ASEN 2004 Aero Lab Milestone 1 - Drag Polar Benchmarking Report

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I. Lift Curve Comparison (C_L vs. α)

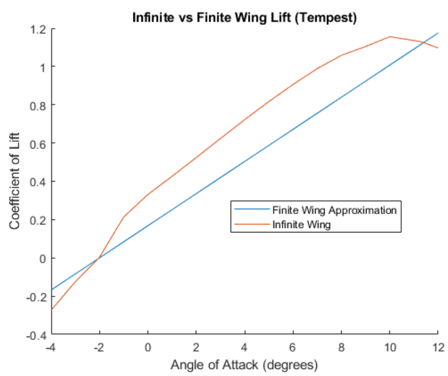

Fig. 1 2-D and 3-D Approximation of C_L vs. α for Tempest UAS

Fig. 2 2-D and 3-D Approximation of C_L vs. α for Boeing 747-200

As expected for both aircraft, the 3-D approximations result in C_L vs. α lines with a lower slope, $a = \frac{dC_L}{d\alpha}$, than the 2-D data, with an unchanged angle of attack at zero lift, $\alpha_{L=0}$, of approximately 2°. This decrease from a_0 to a is a result of the effects of induced drag and downwash on at the wingtips of the finite 3-D wing. This downwash creates pressure differentials, resulting in vortices and reduced lift at angles of attack above $\alpha_{L=0}$. As seen in Fig. 1 and 2, this results in an increase in stall angle and a decrease of the rate at which C_L increases with angle of attack, $\frac{dC_L}{d\alpha}$.

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The following correction equations were used to calculate 3-D finite wing C_L values from the given 2-D infinite wing data.

$$a = \frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{57.3 * a_0}{\pi * e * AR}} \tag{1}$$

$$C_L = a * (\alpha - \alpha_{L=0}) \tag{2}$$

For this analysis, the slope of the 2-D data, a_0, was obtained by averaging the slopes between data points up to the stall angle of attack. An alternative method for obtaining a_0 is to use a best-fit line in Matlab to obtain the slope over the same section of data. Due to the variation in the slopes between each data point in the 2-D data, the method used was determined to be more accurate than the best-fit method. Values for the aspect ratio, AR, were given for each aircraft, while the span efficiency, e, was assumed to be 0.9 for both aircraft.

II. Drag Polar Comparison (C_D vs. C_L)

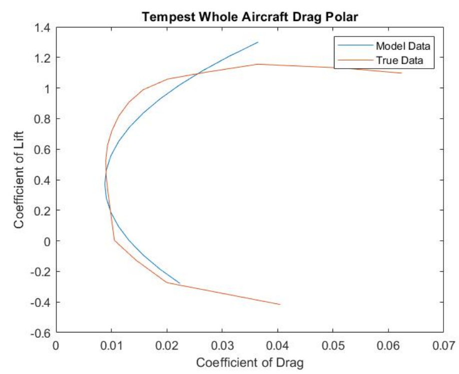

Fig. 3 3-D Finite Wing, Whole Aircraft, and Truth Data Drag Polars for Tempest UAS

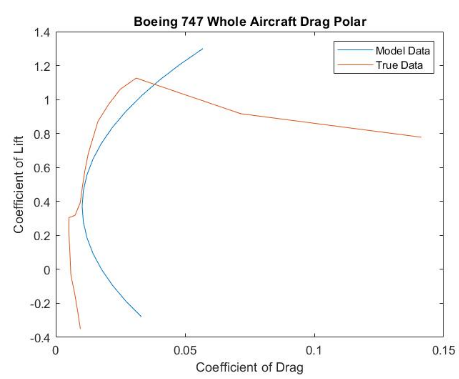

Fig. 4 3-D Finite Wing, Whole Aircraft, and Truth Data Drag Polars for Boeing 747-200

The data shows the relationship between the coefficient of lift and the coefficient of drag for the two different aircraft (Boeing 747 and Tempest). The lift coefficient starts negative as it has negative lift at angles of attack below $\alpha_{L=0}$ and this causes a greater coefficient of drag for both aircraft. For the Tempest model, the minimum drag coefficient occurs when the lift coefficient is about 0.3, while the minimum drag coefficient occurs at about 0.4 for the 747. One possible error is that CAD model was not perfect, so the wetted area of the aircraft were not exact. This can cause a shift in the coefficient of drag depending on if we overestimated or underestimated the amount of wetted area there is. From Fig. 3 and 4, it is clear that the Tempest drag polar model more accurately aligns with the truth data than the Boeing model. The tempest has a smaller area, so the error with the wetted area was most likely less severe and caused the model data to more accurately depict the truth data. The truth data and the model data are similar in shape and follow the same trend. As coefficient of lift drifts farther and farther away from zero, the more coefficient of drag increases. This makes sense as there is more surface area the wind pushes against as the absolute value of the angle increases.

