>3</sup> ksi)</b> | <br><b>(ksi)</b> | <br><b>(ksi)</b> | <br><b>(ksi)</b> | <b>Density<br/>(lb/in<sup>3</sup>)</b> | <b>Cost<br/>(\$/lb)</b> |
|-----------------------------|-------------------------------------|---|---|--|--|-------------------------|
| <b>Aluminum 6061</b>        | 9.9                                 | 42  | 35  | 27   | 0.098                                  | \$8.03                  |
| <b>Steel 4130</b>           | 29.0                                | 90  | 70  | 54   | 0.283                                  | \$8.07                  |
| <b>Nickel Inconel 600</b>   | 30.0                                | 80  | 35  | 51   | 0.304                                  | \$53.78                 |
| <b>Stainless Steel 17-4</b> | 28.5                                | 140   | 115   | 81   | 0.284                                  | \$29.63                 |
| <b>Titanium 6AL-4V</b>      | 16.9                                | 130   | 120   | 80   | 0.16                                   | \$115.36                |

[http://amet-me.mnsu.edu/UserFilesShared/DATA\\_ACQUISITION/mts/MaterialData/MaterialData\\_9391\\_AI-6061.pdf](http://amet-me.mnsu.edu/UserFilesShared/DATA_ACQUISITION/mts/MaterialData/MaterialData_9391_AI-6061.pdf)



5)

- a) The minimum safety factor of the beam due to bending stress is 1.528 and happens at  $x$  equals 0 and at the bottom and top of the beam 5.92 inches away from the centroid.

$$\sigma_x = -M_z * \bar{y} / I_z$$

- b) The minimum safety factor of the beam due to the shear stress is 10.34 and happens at  $x$  equals 0 and at the bottom of the middle rectangle of the beam 5.45 inches away from the centroid.

$$\tau = V * \bar{y}' * A' / (I_z * thickness)$$

- c) The maximum deflection of the beam will occur at the farthest point from the fixed surface, therefore it occurs at  $x = 27.25$  feet. The maximum deflection of the beam is approximately -16 inches, as it will bend downwards.

$$v_{max} = -\omega_0 L^4 / (30EI)$$

- d) The total monetary cost of the beam is approximately \$2642.55 for 27.25 feet of our beam.

6) The aluminum I-beam is prevalent in the given website for various sizes and dimensions. In fact, we discovered an aluminum I-beam with height 10 in, Flange 6 in, and thickness of 0.41 in. This is quite close to our dimensions with slight variation in the flange and web. The product part is #13214 in OnlineMetals.com. For a length of 25 feet, it costs around \$1160. Even before searching for it we expected to find an aluminum I-beam given it is often used in the industry. Although steel is used more in civil engineering, the Aerospace industry uses aluminum.

<https://www.onlinemetals.com/en/buy/aluminum/6-x-10-x-0-25-aluminum-i-beam-6061-t6-extruded-aluminum-association/pid/13214>

7) In terms of weight, wings usually store fuel in the wing and so adding that can make the analysis more realistic. There is also the additional loadings when aircraft maneuver that causes bendings near the wing root. When using our results of beam geometry and material, we think the wing can handle these additional stresses. We also took into account a 1.5 factor safety and these loads probably lie under this safety factor.

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% House Keeping
clc
clear all

% analyze beam geometry for a 1ft by 1ft beam
L = 27.25 * 12; % convert to inches
w_0 = 2001 / 12; % lb/in
% counter
goodFS(1,1) = "Factor of Saftey";
goodFS(1,2) = "Cross Sectional Area (inches^2)";
goodFS(1,3) = "Metal Type";
goodFS(1,4) = "Shape Type";
goodFS(1,5) = "Cost of Design per Inch ($)";
goodFS(1,6) = "Ratio of Cost / Factor of Saftey";
c = 2;
x = [0:10:(27.25*12)];

