ASEN 2003-019 Lab 2
Bouncing Ball

Jack Foster
Andrew Logue
Devon Paris
Siyang Liu

February 12, 2021

Table of Contents

Abstract

The objective of the Bouncing Ball Lab was to determine the coefficient of restitution of the ball through three different methods, with each method utilizing different time and height measurements from the ball's path. An experiment was designed for this lab in order to obtain the necessary measurements, using simple tools. These experiments were each designed such that error was minimized, using techniques such as video tracking software, and a ball dropping system. The error of the experiment was estimated, by three methods, method 1 (Eq.1) being the most prone to error, and method 3 (Eq.3) being the most accurate. Combining the coefficient of restitution calculated by each of the three methods, the average estimate of the ping pong ball's coefficient of restitution was determined to be 0.790, with an average error of ± 0.043. This falls in line very well with the actual coefficient of a standard ping pong ball, which is within the range of 0.8 to 0.694 (Reference 11).

I. Theory

The following equations listed below were used to estimate the elasticity constant of the ball. Equation 1 shows the first method of estimating the elasticity constant, where the height of each bounce of the ball is measured, shown as h_n, where for each bounce n. The second method of estimating the elasticity constant is by measuring the time it takes the ball to complete two bounces, where t_n is the time of the first bounce, and t_{n-1} is the time of the second bounce. This method is shown below in equation 2. Lastly, equation 3 shows the final way to estimate the elasticity constant of the ball, through measuring the time it takes the ball to come to rest (t_s) after being dropped from height h_0. Each equation will be subject to error, as all time and height measurements will not be exact. Method 1, shown in Eq.1, is likely to suffer most from measurement error, as the height of each consecutive bounce will be difficult to determine over n measurements. Both method 2 (Eq.2) and method 3 (Eq.3) only require two measurements, so these two methods are likely to have similar potential to measurement error. However in method 3 (Eq.3), the initial drop height of the ball (h_0) can be very precisely determined, leaving only the time it takes for the ball to come to rest (t_s) to have any significant measurement error. The height of the ball is measured from the ball's center of mass, examples of the center of mass of the ball at different locations are shown in appendix C.

In order to derive the equation for the coefficient of restitution from the height of each bounce (Eq.1), we must consider the formulation for the coefficient based on the velocities of each mass involved in the impact (Eq.4). In reference to appendix A (Figure 1), the first step taken represents the logical assumption that in the impact of the ball and surface, the surface that is impacted will have no significant change to its velocity. It is also valid to assume that the surface is not in motion with respect to the ball, and so the velocity of B can be assumed to be 0. This leaves only the velocities of the ball before and after the bounce. Since it is valid to assume no friction or drag, it can be said that all of the ball's kinetic energy at the apex of each bounce will be entirely converted to potential energy due to conservation of energy. Using this it is possible to solve for the velocity of both (V_A) and (V_A'). After substituting the equivalent values of velocity in terms of their energy, the common multiple divides out of the equation and is left with the solution (Eq.1).

The derivation for coefficient of restitution from the time between each bounce (Eq.2) begins with (Eq.1). Referring to appendix A (Figure 3), it is true to write that the maximum height for each consecutive bounce occurs when time is equal to half. Combining this statement with the kinematic equations for motion, the height of each bounce can be written as its position at half time. From the kinematic equation for position, the initial height and velocity for each can be assumed to be zero. The initial height is zero because each bounce occurs at the lowest point in the trajectory, and the velocity can be assumed to be zero because the ball must change its direction from a downwards velocity to bounce up and at some point it will be zero. This leaves the kinematic equations with only their acceleration term, and since the acceleration in both cases is g downward, they can be divided out of the equation. This can be further simplified by ridding the square root outside the fraction since both the numerator and denominator are terms squared. Further simplification by multiplying out the half of each term leaves us with the quotient of the time for the current bounce and the previous bounce (Eq.2).

When deriving the equation for the coefficient of restitution from the time to rest (Eq.3), it first must be recognized that the time to rest (t_s) can be turned into an infinite geometric series; this is shown in appendix A (Figure 4). Then, using the infinite formula for a geometric series (Eq.5), we can assign a_1 and r to and solver for t_s. This process is also shown in appendix A (Figure 4), which then after isolating

e and substituting for t_0, yields the original method 3 equation (Eq.3) thus the derivation of method 3 is complete.

$$e = \left(\frac{h_n}{h_{n-1}}\right)^{\frac{1}{2}} = \left(\frac{h_n}{h_0}\right)^{\frac{1}{2n}} \tag{1}$$

$$e = \left(\frac{t_n}{t_{n-1}}\right) \tag{2}$$

$$e = \frac{t_s - \sqrt{\frac{2h_0}{g}}}{t_s + \sqrt{\frac{2h_0}{g}}} \tag{3}$$

$$e = \frac{V'_B - V'_A}{V_A - V_B} \tag{4}$$

$$S = \frac{a_1}{1-r} \tag{5}$$

II. Procedures

The experiment (Figures 5-7) created for this lab was designed to eliminate or avoid simple data collection errors. Steps taken to avoid such errors included adding extra light from two points opposite each other facing the measurement stick, and a simple ball dropping system constructed from cardboard. The lights allowed the camera to view both the measuring stick and the ball more clearly, while the ball dropping system was able to reproduce similar ball drops, eliminating most of the error that comes with a human dropping a ball. In order to record the ball, a camera was positioned on a tripod in a fixed position opposite the measuring stick. As the camera never moved throughout the experiment, camera shake was eliminated and the angle of the camera remained constant. The procedure followed in this experiment was to first turn on the camera, then pull the tab in the ball dropper system, allowing the ball to drop. Once the ball had settled, the recording was stopped, and was either saved or discarded depending on the overall quality of the drop. This was repeated until five good quality recordings were made. Lastly, data from the experiment was collected via a tracker program (Reference 2) that pinpointed the location of the ball in an x-y plane based on the recorded video, which was then analyzed manually.