III. Performance Comparisons

Aircraft	$\frac{L}{D}_{max}$	V for Max Glide Range	V for Max Powered Range	V for Max Powered Endurance
Tempest UAS (truth data)	18.8	15.98 $[\frac{km}{h}]$	21.04 $[\frac{km}{h}]$	15.98 $[\frac{km}{h}]$
Tempest UAS (aerodynamic model)	23.3	17.60 $[\frac{km}{h}]$	23.16 $[\frac{km}{h}]$	17.60 $[\frac{km}{h}]$
Boeing 747-200 (truth data)	15.9	930.46 $[\frac{km}{h}]$	1123.8 $[\frac{km}{h}]$	930.46 $[\frac{km}{h}]$
Boeing 747-200 (aerodynamic model)	13.9	867.95 $[\frac{km}{h}]$	1141 $[\frac{km}{h}]$	867.95 $[\frac{km}{h}]$

Table 1 Performance Velocity Comparisons

The following performance comparison is based off of the data table above, using the performance flight conditions provided in the “truth” data and the aerodynamic model data. First examining the maximum lift over drag ratios, which were calculated via Eq. 4, after first determining Oswald’s factor from equation four in reference[1] (shown below as Eq. 3. The Tempest UAS lift to drag ratio derived from the "truth" data was approximately 20 percent lower than the lift to drag ratio derived from the aerodynamic model. However, when examining the lift to drag ratio of the Boeing 747-200, the ratio derived from the provided "truth" data was instead nearly 15 percent higher than that derived from the aerodynamic model. This trend of the figures calculated via the "truth" data being lower than those calculated via the aerodynamic model was additionally present in the velocities for maximum glide range, maximum powered range, and maximum powered endurance for the Tempest UAS aircraft. On average, the Tempest UAS velocities calculated via the model were approximately 9 percent higher than those based on the "truth" data. The trend reversed for the Boeing 747-200 velocities, with the figures based of the truth data remaining higher than those derived from the aerodynamic model. The only outlier to this trend was the velocity for the maximum powered range derived from the model being less than one percent higher than the value calculated from the truth data. The Boeing 747-200 velocities calculated from the "truth" data (excluding the velocity for maximum powered range outlier) were approximately 7 percent higher than those derived from the model. Therefore it can be concluded that the aerodynamic model data produced consistently higher values for the Tempest UAS than those derived from the provided "truth" data, while the Boeing 747-200 aerodynamic model produced consistently lower values with the exception of the velocity for maximum powered range.

$$P = 0.007; Q = 1.05 \cdots e_o = \frac{1}{Q + P\pi(\frac{b^2}{s})} \tag{3}$$

$$\left(\frac{Lift}{Drag}\right)_{max} = \frac{\sqrt{\frac{C_{Dmin}+\pi * e_o(\frac{b^2}{s})(C_{LminD})^2}{\pi * e_o(\frac{b^2}{s})}}}{2 * \frac{C_{Dmin}+(C_{LminD})^2}{\pi * e_o(\frac{b^2}{s})}} \tag{4}$$

$$C_{Do} = C_{Dmin} + k * (C_{LminD})^2 \tag{5}$$

IV. Discussion

The models developed in this lab make several assumptions that decrease the overall accuracy of the results. As discussed in the Lift Curve Comparison section, the span efficiency is assumed to be 0.9 for both aircraft. This value is used in Eq. 1 to convert the 2-D airfoil data into 3-D finite wing data, therefore, a method to calculate span efficiency factor on a per-aircraft basis would result in a more accurate 3-D wing lift curve. Similarly, the method of averaging slopes of the 2-D C_L vs. α data was found to be more appropriate for the given data than using a best-fit line. However, more consistent 2-D data would also allow for increased confidence in this conversion and make the best-fit method results closer to that of the average slope method used in this lab. Additionally, there are several methods and equations used in the literature for calculating Oswald's efficiency factor for a specific aircraft. Through repeated trials, a best equation could be found to calculate this factor for a given aircraft was Eq. 3 shown above. This equation is Obert's model for finding Oswald's factor, which was considered to be the best estimation for the factor because it considered P and Q as constants, therefore the factor could be calculated as a function of the aircraft's aspect ratio. Eq. 5 was used to calculate the parasite drag coefficient, where C_{D_0} was obtained using the minimum coefficient of drag. The coefficient of lift at that minimum coefficient of drag was found from both the truth data and the aerodynamic model. It is important to note that coefficients of lift below zero were ignored as it would cause some mathematical issues in the velocity calculations. This improved Oswald's efficiency factor would subsequently improve the calculation for C_D and the resulting drag polar. Likewise, C_{D_0} has multiple methods of calculating and finding the best method would result in an overall more accurate model.

