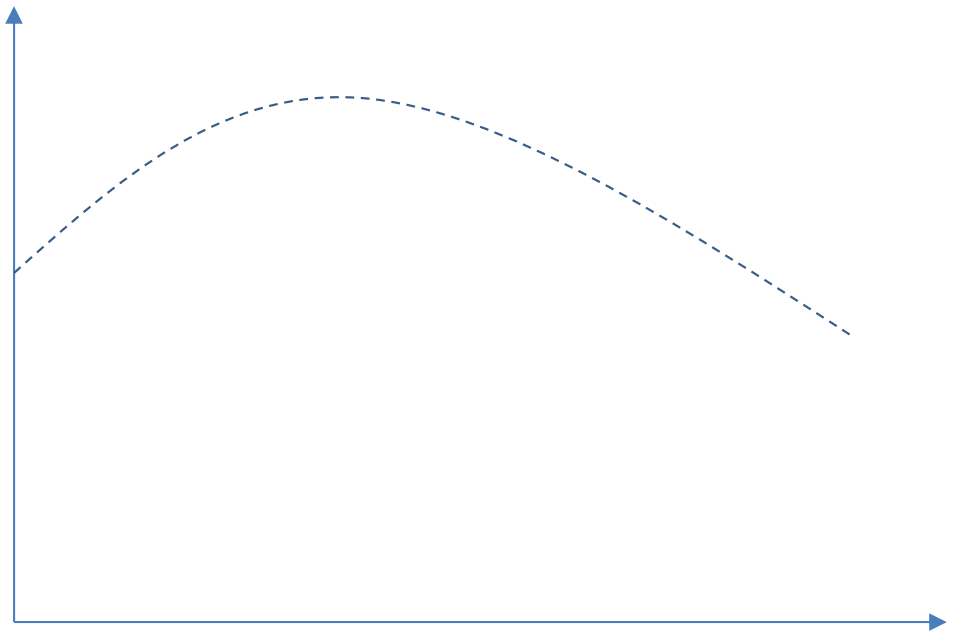
Numerical Solution of ODEs

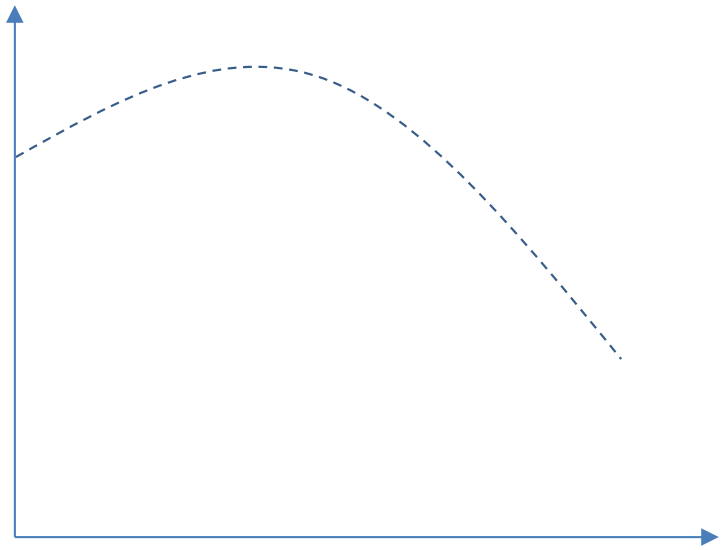
Numerical solution of first order ordinary differential equations

"One step" methods

First step – common to all methods



Euler’s method



EXAMPLE $\frac{dy}{dx} = \frac{x^3 + 1}{y}$ $y(0) = 2$ $0 \leq x \leq 10, \Delta x = 0.5$

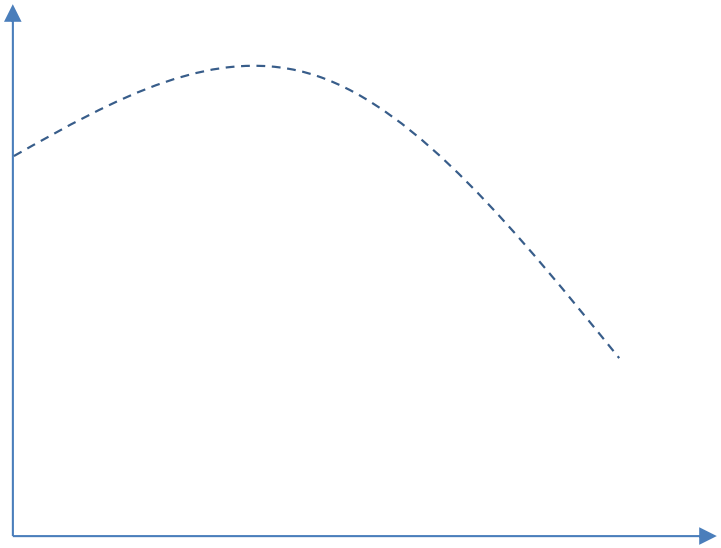
i	x_i	y_i	$\frac{x_i^3 + 1}{y_i}$	y_{i+1}

i	x	y	f(x,y)	y_new	y_exact	error
0	0	2.00	0.50	2.25	2.00	0.00
1	0.5	2.25	0.50	2.50	2.24	0.01
2	1	2.50	0.80	2.90	2.55	0.05
3	1.5	2.90	1.51	3.65	3.09	0.19
4	2	3.65	2.46	4.89	4.00	0.35
17	8.5	50.26	12.24	56.38	51.29	1.03
18	9	56.38	12.95	62.85	57.47	1.09
19	9.5	62.85	13.66	69.68	64.00	1.14
20	10	69.68	14.37	76.86	70.88	1.20

Error estimate

i	x	y	f(x,y)	y_new	y_exact	error
0	0	2.00	0.50	2.25	2.00	0.00
1	0.25	2.25	0.45	2.36	2.12	0.13
2	0.5	2.36	0.48	2.48	2.24	0.12
3	0.75	2.48	0.57	2.63	2.38	0.10
4	1	2.63	0.76	2.82	2.55	0.08
37	9.25	60.13	13.18	63.42	60.69	0.56
38	9.5	63.42	13.53	66.81	64.00	0.57
39	9.75	66.81	13.89	70.28	67.39	0.59
40	10	70.28	14.24	73.84	70.88	0.60

Implicit Euler method



EXAMPLE $\frac{dy}{dx} = \frac{x^3 + 1}{y}$ $y(0) = 2$ $0 \leq x \leq 10, \Delta x = 0.5$

$$y_1 = 2 + 0.5\left(\frac{(0.5)^3 + 1}{y_1}\right)$$

Iterative solution

$$y_2 = y_1 + 0.5\left(\frac{(x_2)^3 + 1}{y_2}\right)$$

x	y_{exact}	y_{Euler}	$y_{Imp.Euler}$
0	2	2	2
1	2.55	2.5	2.63
2	4.00	3.65	4.33
3	7.106	6.58	6.97
4	11.83	11.23	12.02

Notes