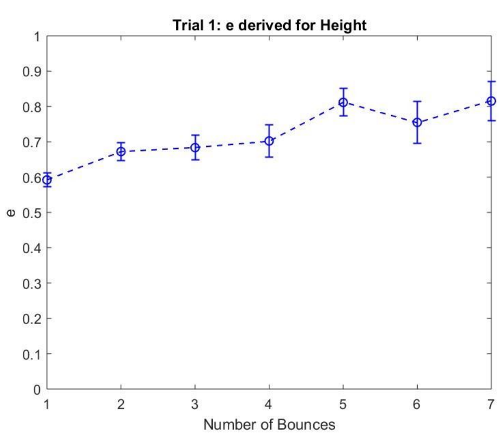
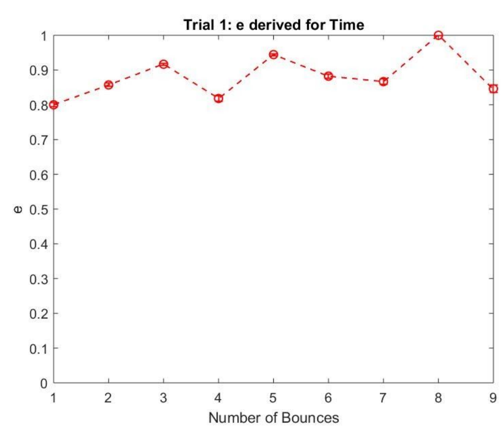
While the experiment was designed to minimize sources of error, certain possible sources of error could not be avoided. For instance, elements such as the ball bouncing closer towards the camera after each bounce could not be controlled; this resulted in some uncertainty when measuring the height of each bounce, as the ball appeared to be higher the closer it was to the camera. This was accounted for in the experiment by subtracting the measured height at the top by the measured height at the bottom in order to calculate the actual bounce height (Eq.6). Additionally, the bottom of each bounce often did not occur exactly on a frame, which resulted in it being difficult to pinpoint the end of a bounce, particularly when the ball was moving fast. The low framerate also resulted in motion blurring of the ball, making the center of mass difficult to locate at high velocity. These uncertainties were quantified by analysing the tools that were used in taking and measuring the data. After quantifying the uncertainty of each measurement, the error analysis method was utilized to determine how the ambiguity in each measurement affected the range of values that the coefficient of restitution could be. Since each measurement had differing degrees of precision, some affected the accuracy of coefficient more than others. For example, when measuring the height, it was sometimes difficult to track the exact location of the center of mass as the motion blur in some frames was greater than in others, to compensate for the vagueness of the measurement an uncertainty slightly larger than the diameter of the ball itself was propagated through to find the error. In method two, the measurements were very much more precise, as the time of the impact could be boiled down to that of a single frame. The error in the final method stems mainly from the end of its motion, when the ball bounces in very rapid succession.

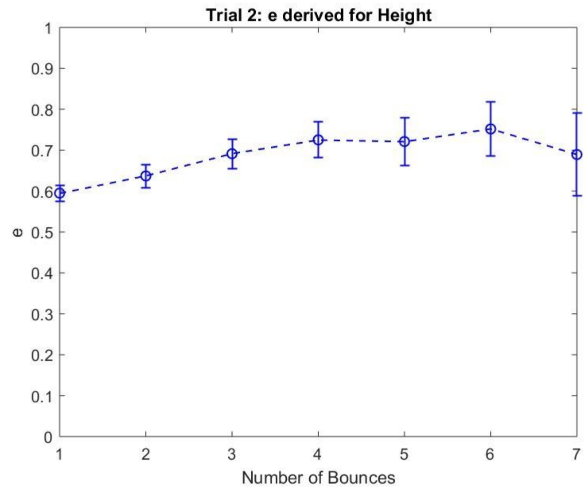
$$h_{actual} = h_{top} - h_{bottom} \tag{6}$$