While these aerodynamic models were applied to full-sized aircraft, all of these methods of calculation and modeling can be applied to the sub-scale glider design in the next part of this lab. The main concerns moving forward are that of obtaining accurate values for the quantities discussed above, namely the efficiency factors and the zero-lift drag coefficient. Since the wetted area calculation was found to be the most accurate when using CAD software, this method is ideal for the glider design analysis and modeling, rather than geometric estimations, resulting in more accurate drag calculations.

V. References

[1] Nita, M, Scholz, D, "*Estimating the Oswald Factor From Basic Aircraft Geometrical Parameters*". 2012.

[2] Mah, J, "*ASEN 2004 Aero Equation Sheet v4.5*". 2020.

[3] Mah, J, "*ASEN 2004 Aero Lab: Foundations of Aircraft Design (MILESTONE 1)*". 2020.

VI. Appendix

Matlab Code

```matlab
%% ASEN 2004 Lab 1
clc;
clear;
close all;
%% Lift Curve Comparison (Ian, Chris)
TempestData = readtable('TempestData.xlsx');
BoeData = readtable('BOEairfoildata.xlsx');
tempest = table2array(TempestData);
boeing = table2array(BoeData);

% First part is for Tempest Data
% avg of slopes from -5 to 10
totalslope = 0;
dh=0;
for i=1:15
    dh = tempest(i+1,2) - tempest(i,2);
    dx = 1;
    totalslope = totalslope + dh;
end
avgslopeinfinity = totalslope/15;
% 3d correction
e=0.9;
a0=avgslopeinfinity;
AR=16.5;
```

```
a=a0/(1+(57.3*a0)/(pi*e*AR));
alpha_L0=-2;
%coefficient of lift to angle comparison (tempest)
figure(1)
hold on
plot(tempest(:,1),a*(tempest(:,1)-alpha_L0));
plot(tempest(:,1),tempest(:,2));
title('Infinite vs Finite Wing Lift (Tempest)');
xlabel('Angle of Attack (degrees)');
ylabel('Coefficient of Lift');
legend('Finite Wing Approximation','Infinite Wing');

% Now Boeing Data
totalslope2 = 0;
dh2 = 0;
for j = 1:17
    dh2 = boeing(j+1,2) - boeing(j,2);
    dx = 1;
    totalslope2 = totalslope2 + dh2;
end
a02 = totalslope2/17;

AR2 = 7;
a2=a02/(1+(57.3*a02)/(pi*e*AR2));
%coefficient of lift to angle comparison (boeing)
figure(2)
hold on
plot(boeing(:,1),a2*(boeing(:,1)-alpha_L0));
plot(boeing(:,1),boeing(:,2));
title('Infinite vs Finite Wing Lift (Boeing)');
xlabel('Angle of Attack (degrees)');
ylabel('Coefficient of Lift');
legend('Finite Wing Approximation','Infinite Wing');

%% Drag Polar Comparison (Tim)
for j=1:2
    AR = [16.5 7];
    e0 = 1.78*(1-.045*AR(j).^(.68))-.64;
    Cfe = .0030;
    k1=1/(pi*e0.*AR(j));
    CLminD = a*(2+2);
    Swet = [2.78 3013.29];
    Sref = [.63 511];
    CDmin = Cfe*(Swet(j)./Sref(j));
    CD0 = CDmin+(k1*CLminD^2);
    k2 = -2*k1*CLminD;
    for i=1:18
    CL(i,j) = a*((i-6)-alpha_L0);
    CD(i,j) = CDmin+(k1*CL(i,j).^2)+(k2.*CL(i,j));
    end
end

figure (3)
plot(CD(:,1),CL(:,1));
```

```
hold on
plot(tempest(:,3),tempest(:,2));
title('Tempest Whole Aircraft Drag Polar');
xlabel('Coefficient of Drag');
ylabel('Coefficient of Lift');
legend('Model Data', 'True Data');
hold off

figure (4)
plot(CD(:,2),CL(:,2));
hold on
plot(boeing(:,3),boeing(:,2));
title('Boeing 747 Whole Aircraft Drag Polar');
xlabel('Coefficient of Drag');
ylabel('Coefficient of Lift');
legend('Model Data', 'True Data');
hold off
```