for j = 1:5
    % get values for each material
    if (j == 1)
        % Aluminum 6061
        j_string = "Aluminum 6061";
        E = 9900; % ksi
        P = 0.098; % lb/in^3
        Strength = 35000; % ksi
        Cost = 8.03;
    elseif (j == 2)
        % Steel 4130
        j_string = "Steel 4130";
        E = 29000; % ksi
        P = 0.283; % lb/in^3
        Strength = 70000; % ksi
        Cost = 8.07;
    elseif (j == 3)
        % Nickel Inconel 600
        j_string = "Nickel Inconel 600";
        E = 30000; % ksi
        P = 0.304; % lb/in^3
        Strength = 35000; % ksi
        Cost = 53.78;
    elseif (j == 4)
        % Stainless Steel 17-4
        j_string = "Stainless Steel 17-4";
        E = 28500; % ksi
        P = 0.284; % lb/in^3
        Strength = 115000; % ksi
        Cost = 29.63;
    elseif (j == 5)
        % Titanium 6AL-4V
        j_string = "Titanium 6AL-4V";
        E = 16900; % ksi
        P = 0.16; % lb/in^3
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        Strength = 120000; % ksi
        Cost = 115.36;
    end
    % calculate the factor of safety
    for i = 1:384
        % for each shape
        for k = 1:5
            if (k == 1)
                % circle
                k_string = "Circle";
                A = pi*(i/64)^2;
                y_bar = i/64;
                I = (pi*(i/32)^4)/64;
                centroid = y_bar;
            elseif (k == 2)
                % rectangle
                k_string = "Rectangle";
                A = (i/32)^2;
                y_bar = i/64;
                I = ((i/32)^4)/24;
                centroid = y_bar;
            elseif (k == 3)
                % hollow square
                k_string = "Hollow square";
                A = 12^2 - ((i-1)/32)^2;
                y_bar = 6 + ((i-1)/32);
                I = (12^4 - ((i-1)/32)^4)/12;
                centroid = y_bar;
            elseif (k == 4)
                % I-beam
                k_string = "I-beam";
                short_side = (i/32)*.04;
                lengt = (i/32);
                middle_rectangle = lengt - (2*short_side);
                lengt=middle_rectangle/2;
                A = (2*(short_side * lengt)) + (middle_rectangle *
short_side);
                y_bar = i/64;
                I = ((lengt * (middle_rectangle
+(2*short_side))^3))/12;
                if A > 10.3232 && A<10.3252 && j==1
                    bestshort=short_side;
                    bestlength=lengt;
                    Abest = A;
                    w = P*A*L;
                    Ibest = I;
                    bestY_bar = y_bar;
                end
                centroid = y_bar;
            elseif (k == 5)
                % T-beam
                k_string = "T-beam";
                short_side = (i/32)/8;
                lengt = (i/32);

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        middle_rectangle = lengt - short_side;
        A = (short_side * lengt) + (middle_rectangle *
short_side);
        centroid = (((lengt^2) / 2) * short_side +
(short_side^2 / 2) * lengt) / A;
        y_bar = (2 * lengt) - short_side - centroid;
        dy_bar = ((lengt - (short_side/2)) + (lengt -
(middle_rectangle/2))) / 2;
        var1 = (lengt - (short_side/2));
        var2 = (lengt - (middle_rectangle/2));
        I_top = ((lengt * short_side^3) / 12) + ((i/32) *
short_side * (dy_bar)^2);
        I_bott = ((short_side * middle_rectangle^3) / 12) +
((i/32) * middle_rectangle * (dy_bar)^2);
        I = I_top + I_bott;
        %disp(I);
    end
    M = ((w_0*(L - x).^3)/(6*L)) + ((-P*A*(L - x).^2) / 2);
    [M, i_max] = max(M);
    Stress = (max(M) * y_bar) / I;
    FS(i,1) = norm(Strength / Stress);

    if (FS(i,1) > 1.5 && FS(i,1) < 1.53)
        goodFS(c,1) = FS(i,1);
        goodFS(c,2) = A;
        goodFS(c,3) = j_string;
        goodFS(c,4) = k_string;
        goodFS(c,5) = Cost * (P * A);
        goodFS(c,6) = (Cost * (P * A)) / FS(i,1);
        goodFS(c,7) = P*A*L;
        c = c+1;
    end
end
end
end

V=(.5*w_0*L)-w;
yy = [0:.01:bestlength];
bestshear = 0;

for i =1:length(yy)
    if yy(i)>bestlength-bestshort
        shear(i) =(V*yy(i)*Abest)/(Ibest*(bestlength));
    else
        shear(i) = (V*yy(i)*Abest)/(Ibest*bestshort);
    end
    if shear(i)>bestshear
        bestshear = shear(i);
    end
end
end
FSshear = 40000/bestshear;

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