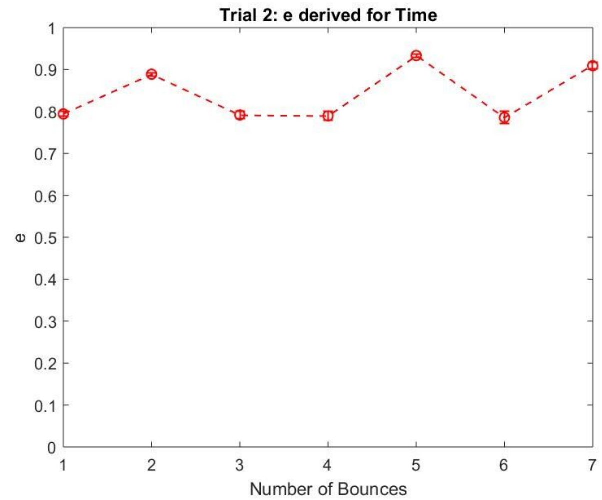
III. Results

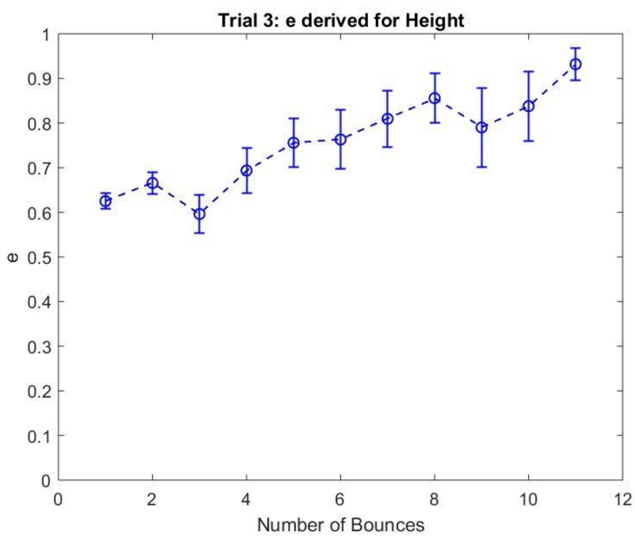
The experiment designed for this lab was performed on the 4th of February 2021, at approximately 5pm west of campus in a lab partner's living room. Given that the ping pong ball used in this experiment was a regulation table tennis ball, it is known that this ball was 2.7 grams and had a diameter of 40 millimeters. The coefficient of restitution of the ping pong ball was calculated through three methods, shown in equations 1-3. Table 1 shows the raw data calculated by MatLab from the measurements taken. The graphs with the blue lines are calculated via (Eq.1), the red line graphs are calculated by (Eq.2), and figure three shows the coefficient of restitution for all trials calculated with (Eq.3). Table two shows the statistical organization of the raw data. For methods one and two, each graph from table one was converted to an average value of the coefficient of restitution through the whole experiment and an average value of the errors. The method three data is extrapolated directly from the graph (figure 1).

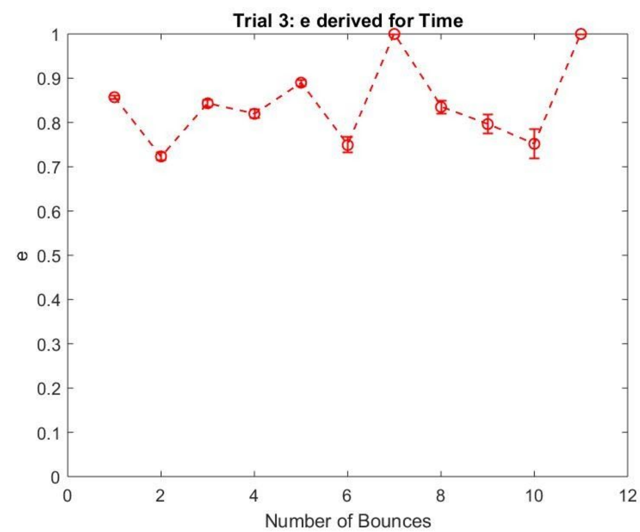
Table 1: Coefficient of Restitution Graphs for Each Bounce and Time Interval With Error

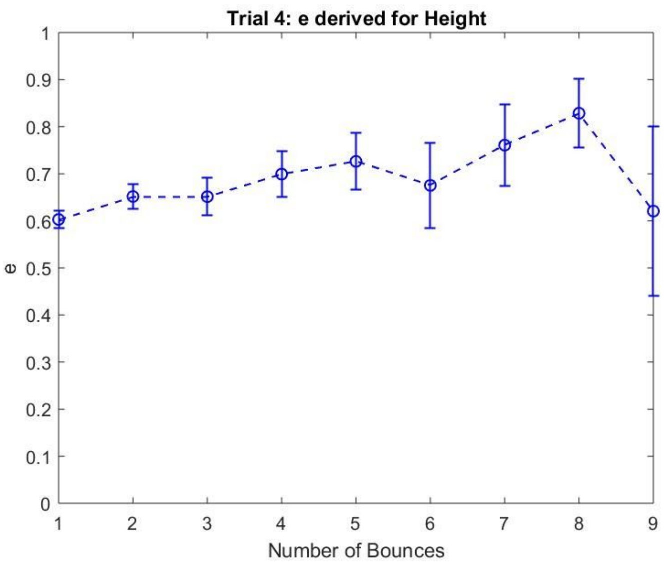
e From Height with Error Bars	e From Time with Error Bars
	

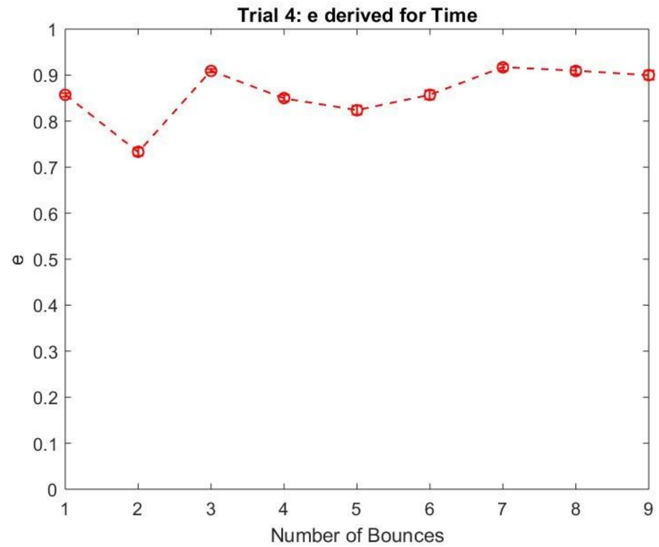

Trial 2: e derived for Height
e
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
1 2 3 4 5 6 7
Number of Bounces

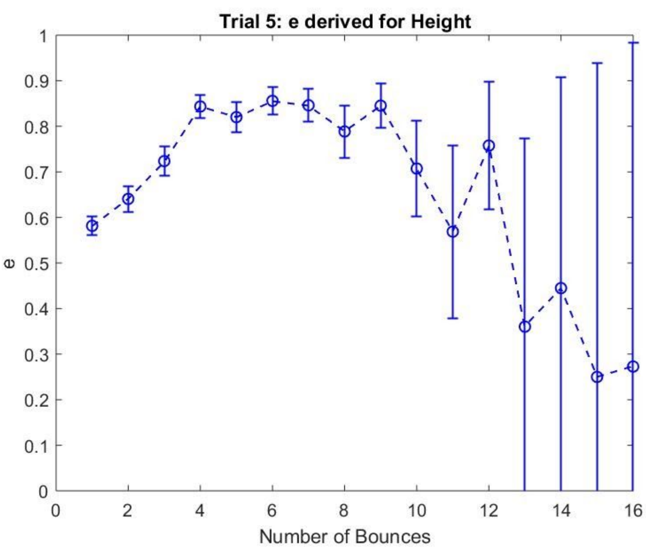

Trial 2: e derived for Time
e
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
1 2 3 4 5 6 7
Number of Bounces