Model Calculations

3. i Boeing 747 (model data)

$(L/D)_{max}$

From our code we extracted the following data:

$e_0 = 0.8392$
$C_{Dmin} = 0.017376$
$C_{LminD} = 0.3357$

@ $(L/D)_{MAX}$ $C_{D0} = C_{Di} = kC_L^2$

$C_{D0} = C_{Dmin} + kC_{LminD}^2$ $k = \frac{1}{\pi(0.8392)6.95}$

$C_{D0} = 0.017376 + (0.0546)(0.3357)^2$ $k = 0.0546$

$C_{D0} = 0.0235$ $C_D = 2C_{D0}$

$C_D = 2(0.0235) = 0.04705$

$C_L = \sqrt{\frac{C_{D0}}{k}} = 0.656$

$(L/D)_{max} = (C_L/C_D)_{max} = \frac{0.656}{0.0471} = \boxed{13.9}$

ii. V @ $(L/D)_{max}$ is max glide range

For the Boeing 747
$\rho_\infty = 0.000738$ slug/ft³ @ 35K STD atm
$W = 833000$ lb
$S = 5500$ ft²

$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(833000)}{0.000738(0.656)(5500)}}$

$V = 790.999$ ft/s
⇓
$\boxed{V = 867.95 \text{ km/hr}}$

iii V @ R_{max} (powered) → jet

$C_{D0} = 3C_{Di} = 3kC_L^2$

$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.0235}{3(0.0546)}} = 0.379$

$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(833000)}{0.000738(0.379)(5500)}}$

$V = 1040.7$ ft/s

$\boxed{V = 1141 \text{ km/h}}$

iv V @ max jet endurance

$\boxed{V_{(L/D)max} = 867.95 \text{ km/hr}}$

Scanned with CamScanner

Scanned with CamScanner

3i Tempest (model data)

$(L/D)_{max}$

From our code we extracted the following data:

$e_0 = 0.6011$
$C_{Dmin} = 0.0107$
$C_{LminD} = 0.3357$

$C_{D0} = C_{Dmin} + kC_{LminD}^2$ $k = \frac{1}{\pi(e_0)AR}$

$C_{D0} = 0.0107 + 0.03217(0.3357)^2$ $k = \frac{1}{\pi(0.6011)(16.46)}$

$C_{D0} = 0.0143$ $k = 0.03217$

$C_L = \sqrt{\frac{C_{D0}}{k}} = \sqrt{\frac{0.0143}{0.03217}} = 0.667$ $C_D = 2C_{D0}$
$= 2(0.0143)$
$C_D = 0.0286$

$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \boxed{23.3}$

ii. V @ $(L/D)_{max}$

For the Tempest
$W = 6.4$ kg
$\rho_\infty = 1.0581$ kg/m³
$S = 0.63$ m²

$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(6.4)}{1.0581(0.667)(0.63)}}$

$V = 5.365$ m/s

$\boxed{V = 17.60 \text{ km/hr}}$

[same for iv]

iii V @ R_{max} for jet

$C_{D0} = 3kC_L^2$

$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.0143}{3(0.03217)}} = 0.385$

$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(6.4)}{1.0581(0.385)0.63}}$

$V = 7.06$ m/s

$\boxed{V = 23.16 \text{ km/hr}}$

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Scanned with CamScanner

Truth Data Calculations

2004 Lab 1.

3.i For Boeing 747 (Truth data)

$(L/D)_{max}$.

We found Oswald's factor using eq (4) (Obert) from "ESTIMATING THE OSWALD FACTOR FROM BASIC AIRCRAFT GEOMETRICAL PARAMETERS"

$$e_0 = \frac{1}{Q + P\pi AR}$$

where Q & P are constants: P = 0.007, Q = 1.05

$$AR = \frac{b^2}{S} = \frac{(59.6)^2}{511} = 6.95$$

$$\therefore \ e_0 = \frac{1}{1.05 + 0.007\pi(6.95)} = 0.831$$

@ $(L/D)_{max}$ $C_{D0} = C_{Di} = KC_L^2$ and $C_D = C_{D0} + C_{Di}$

$C_D = 2C_{Di}$ or

$C_D = 2C_{D0}$

Find C_{D0}

$$C_{D0} = C_{Dmin} + K_1 C_{L\,min\,D}^2 \qquad K_1 = \frac{1}{\pi e_0 AR}$$

From data provided: $K_1 = \frac{1}{\pi(0.831)(6.95)} = 0.0551$

$C_{Dmin} = 0.0180$ $C_{L\,min\,D} = 0.0039$

$$C_{D0} = 0.0180 + (0.0551)(0.0039)^2$$

$\boxed{C_{D0} = 0.0180}$ checks out

Now we have e_0, K and C_{D0} and we know that $C_{D0} = KC_L^2$ now solve for C_L