Trial 3: e derived for Height
e
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 2 4 6 8 10 12
Number of Bounces


Trial 3: e derived for Time
e
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 2 4 6 8 10 12
Number of Bounces


Trial 4: e derived for Height
e
Number of Bounces


Trial 4: e derived for Time
e
Number of Bounces


Trial 5: e derived for Height
e
Number of Bounces

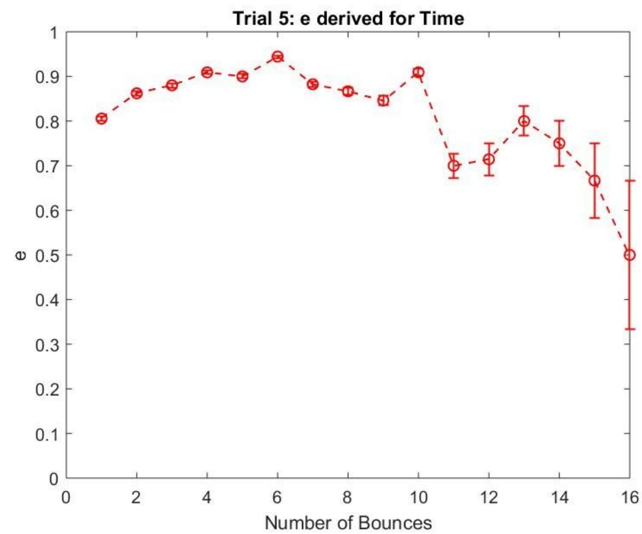

Trial 5: e derived for Time
e
Number of Bounces

Figure 1: Total Time Calculation With Error Bars - All Trials

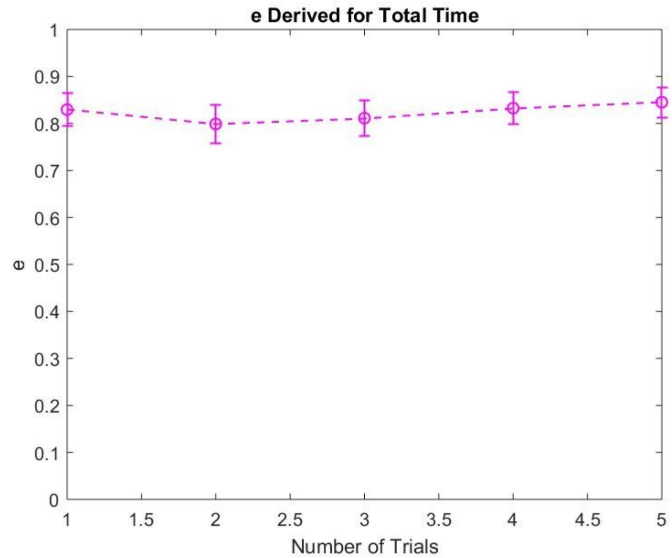


Table 2: Mean coefficient of restitution from each method and error

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
Method 1 *e*	0.719	0.687	0.757	0.691	0.644	0.700
Error	± 0.040	± 0.051	± 0.053	± 0.069	± 0.19	± 0.081
Method 2 *e*	0.881	0.842	0.842	0.862	0.809	0.847
Error	± 0.0056	± 0.0078	± 0.011	± 0.0073	± 0.029	± 0.012
Method 3 *e*	0.829	0.800	0.811	0.833	0.845	0.824
Error	± 0.035	± 0.041	± 0.039	± 0.035	± 0.032	± 0.036

IV. Performance Analysis

Through our experimental phase, each different trial and method of analysing the coefficient of restitution provided slightly differing results. This is partly due to the assumptions made in our calculations, but it can also be attributed to differences in measurement error and the data considered during each method. The method used to quantify the uncertainty in measurements was through an analysis of each tool used to quantify the data in the first place. For method one, the quantified uncertainty chosen was five centimeters. This was chosen because the data for the height measurements comes directly from the tracker software; moreover this software produces slightly larger uncertainties for the height measurements due to the fact that the center of mass must be tracked manually. The large uncertainty stems mainly from the blurred motion of the ball making it difficult to pinpoint the exact location of the center of mass. Other reasons for the large error with height measurements include the side to side and forward motion of the ball, it was very hard to get the ball to bounce straight up and down. This was combated by doing more trials than necessary and using only the best ones. In the second method, coefficient of restitution from time between bounces, the uncertainty was a lot easier to quantify due to the tracking software. We were able to view its motion progressing a single frame at a time (0.02 seconds each frame), and so the error in each measurement ended up being the time for a single frame. This is because the frame by frame motion allows you to easily see when it changes its direction, but it does not always reveal the exact moment the ball touched the ground. It must have been at some point during the frame in which its direction changed, thus the quantified uncertainty in measuring the time between bounces is simply the amount of time for each frame. In the final method, coefficient of restitution from the total time, uncertainty had to be quantified as larger than just the time of a single frame. Towards the end of its motion, the ball comes to a series of very rapid and low height bounces, and then generally begins to roll. Because of this it is very difficult to tell exactly which frame the ball ceases bouncing and starts to roll. The chosen value for the quantified uncertainty of method three ended up being 0.1 seconds, much larger than the previous method. After assessing our measuring tools and choosing values that reflected their capabilities, the uncertainty values were propagated through our calculations using a simple error analysis in order to quantify the final error in the coefficient of restitution for each time it was calculated. This was achieved by running the code with the actual measurement values as well as the values with their quantified uncertainties. This produced the actual value and the maximum amount that the value could deviate from actual with its uncertainty. This value was then further used to create the error bars on each graph in order to visualize the amount of error in each calculation, identify trends in the experiment, and to choose which method seems most accurate.

Upon an in depth analysis of each graph, a few emergent trends became evident. In both methods one and two, the amount of error in the calculation increases with the number of bounces. This is mainly due to the same reason that the quantified uncertainty in method three is comparatively large, towards the end of its motion, the ball takes increasingly small bounces. Due to this phenomenon, it becomes more difficult to get accurate values of the height and time between each small bounce in addition to the fact that the same uncertainty values will give larger deviations in such smaller measurements, as is demonstrated in trial five. The total error through trial five is significantly higher than other trials of methods one and two because data was taken for a larger number of bounces. In the height method, the values of the coefficient of restitution tend to start low and trend upwards towards the end, as shown in table 3, this caused the final value for that method to be significantly lower than the others and have the highest amount of error. Data from method two tended to be more erratic and random, with some of the estimated coefficients being equal to one, a mere impossibility in the real world situation being modelled. This happened due to the fact that for some of the smaller bounces, the time between each bounce is so similar that it took the same number of frames in each bounce, resulting in the "same" time from measurement and outputting a coefficient of one.

After analyzing each method's sources of uncertainty and thinking critically about how the experimental procedure may have affected the results, it is important to discuss which method is preferred for calculating the coefficient of restitution. In table 3, average values from all the trials were used to determine a final value from each method. From the raw data and the organized averages, it becomes apparent that the total time model gives the most valid and correct data. Taking all things into consideration, the total time model may not have the lowest uncertainty in measurement, but it consistently gives the most precise results. The grouping in figure 1 is much more compact than any of the other graphs included from table 1. The reason for its consistency is that it only depends on a single data value, unlike the other two methods which take data throughout the balls full motion. This is important because a lot of the factors that obscured the results in the previous methods did not have an affect on the total time. It did not matter whether the ball bounced slightly left, right, or forward, it only mattered when the ball stopped. While it may have been challenging to get this number exactly, it was close enough to be able to provide consistent results with only a 4.37% error across all five trials. To get an exact value of the coefficient of restitution, the mean of the coefficients in table 3 was taken along with the errors to produce a final estimation for the ping pong ball's elasticity constant.

Table 3: Final Coefficient of Restitution Values

Method	Coefficient of Restitution	Error
Bounce Height	0.700	± 0.081
Time Between Bounces	0.847	± 0.012
Total Time	0.824	± 0.036

V. Conclusion

In this lab, an experiment was designed to collect both time and height data from the bounces of a ping pong ball, then use that data to estimate the ball's coefficient of restitution. Throughout the design process we focused on ways to minimize possible errors in the experiment, and learned about the impact error has when measuring data. We determined that there were many sources of error in our experiment such as measuring the exact height of the ball, determining when the ball stopped moving, finding the exact center of mass of the ball and dropping it in such a way where it has no spin or forward velocity. Improvements to the design process are needed in order to eliminate or minimize these errors, a major chance that could be made would be recording at a higher frame rate to minimize motion blurring of the ping pong ball. This would result in being able to pinpoint the center of mass of the ping pong ball throughout each bounce, and better tell when the ball comes to rest. Secondly, we would like to improve our ball drop system, in an ideal situation we would like to create a better release system that minimizes the spin given to the ball and ideally use a vacuum tube to minimize the effects of friction so as well as use a laser tracking device to get more accurate height measurements and more data. From this experiment our group learned that when doing testing there are many other sources of information relative to the way in which the experiment was done that need to be evaluated and accounted for in order to collect accurate data. In doing so we made many different prototypes for dropping our ball, ultimately using the set up that can be seen under appendix C in figure 4, as it gave up the most vertical drop. Another conclusion that we made was that something as simple as dropping a ball and measuring its max height per bounce and time in between bounces needs to be done with much as much precision as possible to get valid results.

VI. References

[1] Brown, D, Hanson, R, Christian, W, "*Tracker*". 2020.

[2] Hodgkinson, R, "*ASEN_2003_LAB_2_Bouncing_Ball_2021*". Jan, 2021.

[3] Hodgkinson, R, "*Slide Deck*". Jan, 2021.

[4] Hodgkinson, R, "*Technical Writing*". Jan, 2021.

[5] Frew, E, Larson, K, "*Measurement Errors and Error Analysis*". Jan, 2016.

[6] Hodgkinson, R, "*Weighted Averages*". Jan, 2021.

[7] Hodgkinson, R, "*Method1_Data*". Jan, 2021.

[8] Hodgkinson, R, "*Method2_Data*". Jan, 2021.

[9] Hodgkinson, R, "*Method3_Data*". Jan, 2021.

[10] Hodgkinson, R, "*Bouncing_Ball_Logger_Pro*". Jan, 2021.

[10] Styer, M, "*Mathematical Modeling of the Coefficient of Restitution*". 2007.