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.0180}{0.0551}} = 0.571$$

And we also know that $C_D = 2C_{D0}$

$$C_D = 2(0.0180) = 0.0360$$

Moreover $(L/D)_{max} = (C_L/C_D)_{max}$

$$\text{So} \quad (L/D)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \frac{0.571}{0.0360} = 15.86$$

$$\boxed{(L/D)_{max} = 15.9}$$

ii V@ $(L/D)_{max}$ (Max glide range)

Because we assume SLUF L=W

$$C_L = \frac{W}{q_\infty S} = \frac{W}{\frac{1}{2}\rho_\infty V^2 S}$$

For Boeing 747

$\rho_\infty = 0.000738$ slug/ft³ @ 35,000 ft STD atm

W = 833,000 lb

S = 5500 ft²

Solve for V

$$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(833{,}000)}{0.000738(0.571)(5500)}}$$

V = 847.83 ft/s

V = 258.46 m/s

$$\boxed{V = 930.46 \text{ km/hr}}$$

iii V@ R_{MAX} Powered range (Because it is a jet)

$$C_{D0} = 3C_{Di} = 3KC_L^2$$

1st find C_L → $C_L = \sqrt{\frac{C_{D0}}{3K}} = \sqrt{\frac{0.0180}{3(0.0551)}}$

$C_L = 0.329989$

$$V = \sqrt{\frac{2W}{\rho_\infty C_L S}}$$

$$V = \sqrt{\frac{2(833000)}{0.000738(0.32999)(5500)}}$$

$\boxed{V = 1115.3 \text{ ft/s} \rightarrow 1223.8 \text{ km/hr}}$

iv. V @ max jet endurance

same as $V_{L/D\,max}$ => $\boxed{930.46 \text{ km/hr}}$

3. For Tempest. (Truth data)

$$\text{i} \quad e_0 = \frac{1}{Q + P\pi AR} = \frac{1}{1.05 + 0.007\pi(16.46)} = 0.708$$

$$AR = \frac{b^2}{S} = \frac{(3.22)^2}{0.63} = 16.46$$

$(L/D)_{max}$ → $C_{D0} = C_{Di} = KC_L^2$

$$C_{D0} = C_{Dmin} + K_1 C_{L\,min\,D}^2$$

From data: $C_{Dmin} = 0.025864$, $C_{L\,min\,D} = 0.010503$

$$K_1 = \frac{1}{\pi e_0 AR} = \frac{1}{\pi(0.708)(16.46)} = 0.0273$$

$$C_{D0} = 0.025864 + (0.0273)(0.010503)^2$$

$C_{D0} = 0.02587$ (makes sense)

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.02587}{0.0273}} = 0.9734$$

$C_D = C_{D0} + C_{Di}$ => $C_{D0} = C_{Di}$

$\therefore$ $C_D = 2C_{D0} = 0.05174$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \left(\frac{0.9734}{0.05174}\right) = \boxed{18.8}$$

ii Max glide range is V@ $(L/D)_{max}$ ($C_{D0} = C_{Di}$)

Assume SLUF, L = W

$$\text{So} \quad C_L = \frac{W}{q_\infty S}$$

For tempest:

GTOW => 6.4 kg

$\rho_\infty = 1.0581$ kg/m³ @ 1.5 km

S = 0.63 m²

$$q_\infty = \frac{1}{2}\rho_\infty V^2$$

Solve for V using C_L when $(L/D)_{max}$

$$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(6.4)}{1.0581(0.9734)(0.63)}}$$

V = 4.44 m/s

$$\boxed{V = 15.98 \text{ km/hr}}$$

iii V @ R_{max} (Powered) Because it's a jet => $C_{D0} = 3C_{Di}$

$C_{D0} = 3KC_L^2$

$$C_L = \sqrt{\frac{C_{D0}}{3K}} = \sqrt{\frac{0.02587}{3(0.0273)}} = 0.562$$

$$V = \sqrt{\frac{2W}{\rho_\infty C_L S}} = \sqrt{\frac{2(6.4)}{1.0581(0.562)(0.83)}}$$

$$V = 5.845 \ m/s$$

$$\boxed{V = 21.04 \ km/hr}$$

iv. V @ max endurance for jet is same as $V(L/D)_{max}$.

$$\boxed{V = 15.98 \ km/hr}$$