VII. Appendix A

Figure 2: Height of Bounce - e Derivation

ASEN 2003

Bouncing Ball Lab

Height of Bounce: e Derivation

$$e = \frac{V_B' - V_A'}{V_A - V_B}$$

① $V_B' = V_B$

- The table does not gain any significant Velocity from the impact of the ball

$$e = \frac{V_B - V_A'}{V_A - V_B}$$

② $V_B = 0$

- The table is not moving with respect to the ball

$$e = \frac{-V_A'}{V_A}$$

③ $\frac{1}{2} m (V_A')^2 = mgh_n$

$V_A' = \sqrt{2gh_n}$

④ $\frac{1}{2} m (V_A)^2 = mgh_{n-1}$

$V_A = \sqrt{2gh_{n-1}}$

- Express the Velocity of the ball before & after impact in terms of its kinetic & potential energy

$$e = \frac{\sqrt{2gh_n}}{\sqrt{2gh_{n-1}}}$$

⑤ Simplify

$$e = \left[\frac{h_n}{h_{n-1}}\right]^{1/2} = \left[\frac{h_n}{h_0}\right]^{1/2n}$$

Figure 3: Time Between Bounces - e Derivation

ASEN 2003

Bouncing Ball Lab

Time Between Bounces: e Derivation

$$e = \left[\frac{h_n}{h_{n-1}}\right]^{1/2}$$

① $h_n = Y_0 + V_0 \frac{t_n}{2} + \frac{1}{2} a \left(\frac{t_n}{2}\right)^2$

• max height of bounce at $t/2$ through its motion

② $h_{n-1} = Y_0 + V_0 \frac{t_{n-1}}{2} + \frac{1}{2} a \left(\frac{t_{n-1}}{2}\right)^2$

• Same as ①

$$e = \left[\frac{Y_0 + V_0 \frac{t_n}{2} + \frac{1}{2} a \left(\frac{t_n}{2}\right)^2}{Y_0 + V_0 \frac{t_{n-1}}{2} + \frac{1}{2} a \left(\frac{t_{n-1}}{2}\right)^2}\right]^{1/2}$$

③ $Y_0 = 0 \quad V_0 = 0$

• Bounce height starts at 0

• Velocity must change directi and at some point will be 0 at $Y = 0$

④ $a = g$

⑤ Simplify

$$e = \left[\frac{\frac{1}{2} a \left(\frac{t_n}{2}\right)^2}{\frac{1}{2} a \left(\frac{t_{n-1}}{2}\right)^2}\right]^{1/2}$$

$$e = \left[\frac{\left(\frac{t_n}{2}\right)^2}{\left(\frac{t_{n-1}}{2}\right)^2}\right]^{1/2}$$

$$e = \left[\frac{t_n}{t_{n-1}}\right]$$

Figure 4: Time to Rest - e Derivation

ASEN 2003: Bouncing Ball Lab

Method 3: Time to Stop: e Derivation

$$e = \frac{t_s - \sqrt{2h_0/g}}{t_s + \sqrt{2h_0/g}} \qquad t_s = \sum_{n=1}^{\infty} t_n + t_0/2 \qquad t_n = e^n t_0$$

$$\therefore t_s = \sum_{n=1}^{\infty} e^n t_0 + t_0/2$$

for infinite geometric series: $S = \frac{a_1}{1-r}$

let $a_1 = e t_0$ and $r = e$ then: $S = \frac{e t_0}{1-e}$

$$\therefore t_s = S + t_0/2 \rightarrow t_s - t_0/2 = \frac{e t_0}{1-e}$$

$$(1-e)(t_s - t_0/2) = \left(\frac{e t_0}{\cancel{1-e}}\right)\cancel{(1-e)}$$

$$\therefore e t_0 = t_s - \frac{t_0}{2} - e t_s + \frac{e t_0}{2}$$

Isolate e:

$$e t_0 + e t_s - \frac{e t_0}{2} = t_s - \frac{t_0}{2}$$

$$\frac{e\cancel{\left(t_0 + t_s - \frac{t_0}{2}\right)}}{\cancel{t_0 + t_s - \frac{t_0}{2}}} = \frac{t_s - \frac{t_0}{2}}{t_0 + t_s - \frac{t_0}{2}} \rightarrow e = \frac{t_s - \frac{t_0}{2}}{\frac{t_0}{2} + t_s}$$

Since we know that $t_0 = 2\sqrt{\frac{2h_0}{g}}$

$$\therefore e = \frac{t_s - \cancel{2}\frac{\sqrt{2h_0/g}}{\cancel{2}}}{\cancel{2}\frac{\sqrt{2h_0/g}}{\cancel{2}} + t_s} \rightarrow \boxed{e = \frac{t_s - \sqrt{2h_0/g}}{t_s + \sqrt{2h_0/g}}}$$

VIII. Appendix B

```
% ASEN 2003 Roller Coaster Project Code
% Lab Section 019
% Andrew Logue, Devon Paris, Jack Foster, Siyang Liu
% This code will take in data describing the path of a bouncing ball and
%   use equations of motion to find the coefficient of restitution for the
%   bouncing motion. This will be done in 3 methods, the first using the
%   height of each consecutive bounce, again using the time between
%   bounces, and finally using the total time needed to stop bouncing
%% Clearing Commands
clc
clear all
close all
%% Data Inputs
% Initial Height of Drop
h0 = 1;     % (m)
% Error in Measured Height
hr = 0.05;   % (m)
% Error in Time Between Bounces
tbr = 0.02; % (s)
% Error in Final Time
tfr = 0.1; % (s)
% Trial 1
T1Raw = readmatrix("BBTrial1.xlsx");    % (m)
T1_total = T1Raw(1,1);
T1_times = T1Raw(:,2);
T1_height = T1Raw(1:8,3);
% Trial 2
T2Raw = readmatrix("BBTrial2.xlsx");    % (m)
T2_total = T2Raw(1,1);
T2_times = T2Raw(:,2);
T2_height = T2Raw(:,3);
% Trial 3
T3Raw = readmatrix("BBTrial3.xlsx");    % (m)
T3_total = T3Raw(1,1);
T3_times = T3Raw(:,2);
T3_height = T3Raw(:,3);
% Trial 4
T4Raw = readmatrix("BBTrial4.xlsx");    % (m)
T4_total = T4Raw(1,1);
T4_times = T4Raw(:,2);
T4_height = T4Raw(:,3);
% Trial 5
T5Raw = readmatrix("BBTrial5.xlsx");    % (m)
```

```
T5_total = T5Raw(1,1);
T5_times = T5Raw(:,2);
T5_height = T5Raw(:,3);
%% e From Bounce Height
for j=1:size(T1_height,1)-1
    eT1_height(j) = (T1_height(j+1)/T1_height(j));
    eT1_hr(j) = ((T1_height(j+1)+hr)/(T1_height(j)+hr))-eT1_height(j);
    eT1_hpr(j) = 100*eT1_hr(j)/eT1_height(j);
end
for j=1:size(T2_height,1)-1
    eT2_height(j) = (T2_height(j+1)/T2_height(j));
    eT2_hr(j) = ((T2_height(j+1)+hr)/(T2_height(j)+hr))-eT2_height(j);
    eT2_hpr(j) = 100*eT2_hr(j)/eT2_height(j);
end
for j=1:size(T3_height,1)-1
    eT3_height(j) = (T3_height(j+1)/T3_height(j));
    eT3_hr(j) = ((T3_height(j+1)+hr)/(T3_height(j)+hr))-eT3_height(j);
    eT3_hpr(j) = 100*eT3_hr(j)/eT3_height(j);
end
for j=1:size(T4_height,1)-1
    eT4_height(j) = (T4_height(j+1)/T4_height(j));
    eT4_hr(j) = ((T4_height(j+1)+hr)/(T4_height(j)+hr))-eT4_height(j);
    eT4_hpr(j) = 100*eT4_hr(j)/eT4_height(j);
end
for j=1:size(T5_height,1)-1
    eT5_height(j) = (T5_height(j+1)/T5_height(j));
    eT5_hr(j) = ((T5_height(j+1)+hr)/(T5_height(j)+hr))-eT5_height(j);
    eT5_hpr(j) = 100*eT5_hr(j)/eT5_height(j);
end
%% e From Time Between Bounces
for j=1:size(T1_times,1)-1
    eT1_times(j) = (T1_times(j+1)/T1_times(j));
    eT1_tbr(j) = ((T1_times(j+1)+tbr)/(T1_times(j)+tbr))-eT1_times(j);
    eT1_tbpr(j) = 100*eT1_tbr(j)/eT1_times(j);
end
for j=1:size(T2_times,1)-1
    eT2_times(j) = (T2_times(j+1)/T2_times(j));
    eT2_tbr(j) = ((T2_times(j+1)+tbr)/(T2_times(j)+tbr))-eT2_times(j);
    eT2_tbpr(j) = 100*eT2_tbr(j)/eT2_times(j);
end
for j=1:size(T3_times,1)-1
    eT3_times(j) = (T3_times(j+1)/T3_times(j));
    eT3_tbr(j) = ((T3_times(j+1)+tbr)/(T3_times(j)+tbr))-eT3_times(j);
    eT3_tbpr(j) = 100*eT3_tbr(j)/eT3_times(j);
```

```
end
for j=1:size(T4_times,1)-1
    eT4_times(j) = (T4_times(j+1)/T4_times(j));
    eT4_tbr(j) = ((T4_times(j+1)+tbr)/(T4_times(j)+tbr))-eT4_times(j);
    eT4_tbpr(j) = 100*eT4_tbr(j)/eT4_times(j);
end
for j=1:size(T5_times,1)-1
    eT5_times(j) = (T5_times(j+1)/T5_times(j));
    eT5_tbr(j) = ((T5_times(j+1)+tbr)/(T5_times(j)+tbr))-eT5_times(j);
    eT5_tbpr(j) = 100*eT5_tbr(j)/eT5_times(j);
end
%% e From Total Time
q = sqrt(2*h0/9.81);
eTT1 = (T1_total-q)/(T1_total+q);
eTT1_r = (T1_total+tfr-q)/(T1_total-tf r+q)-eTT1;
eTT1_pr = eTT1_r*100/eTT1;
eTT2 = (T2_total-q)/(T2_total+q);
eTT2_r = (T2_total+tfr-q)/(T2_total-tfr+q)-eTT2;
eTT2_pr = eTT2_r*100/eTT2;
eTT3 = (T3_total-q)/(T3_total+q);
eTT3_r = (T3_total+tfr-q)/(T3_total-tfr+q)-eTT3;
eTT3_pr = eTT3_r*100/eTT3;
eTT4 = (T4_total-q)/(T4_total+q);
eTT4_r = (T4_total+tfr-q)/(T4_total-tfr+q)-eTT4;
eTT4_pr = eTT4_r*100/eTT4;
eTT5 = (T5_total-q)/(T5_total+q);
eTT5_r = (T5_total+tfr-q)/(T5_total-tfr+q)-eTT5;
eTT5_pr = eTT5_r*100/eTT5;

%% Plots
%T1
figure
errorbar(1:size(eT1_height,2),eT1_height, eT1_hr, 'b--o','LineWidth',1)
title('Trial 1: e derived for Height')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])

figure
errorbar(1:size(eT1_times,2),eT1_times, eT1_tbr, 'r--o','LineWidth',1)
title('Trial 1: e derived for Time')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])
```

```
%T2
figure
errorbar(1:size(eT2_height,2),eT2_height, eT2_hr, 'b--o','LineWidth',1)
title('Trial 2: e derived for Height')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])

figure
errorbar(1:size(eT2_times,2),eT2_times, eT2_tbr, 'r--o','LineWidth',1)
title('Trial 2: e derived for Time')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])
%T3
figure
errorbar(1:size(eT3_height,2),eT3_height, eT3_hr, 'b--o','LineWidth',1)
title('Trial 3: e derived for Height')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])

figure
errorbar(1:size(eT3_times,2),eT3_times, eT3_tbr, 'r--o','LineWidth',1)
title('Trial 3: e derived for Time')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])
%T4
figure
errorbar(1:size(eT4_height,2),eT4_height, eT4_hr, 'b--o','LineWidth',1)
title('Trial 4: e derived for Height')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])

figure
errorbar(1:size(eT4_times,2),eT4_times, eT4_tbr, 'r--o','LineWidth',1)
title('Trial 4: e derived for Time')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])
%T5
figure
```

```
errorbar(1:size(eT5_height,2),eT5_height, eT5_hr, 'b--o','LineWidth',1)
title('Trial 5: e derived for Height')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])

figure
errorbar(1:size(eT5_times,2),eT5_times, eT5_tbr, 'r--o','LineWidth',1)
title('Trial 5: e derived for Time')
xlabel('Number of Bounces')
ylabel('e')
ylim([0,1])
%ToT
figure
ett=[eTT1,eTT2,eTT3,eTT4,eTT5];
ettr=[eTT1_r,eTT2_r,eTT3_r,eTT4_r,eTT5_r];
errorbar(1:size(ett,2),ett, ettr, 'm--o','LineWidth',1)
title('e Derived for Total Time')
xlabel('Number of Trials')
ylabel('e')
ylim([0,1])

eM1T1 = sum(eT1_height)/size(eT1_height,2);
eM1T1_r = sum(eT1_hr)/size(eT1_hr,2);
eM1T2 = sum(eT2_height)/size(eT2_height,2);
eM1T2_r = sum(eT2_hr)/size(eT2_hr,2);
eM1T3 = sum(eT3_height)/size(eT3_height,2);
eM1T3_r = sum(eT3_hr)/size(eT3_hr,2);
eM1T4 = sum(eT4_height)/size(eT4_height,2);
eM1T4_r = sum(eT4_hr)/size(eT4_hr,2);
eM1T5 = sum(eT5_height)/size(eT5_height,2);
eM1T5_r = sum(eT5_hr)/size(eT5_hr,2);
eM2T1 = sum(eT1_times)/size(eT1_times,2);
eM2T1_r = sum(eT1_tbr)/size(eT1_tbr,2);
eM2T2 = sum(eT2_times)/size(eT2_times,2);
eM2T2_r = sum(eT2_tbr)/size(eT2_tbr,2);
eM2T3 = sum(eT3_times)/size(eT3_times,2);
eM2T3_r = sum(eT3_tbr)/size(eT3_tbr,2);
eM2T4 = sum(eT4_times)/size(eT4_times,2);
eM2T4_r = sum(eT4_tbr)/size(eT4_tbr,2);
eM2T5 = sum(eT5_times)/size(eT5_times,2);
eM2T5_r = sum(eT5_tbr)/size(eT5_tbr,2);
```

IX. Appendix C

Figure 5: Experiment setup

Figure 6: Center of Mass of Ping Pong Ball at Different Locations

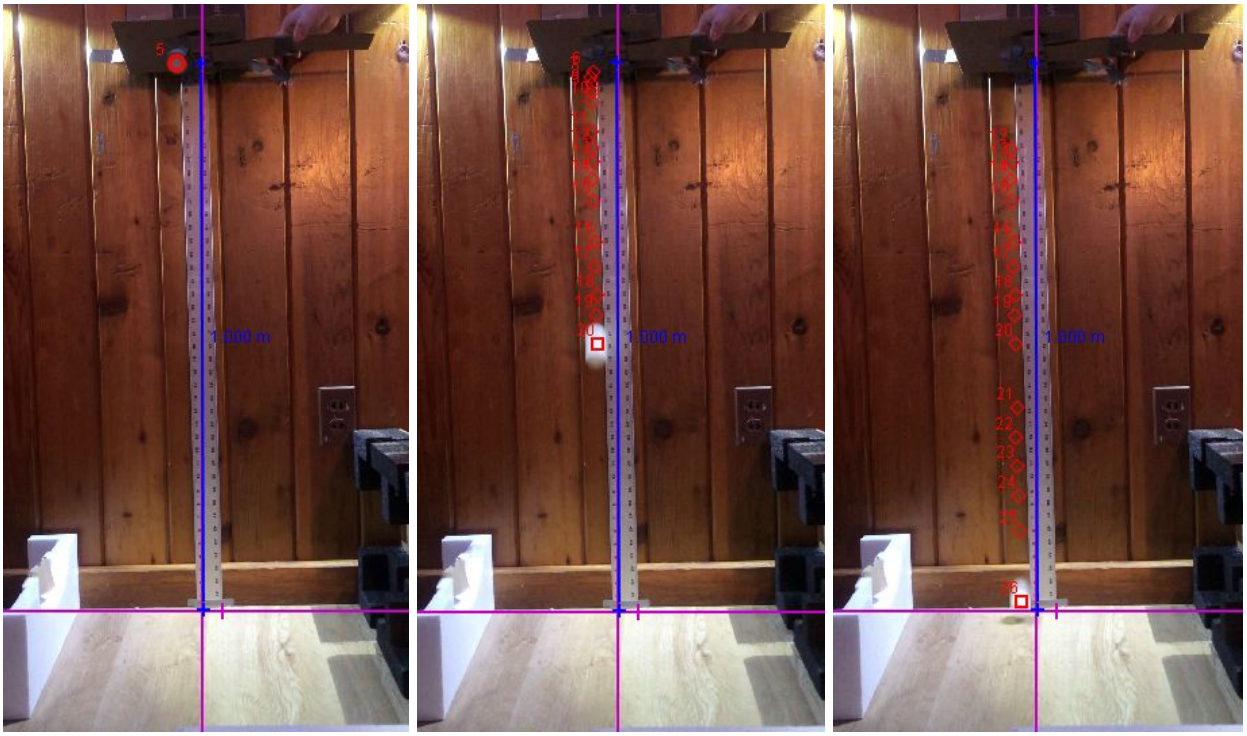


Figure 7: Height of the Center of Mass of the Ping Pong Ball Relative to Time